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ORIGINAL PAPER

# Commitment without reputation: renegotiation-proof contracts under asymmetric information

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**Abstract** This paper characterizes equilibrium outcomes of extensive form games with incomplete information in which players sign renegotiable contracts with third parties. Our aim is to understand the extent to which third-party contracts can be used as commitment devices when it is impossible to commit not to renegotiate them. We characterize renegotiation-proof contracts and strategies for extensive form games with incomplete information and apply our results to two-stage games. If contracts are observable, then the second mover obtains the best possible payoff given that she plays an incentive compatible and renegotiation-proof strategy and the first mover best responds. If contracts are unobservable, then any Bayesian Nash equilibrium outcome of the original game in which the second mover plays an incentive compatible and renegotiation-proof strategy can be supported. We apply our results to Stackelberg competition and show that renegotiation-proofness imposes a very simple restriction.

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## 1 Introduction

Could an incumbent firm deter entry by contracting with third parties, such as a bank or a labor union? Could a government credibly commit to fiscal policy through a contract with a supranational body? More generally can contracts with third parties change the outcome of a game to the advantage of the contracting player? When contracts are non-renegotiable, the answer to this question is in general yes.[1] An incumbent can prevent entry by writing a contract that specifies a large enough payment to a third party if it fails to punish the entrant. A country can obtain entry into European Union or better borrowing terms by writing a contract that specifies a fine if it runs high budget deficits.

Unfortunately, the very same reason that makes contracts useful as commitment devices, also makes them susceptible to renegotiation and calls their credibility into question. If entry occurs, for example, the incumbent and the third party have an incentive to renegotiate the contract because punishing the entrant reduces the total surplus available to them. In fact, this is true more generally. If renegotiation takes place without any frictions, the contracting party will always best respond to other players, nullifying the commitment power of contracts. This problem is well known and has led to well founded skepticism about the robustness of results obtained with non-renegotiable contracts.

Dewatripont (1988) was the first to show that this skepticism may be unwarranted in certain situations. He analyzed an entry game and showed that the incumbent can deter entry even with renegotiable contracts, as long as there is asymmetric information between the incumbent and the third party at the renegotiation stage and the contracts are publicly *observable*. Dewatripont's result is important because it reinstates the commitment value of contracts within a commonly used model.

In this paper, we follow Dewatripont's lead and extend his analysis beyond the entry model to general extensive form games with incomplete information. We show that his message applies more generally. As long as one can find a third party over whom the contracting party has an informational advantage, contracts can be used as commitment devices even when they are renegotiable. More specifically, our analysis achieves three main objectives. First, we can use our results to determine whether existing results on commitment through contracts are robust to renegotiation. Second, we can use our results as a guide to "design" renegotiation-proof contracts that achieve some strategic objective. Third, we can characterize the outcomes that can be supported with renegotiable contracts for both observable and *unobservable* contracts.

[1] See, among many others, Vickers (1985), Fershtman and Judd (1987), Sklivas (1987), Koçkesen et al. (2000), Brander and Lewis (1986), Bolton and Scharfstein (1990), Snyder (1996), Spencer and Brander (1983), Brander and Spencer (1985), Eaton and Grossman (1986), Walsh (1995).

In the main body of the paper we analyze a two-stage game, which we call the *original game*, and allow the second mover to write a contract with a neutral third party. We model renegotiation as a game form: The contracting party (the incumbent or the government), who has private information about the state of the world, can make a renegotiation offer to the (uninformed) third party (the labor union or the supranational body). This game form is different from the one chosen by Dewatripont and it simplifies the characterization of renegotiation-proof contracts.[2] We analyze the renegotiation-proof Perfect Bayesian equilibrium of the game with contracts, and this leads us to the following definition of renegotiation-proof contracts (see Definition 5): A contract is renegotiation-proof if it is optimal for the third party to reject any renegotiation offer that is found profitable by the contracting party. This can happen in equilibrium if the third party puts a high probability on a "blocking type" of the contracting party, i.e., a type which, under the renegotiated contract, would not transfer more to the third party than under the old contract. However, using this definition directly is not very easy. In Sect. 3 we present more operational characterizations of renegotiation-proof strategies, which are adaptations of some recent results in Gerratana and Koçkesen (2012) to our setting.

In Sect. 4 we characterize the equilibrium outcomes of games with contracts. We allow contracts to be observable or unobservable (by other players) and renegotiable or non-renegotiable. If contracts are observable and non-renegotiable, then the contracting player obtains her Stackelberg payoff (of the original game), i.e., the best payoff that she can achieve given that she plays an incentive compatible strategy and the first mover plays a best response. If they are unobservable and non-renegotiable, then any Bayesian Nash equilibrium outcome of the original game in which the second mover plays an incentive compatible strategy can be supported. We show that the possibility of renegotiation affects the games with observable and with unobservable contracts in the same way: In both cases the only additional restriction is that the contracting player's strategy is renegotiation-proof.

In Sect. 5, we illustrate our results by applying them to a quantity competition and entry-deterrence game. This game is the canonical model in which the second mover has a strategic disadvantage and hence may benefit from commitment via third-party contracts. Furthermore, it allows us to compare our findings with those of Dewatripont (1988) (see Proposition 9 in Sect. 5) and identify the contribution of the current paper over Gerratana and Koçkesen (2012), which analyzes the same question in a different model (see Sect. 1.1). We show that, when applied to this model, renegotiation-proofness imposes a very simple restriction: The harshest credible punishment that the incumbent (follower) can inflict upon the entrant (leader) is to best respond in the worst state of the world, i.e., when the incumbent's unit cost is highest, and flood the market in all other states.

In fact, this is true in a more general class of games in which the first mover's payoff is monotone increasing (or decreasing) in the second mover's action. For this class of games, which includes many interesting economic environments such as sequential quantity and price competition, monopolistic screening, and ultimatum bargaining,

[2] See Sect. 5 (Proposition 9) for a comparison of the two renegotiation protocols.

renegotiation-proofness imposes the same simple restriction: The harshest credible punishment that player 2 can inflict upon player 1 always requires player 2 to best respond after the highest (or lowest) state of the world and to choose the lowest (highest) action for all the other states. We further discuss the intuition behind this result in Sect. 5 after analyzing the quantity competition game and refer the reader to Gerratana and Koçkesen (2013) for a complete analysis.

In Sect. 6, we discuss how in results of Sect. 3 can be generalized in several dimensions. These generalizations are useful in applications where the third party is not neutral or those that cannot be modeled as two-stage games.

### 1.1 Relationship to the literature

This paper contributes to the literature on the the strategic effects of third-party contracts. The role of third-party contracts are maximal when they are both observable and non-renegotiable. In fact, there are several "folk theorem" type results for different classes of games (see Fershtman et al. 1991, Polo and Tedeschi 2000, and Katz 2006). The effects of *unobservable and non-renegotiable* third-party contracts are also well-understood: Nash equilibrium outcomes of a game with and without third-party contracts are identical (Katz 1991). In fact, all (and only) Nash equilibrium outcomes of the original game can be supported as a sequential equilibrium outcome of the game with unobservable and non-renegotiable contracts (Koçkesen and Ok 2004, Koçkesen 2007).[3]

The strategic role of renegotiable contracts is less understood and the pioneering contribution is provided by Dewatripont (1988). As we explained above, one contribution of our paper is to show that commitment effects exist in arbitrary two-stage games. Furthermore, we show that commitment effects exist even if contracts are unobservable and hence they exist also if we allow the contracts to be renegotiated immediately after they are signed.[4]

Caillaud et al. (1995) analyzes a game between two principal-agent hierarchies. In the first stage of their game each principal decides whether to publicly offer a contract to the agent; in the second stage each principal offers a secret contract to the agent, which, if accepted, overwrites the public contract that might have been offered in stage 1; in the third stage each agent receives a payoff relevant information, decides whether to quit, and if he does not quit, he plays a normal form game with the other agent. Their main question is whether there exist equilibria of this game in which the principals choose not to offer a public contract in stage 1. If the answer to this question is no, then the interpretation is that contracts have commitment value. They show that contracts have commitment value if the market game stage is of Cournot

[3] Prat and Rustichini (2003) and Jackson and Wilkie (2005) analyze related models in which players can write action contingent contracts before the game is played. Unlike the current paper, in these papers contractual relationships are not exclusive and the focus is on the efficiency properties of the equilibrium set. Also related is Bhaskar (2009), in which players need to pay a price to a supplier in order to play certain actions that are controlled by the supplier.

[4] Allowing for secret renegotiation right after the contract is signed has no effect on the results if the game is with unobservable contracts and reduces the case of observable contracts to unobservable ones.

type, but not if it is of Bertrand type. Moreover, when contracts have commitment value, they reduce the payoff of the contracting parties. The received message of Caillaud et al. (1995), in comparison with Dewatripont (1988), is that by allowing secret renegotiation, Caillaud et al. (1995) enhanced the realism of the model and clarified the role of strategic contracting.[5] The crucial difference between our model and Caillaud et al. is that they assume that agents play a simultaneous move game (and principals offer contracts to the agents simultaneously) whereas we focus on sequential move games. Therefore, one contribution of our paper is to show that the differences between the results of Caillaud et al. (1995) and Dewatripont (1988) do not depend on allowing secret renegotiation right after the contracts are signed, but instead depend on the fact that Dewatripont (1988) studies a sequential move game while Caillaud et al. (1995) a simultaneous move game.

The current paper follows Gerratana and Koçkesen (2012) (from now on GK (2012)), which also studies the effects of renegotiation-proof third-party contracts in two-stage games. Some aspects of the analysis in the two papers are similar and use similar tools, namely theorems of the alternative. Indeed, results on renegotiation-proof strategies in Sect. 3 are exact analogs of their counterparts in GK (2012). However, the games to which these results are applied are completely different: In GK (2012), the original game is with complete information and the asymmetry of information between the contracting player (second mover) and the third party is due to the assumed inability of the third party to observe the action of the first mover. The current paper considers games in which the contracting player has private information and therefore there is no need to add another layer of asymmetric information between that player and the third party.

The modeling choice in the current paper is more standard and allows us to determine whether some of the well-known results on commitment through contracts are robust to renegotiation. This is not possible in GK (2012), as it considers original games with complete information.

Perhaps the best way to illustrate the differences between the current paper and GK (2012) is to consider an application that we will analyze in more detail in Sect. 5.

