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Comment on "Why Selective Colleges Should Become Less Selective–And Get Better Students" (by Barry Schwartz)

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1 Introduction

Schwartz proposes that very selective colleges (Harvard, Stanford, etc.) change their admissions procedures by selecting randomly from a pool of candidates deemed "good enough". The main argument for this proposal is the following: competition for admission to the most selective colleges has been increasing. As a result, students have been multiplying their efforts to provide better credentials to the admission committees, and in doing so, they have been sacrificing risk-taking, intellectual curiosity and have been suffering emotional harm. Schwartz argues that the proposed policy would have the advantages of "reliev[ing] the pressure on high-achieving students to be even higher-achieving students" and would "free students up to do the things they were really passionate about while in high school". Schwartz is concerned not only with the students' welfare that result from the "college admission game", but also with the ex-post quality of the students admitted: according to Schwartz selective colleges "(...) are admitting students who have done things for the wrong reasons, and who are likely to be disappointing in college".

In this comment, I present a formal model to describe the strategic choice that students face when they have to decide how much effort to put into providing better credentials to the admissions committee. It is a simple model of screening in which the college cannot observe the quality of students but only a score that each student produces by making a costly (and possibly wasteful) effort. Higher quality students have a lower cost to produce the observable score.[1] Any admissions rule based on the observed students' score, induces a game of incomplete information among prospective students. In these games, it is assumed that the college knows the distribution of quality in the population of students. I study three different games: the one induced by the status quo admission rules (the status-quo college admission game), the game induced by the Schwartz proposal (the original Schwartz proposal game) and the game induced by a modification of the Schwartz proposal (the modified Schwartz proposal game). In the status quo college admission game, each student knows the capacity of the college (i.e. how many students the college will admit) and the probability distribution of other students' cost. Each student will choose the level of effort to maximize her expected payoff: the trade-off that each student faces is between increasing the probability of being admitted to the college and the cost of the effort necessary to increase her score. In order to reduce the wasteful effort made by high achieving students, Schwartz envisions a rule where the college establishes a threshold for "good enough". Students decide whether to produce the score set by the college. Again, I assume that the students take this decision knowing the probability distribution of costs of the other students. This enables each student to determine the expected equilibrium number of students who reach the threshold and therefore their expected payoff of the effort (the more

[1] This assumption is standard in signaling and screening models since Spence (1970).

students reach the "good enough" threshold the lower the expected payoff from passing). In the modified rule, the college establishes the number of students who will be eligible for random selection. For example, a selective college with capacity of 2,000 students can announce that that the best 4,000 students (in terms of the observable score) will participate in the lottery phase of admission. I assume that the students know both the capacity of the college and the number of students that will be eligible for random selection. I also keep assuming that students choose the score knowing the probability distribution of other students' cost. The trade-off that each student faces is again between increasing the probability to be admitted and the cost of the effort necessary to increase her score. However, in the modified rule, as in the original Schwartz proposal, the randomization stage reduces the high achieving students' incentive to produce wasteful effort. In fact, the reward for increasing the effort is not being admitted to college, but being selected to the randomization stage. Is there a sense in which the modified rule and the Schwartz rule are equivalent (like a tariff is equivalent to a quota)? For each modified rule, there is an original Schwartz rule that results in the same expected number of students being selected for the randomization. However, the students' equilibrium strategies in the two games can never be the same: the equilibrium strategy in the original Schwarz proposal game is always a step function (either a student chooses to produce a score of zero or a score equal to the threshold set by the college), while the equilibrium strategy in the modified proposal game is qualitatively similar to the equilibrium strategy in the status quo game.

