APMA E4300 – Spring 2009

Assignment 5, Due Tue. Apr. 7

There are two pages in this assignment.

1. Ascher and Greif, Chapter 10, Exercises 3 and 7.

2. Given Runge's function $f(t) = 1/(1+25t^2)$ in the interval $-1 \leq t \leq 1$, what is the interpolating polynomial (in the monomial basis) for the interpolant using 11 equidistant points? What is the interpolating polynomial (in the monomial basis) for the interpolant using 11 Chebyshev points? (You may use Matlab, but you do not need to show your Matlab work.)

3. Complexity of interpolation. Suppose you have a function and you want to interpolate it using 10 points. How many multiplications are needed to construct the interpolating polynomial using monomial, Lagrange, and Newton interpolation? How many multiplications are needed to compute the value of the polynomial at a given t in each of the three cases? Show all your results in a table. (Hint: see Ascher and Grief, Chapter 10.)

4. Barycentric Lagrange Interpolation. There is an alternative way to construct and evaluate the Lagrange interpolant. Following our notation for Lagrange interpolation (or refer to Heath), define
$$l(t) = (t-t_1)(t-t_2)\cdots(t-t_n)$$
and
$$w_j = \frac{1}{\prod\limits_{k=1,\, k\neq j}^{n} (t_j - t_k)}$$
Show that the polynomial interpolant can be written as
$$p(t) = l(t) \sum_{j=1}^{n} \frac{w_j}{t-t_j} y_j$$
Suppose you have a function and you want to interpolate it using 10 points. How many multiplications are needed to construct the barycentric Lagrange polynomial $p(t)$? Suppose you want to evaluate $p(t)$ at a large number of points t. If you organize the computation efficiently, how many multiplications are needed to compute the value of the polynomial for each value of t? Note that the w_j do not depend on t.

5. Show that the piecewise cubic Hermite polynomial between two points $t_0 = 0$ and $t_1 = 1$ with values y_0 and y_1 and derivatives m_0 and m_1 can be written as
$$p(t) = (2t^3 - 3t^2 + 1)y_0 + (t^3 - 2t^2 + t)m_0 + (-2t^3 + 3t^2)y_1 + (t^3 - t^2)m_1$$
The quantities in brackets can be considered a basis for the piecewise cubic Hermite interpolant.

Show how you can use your result above to write the interpolant for $t_0 = 1$, $t_1 = 3$, $y_0 = 1$, $y_1 = 5$, $m_0 = 1$, $m_1 = 2$. Note in particular that t_0 and t_1 are not the same as before.

6. Consider the three points (1, 0.1), (2, 0.9), (3, 2).

   (a) Find the two *natural* cubic splines that interpolate this data of the form

$$\begin{aligned} p_1(t) &= \alpha_1 + \alpha_2 t + \alpha_3 t^2 + \alpha_4 t^3, \quad 1 \leq t \leq 2 \\ p_2(t) &= \beta_1 + \beta_2 t + \beta_3 t^2 + \beta_4 t^3, \quad 2 \leq t \leq 3 \end{aligned}$$

   (b) Now find the two *natural* cubic splines using the form

$$\begin{aligned} q_1(t) &= \alpha_1 + \alpha_2(t-1) + \alpha_3(t-1)^2 + \alpha_4(t-1)^3, \quad 1 \leq t \leq 2 \\ q_2(t) &= \beta_1 + \beta_2(t-2) + \beta_3(t-2)^2 + \beta_4(t-2)^3, \quad 2 \leq t \leq 3 \end{aligned}$$

   (c) Check that your answers in part (a) and part (b) give you the same polynomials.

   (d) Which form do you feel is easier for hand computation? Which form do you feel is easier for computer implementation?