#### *1.1.1 An application: quantity competition and entry-deterrence*

Consider a Stackelberg competition in which firm 1 moves first by choosing an output level $q_1 \in Q_1$ and firm 2, after observing $q_1$, chooses its own output level $q_2 \in Q_2$. Inverse demand function is given by $P(q_1, q_2) = \max\{0, \alpha - q_1 - q_2\}$, where $\alpha > 0$, and we assume $Q_i$ is a rich enough finite subset of $\mathbb{R}_+$ whose largest element is $\alpha$.[6] Cost function of firm 1 is $C_1(q_1) = cq_1$, where $c$ is common knowledge, whereas the cost function of firm 2 is $C_2(q_2) = \theta q_2$. We assume that $\theta \in \{\theta^1, \theta^2, \ldots, \theta^n\}$, where $n \geq 2$, is private information of firm 2 and $\theta^1 < \theta^2 < \cdots < \theta^n$. Firm 1 believes that the probability of $\theta^i$ is given by $p(\theta^i)$ and for ease of exposition we assume that expected value of $\theta$ is equal to $c$. The profit function of firm $i$ is given

5 See Bolton and Dewtripont (2005) pages 631–636.

6 We introduce this assumption so that player 2 can choose a high enough output level to drive the price to zero.

by $\pi_i(q_1, q_2, \theta) = P(q_1, q_2)q_i - C_i(q_i)$ and we assume that both firms are profit maximizers.

As we have already mentioned, the first difference is that, while in GK (2012) the original game is a game with complete information, i.e., firm 2's cost function is common knowledge, in the current paper it is firm 2's private information. Secondly, and more interestingly, this difference in modeling has important consequences with regard to the effects of renegotiation. In the class of games for which GK (2012) obtain sharp results, renegotiation is "irrelevant," that is, the outcomes of games with unobservable and non-renegotiable contracts are robust to the introduction of renegotiation (see Corollary 1 in GK 2012). This class contains the complete information version of the quantity competition game introduced above. GK (2012) shows that in this game an outcome can be supported with *unobservable* third-party contracts if and only if firm 1's profit is non-negative, its output is at least as high as the Cournot Nash equilibrium output, and the follower's output is a best response to that, irrespective of whether the contracts are *non-renegotiable* or *renegotiable*. In particular, entry cannot be deterred with either non-renegotiable or renegotiable contracts (see Section 6.1 in GK 2012).

This is not true in the current paper. Section 5 shows that any outcome in which firm 1 obtains non-negative profit and firm 2 best responds to firm 1's quantity choice can be supported with *unobservable and non-renegotiable contracts*. In particular entry can be deterred. If, however, the contracts are *unobservable and renegotiable*, then under certain conditions (Condition (5) in Sect. 5) the lower bound on firm 1's profit is positive, which implies that entry cannot be deterred with renegotiable contracts. In other words, renegotiation, in general, has a bite. Therefore, one contribution of our paper is to show that the "irrelevance of renegotiation" result obtained in GK (2012) is specific to games with complete information and does not come from the definition of renegotiation-proofness.

Thirdly, GK (2012) considers only the case of unobservable contracts, whereas the current paper analyzes observable contracts as well. This allows us clarify precisely the distinction between observable and unobservable contracts with and without renegotiation. For example, in the quantity competition game, entry-deterrence is the unique equilibrium outcome under *observable and non-renegotiable contracts*, while it is the unique outcome under *renegotiable* contracts if and only if condition (5) in Sect. 5 is not true. If, on the other hand, contracts are unobservable, the set of equilibrium outcomes is larger.

Finally, we show that it is possible to extend the main results on characterization of renegotiation-proof contracts and strategies in GK (2012) in two non-trivial directions (see Sect. 6). The first extension is to arbitrary extensive form games that satisfy an increasing differences property, examples of which include the chain store and repeated bargaining games. The second extension is to allow for non-neutral third parties, i.e., a third party who cares not only about the transfer he receives (or pays out) but also about the outcome of the original game. One could think of many situations in which this would be a more suitable assumption than a neutral third party. For example, the European Union, in its contractual relationships with Airbus, would be interested in the outcome of the competition between Airbus and Boeing.

## 2 The model

Our aim is to understand the effects of renegotiation-proof third-party contracts in extensive form games. In this section, we will do this in a particularly simple environment, namely two-stage games with private information, which we call the *original game*. The main reason we present our results for two-stage games is ease of exposition. Still, we should note that many models in economics such as the entry game, the Stackelberg game, and monopolistic screening belong to this class of games. Furthermore, we show in Sect. 6 that our main results extend to arbitrary extensive form games with incomplete information as long as they satisfy an increasing differences property.

We allow one of the players to sign a contract with a third party before the original game begins and call this new game the *game with third-party contracts*. The contract specifies a transfer between the player and the third party as a function of the contractible outcomes of the original game. The crucial aspect of our model is the presence of asymmetric information between this player and the third party during the renegotiation phase.

More precisely, we define the *original game*, denoted $G$, as follows: Nature chooses $\theta \in \Theta$ according to probability distribution $p \in \Delta(\Theta)$. After the move of Nature, player 1, without observing $\theta$, chooses $a_1 \in A_1$. Lastly, player 2 observes $(\theta, a_1)$ and chooses $a_2 \in A_2$. We assume that $A_1$, $A_2$, and $\Theta$ are finite and let $p(\theta)$ denote the probability of Nature choosing $\theta$. Payoff function of player $i \in \{1, 2\}$ is given by $u_i : A \times \Theta \to \mathbb{R}$, where $A = A_1 \times A_2$.

The *game with third-party contracts* is a three player extensive form game described by the following sequence of events: Player 2 offers a contract $f : A \to \mathbb{R}$ to a third party. The third party accepts (denoted $y$) or rejects (denoted $n$) the contract. In case of rejection the game ends and the third party receives a fixed payoff of $\delta \in \mathbb{R}$ while player 2 receives $-\infty$.[7] In case of acceptance, player 1 and 2 play the original game. We assume that throughout the entire game $\theta$ remains the private information of player 2. We should note that the contracting phase takes place before player 2 learns $\theta$.

Since offering a contract that is rejected yields player 2 a very small payoff, the contract offer will be accepted in all equilibria. Therefore, for simplicity, we omit the third party's acceptance decision from histories and represent an outcome of the game with third-party contracts as $(f, \theta, a_1, a_2)$. The payoff functions in the game with contracts are given by $v_1(f, a_1, a_2, \theta) = u_1(a_1, a_2, \theta)$, $v_2(f, a_1, a_2, \theta) = u_2(a_1, a_2, \theta) - f(a_1, a_2)$, $v_3(f, a_1, a_2, \theta) = f(a_1, a_2)$, where $v_3$ is the payoff function of the third party. Note that the payoff function of the third party assumes that he is neutral towards the outcome of the game, i.e., he cares only about the transfer. In Sect. 6 we discuss what happens if we allow the third party to have intrinsic preferences over the outcomes of the original game.

The game is with renegotiable contracts if the contracting parties can renegotiate the contract after player 1 plays $a_1$ and before player 2 chooses $a_2$. We assume that player 2, who is the informed party, initiates the renegotiation process by offering

[7] This assumption is made only to eliminate equilibria in which no contract has been signed and can easily be relaxed.

a new contract, which the third party may accept or reject. If the third party rejects the renegotiation offer $g$, then player 2 chooses $a_2 \in A_2$ and the outcome is payoff equivalent to $(f, \theta, a_1, a_2)$. If he accepts, then player 2 chooses $a_2 \in A_2$ and the outcome is payoff equivalent to $(g, \theta, a_1, a_2)$.

We say that the game is with observable contracts if the initial contract is observed by player 1. Otherwise, we say that the game is with unobservable contracts. In other words, there are four possible games with third-party contracts depending upon whether the contract is renegotiable or non-renegotiable and observable or unobservable. Given an original game $G$, we will denote the game with non-renegotiable and observable contracts with $\Gamma_{NO}(G)$, non-renegotiable and unobservable contracts with $\Gamma_{NU}(G)$, renegotiable and observable contracts with $\Gamma_{RO}(G)$, and renegotiable and unobservable contracts with $\Gamma_{RU}(G)$.

A *behavior strategy* for player $i \in \{1, 2, 3\}$ is defined as a set of probability measures $\beta_i \equiv \{\beta_i(I) : I \in \mathscr{I}_i\}$, where $\mathscr{I}_i$ is the set of information sets of player $i$ and $\beta_i(I)$ is defined on the set of actions available at information set $I$. One may write $\beta_i(h)$ for $\beta_i(I)$ for any history $h \in I$. By a *system of beliefs*, we mean a set $\mu \equiv \{\mu(I) : I \in \mathscr{I}_i$ for some $i\}$, where $\mu(I)$ is a probability measure on $I$. A pair $(\beta, \mu)$ is called an *assessment*. An assessment $(\beta, \mu)$ is said to be a *perfect Bayesian equilibrium* (PBE) if (1) each player's strategy is optimal at every information set given her beliefs and the other players' strategies; and (2) beliefs at every information set are consistent with observed histories and strategies.[8]

We will limit our analysis to pure behavior strategies, and hence a strategy profile of the original game $G$ is given by $(b_1, b_2) \in A_1 \times A_2^{A_1 \times \Theta}$.[9] For any behavior strategy profile $(b_1, b_2)$ of $G$, define the expected payoff of player $i = 1, 2$ as $U_i(b_1, b_2) = \sum_{\theta \in \Theta} p(\theta) u_i(b_1, b_2(b_1, \theta), \theta)$ and the best response correspondences as $BR_1(b_2) = \text{argmax}_{a_1 \in A_1} U_1(a_1, b_2)$ for all $b_2 \in A_2^{A_1 \times \Theta}$ and $BR_2(a_1, \theta) = \text{argmax}_{a_2 \in A_2} u_2(a_1, a_2, \theta)$ for all $(a_1, \theta) \in A_1 \times \Theta$. We say that a strategy profile $(b_1^*, b_2^*)$ is a *Bayesian Nash equilibrium* of $G$ if $b_1^* \in BR_1(b_2^*)$ and $b_2^*(b_1^*, \theta) \in BR_2(b_1^*, \theta)$ for all $\theta$. The difference between a perfect Bayesian equilibrium and a Bayesian Nash equilibrium, of course, is that the former requires player 2 to best respond to every action of player 1, whereas the latter requires best response to only the equilibrium action. Therefore, every perfect Bayesian equilibrium is a Bayesian Nash equilibrium but not conversely.

For any behavior strategy profile $(b_1, b_2)$ in $G$, we say that an assessment $(\beta, \mu)$ in $\Gamma_k(G), k = NO, NU, RO, RU$, *induces* $(b_1, b_2)$ if in $\Gamma_k(G)$ player 1 plays according to $b_1$ and, after the equilibrium contract, player 2 plays according to $b_2$.[10]

Our ultimate aim is to characterize renegotiation-proof equilibria, in which the equilibrium contract is not renegotiated after any history.[11] More precisely,

[8] See Fudenberg and Tirole (1991) for a precise definition of perfect Bayesian equilibrium.

[9] In Sect. 6 we relax this and allow also mixed strategies. This introduces some technical difficulties but our main results go through.

[10] Note that in $\Gamma_{RO}(G)$ and $\Gamma_{RU}(G)$, player 2 may choose an action $a_2 \in A_2$ either without renegotiating the initial contract or after attempting renegotiation.

[11] We follow the previous literature in our definition of renegotiation-proof equilibrium. See, for example, Maskin and Tirole (1992) and Beaudry and Poitevin (1995).