It turns out that the original Schwartz proposal and the modified one have very similar properties: both proposals induce a reduction of the score produced by some students (the ones who produce the highest score with the current system) and an increase of the score produced by others (the ones who currently produce a lower score). Moreover, some students will incur in a reduction of well-being, even if the overall welfare increases with both versions of the proposal.[2] What about the effect of these two proposals on the quality of the students who are admitted to the selective college? For this purpose, it is useful to distinguish between ex-ante and ex-post quality of students. By ex-ante quality of the students, I mean the quality of the students in an ideal world in which students are not induced by the college admission race to waste their time and talent in tasks that serve only the purpose to be admitted to college. By ex-post quality, I mean the quality of students who have spent time and talent to provide better credential to be admitted to college and in doing so, they "burn out", reducing their ability to be creative and inspired students once in college. Unsurprisingly, both the original Schwartz proposal and the modified one result in a lower ex-ante quality of the students admitted to the selective college. Assessing the effect of the proposals on the ex-post quality is more complicated and I believe it requires more work to

[2] While this statement is true for the numerical simulations that I performed, it is possible in principle that overall welfare goes down in both versions of the proposal.

understand how the effort to provide credential affects the ability of the students who end up in a selective college. On one hand, the students who produce a lower score reduce their effort and will have higher ex-post quality than with the status quo. On the other hand, the students who under both proposals produce a higher score increase the level of effort and reduce the average ex-post quality of admitted students for two reasons: first, under both proposals, those students have a higher probability to be admitted to college than in the status quo and they are students with relatively lower ex-ante ability. Moreover, since they increase the effort, they might end up being worse students than under the status quo.

2 Model

There is a single selective college that has a given capacity (*P* spots per year) and there are *K* students who compete for admission to the selective college ($K > P$). In the college admission game, each player *i* makes an effort x_i that we interpret as the effort to provide credentials to impress the college admission officers. Students differ in quality and the college preferences are represented by a utility function increasing in the quality of each student admitted. The quality of student *i* is private information to *i*, but the college can observe the score provided, x_i. Attending the college is valuable to the students, but the effort spent in providing the credentials is costly to them. The quality of student *i* is perfectly (and inversely) correlated with the cost of effort. The payoff of player *i* is equal to $\overline{U} - c_i\gamma(x_i)$ if the student is admitted to college and $-c_i\gamma(x_i)$ if the student is not admitted to college, where c_i is the cost-quality parameter for student *i* and the function $\gamma(\cdot)$ is strictly increasing in x_i. Finally, it is assumed that the cost-quality parameters c_i are drawn independently to each other from an interval [*m*, 1] according to the distribution function *F* (with everywhere continuous density *f*).

The assumptions of the model are standard in the literature of signaling and screening, and in particular the literature on contests with asymmetric information.[3] However it is important to clarify a matter of interpretation: in order to simplify the analysis, there is no distinction in the model between "effort" and "score". And so, the assumption that for higher quality students it is cheaper to produce a higher score can be translated in the assumption that for higher quality students it is cheaper to exert a higher effort that results in a higher score. When it comes to interpreting the effects of the different proposals on the wasteful effort exerted by the students, one must be careful in understanding what the model implies.

[3] The model is in fact identical to Moldovanu and Sela (2001), so that some of the results (proposition 1) follow immediately from it.

2.1 The status quo game

In the status quo game, the college admits the first P students ranked according to their score. The timing of the status quo game is the following: at stage 1, the student ability is realized (a move of nature). Each student observes her own ability, c_i, but not the ability of the other students. Also, students know the capacity of the college P and the probability distribution of the cost parameter F. In the second stage, each student i chooses the score level x_i. In the third stage, the selective college observes the score of all the students (but not their abilities) and admits the first P students ranked according to their score level chosen. A pure strategy for student i is a function that maps the cost-ability of student i to the score level chosen by student i. Denote with $x_{sq}(c)$ the symmetric, pure-strategy Bayesian Nash equilibrium of the status quo game.

2.2 The modified Schwartz proposal game

The modified Schwartz proposal game can be described as a new game. At stage 1 the college announces the value of $\alpha \geq 1$, a randomization factor. In the second stage, the students' ability is realized. Each student observes her own ability c_i, but not the ability of the other students. Also, students know the capacity of the college P, the randomization factor α and the probability distribution of the cost parameter F. At stage 3, each student chooses the score level x_i. At stage 4, the college chooses P students randomly over the set of students with the best αP scores. Note that if α is set to be equal to 1, this game is identical to the status quo game. The higher the α, the higher the role of randomization in the selection of the students. For each α chosen by the selective college, denote with G_α, the Bayesian game among the prospective students. For each G_α, a pure strategy for student i is a function that maps the cost-ability of student i to the effort level chosen by student i. Denote with $x_{mp(\alpha)}(c)$ the symmetric, pure-strategy Bayesian Nash equilibrium of the modified proposal game G_α.

