

APMA E4300 – SPRING 2009

ASSIGNMENT 2

1. Fixed-point iteration.

(a) $x^3 - x - 1 = 0$

(b) $e^{-x} - \cos x = 0$

For each of the two equations above, find three different fixed-point methods (specified by the $g(x)$ function and initial guess), including one that converges and one that does not converge, if possible. Show that you checked your answers with Matlab.

2. Kepler's equation for planetary orbits

$$M = E - e \sin E$$

relates the mean anomaly M to the eccentricity anomaly E of an elliptical orbit of eccentricity e . For this problem, use $M = 24.851090$ and $e = 0.1$.

(a) Use Matlab's root finder `fzero` to determine E . Discuss how you choose the stopping criterion and other input parameters to `fzero`.

(b) Compute E using the formula

$$E = M + 2 \sum_{m=1}^{\infty} \frac{1}{m} J_m(me) \sin(mM)$$

where $J_m(x)$ is the Bessel function of the first kind of order m . Use Matlab's built-in Bessel function. Discuss how to choose the number of terms.

(c) Compute E using the formula above, but instead of using Matlab's Bessel function, use

$$J_m(me) = \sum_{n=0}^{\infty} \frac{(-1)^n (me/2)^{2n+m}}{n!(m+n)!}.$$

Discuss how to choose the number of terms.

(d) Compare the computational effort and accuracy of (a), (b), and (c). (See Moler, Question 4.15 for a similar problem.)

3. Consider the following two high-order methods:

(a) Third order method:

$$\begin{aligned} z_k &= x_k - f(x_k)/f'(x_k) \\ x_{k+1} &= x_k - (f(x_k) + f(z_k))/f'(x_k) \end{aligned}$$

(b) Fourth order method:

$$x_{k+1} = x_k - v(x_k)$$

where

$$v(x) = u(x) \frac{f(x - u(x)) - f(x)}{2f(x - u(x)) - f(x)}$$
$$u(x) = \frac{f(x)}{f'(x)}$$

Implement these two methods in Matlab and compare their convergence rates to each other and to the convergence rate of Newton's method. (Use your own choice of test functions and show your work.) Explain which of the three methods is preferable when you take into account both the convergence rate and the computational effort (e.g., the number of function evaluations)? What are other pros and cons of these two high-order methods?

4. Newton's method and multiple roots.

(a) Construct a function $f(x)$ which has a double root, i.e., there are two roots at s and we also have $f'(s) = 0$.

(b) Implement Newton's method to solve for this root using a nearby initial guess. Print the ratio of the absolute errors e_{n+1}/e_n at each iterate. What do you conclude about the convergence rate?

(c) In Newton's method, $g(x) = x - f(x)/f'(x)$. When s is a double root, show that

$$\lim_{x \rightarrow s} g'(x) = 1/2.$$

(Hint: Use l'Hôpital's rule.) What does this imply about the convergence rate?

(d) Consider the Newton-like method

$$x_{n+1} = x_n - 2f(x)/f'(x)$$

which differs from Newton's method by a factor of 2 on $f(x)$. Implement this method and solve for the double root using your original problem. Show that the convergence rate is quadratic.

(e) How does this method converge to simple roots? (Show this experimentally using Matlab.)

(f) Prove that the convergence rate of this method is quadratic for a double root s , i.e.,

$$\lim_{x \rightarrow s} g'(x) = 0.$$

(g) Comment on any roundoff error effects, if any, that you observe in your tests.