**Definition 1** (*Renegotiation-Proof Equilibrium*) A perfect Bayesian equilibrium $(\beta^*, \mu^*)$ of $\Gamma_{RO}(G)$ and $\Gamma_{RU}(G)$ is *renegotiation-proof* if the equilibrium contract is not renegotiated after any $a_1 \in A_1$ and $\theta \in \Theta$.

We say that a strategy profile $(b_1, b_2)$ of the original game $G$ can be *supported with observable and non-renegotiable contracts* if there exists a perfect Bayesian equilibrium of $\Gamma_{NO}(G)$ that induces $(b_1, b_2)$. Similarly, a strategy profile $(b_1, b_2)$ of the original game $G$ can be *supported with observable renegotiation-proof contracts* if there exists a renegotiation-proof perfect Bayesian equilibrium of $\Gamma_{RO}(G)$ that induces $(b_1, b_2)$. Similarly for unobservable and non-renegotiable and unobservable renegotiation-proof contracts.

One important question is whether we are "missing" equilibria by restricting the analysis to renegotiation-proof equilibria. The following result shows that the answer is no.

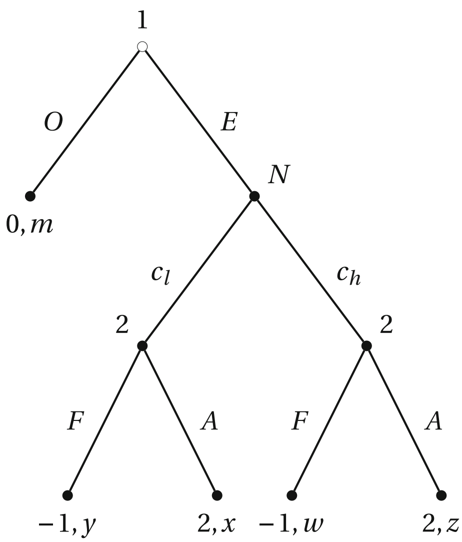
**Proposition 1** (Renegotiation-Proofness Principle) *If there is a perfect Bayesian equilibrium of the game $\Gamma_{RO}(G)$ (resp. $\Gamma_{RU}(G)$) that induces a strategy profile $(b_1, b_2)$ of the original game G, then there exists a renegotiation-proof perfect Bayesian equilibrium of $\Gamma_{RO}(G)$ (resp. $\Gamma_{RU}(G)$) that induces the same strategy profile $(b_1, b_2)$.*

*Proof* In Sect. 7. □

### 2.1 An example: entry deterrence

In order to illustrate our main query as well as some of our results, we introduce a very simple entry game in this section (see Fig. 1). Player 1 is a potential entrant, who may enter ($E$) or stay out ($O$) and player 2, who is the incumbent, may fight ($F$) or accommodate ($A$) entry.

**Fig. 1** Entry game



We assume that fighting is costly, and it is costlier for the high cost incumbent (type $c_h$) than for the low cost (type $c_l$): $z - w > x - y > 0$. The entrant believes that the incumbent's type is low cost with probability $p \in (0, 1)$.

The unique perfect Bayesian equilibrium (PBE) of this game is $(E, AA)$, i.e., the entrant enters and both types of the incumbent accommodate. We assume that the monopoly profit is larger than the highest possible profit following entry, i.e., $m > x$. In other words, the incumbent would benefit from deterring entry, and one way of achieving this would be to sign a contract with a third party that makes fighting optimal. For example, the following contract makes playing $FF$ optimal: $f(F) = \delta$, $f(A) = \delta + (z - w)$. Is such a contract renegotiation-proof? If not, can entry still be deterred with renegotiation-proof contracts?

In what follows we will answer these questions and also characterize the equilibrium outcomes that can be supported with third-party contracts under different assumptions regarding their observability and renegotiation-proofness.

## 3 Renegotiation-proof contracts

In this section we will present results that help identify the set of outcomes of any original game that can be supported with renegotiation-proof contracts. As we have mentioned before, these results are straightforward adaptations of some of the results in Gerratana and Koçkesen (2012) to the current setting. Therefore, we will be brief in their presentation and relegate their proofs to the online supplement to this paper.[12]

In order to decide whether to accept a new contract offer in the renegotiation phase of the game with renegotiable contracts, the third party forms beliefs regarding player 2's strategy under the new contract and compares his payoffs from the old and the new contracts. In equilibrium, these beliefs must be such that player 2's strategy is sequentially rational, i.e., incentive compatible, under the new contract. Let the contract space be $\mathscr{C} = \mathbb{R}^{A_1 \times A_2}$ and define incentive compatibility as a property of any contract-strategy pair $(f, b_2) \in \mathscr{C} \times A_2^{A_1 \times \Theta}$.

**Definition 2** (*Incentive Compatibility*) $(f, b_2) \in \mathscr{C} \times A_2^{A_1 \times \Theta}$ is *incentive compatible* if

$$u_2(a_1, b_2(a_1, \theta), \theta) - f(a_1, b_2(a_1, \theta)) \geq u_2(a_1, b_2\left(a_1, \theta'\right), \theta) \\ - f\left(a_1, b_2\left(a_1, \theta'\right)\right) \text{ for all } a_1 \in A_1 \text{ and } \theta, \theta' \in \Theta.$$

We say that a strategy $b_2$ is incentive compatible if there is a contract $f$ such that $(f, b_2)$ is incentive compatible. We can obtain a sharp characterization of incentive compatible strategies if we impose more structure on the original game. To this end, let $\succsim_\theta$ be a linear order on $\Theta$ and $\succsim_2$ a linear order on $A_2$, and denote their asymmetric parts by $\succ_\theta$ and $\succ_2$, respectively.

[12] We cite this supplement as Gerratana and Koçkesen (2015) in the text and it is available at http://home.ku.edu.tr/~lkockesen/research/commit_wo_rep_11_omitted_proofs.

**Definition 3** (*Increasing Differences*) $u_2 : A_1 \times A_2 \times \Theta \to \mathbb{R}$ is said to have *increasing differences* in $(\succsim_\theta, \succsim_2)$ if for all $a_1 \in A_1$, $\theta \succsim_\theta \theta'$ and $a_2 \succsim_2 a_2'$ imply that $u_2(a_1, a_2, \theta) - u_2(a_1, a_2, \theta') \geq u_2(a_1, a_2', \theta) - u_2(a_1, a_2', \theta')$. It is said to have *strictly increasing differences* if $\theta \succ_\theta \theta'$ and $a_2 \succ_2 a_2'$ imply that $u_2(a_1, a_2, \theta) - u_2(a_1, a_2, \theta') > u_2(a_1, a_2', \theta) - u_2(a_1, a_2', \theta')$.

**Definition 4** (*Increasing Strategies*) $b_2 : A_1 \times \Theta \to A_2$ is called *increasing* in $(\succsim_\theta, \succsim_2)$ if for all $a_1 \in A_1$, $\theta \succsim_\theta \theta'$ implies that $b_2(a_1, \theta) \succsim_2 b_2(a_1, \theta')$. Denote the set of all increasing $b_2$ by $B_2^+$.

For the rest of the paper, we restrict attention to games in which there exists a linear order on $\Theta$ and a linear order on $A_2$ such that $u_2$ has strictly increasing differences in $(\succsim_\theta, \succsim_2)$. We further comment on the role played by the increasing differences property in Sect. 6.

Standard arguments show that under increasing differences, incentive compatibility implies that $b_2$ is increasing. The following proposition states this result and shows that its converse also holds.

**Proposition 2** *If $u_2 : A_1 \times A_2 \times \Theta \to \mathbb{R}$ has strictly increasing differences, then a strategy $b_2 : A_1 \times \Theta \to A_2$ is incentive compatible if and only if it is increasing.*

*Proof* See Gerratana and Koçkesen (2015). $\square$

We next define our renegotiation-proofness concept, which follows from the definition of renegotiation-proof perfect Bayesian equilibrium (Definition 1).

**Definition 5** (*Renegotiation-Proofness*) We say that $(f, b_2^*) \in \mathscr{C} \times A_2^{A_1 \times \Theta}$ is *renegotiation-proof* if for all $a_1 \in A_1$ and $\theta \in \Theta$ for which there exists an incentive compatible $(g, b_2)$ such that

$$u_2\left(a_1, b_2(a_1, \theta), \theta\right) - g\left(a_1, b_2(a_1, \theta)\right) > u_2\left(a_1, b_2^*(a_1, \theta), \theta\right) - f\left(a_1, b_2^*(a_1, \theta)\right) \tag{1}$$

there exists a $\theta' \in \Theta$ such that

$$f\left(a_1, b_2^*\left(a_1, \theta'\right)\right) \geq g\left(a_1, b_2\left(a_1, \theta'\right)\right) \tag{2}$$

In words, if, for some $(\theta, a_1)$, there is a contract $g$ and an incentive compatible continuation play $b_2$ such that player 2 prefers $g$ over $f$ (i.e., (1) holds), there must exist a belief of the third party (over $\theta$) under which it is optimal to reject $g$, which is implied by (2).[13] Finally, we define a renegotiation-proof strategy as follows.

**Definition 6** (*Renegotiation-Proof Strategy*) A strategy $b_2 \in A_2^{A_1 \times \Theta}$ is *renegotiation-proof* if there exists an $f \in \mathscr{C}$ such that $(f, b_2)$ is incentive compatible and renegotiation-proof. Denote the set of all renegotiation-proof strategies by $B_2^R$.

[13] This definition allows beliefs to be arbitrary following an off-the-equilibrium renegotiation offer. An alternative definition would be to require the beliefs to satisfy intuitive criterion. In Sect. 6 we argue that our results go through with minor modifications when we adopt this stronger version.

Definitions 5 and 6 are indeed the correct definitions to work with, in the sense that they identify the conditions that any contract and strategy must satisfy to be part of a renegotiation-proof perfect Bayesian equilibrium of $\Gamma_{RO}(G)$ or $\Gamma_{RU}(G)$. Indeed, if a strategy $b_2$ of the original game is not renegotiation-proof, then there is no perfect Bayesian equilibrium (of the game with renegotiable contracts) in which a contract $f$ is offered and $b_2$ is played without renegotiating $f$. This simply follows from the fact that if $(f, b_2)$ is not renegotiation-proof, then there is $(a_1, \theta)$ and a contract $g$ that would be accepted for any belief of the third party at the renegotiation stage and increase player 2's payoff. In other words, $f$ will be renegotiated after $(a_1, \theta)$ and therefore the equilibrium is not renegotiation-proof. In fact, the converse of that statement also holds: If $b_2$ is renegotiation-proof, we can construct a perfect Bayesian equilibrium of the game with renegotiable contracts in which the equilibrium contract is not renegotiated after any $a_1$ and $\theta$. Of course, the equilibrium contract and $b_2$ will have to satisfy other conditions in order to be part of an equilibrium, but these would depend on whether the contracts are observable or unobservable, an issue which we will address in Sect. 4.

Applying these definitions in order to characterize renegotiation proof equilibrium outcomes is far from trivial. In order to circumvent this problem, GK (2012) utilizes theorems of alternatives and develops much more operational characterizations of renegotiation-proof contracts and strategies. We will next present two such results that we make direct reference to in Sect. 5.