Notice, that while the modified proposal game is a game in which the college is a player, I am focusing attention on the subgames G_α; that is, on games in which the college has already chosen the parameter α and so the only players are the K prospective students.

2.3 Comparison between status quo and modified Schwartz proposal games

What would be the consequences of the modified Schwartz proposal on the students' effort (i.e. their score), their welfare, and on the quality of the students admitted to the selective college? The difference between the two games is that in the status quo game the students compete for P prizes, each of them valued $\overline{U}$ while in the modified games students compete for more prizes (αP), each of them valued only $\frac{1}{\alpha}\overline{U}$. The model is a particular case of Moldovanu and Sela (2001) that studies contests with asymmetric information and finds how the equilibrium effort function depends on the number and sizes of prizes.[4] In particular, using their results we have the following:

Proposition 1.

1. *The equilibrium effort strategy $x(c_i)$ is decreasing in c_i; that is, higher-ability students produce higher scores. This is true for both the status quo game and the modified Schwartz proposal game.*

2. *Students with higher ability produce lower scores with the new admittance rule respect to the status quo. The opposite is true for students with lower ability.*

3. *If the cost of producing the score is linear or concave, the total score $\int_m^1 x(c)\,dF(c)$ is lower under the Schwartz proposal game than under the status quo game.*

The proof of proposition 1 follows immediately from Moldovanu and Sela (2001). How can we interpret these results? First, higher quality students produce a lower score with the modified proposal. This implies that they exert less wasteful effort, and for this reason, their utility increases and they might become better college students, if admitted. However, their probability of being admitted could decrease (this is certainly true for the very top students) and for this reason their expected utility decreases. In fact, at least in the numerical example presented below, the very top students experience a decrease in welfare, while the rest of students who produce a lower score as a result of the policy have a higher expected utility. What about the students who produce a higher score as a result of the policy? They increase their effort and so they might become worse students and experience a decrease in expected utility. On the other hand, their probability of being admitted to college increases and this increases their expected utility. Again, in the numerical examples I considered, those students experience an increase in expected utility.

What are the implications of the result on the reduction of the total score produced by the students? It implies a reduction in the total cost of producing the score and therefore an increase in total welfare for the numerical examples I considered. I conjecture that this would hold for most plausible choices of the probability distribution, *F*.

Finally, what happens if the cost of producing the score is a convex function? This, in fact seems the most reasonable assumption to make, and Moldovanu and Sela (2001) show that with this assumption we could have that the total score might increase with this policy. However, if the total cost decreases in the linear case, like it is the case in the numerical

[4] As has been noted, this model is isomorphic to a private value all-pay auction with several prizes. See Moldovnu and Sela (2001) for a relation with this literature and the larger literature of contests.

examples I examined, then it will decrease also for a convex cost function (see online appendix).

2.4 Numerical example part 1

I now introduce a numerical example to illustrate the results discussed above: there are 10 students competing for one spot in the selective college. Assume that the students' ability is uniformly distributed over the interval $[\frac{1}{2}, 1]$, and the cost of producing the score is linear. That is, assume $K = 10$, $P = 1$, $F(c) = 2c - 1$ and $\gamma(x) = 1$ (we also normalize $\overline{U} = 1$). Finally assume $\alpha = 2$; that is, if the college is admitting students following the modified Schwartz proposal, it is choosing randomly between the two students with the highest score. With these parameters, we can calculate the equilibrium score function with the status quo (denoted $x_{sq}(c)$) and the modified Schwartz proposal (denoted $x_{mp}(c)$). Applying proposition 1 of Moldovanu and Sela (2001) we have that if the college uses the status quo admittance rule the equilibrium score function is[5]

$$x_{sq}(c) = 9\int_c^1 \frac{1}{a}(2\text{-}2a)^8 \times 2da \qquad (1)$$

On the other end, if the admittance rule is the modified Schwartz proposal with $\alpha = 2$, the equilibrium score function is

$$x_{mp}(c) = \frac{1}{2}\left(9\int_c^1 \frac{1}{a}(2\text{-}2a)^8 \times 2da\right) + \frac{1}{2}\left(9\int_c^1 \frac{1}{a}(2\text{-}2a)^7 \times [9(2a\text{-}1)\text{-}1]2da\right) \qquad (2)$$

[5] See the online supplement for a derivation of the equations (1) and (2).