Let the number of elements in $\Theta$ be equal to $n$ and order its elements so that $\theta^n \succsim_\theta \theta^{n-1} \succsim_\theta \cdots \theta^2 \succsim_\theta \theta^1$.

**Definition 7** For any $b_2 \in A_2^{A_1 \times \Theta}$ we say that $(a_1, i)$, $i \in \{1, 2, \ldots, n\}$ has *right deviation* (*left deviation*) at $b_2$ if there exists an $a_2 \in A_2$ such that $a_2 \succsim_2 b_2(a_1, \theta^i)$ ($b_2(a_1, \theta^i) \succsim_2 a_2$) and $u_2(a_1, a_2, \theta^i) > u_2(a_1, b_2(a_1, \theta^i), \theta^i)$. Otherwise, we say that $i$ has *no right deviation* (*no left deviation*) at $b_2$.

In other words, given a strategy $b_2$ and an action $a_1$, a type $\theta$ has right (left) deviation if there is a higher (lower) action than $b_2(a_1, \theta)$ that leads to a strictly bigger surplus.

For any $b_2 \in A_2^{A_1 \times \Theta}$ and $(a_1, i)$, $i \in \{1, \cdot \cdot \cdot, n\}$, that has right deviation at $b_2$, define

$$\begin{aligned} R(a_1, i) = \Big\{ & k > i : b_2\left(a_1, \theta^k\right) \in BR_2\left(a_1, \theta^k\right) \text{ and} \\ & i < j < k \;\text{ implies that } (a_1, j) \text{ has no left deviation at } b_2 \Big\} \end{aligned}$$

Similarly, for any $(a_1, i)$ with $i \in \{1, \cdot \cdot \cdot, n\}$, that has a left deviation at $b_2$, define

$$\begin{aligned} L(a_1, i) = \Big\{ & k < i : b_2\left(a_1, \theta^k\right) \in BR_2\left(a_1, \theta^k\right) \text{ and} \\ & k < j < i \;\text{ implies that } (a_1, j) \text{ has no right deviation at } b_2 \Big\}, \end{aligned}$$

The next result shows that a necessary condition for a strategy to be renegotiation-proof is that the highest type cannot have a right deviation and the lowest type cannot have a left deviation:

**Proposition 3** *If* $b_2 \in A_2^{A_1 \times \Theta}$ *is renegotiation-proof, then* $(a_1, \theta^n)$ *has no right deviation and* $(a_1, \theta^1)$ *has no left deviation at* $b_2$ *for any* $a_1 \in A_1$.

*Proof* In Sect. 7. □

The following provides a sufficient condition for being renegotiation-proof strategy:

**Proposition 4** $b_2 \in A_2^{A_1 \times \Theta}$ *is renegotiation-proof if for any* $(a_1, i_1)$ *that has right deviation and any* $(a_1, i_2)$ *that has left deviation at* $b_2$ *the following conditions are true: (1)* $R(a_1, i_1) \neq \emptyset$ *and* $L(a_1, i_2) \neq \emptyset$*; (2)* $i_1 < i_2$ *implies* $R(a_1, i_1) \cap L(a_1, i_2) \neq \emptyset$.

*Proof* In Sect. 7. □

Fix $a_1 \in A_1$ and let $br_2$ be any selection from the best response correspondence of player 2, i.e., $br_2(a_1, \theta^i) \in BR_2(a_1, \theta^i)$ for all $i \in \{1, 2, \ldots, n\}$. Proposition 3 tells us something about the relation between a renegotiation-proof strategy $b_2$ and the best response correspondence $br_2$. In particular, Proposition 3 implies that $br_2(a_1, \theta^1) \succsim_2 b_2(a_1, \theta^1)$ and $b_2(a_1, \theta^n) \succsim_2 br_2(a_1, \theta^n)$ for any renegotiation-proof $b_2$, i.e., the lowest type's action must be (weakly) smaller and the highest type's action must be (weakly) larger than the best response. Although, it does not directly follow from Proposition 3, we can also show that a renegotiation-proof strategy must be best responding for at least one type (see Corollary 5 on page 23).[14]

The sufficient conditions in Proposition 4 requires that whenever there is a type $\theta$ that has a right (resp., left) deviation, there exists a larger (resp., smaller) type $\theta'$ that best responds. Moreover, no type between $\theta$ and $\theta'$ must have left (resp., right) deviation. This, for example, implies that a strategy in which the highest (resp., lowest) type best responds and all the other types play the lowest (resp., highest) action is renegotiation-proof.

In order to say more about the restrictions brought about by renegotiation-proofness, we need to impose more structure on the original game. In Sect. 5, we will do so by applying our results to the quantity competition and entry deterrence application introduced in Sect. 1.1. Furthermore, we will argue that similar results hold in a larger class of games in which the first player's payoff is monotone in player 2's action. For now, we use our running example to illustrate our results so far and give some intuition.

### 3.1 Example: entry deterrence (continued from Sect. 2.1)

In this example Propositions 3 and 4 can be used to fully characterize the set of incentive compatible and renegotiation-proof strategies. However, in order to give some intuition, we will first use brute force to do so, after which we will apply Propositions 3 and 4 to reach the same conclusion.

Let us first understand the implications of incentive compatibility, i.e., strategies that a third-party contract can induce in this game. We already showed that the contract

[14] However, it is not difficult to construct examples where a strategy satisfies Proposition 3 and Corollary 5 and yet fails to be renegotiation-proof.

$f(F) = \delta$, $f(A) = \delta + (z - w)$ induces strategy $FF$. How about $AF$? A contract that induces $AF$ must satisfy the incentive compatibility constraints $x - f(A) \geq y - f(F)$ and $w - f(F) \geq z - f(A)$, which leads to a contradiction since we assumed $z - w > x - y$. Therefore, there is no contract that supports $AF$. It is easy to check that the following contract supports $FA$: $f(F) = \delta$, $f(A) = \delta + (x - y)$ and the constant contract $f(F) = f(A) = \delta$ supports $AA$.

In other words, even without renegotiation, there are restrictions on the strategies that can be supported with third-party contracts. As it must be apparent, these restrictions come from the incentive compatibility constraints and the assumption that $z - w > x - y$. In fact, if we order types as $c_h \succ_\theta c_l$ and actions as $A \succ_2 F$, then player 2's payoff function exhibits strictly increasing differences, which is inherited by her net payoff function obtained after the transfer to (or from) the third party is taken into account. Then, a well-known result from contract theory implies that only increasing strategies, i.e., $FF$, $FA$, and $AA$, can be supported. Proposition 2 states exactly this necessary condition and also shows that it is sufficient.

Which ones among these strategies are renegotiation-proof? $AA$ is clearly renegotiation-proof because both types are best responding and renegotiation cannot lead to an increase in the total surplus available to player 2 and the third party. How about $FF$? Suppose there is an incentive compatible and renegotiation-proof contract $f$ that supports $FF$ and consider the contract $g(F) = g(A) = f(F) + (x - y)/2$. Under $g$, the optimal strategy is $AA$. Furthermore, since $x - g(A) > y - f(F)$, type $c_l$ of player 2 is better off under this contract, and since $g(A) > f(F)$, the third party is better off as well. This implies that $f$ is not renegotiation-proof, because type $c_l$ could deviate and offer the alternative contract $g$ and the third party would accept it under any belief. Therefore, $FF$ is not renegotiation-proof.

In contrast, we can show that $FA$ is renegotiation-proof. Let $f$ be a contract that supports $FA$ and note that type $c_l$ is not best responding and her incentive compatibility implies that $y - f(F) \geq x - f(A)$. If she offers a new contract $g$ that will make her better off by playing $A$, it must be the case that $x - g(A) > y - f(F)$. These two conditions imply that $g(A) < f(A)$, i.e., the transfer to the third party after $A$ must decrease. This, of course, is completely intuitive: In order to make switching from $F$ to $A$ profitable, transfer after $A$ must decrease. But then type $c_h$ would find contract $g$ strictly better than $f$ as well, because $z - g(A) > z - f(A)$. Therefore, the third party may reasonably believe that the renegotiation offer has been made by type $c_h$, in which case rejecting the offer is in fact optimal. In sum, the set of renegotiation-proof strategies is $\{FA, AA\}$.

Let us now apply Propositions 3 and 4 directly to reach the same conclusion. Proposition 3 implies that $FF$ is not renegotiation-proof because the highest type. i.e., $c_h$, has a right deviation (to $A$). Proposition 4 implies that $AA$ is renegotiation-proof because there is no left or right deviation for either type, and $FA$ is renegotiation-proof because there is no left or right deviation for type $c_h$ while there is only right deviation for $c_l$ and $R(E, c_l) = \{c_h\} \neq \emptyset$.

In other words, renegotiation-proofness in this example is satisfied whenever the high cost type best responds. Also note that for the high cost type, not best responding is costlier, i.e., $z - w > x - y$. Credible commitment, in this example, requires best responding when it is very costly not to do so. This same feature arises in the quantity competition game that we consider in Sect. 5 as well as a more general class of games in which player 1's payoff is monotone in player 2's action.

## 4 Equilibrium outcomes of games with contracts

There may be legal or technological constraints that might render contracts non-renegotiable and therefore outcomes that can be supported by non-renegotiable contracts are of interest on their own. Furthermore, understanding non-renegotiable contracts will help situate our results within the literature and allow us to isolate the effects of renegotiation. Similarly, and irrespective of whether a contract is renegotiable, there may be valid reasons why a contract maybe observable or unobservable. Legal contracts between a firm and a bank, or a government and an international body, and many compensation contracts are observable yet subject to renegotiation if the parties find it in their interest to do so. Other contracts can be either secret or subject to renegotiation before the game begins, i.e., they can be unobservable. In this section we will present results regarding the outcomes that can be supported under different assumptions about the contracts.

### 4.1 Observable contracts

Let us assume that the contract signed between player 2 and the third party before the game begins is observable to player 1 but may or may not be renegotiated after player 1 moves in the game.

#### *4.1.1 Non-renegotiable contracts*

If the contracts are observable but not renegotiable, then we can show that player 2 can obtain the best payoff possible given that she plays an increasing strategy and player 1 best responds. More precisely, define the best Stackelberg payoff of player 2 as $\bar{U}_2^B = \max_{b_2 \in B_2^+} \max_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)$ and the worst Stackelberg payoff as $\bar{U}_2^W = \max_{b_2 \in B_2^+} \min_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)$.

**Proposition 5** *If contracts are observable and non-renegotiable, then (1) $\bar{U}_2^B - \delta$ can be supported and (2) $\bar{U}_2^W - \delta$ is the smallest payoff that can be supported.*

*Proof* In Sect. 7. □

The proof of part (1) is quite easy. In the definition of the best Stackelberg payoff, player 2 is playing the best increasing strategy, say $b_2^*$, given that player 1 is playing a best response that is most favorable for player 2. Proposition 2 implies that $b_2^*$ is incentive compatible, i.e., there is a contract, say $f^*$, that makes it optimal to play. It is easy to show that there is a perfect Bayesian equilibrium of the game with observable and non-renegotiable contracts in which player 2 offers $f^*$ with expected value $\delta$, player 1 plays the most favorable best response to that, say $b_1^*$, and player 2 plays

$b_2^*(b_1^*, \theta)$ after $(f^*, b_1^*, \theta)$. Expected payoff of player 2 in such an equilibrium is $\bar{U}_2^B - \delta$.

The intuition behind part (2) is as follows. Let $\hat{b}_{2,a_1}$ argmin$_{b_2 \in B_2^+} U(a_1, b_2)$ for any $a_1 \in A_1$. In other words, for any $a_1$, $\hat{b}_{2,a_1}$ is the worst increasing strategy for player 1 that player 2 can play. The fact that $\hat{b}_{2,a_1}$ is increasing can be used to show that there is a contract that makes it *uniquely* optimal to play. Now let $b_1^*(b_2) \in \text{argmin}_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)$, $b_2^* \in \text{argmax}_{b_2 \in B_2^+} U_2(b_1^*(b_2), b_2)$, and $a_1^* = b_1^*(b_2^*)$. Note that $U_2(a_1^*, b_1^*) = \bar{U}_2^W$ and suppose, for contradiction, that player 2 gets a payoff that is strictly smaller than $\bar{U}_2^W - \delta$. We show that there exists a contract that makes it *uniquely* optimal to play $b_2^*(a_1^*, \theta)$ after $a_1^*$ and $\hat{b}_{2,a_1}(a_1, \theta)$ after any other $a_1$. If Player 2 offers this contract, player 1 must play a best response to $b_2^*$. This is because for any $a_1 \notin BR_1(b_1^*)$, we have $U_1(br_1(b_2^*), b_2^*) > U_1(a_1, b_2^*) \geq U_1(a_1, \hat{b}_{2,a_1})$. Therefore, deviation to such a contract yields a gross payoff of at least $U_2(a_1^*, b_1^*) = \bar{U}_2^W$ and a net payoff arbitrarily close to $\bar{U}_2^W - \delta$, a contradiction.

Of course, the result becomes a full characterization if player 1's best response correspondence is single-valued, i.e., $BR_1(b_2)$ is a singleton for any $b_2 \in B_2^+$: The unique equilibrium payoff of player 2 that can be supported with observable and non-renegotiable contracts is $\bar{U}_2^B - \delta$.

### *4.1.2 Renegotiable contracts*

If the contracts are observable and renegotiable, then player 2 can again achieve her Stackelberg payoff, except that the definition of this payoff must reflect the fact that player 2 plays a renegotiation-proof strategy. Define the best and worst renegotiation-proof Stackelberg payoffs of player 2 as $\bar{U}_2^{BR} = \max_{b_2 \in B_2^R} \max_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)$ and the worst Stackelberg payoff as $\bar{U}_2^{WR} = \max_{b_2 \in B_2^R} \min_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)$ and note that the difference in the definitions comes from the fact that player 2 has to play a renegotiation-proof strategy.

**Proposition 6** *If contracts are observable and renegotiable, then (1) $\bar{U}_2^{BR} - \delta$ can be supported and (2) $\bar{U}_2^{WR} - \delta$ is the smallest payoff that can be supported.*

*Proof* In Sect. 7. □

The proof of part (1) also constructs an equilibrium in which player 2 receives the best Stackelberg payoff that she can get by playing a renegotiation-proof strategy. There is however a complication in the proof compared with the proof of Proposition 5. When contracts are non-renegotiable any deviation from the contract that induces the best Stackelberg outcome under increasing strategies must still induce an increasing strategy. This implies that no deviation can yield a higher payoff. When contracts are renegotiable, a deviation may or may not induce a renegotiation-proof strategy and hence we cannot tell whether such a deviation can yield a payoff that is strictly higher than the best Stackelberg payoff that can be obtained by a renegotiation-proof strategy. In the proof, we construct an equilibrium in which any deviation obtained via renegotiation can also be obtained via a renegotiation-proof strategy, and this gives us the desired result.

The above results provide sharp predictions for equilibrium outcomes of the games with observable contracts. In particular, they show that third-party contracts play the role of a commitment device to the extent that player 2's strategy respects the constraints brought about by incentive compatibility, in the case of non-renegotiable contracts, and renegotiation-proofness, in the case of renegotiable contracts. The implications of these results in terms of the equilibrium outcomes depend on the specifics of the original game. We present some of these implications in our running example and further in Sect. 5.

#### *4.1.3 Example: entry deterrence (continued)*

Assume that $p \neq 2/3$ so that player 1's best response correspondence is single-valued:

$$br_1(FF) = O, br_1(AA) = E, br_1(FA) = \begin{cases} O, & p > 2/3 \\ E, & p < 2/3 \end{cases}$$

Remember that the set of incentive compatible strategies is $B_2^+ = \{FF, FA, AA\}$ and the set of renegotiation-proof strategies is $B_2^R = \{FA, AA\}$. Therefore, the Stackelberg payoff of player 2 given that she plays an incentive compatible strategy is $m$, which she achieves by playing $FF$. Proposition 5 implies that this is the unique payoff that can be supported with observable and non-renegotiable contracts. In other words, entry-deterrence is the unique equilibrium outcome. How about with RP contracts? If $p > 2/3$, then the Stackelberg payoff is $m$, obtained by playing $FA$, whereas if $p < 2/3$, $FA$ does not deter entry and the best that player 2 can do in this case is to play $AA$, with payoff $px + (1 - p)z$. In other words, if $p > 2/3$ unique equilibrium outcome is entry-deterrence and if $p < 2/3$ unique equilibrium outcome is entry and accommodate.

### 4.2 Unobservable contracts

We now assume that the initial contract between player 2 and the third party is not observable to player 1. Again there are two possibilities: the contract could be renegotiable or non-renegotiable.

#### *4.2.1 Non-renegotiable contracts*

If contracts are non-renegotiable, we have the following characterization.

**Proposition 7** *A strategy profile $(b_1^*, b_2^*)$ of the original game $G$ can be supported with unobservable and non-renegotiable contracts if and only if $(b_1^*, b_2^*)$ is a Bayesian Nash equilibrium of $G$ and $b_2^*$ is increasing.*

*Proof* In Sect. 7. □

This result shows that unobservable third-party contracts potentially enlarges the set of outcomes that can arise in equilibrium. Furthermore, while earlier papers showed that, when there is no asymmetric information, any Nash equilibrium of the original game can be supported with unobservable contracts, this result shows that only the subset of Bayesian Nash equilibria in which the second player plays an increasing strategy can be supported if, instead, there is asymmetric information.

This result also has an immediate corollary in terms of the outcomes that can be supported. For any strategy profile $(b_1, b_2) \in A_1 \times A_2^{A_1 \times \Theta}$, we define an *outcome* $(a_1, a_2) \in A_1 \times A_2^{\Theta}$ of $G$ as $a_1 = b_1$ and $a_2(\theta) = b_2(b_1, \theta)$. Define the individually rational payoff of player 1 as

$$\underline{U}_1^+ = \max_{a_1 \in A_1} \min_{b_2 \in B_2^+} U_1(a_1, b_2). \tag{3}$$

This is the best payoff player 1 can guarantee for herself in game $G$, given that player 2 plays an increasing strategy.[15] The following easily follows from Proposition 7.

**Corollary 1** *An outcome $(a_1^*, a_2^*)$ of the original game G can be supported with unobservable and non-renegotiable contracts if and only if (1) $a_2^*(\theta) \in BR_2(a_1^*, \theta)$ for all $\theta$ and (2) $U_1(a_1^*, a_2^*) \geq \underline{U}_1^+$.*

Again, note that, in general, outcomes that are not perfect Bayesian equilibrium outcomes of the original game can also be supported. This can be achieved by writing a contract that leads player 2 to punish player 1 when he deviates from his equilibrium action. Since contracts cannot be conditioned on $\theta$ and $u_2$ has increasing differences, player 2 can only use punishment strategies that are increasing in $\theta$. The best that player 1 can do by deviating is therefore given by $\underline{U}_1^+$, and his equilibrium payoff cannot be smaller than this payoff. This is condition (2). Condition (1), on the other hand, simply follows from the requirement that only Bayesian Nash equilibrium outcomes can be supported, and hence, player 2 must be best responding along the equilibrium path.

Note that if $\theta$ were contractible as well, we would not need to limit the punishment strategies to be increasing. In this case, condition (2) would have the individually rational payoff defined as $\max_{a_1 \in A_1} \min_{b_2 \in A_2^{A_1 \times \Theta}} U_1(a_1, b_2)$. In that case, the result would be the exact analog of those in models without asymmetric information, i.e., Koçkesen and Ok (2004) and Koçkesen (2007).

We should also note that there are interesting environments in which non-contractibility of $\theta$ does not restrict the set of outcomes that can be supported with non-renegotiable contracts. For example if player 1's payoff does not depend on $\theta$, then the punishment does not have to depend on $\theta$ either. Therefore, one can simply use a constant punishment after each deviation, which would be increasing by construction. Similarly, if $u_1$ is increasing (or decreasing) in $a_2$, then after any $a_1$, the harshest punishment is the lowest (or highest) $a_2$, which is constant and hence increasing.

[15] We should also note that this is different from the definition of individually rational payoff used in the repeated games literature, which is the minmax payoff rather than the maxmin payoff. The maxmin payoff is at most equal to the minmax payoff.

*4.2.2 Renegotiable contracts*

Suppose now that the contracts are unobservable and renegotiable. The counterpart to Proposition 7 is the following:

**Proposition 8** *A strategy profile* $(b_1^*, b_2^*)$ *of the original game G can be supported with unobservable and renegotiation-proof contracts if and only if* $(b_1^*, b_2^*)$ *is a Bayesian Nash equilibrium of G and* $b_2^*$ *is increasing and renegotiation-proof.*

*Proof* In Sect. 7. □

This result too has an immediate corollary. Define the best payoff player 1 can guarantee for herself in game $G$, given that player 2 plays a renegotiation-proof strategy as

$$\underline{U}_1^R = \max_{a_1 \in A_1} \min_{b_2 \in B_2^R} U_1(a_1, b_2). \tag{4}$$

We have the following corollary.

**Corollary 2** *An outcome* $(a_1^*, a_2^*)$ *of the original game G can be supported with unobservable and renegotiation-proof contracts if and only if (1)* $a_2^*(\theta) \in BR_2(a_1^*, \theta)$ *for all* $\theta$ *and (2)* $U_1(a_1^*, a_2^*) \geq \underline{U}_1^R$.