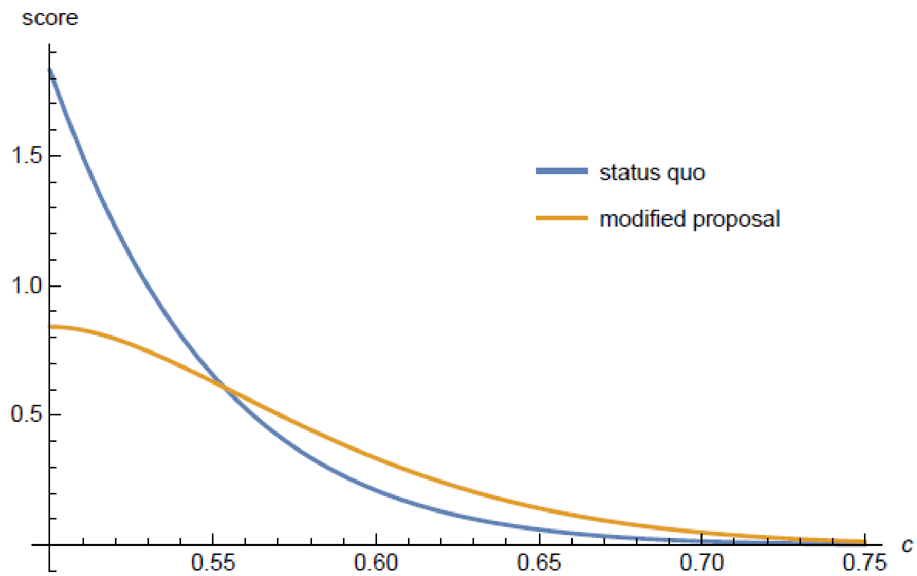

Figure 1: Score function with status quo and with modified proposal

Figure 1 illustrates both score functions. Notice that in Figure 1, I only represent the top 50% of the candidates, that is students with $\frac{1}{2} \leq c \leq \frac{3}{4}$ because for the bottom 50% students, the score function is very close to zero for both admittance rules. Also notice that approximately the top 10.6% of the students decrease their score if the college moves from status quo to randomization over the two top students, while the remaining 89.4% of the students increase their effort, even if most of them don't increase it by much.

Figure 2 illustrates the students' welfare in the status quo and with the modified Schwartz proposal game.