*4.2.3 Example: entry deterrence (continued)*

Individually rational payoffs of player 1 are given by

$$\underline{U}_1^+ = \max_{a_1 \in A_1} \min_{b_2 \in B_2^+} U_1(a_1, b_2) = U_1(O, FF) = 0$$

$$\underline{U}_1^R = \max_{a_1 \in A_1} \min_{b_2 \in B_2^R} U_1(a_1, b_2) = \begin{cases} U_1(O, FA) = 0, & p > 2/3 \\ U_1(E, AA) = 2 - 3p, & p < 2/3 \end{cases}$$

Corollary 1 implies that $(O, FF)$ and $(E, AA)$ can both be supported with unobservable and non-renegotiable contracts. Corollary 2 implies that if $p > 2/3$ both $(O, FA)$ and $(E, AA)$ can be supported with unobservable and RP contracts, whereas if $p < 2/3$ only $(E, AA)$ can be supported.

## 5 An application: quantity competition and entry-deterrence

Consider the quantity competition game described in Sect. 1.1.1. In order to ensure positive output levels in equilibrium, assume that $\alpha > 2\theta^n - c$, in which case the (Stackelberg) equilibrium outcome of this game is given by

$$\left(q_1^s, q_2^s(\theta)\right) = \left(\frac{\alpha - c}{2}, \frac{\alpha - 2\theta + c}{4}\right)$$

Define the game $G$ as follows: Let $A_1 = Q_1$ and $A_2 = \{-q_2 : q_2 \in Q_2\}$ and define $\succsim_i$ on $A_i$ as $a_i \succsim_i a_i' \Leftrightarrow a_i \geq a_i'$ and $\succsim_\theta$ as $\theta \succsim_\theta \theta' \Leftrightarrow \theta \geq \theta'$. Let the payoff function of player $i$ be given by $u_i(a_1, a_2, \theta) = \pi_i(a_1, -a_2, \theta)$, for any $(a_1, a_2) \in A_1 \times A_2$. The game $G$ is strategically equivalent to the Stackelberg game defined in the previous paragraph. It is also easy to show that $u_2$ has strictly increasing differences in $(a_2, \theta)$ and $u_1$ is increasing in $a_2$.

Let us first assume that contracts are *unobservable*. In order to apply Corollary 1, we need to calculate the individually rational payoff of player 1, i.e., $\underline{U}_1^+$ as defined in equation (3). The harshest punishment firm 2 can inflict is to drive the price down to zero by producing $\alpha$ for any type $\theta$. Since this is a constant (and hence an increasing) strategy, it follows that $\underline{U}_1^+ = 0$. In other words, any outcome $(a_1^*, a_2^*(\theta))$ such that firm 2 best responds to $a_1^*$ and firm 1 gets at least zero profit can be supported with non-renegotiable contracts. In particular, entry can be deterred with non-renegotiable contracts.

Can entry be deterred with renegotiation-proof contracts? In order to apply Corollary 2, we need to first calculate player 1's individually rational payoff given that player 2 plays a renegotiation-proof strategy, i.e., $\underline{U}_1^R$ as defined in equation (4). Proposition 3 and 4 imply that the harshest renegotiation-proof punishment is obtained when the highest type of player 2 best responds while the other types choose the lowest $a_2$, i.e., $a_2 = -\alpha$. Player 1's expected payoff when player 2 plays this strategy is given by $\frac{1}{2} p(\theta^n) (\alpha + \theta^n - a_1) a_1 - c a_1$. Its maximum, i.e., player 1's individually rational payoff, is therefore equal to

$$\underline{U}_1^R = \begin{cases} 0, & p\left(\theta^n\right)\left(\alpha + \theta^n\right) - 2c \leq 0 \\ \frac{(p(\theta^n)(\alpha+\theta^n)-2c)^2}{8p(\theta^n)}, & p(\theta^n)\left(\alpha + \theta^n\right) - 2c > 0 \end{cases}$$

Condition (1) of Corollary 2 requires that $a_2^*(\theta) = \frac{a_1^*+\theta-\alpha}{2}$ for all $\theta$, and hence $U_1(a_1^*, a_2^*) = \frac{1}{2}(\alpha - c - a_1^*)a_1^*$. Therefore, by condition (2), any outcome such that $\frac{1}{2}(\alpha - c - a_1^*)a_1^* \geq \underline{U}_1^R$ can be supported.

Also note that if

$$p\left(\theta^n\right)\left(\alpha + \theta^n\right) - 2c > 0, \tag{5}$$

then $\underline{U}_1^R$ is strictly positive, which implies that entry cannot be deterred. Therefore, we have the following result:

**Corollary 3** *Entry can be deterred with unobservable and non-renegotiable contracts. It can be deterred with unobservable and renegotiation-proof contracts if and only if* $p(\theta^n)(\alpha + \theta^n) - 2c \leq 0$.

Now let us assume that contracts are *observable*. The best payoff that player 2 can obtain in the original game is the monopoly outcome, i.e., $a_1^* = 0$ and $a_2^*(\theta) = (\theta - \alpha)/2$. If contracts are non-renegotiable, then Player 2 can obtain this outcome in exactly the same way as with unobservable contracts: If player 1 plays any $a_1 > 0$, punish him by flooding the market, i.e., choose $a_2 = -\alpha$. In other words, with observable and non-renegotiable contracts the unique outcome is the monopoly (entry-deterrence) outcome.

Could player 2 achieve the monopoly outcome with renegotiation-proof contracts? The above analysis implies that the answer is yes as long as $\underline{U}_1^R = 0$, i.e., $p(\theta^n)(\alpha + \theta^n) - 2c \leq 0$. It is easy to see that if this condition holds, then the unique equilibrium outcome that can be achieved with observable and renegotiation-proof contracts is the monopoly outcome.

**Corollary 4** *Unique equilibrium outcome is entry deterrence with observable and non-renegotiable contracts. It is the unique equilibrium outcome with observable and renegotiable contracts if and only if* $p(\theta^n)(\alpha + \theta^n) - 2c \leq 0$.

We would like to emphasize that the only restriction renegotiation-proofness imposes on the equilibrium outcome is that, if entry happens, player 2 best responds when the cost is the highest, i.e., in the "worst state of the world," while she floods the market for lower costs. As we have mentioned before, this turns out to be a general property in a more general class of games in which player 1's payoff is monotone increasing or decreasing in player 2's action.

The main intuition is as follows. Suppose, for concreteness, that player 1's payoff is increasing in $a_2$. If contracts are non-renegotiable, then player 2 can obtain a favorable outcome by punishing player 1 with the smallest $a_2$ whenever he plays an unfavorable action. Since a constant strategy is increasing, incentive compatibility does not impose any further restrictions on the outcomes that can be supported with non-renegotiable contracts. On the other hand, renegotiation-proofness imposes a very specific type of constraint on the kind of punishment player 2 can inflict upon player 1. Proposition 3 implies that the highest type of player 2 must have no right deviation. Since player 1's payoff is increasing in $a_2$, the worst punishment the highest type can inflict is to play a best response. Proposition 4, on the other hand, implies that the strategy where the highest type best responds while the other types play the smallest action is renegotiation-proof, which is therefore the harshest credible punishment player 2 can impose upon player 1. This, in turn, implies that the additional restriction renegotiation-proofness brings about depends on the probability of the highest type: The lower this probability, the less severe the effect of renegotiation. We refer the reader to Gerratana and Koçkesen (2013) for a complete analysis of this class of games.

Dewatripont (1988) has analyzed a similar entry game and showed that entry can be deterred with renegotiation-proof contracts under certain conditions. His conditions are different from ours because he uses a different renegotiation-proofness concept, namely durability, first introduced by Holmstrom and Myerson (1983). A decision rule is durable if and only if the parties involved would never unanimously approve a change from this decision rule to any other decision rule. Holmstrom and Myerson showed that this is equivalent to interim incentive efficiency when there is only one player with private information. In our context, only player 2 has private information and hence a contract-strategy pair $(f, b_2^*)$ is interim incentive efficient (and therefore durable) if and only if there is no $a_1 \in A_1$ and an incentive compatible $(g, b_2)$ such that after $a_1$ every type of player 2 and the third party do better under $(g, b_2)$, with at least one doing strictly better.

We have a characterization of durable strategies for the two-type case, i.e., when $\Theta = \{\theta^1, \theta^2\}$, and even in that case, the relationship between our concept of renegotiation-proofness and durability turns out to be quite subtle. It is not difficult to show that neither concept implies the other one in general. However, in the entry-deterrence game it can be shown that durability implies renegotiation-proofness and we have the following result.

**Proposition 9** *In the entry-deterrence game with two types, if $p_1(\theta^2+\alpha) > (\theta^2-\theta^1)$, then entry can be deterred with renegotiation-proof contracts but not with durable contracts.*

*Proof* Available upon request. □

Remember that the harshest renegotiation-proof punishment strategy of the incumbent is to flood the market if entry occurs, except for the highest type (type $\theta^2$), who has to best respond. Durability still requires that the highest type best responds. The difference is that flooding the market for type $\theta^1$ is not a durable strategy: There is a restriction on how much the incumbent can produce in response to entry, which is condition (d) of Proposition 1 in Dewatripont (1988).

## 6 Further remarks and extensions

### 6.1 The role of increasing differences

Increasing differences assumption in Definition 3 has two distinct roles in our analysis and results: (1) It plays a crucial role in characterizing renegotiation-proof strategies, i.e., Propositions 3 and 4. Proofs of these results use Lemmas 1, 2, and 3 in Sect. 7, whose proofs utilize increasing differences property; (2) It allows us to make more precise statements about the outcomes that can be supported with renegotiation-proof contracts. More precisely, Corollaries 1 and 2 and their applications in Sect. 5 directly use the fact that increasing differences property is equivalent to increasing strategies. In other words, restricting the environment in this manner gives us quite a bit of power in identifying renegotiation-proof equilibrium outcomes.

We next briefly discuss how our main characterization results in Sect. 3, namely Propositions 3 and 4, can be generalized in several directions. The precise statements of the arguments and their proofs involve quite a bit of notation and technicalities, which we omit here and refer the reader to Gerratana and Koçkesen (2013).

### 6.2 General extensive form games

Instead of two-stage games in which only player 2 can contract with a third party, we could consider an arbitrary finite extensive form game with incomplete information and perfect recall and allow every player to contract with a third party. The only restriction we impose is a generalization of the increasing differences assumption in the following way. Fix a player $i$ and let her (linearly ordered) type set be $\Theta_i$. Let $S_i|_I$ be the restriction of player $i$'s pure strategies to information sets of player $i$ that follows (and includes) information set $I$. We require that player $i$'s payoff function has increasing differences in $(s_i, \theta_i) \in S_i|_I \times \Theta_i$ at every information set $I$ of player $i$, irrespective of how the other players play and what their types are. Examples of games with increasing differences include repeated ultimatum bargaining and chain store games. Our main results can be generalized to include this class of games and they are valid in mixed strategy equilibria as well.

### 6.3 Interested third party

In our model we assumed that the third party has no interest in the outcome of the original game other than the transfer from (or to) player 2. Obviously, this is not always realistic. For example, the European Union in its contractual relationships with Airbus would be interested in the entry game played by Airbus and Boeing.