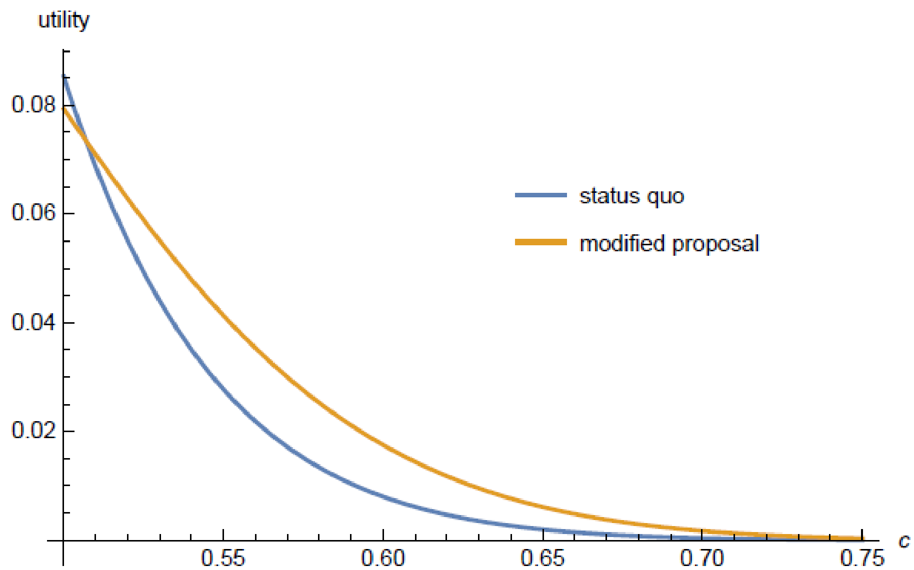

Figure 2: Students' welfare with status quo and modified proposal

The very top students (approximately 1.4% of the student population) experience a decrease in welfare going from the status quo to the modified proposal, while the rest of the students are better off with the modified proposal.[6] Finally the total students' welfare is higher with the modified Schwartz proposal than with the status quo.

2.5 The original Schwartz proposal

In the original Schwartz proposal, the selective college sets a known threshold x_T and chooses randomly above all the students who produce a score above the threshold. The original Schwartz proposal game can be described as follows: In the first stage the college announces the threshold x_T. In the second stage, the student ability is realized (a move of nature). Each student observes her own ability c_i, but not the ability of the other students. Also, students know the capacity of the college P, the threshold set by the college x_T and the probability distribution of the cost parameter F. In the third stage, each student i chooses the score level

[6] Even if for more than 50% of the students the change is minimal. Those are the students who have virtually zero probability of being admitted and choose a score very close to zero with both admittance rules.

x_i. Finally the college observes the score level produced by all the students and chooses randomly P students among the ones with $x_i \geq x_T$.[7]

For each x_T chosen by the selective college, denote with G_{x_T} the Bayesian game among the prospective students. For each G_{x_T} a pure strategy for student i is a function that maps the cost-ability of student i to the score level chosen by student i. Denote with $x_{SP(x_T)}(c)$ the symmetric, pure-strategy Bayesian Nash equilibrium of the original Schwartz proposal game G_{x_T}. Again notice, that while in the original Schwartz proposal game, the college is a player, I am focusing attention to the subgames G_{x_T}, that is on games in which the college has already chosen the parameter x_T and so the only players are the K prospective students.

Given the admittance rule of the college, a student will choose either score $x_i = x_T$ or score $x_i = 0$. Clearly, if a student with cost-quality parameter c_i chooses $x_i = x_T$, all the students with $c < c_i$ will also choose $x = x_T$. Therefore, the equilibrium score function $x_{SP(x_T)}(c)$ of this game has the following structure:

$$x_{SP(x_T)}(c) = \begin{cases} x_T, & c \leq c' \\ 0, & c > c' \end{cases} \tag{3}$$

The student with cost parameter c' can be thought as the marginal student, she is indifferent between choosing score x_T and choosing score $x = 0$ and she has the highest cost of producing the score (and therefore the lowest ex-ante quality) among the students that do provide the minimal credential that the college requires. Moreover the expected utility of the marginal student is very close to zero and it should be considered zero for policy analysis purposes.[8] What are the effects of the original Schwartz proposal on the students' scores? Denote with $c' = f(x_T)$ the function from the threshold set by the college to the cost parameter of the marginal student. The function f is decreasing, the lower the threshold x_T, the higher the cost parameter c' for the marginal student. Denote with $\overline{x}_T$ the level of score such that $\overline{x}_T = x_{sq}(m)$ (that is, $\overline{x}_T$ is the level of score chosen by the student with lowest possible cost parameter in the status quo college admission game) and with $\underline{x}_T$ the level of score such

[7] In order to define the game properly, we need to specify what happens if the number of students with $x_i \geq x_T$ is less than P. In this case, we assume that the college admits all the students with $x_i \geq x_T$ and chooses randomly over the remaining students irrespective of the effort x_i.

[8] The reason for which is not exactly zero, is that there is a positive probability that the number of students that choose $x = x_T$ is less than P. Therefore in this model, a student that chooses $x = 0$, has an expected utility larger than zero and so the marginal student will have the same expected utility of the students who choose zero score.

that $1 = f(\underline{x}_T)$ (that is $\underline{x}_T$ is the the level of the threshold such that the student with the highest possible cost parameter is indifferent between choosing $x = \overline{x}_T$ and choosing $x = 0$). We have the following

Proposition 2. If the threshold set by the college x_T is such that $\underline{x}_T < x_T < \overline{x}_T$, then there exist c_1 and c_2 with $m \leq c_1 \leq c_2 \leq 1$, such that

1. *Students with cost parameter c, with $m \leq c < c_1$ and with $c_2 < c < 1$, produce a lower score with the original Schwartz respect to the status quo, that is $x_{SP(x_T)}(c) < x_{sq}(c)$*

2. *Students with cost parameter c, with $c_1 < c < c_2$, produce a higher score with the original Schwartz respect to the status quo, that is $x_{SP(x_T)}(c) > x_{sq}(c)$.*

The proof of Proposition 2 is in the online appendix. Proposition 2 clarifies the effect of the Schwartz proposal on the score made by the students. At one extreme, the very high quality students, who with the status quo are producing a score above the threshold set by the college, reduce the score to the level x_T. At the other extreme, there will be students, the low quality ones, who reduce the score to zero. However, there is a middle range of students who prefer to increase the score to x_T and so have a chance at college acceptance rather than choosing zero score.

2.6 Numerical example part 2

To illustrate the results of Proposition 2 and to analyze the effects of the original Schwartz proposal on the students welfare, I turn to the same numerical example used above (that is $P = 1$, $K = 10$, $m = \frac{1}{2}$, $F(c) = 2c - 1$, $\gamma(x) = 1$ and $\overline{U} = 1$). I further assume that the selective college announces a threshold $x_T = 0.552338$. Such a threshold induces a marginal student with cost parameter equal to $c' = \frac{22}{48}$ and an average ex-ante quality for the admitted students in line with the average ex-ante quality obtained in the modified proposal with $\alpha = 2$.[9] Figure 3 illustrates the score function with the status quo college admission game, the original Schwartz proposal and the modified proposal.

[9] The average ex-ante quality in the modified proposal game is $\frac{25}{44}$. See the online appendix for the calculations to obtain the threshold $x_T = 0.552338$.

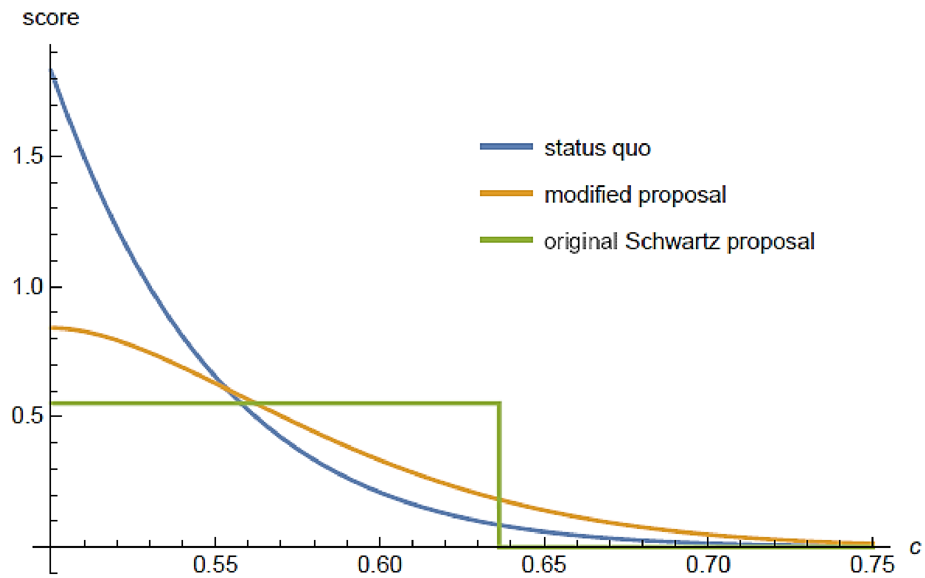

Figure 3: Score function with status quo, the original Schwartz proposal and the modified proposal

The students in the top 12% of the distribution of quality reduce the score to x_T, while the students in the bottom 74% of the population reduce the score to $x = 0$. However the students in the middle range between these two groups increase their effort to x_T.