Let $u_3(a_1, a_2, \theta)$ be the third party's payoff function so that under contract $f$ his payoff would be $u_3(a_1, a_2, \theta) + f(a_1, a_2)$. We need to change the definition of renegotiation-proofness as follows: A contract strategy pair $(f, b_2^*)$ is renegotiation-proof if for all $a_1 \in A_1$ and $\theta \in \Theta$ for which there exists an incentive compatible $(g, b_2)$ such that $u_2(a_1, b_2(a_1, \theta), \theta) - g(a_1, b_2(a_1, \theta)) > u_2(a_1, b_2^*(a_1, \theta), \theta) - f(a_1, b_2^*(a_1, \theta))$, there exists a $\theta' \in \Theta$ such that $u_3(a_1, b_2^*(a_1, \theta'), \theta') + f(a_1, b_2^*(a_1, \theta')) \geq u_3(a_1, b_2(a_1, \theta'), \theta') + g(a_1, b_2(a_1, \theta'))$. Once we employ the new definition of renegotiation-proofness, results is Sect. 3 generalize in an intuitive manner.

Note that in the model with neutral third party a renegotiation opportunity arises whenever there is an increasing strategy that increases player 2's payoff $u_2(a_1, a_2, \theta)$. When the third party is no longer neutral, total surplus available becomes $u_2(a_1, a_2, \theta) + u_3(a_1, a_2, \theta)$. Accordingly, a renegotiation opportunity arises whenever there is an increasing strategy that increases total surplus $u_2(a_1, a_2, \theta) + u_3(a_1, a_2, \theta)$. This might in fact help a contract become renegotiation-proof, which would be the case, for example, if the third party and player 2 have completely opposite preferences.

### 6.4 Refinements of perfect Bayesian equilibrium

Our definition of renegotiation-proofness follows directly from the assumed game form for the renegotiation procedure, i.e., player 2, who is the informed party, makes a new contract offer and the third party, who is uninformed, accepts or rejects. In a renegotiation-proof equilibrium, the contract is never renegotiated, and therefore any renegotiation offer is an out-of-equilibrium event. This allows us to specify the beliefs of the third party freely after a new contract offer. This may be found unreasonable and a more plausible alternative could be to require beliefs satisfy the conditions specified in the intuitive criterion introduced by Cho and Kreps (1987). We can show that our results is Sect. 3 go through with minor modifications under this stronger notion of equilibrium.

## 7 Proofs

In the game with non-renegotiable contracts, player 2 has an information set at the beginning of the game, which we identify with the null history $\emptyset$, and an information set for each $(f, \theta, a_1) \in \mathscr{C} \times \Theta \times A_1$, where $\mathscr{C} = \mathbb{R}^{A_1 \times A_2}$. Player 3 has an information set for each $f \in \mathscr{C}$. If contracts are unobservable, then player 1 has only one information set, given by $\mathscr{C}$. If contracts are observable, then player 1 has an information set for each $f \in \mathscr{C}$. In the game with renegotiable contracts, player 2 has additional information sets corresponding to each history $(f, \theta, a_1, g, y)$ and $(f, \theta, a_1, g, n)$ and player 3 has an additional information set of each $(f, a_1, g)$, which we denote by $I_3(f, a_1, g)$.

*Proof of Proposition 1* Take a perfect Bayesian equilibrium (PBE) $(\beta, \mu)$ of the game $\Gamma_{RO}(G)$ that induces the strategy profile $(b_1^*, b_2^*)$ of $G$ and let $f^*$ be the contract offered by player 2 to the third party in the first stage, that is $\beta_2(\emptyset) = f^*$. For each $a_1$, partition the set $\Theta$ in two sets, $\Theta_{a_1}^R$ and $\Theta_{a_1}^{NR} = \Theta - \Theta_{a_1}^R$, where $\theta \in \Theta_{a_1}^R$ if the contract $f^*$ is renegotiated after $(a_1, \theta)$; that is $\theta \in \Theta_{a_1}^R$ if $\beta_2(f^*, \theta, a_1) = g$ for some $g \in \mathscr{C}$ and $\beta_3(f^*, \theta, a_1, g) = y$. If $\Theta_{a_1} = \emptyset$ for all $a_1$, then $(\beta, \mu)$ is renegotiation-proof and we are done. For each $a_1$ and $\theta \in \Theta_{a_1}^R$ denote with $g_{(\theta,a_1)}$ the contract that is offered and accepted after $(f^*, \theta, a_1)$, that is $\beta_2(f^*, \theta, a_1) = g_{(\theta,a_1)}$ and $\beta_3(f^*, \theta, a_1, g_{(\theta,a_1)}) = y$. Also, for each $a_1$ and $\theta \in \Theta_{a_1}^R$ denote with $b_{g_{(\theta,a_1)}}$ the strategy that player 2 chooses after the contract $g_{(\theta,a_1)}$ is offered and accepted, i.e., $\beta_2(f^*, \theta, a_1, g_{(\theta,a_1)}, y) = b_{g_{(\theta,a_1)}}$. For each $a_1$ and $\theta \in \Theta_{a_1}^{NR}$, denote with $b_{f^*}(a_1, \theta)$ the strategy that player 2 chooses after $(f^*, a_1, \theta)$, that is $\beta_2(f^*, \theta, a_1) = b_{f^*}(a_1, \theta)$. Define

$$h^*(a_1, a_2) = \begin{cases} f^*(a_1, a_2), & \text{if } \exists \theta \in \Theta_{a_1}^{NR} \text{ such that } a_2 = b_{f^*}(a_1, \theta) \\ g_{(\theta,a_1)}(a_1, a_2), & \text{if } \exists \theta \in \Theta_{a_1}^{R} \text{ such that } a_2 = b_{g_{(\theta,a_1)}}(a_1, \theta) \\ \infty & \text{otherwise} \end{cases}$$

$$b_{h^*}(a_1, \theta) = \begin{cases} b_{f^*}(a_1, \theta), & \text{if } \theta \in \Theta_{a_1}^{NR} \\ b_{g_{(\theta,a_1)}}(a_1, \theta), & \text{if } \theta \in \Theta_{a_1}^{R} \end{cases}$$

Consider the following assessment, denoted $(\beta', \mu')$, in which player 2 offers $h^*$ and plays according to $b_{h^*}$ without attempting to renegotiate. Player 1 has the same beliefs as in $(\beta, \mu)$, $\beta_1'(h^*) = \beta_1(f^*)$, and $\beta_1'(f) = \beta_1(f)$ for $f \neq h^*$; Player 3 has the same beliefs as in $(\beta, \mu)$, $\beta_3'(h^*) = y$, and $\beta_3'(I) = \beta_3(I)$ at any other information set $I$; $\beta_2'(\emptyset) = h^*$, $\beta_2'(h^*, \theta, a_1) = b_{h^*}(a_1, \theta)$, and $\beta_2'(I) = \beta_2(I)$ at any the other information set $I$.

We will show that $(\beta', \mu')$ is a renegotiation-proof PBE of $\Gamma_{RO}(G)$ and induces $(b_1^*, b_2^*)$. Since $\beta_1'(h^*) = b_1^*$ and $\beta_2'(h^*, \theta, a_1) = b_{h^*}(a_1, \theta) = b_2^*(a_1, \theta)$, the assessment $(\beta', \mu')$ induces $(b_1^*, b_2^*)$, and since $h^*$ is not renegotiated after any $(a_1, \theta)$, it is renegotiation-proof.

In order to prove that $(\beta', \mu')$ is a PBE, we first show that $h^*(a_1, a_2)$ is well defined. Indeed, if there exist $a_1 \in A_1$, $\theta \in \Theta_{a_1}^R$ and a $\theta' \in \Theta_{a_1}^{NR}$ such that $b_{f^*}(a_1, \theta') = b_{g_{(\theta,a_1)}}(a_1, \theta) = a_2$, then $f^*(a_1, a_2) = g_{(\theta,a_1)}(a_1, a_2)$. Suppose, for contradiction, that $f^*(a_1, a_2) \neq g_{(\theta,a_1)}(a_1, a_2)$. Assume first that $f^*(a_1, a_2) > g_{(\theta,a_1)}(a_1, a_2)$. In the PBE $(\beta, \mu)$, after $(f^*, a_1, \theta')$, player 2 chooses $b_{f^*}(a_1, \theta') = a_2$ and gets a payoff of $u_2(a_1, a_2) - f^*(a_1, a_2)$. However by choosing $g_{(\theta,a_1)}$ and then playing $a_2$, player 2 would get $u_2(a_1, a_2) - g_{(\theta,a_1)}(a_1, a_2) > u_2(a_1, a_2) - f^*(a_1, a_2)$.[16] This implies that player 2 has a profitable deviation, contradicting that $(\beta, \mu)$ is a PBE.

Assume now that $f^*(a_1, a_2) < g_{(\theta,a_1)}(a_1, a_2)$. In $(\beta, \mu)$, after $(f^*, a_1, \theta)$, player 2 renegotiates by offering $g_{(\theta,a_1)}$, which is accepted, and 2 chooses $a_2$. She receives a payoff of $u_2(a_1, a_2, \theta) - g_{(\theta,a_1)}(a_1, a_2)$, which is smaller than the payoff she could receive by playing $a_2$. Therefore player 2 has a profitable deviation contradicting that $(\beta, \mu)$ is a PBE.

Next, we prove that $(h^*, b_{h^*})$ is incentive compatible, that is, for all $a_1$ and $\theta$:

$$\begin{aligned} &u_2(a_1, b_{h^*}(a_1, \theta), \theta) - h^*(a_1, b_{h^*}(a_1, \theta)) \\ &\quad \geq u_2(a_1, a_2, \theta) - h^*(a_1, a_2) \quad \text{for any } a_2 \in A_2 \end{aligned} \tag{6}$$

First, assume $\theta \in \Theta^{NR}_{a_1}$ so that $b_{h^*}(a_1, \theta) = b_{f^*}(a_1, \theta)$. If $a_2$ is such that $\exists \theta' \in \Theta^{NR}_{a_1}$, with $a_2 = b_{h^*}(a_1, \theta') = b_{f^*}(a_1, \theta')$, then (6) becomes

$$\begin{aligned} &u_2\left(a_1, b_{f^*}(a_1, \theta), \theta\right) - f^*\left(a_1, b_{f^*}(a_1, \theta)\right) \geq u_2\left(a_1, b_{f^*}\left(a_1, \theta'\right), \theta\right) \\ &\quad - f^*\left(a_1, b_{f*}\left(a_1, \theta'\right)\right) \end{aligned}$$

which holds by optimality of $b_{f^*}(a_1, \theta)$. If $a_2$ is such that there exist $\theta' \in \Theta^R_{a_1}$ such that $a_2 = b_{h^*}(a_1, \theta') = b_{g_{(\theta',a_1)}}(a_1, \theta')$, then (6) holds because otherwise after $(a_1, \theta)$ player 2 could offer $g_{(\theta',a_1)}$ and once accepted, play $a_2 = b_{g_{(\theta',a_1)}}(a_1, \theta')$. This yields a payoff of $u_2(a_1, a_2, \theta) - g_{(\theta',a_1)}(a_1, a_2) > u_2(a_1, b_{f^*}(a_1, \theta), \theta) - f^*(a_1, b_{f^*}(a_1, \theta))$, implying that player 2 has a profitable deviation after $(f^*, \theta, a_1)$ and contradicting that $(\beta, \mu)$ is a perfect Bayesian equilibrium.[17] Finally, if there is no $\theta' \in \Theta$, with $a_2 = b_{h^*}(a_1, \theta')$, then (6) holds trivially.