Figure 4 illustrates the students' welfare with the status quo college admission game, the original Schwartz proposal and the modified proposal.

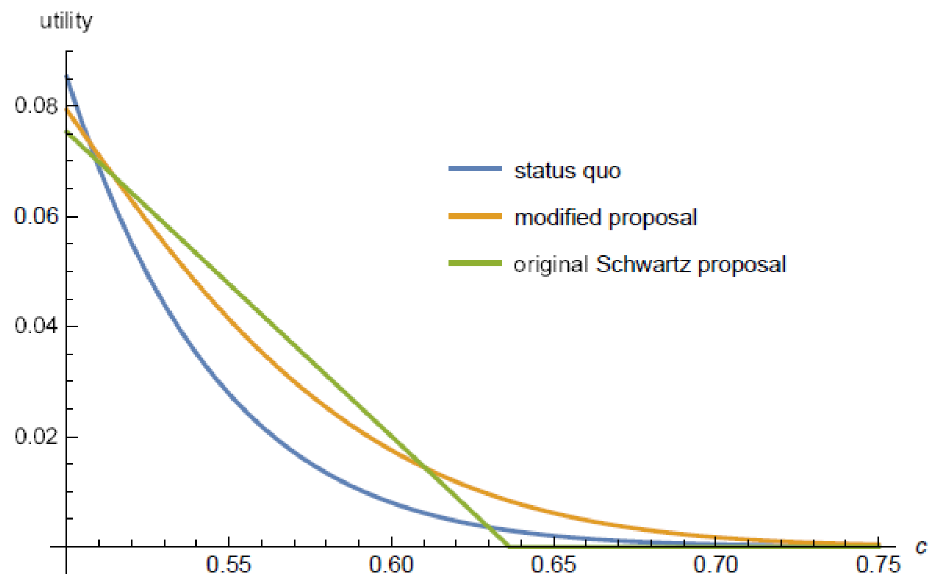

Figure 4: Students' welfare with status quo, the original Schwartz proposal and the modified proposal

We can divide the population of students in 5 groups. In the first group ($0.5 < c < 0.51$) the very top students experience a decrease in expected utility going from the status quo to the Schwartz proposal. For these students, the decrease in cost of producing the score does not compensate for the reduction in the probability to be admitted to college. In the second group (with c approximately $0.51 < c < 0.56$) students reduce the score and experience an increase in expected utility respect to the status quo. In the third group, (approximately $0.56 < c < 0.61$) students increase the score and experience an increase in expected utility respect to the status quo. For these students, the increase in the probability of being admitted to college more than compensates for the increase of the cost of producing the score. In the fourth group (approximately $0.61 < c < 0.63$) students increase the score and experience a decrease in expected utility respect to the status quo. Finally in the fifth group (with $c > 0.63$), the students reduce the score to zero and experience a decrease in expected utility. In this example, the overall welfare is higher with the Schwartz proposal than with the status quo and I suspect this is true more generally.

3 Conclusions

The simple model presented here abstracts from many of the considerations that Schwartz makes in his article.[10] And yet, I believe, it clarifies the merits and limitations of introducing some randomization in the college admission process. For example, Schwartz reports that "people argue that all a proposal like mine will do is focus competition on getting to the right side of the cut-off line between good enough and not". According to Proposition 2, this is not true: for any reasonable choice of the threshold (that is for $x_T < x_{sq}(m)$) with the Schwartz original proposal, some students will reduce their score to x_T. On the other hand, it is also true that some other students will increase their score to reach the threshold set by the college and in doing so, they will bring their level of utility close to their "reservation utility" (the utility of not attending college and not producing any score) and they will reduce their ex-post quality. Another conclusion of the model is that both proposals will have some "losers" among the student population. For the modified proposal, these are the very top quality students, while for the original Schwartz proposal they are both the very high top quality students and students below a certain quality level.

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[10] For example, I assume that the admission professionals are able to distinguish between any two score levels x and x', with $x \neq x'$.