Next, assume $\theta \in \Theta^R_{a_1}$ so that $b_{h^*}(a_1, \theta) = b_{g_{(\theta,a_1)}}(a_1, \theta)$. If $a_2$ is such that $\exists \theta' \in \Theta^{NR}_{a_1}$, with $a_2 = b_{h^*}(a_1, \theta') = b_{f^*}(a_1, \theta')$, then (6) holds, because, otherwise after $(a_1, \theta)$ player 2 could choose $a_2 = b_{f^*}(a_1, \theta')$ (without renegotiating $f^*$) and could get a payoff of

$$\begin{aligned} &u_2\left(a_1, b_{f^*}\left(a_1, \theta'\right), \theta\right) - f^*\left(a_1, b_{f^*}\left(a_1, \theta'\right)\right) > u_2\left(a_1, b_{g_{(\theta,a_1)}}(a_1, \theta), \theta\right) \\ &\quad - g_{(\theta,a_1)}\left(a_1, b_{g_{(\theta,a_1)}}(a_1, \theta)\right). \end{aligned}$$

This implies that player 2 has a profitable deviation after history $(f, \theta, a_1)$, contradicting that $(\beta, \mu)$ is a PBE. If $a_2$ is such that there exists a $\theta' \in \Theta^R_{a_1}$ with $a_2 = b_{h^*}(a_1, \theta') = b_{g_{(\theta',a_1)}}(a_1, \theta')$, then (6) holds, because, otherwise player 2 could offer $g_{(\theta',a_1)}$ after $(a_1, \theta)$ and once accepted play $a_2 = b_{g_{(\theta',a_1)}}(a_1, \theta')$. This yields

16 Note that $g_{(\theta,a_1)}(a_1, a_2)$ is accepted after $a_1$ in $(\beta, \mu)$ since $\theta \in \Theta^R_{a_1}$ by hypothesis.

17 Note that $g_{(\theta',a_1)}$ is accepted by construction.

$$u_2(a_1, a_2, \theta) - g_{(\theta', a_1)}(a_1, a_2) > u_2\left(a_1, b_{g_{(\theta,a_1)}}(a_1, \theta), \theta\right)$$
$$-g_{(\theta,a_1)}\left(a_1, b_{g_{(\theta,a_1)}}(a_1, \theta)\right)$$

and shows that player 2 has a profitable deviation after history $(f, \theta, a_1)$, contradicting that $(\beta, \mu)$ is a PBE.[18] Finally, if there is no $\theta' \in \Theta$, with $a_2 = b_{h^*}(a_1, \theta')$, then (6) holds trivially.

We now verify that $(\beta', \mu')$ is indeed a perfect Bayesian equilibrium of $\Gamma_{RO}(G)$. For player 1, $\beta_1'(h^*) = \beta_1(f^*)$ is optimal because for any $a_1$, $h^*$ and $f^*$ induce the same outcome. Similarly, for any $f \neq h^*$, $\beta_{-1}'$ and $\beta_{-1}$ induce the same continuation play, which implies that $\beta_1'(f) = \beta(f)$ is optimal. For player 3, $\beta_3(h^*) = y$ is optimal because $\beta_3(f^*) = y$ is optimal and $h^*$ and $f^*$ induce the same continuation play; $\beta_3'(f, a_1, g) = \beta_3(f, a_1, g)$ is optimal because $\beta_2'(f, \theta, a_1, g, x) = \beta_2(f, \theta, a_1, g, x)$, for $x = y, n$.

For player 2, $\beta_2'(f, \theta, a_1, g, x) = \beta_2(f, \theta, a_1, g, x)$, for $x = y, n$, is optimal by construction. Similarly, for any $f \neq h^*$, $\beta_2'(f, \theta, a_1) = \beta_2(f, \theta, a_1)$ is optimal since continuation plays after $(f, \theta, a_1)$ are the same under $\beta$ and $\beta'$. Now consider optimality of $\beta_2'(h^*, \theta, a_1) = b_{h^*}(a_1, \theta)$. Suppose first that $\theta \in \Theta_{a_1}^{NR}$. Incentive compatibility of $(h^*, b_{h^*})$ implies that there is no profitable deviation to a different action $a_2$. There cannot be a profitable deviation to offering a contract either, because continuation play after any such contract is the same under $\beta$ and $\beta'$ and hence if there was such a contract, then playing according to $b_{f^*}(a_1, \theta)$ would not be optimal under $\beta$. Now suppose $\theta \in \Theta_{a_1}^{R}$. Again, incentive compatibility of $(h^*, b_{h^*})$ implies that there is no profitable deviation to a different action $a_2$. Suppose, for contradiction, that there is a profitable deviation to offering a contract $g'$. If $g'$ is rejected, then incentive compatibility of $(h^*, b_{h^*})$ is contradicted. If $g'$ is accepted, then it is also accepted in under $\beta$. If this were to bring a higher payoff, then renegotiating to $g_{(\theta,a_1)}$ would not be optimal in $\beta$. Finally, $\beta_2'(\emptyset) = h^*$ is optimal because $h^*$ and $f^*$ yield the same expected payoffs and any $f \neq h^*$ induces the same continuation play under $\beta'$ and $\beta$. Consistency of beliefs follows easily.

The proof for a perfect Bayesian equilibrium $(\beta, \mu)$ of the game $\Gamma_{RU}(G)$ is virtually identical and omitted. □

Before we proceed to the proofs of Propositions 3 and 4 we need some results that have exact analogs in Gerratana and Koçkesen (2012). In order to save space, we only state these results in this section and refer the interested reader to Gerratana and Koçkesen (2012, 2015) for further details. We first need a few definitions.

Let $e_i$ be the $i$ th standard basis row vector for $\mathbb{R}^n$ and define the row vector $d_i = e_i - e_{i+1}$, $i = 1, 2, \ldots, n-1$. Let $D$ be the $2(n-1) \times n$ matrix whose row $2i-1$ is $d_i$ and row $2i$ is $-d_i$, $i = 1, \ldots, n-1$. For any $a_1 \in A_1$ and $b_2 \in A_2^{A_1 \times \Theta}$ define $\vec{U}_2(a_1, b_2)$ as a column vector with $2(n-1)$ components, where component $2i-1$ is given by $u_2(a_1, b_2(a_1, \theta^i), \theta^i) - u_2(a_1, b_2(a_1, \theta^{i+1}), \theta^i)$ and component $2i$ is given by $u_2(a_1, b_2(a_1, \theta^{i+1}), \theta^{i+1}) - u_2(a_1, b_2(a_1, \theta^i), \theta^{i+1})$, $i = 1, 2, \ldots, n-1$.

[18] Again note that $g_{(\theta',a_1)}$ is accepted under $(\beta, \mu)$ by construction.

**Notation 1** Given two vectors $x, y \in R^n$

1. $x \geq y$ if and only if $x_i \geq y_i$, for all $i = 1, 2, \ldots, n$;
2. $x > y$ if and only if $x_i \geq y_i$, for all $i = 1, 2, \ldots, n$ and $x \neq y$;
3. $x \gg y$ if and only if $x_i > y_i$, for all $i = 1, 2, \ldots, n$.

Similarly for $\leq$, $<$, and $\ll$.

For any $a_1 \in A_1, b_2 \in A_2^{A_1 \times \Theta}$ and $f \in \mathscr{C}$, let $f(a_1, b_2)$ be the column vector with $n$ components, where $i^{\text{th}}$ component is given by $f(a_1, b_2(a_1, \theta^i)), i = 1, 2, \ldots, n$. For any strategy profile $(b_1, b_2)$ of $G$ define the expected transfer from player 2 to the third party as $F(b_1, b_2) = \sum_{\theta \in \Theta} p(\theta) f(b_1, b_2(b_1, \theta))$.

Note that condition (2) in Definition 5 is satisfied trivially if the strategy $b_2$ does not lead to a higher surplus for the contracting parties after $(a_1, \theta)$. In other words, for each $a_1$ and $i = 1, \ldots n$, we need to check renegotiation-proofness of $(f, b_2^*)$ only against strategies that belong to the following set:

$$\mathfrak{B}\left(a_1, i, b_2^*\right) = \left\{b_2 \in A_2^{A_1 \times \Theta} : b_2 \text{ is increasing and } u_2\left(a_1, b_2\left(a_1, \theta^i\right), \theta^i\right) \right.$$
$$\left. > u_2\left(a_1, b_2^*\left(a_1, \theta^i\right), \theta^i\right)\right\}. \tag{7}$$

The first result that we present characterizes renegotiation-proof contracts and strategies.

**Lemma 1** *$(f, b_2^*) \in \mathscr{C} \times A_2^{A_1 \times \Theta}$ is renegotiation-proof if and only if for any $a_1 \in A_1$, $i \in \{1, 2, \ldots, n\}$, and $b_2 \in \mathfrak{B}(a_1, i, b_2^*)$ there exists a $k \in \{1, 2, \ldots, i-1\}$ such that*

$$u_2\left(a_1, b_2\left(a_1, \theta^i\right), \theta^i\right) - u_2\left(a_1, b_2^*\left(a_1, \theta^i\right), \theta^i\right)$$
$$+ \sum_{j=k}^{i-1} \vec{U}_2(a_1, b_2)_{2j-1} \leq f(a_1)_k - f(a_1)_i \tag{8}$$

*or there exists an $l \in \{i+1, i+2, \ldots, n\}$ such that*

$$u_2\left(a_1, b_2\left(a_1, \theta^i\right), \theta^i\right) - u_2\left(a_1, b_2^*\left(a_1, \theta^i\right), \theta^i\right)$$
$$+ \sum_{j=i+1}^{l} \vec{U}_2\left(a_1, b_2\right)_{2(j-1)} \leq f\left(a_1\right)_l - f\left(a_1\right)_i \tag{9}$$

*Proof* Similar to the proof of Theorem 2 in Gerratana and Koçkesen (2012). For details see Gerratana and Koçkesen (2015). □

Our next step is to find conditions for a strategy $b_2^*$ to be supported with renegotiation-proof contracts. Remember that for a contract and a strategy to be renegotiation-proof, there must be a type who will not transfer more to the third party under the new contract offer and the associated strategy (This is type $\theta'$ in Definition 5). We show that such a type must be a *blocking type*, which we define as follows.

**Definition 8** For any $a_1$, $i = 1, \ldots, n$ and $b_2 \in \mathfrak{B}(a_1, i, b_2^*)$ we say that $m(b_2) \in \{1, 2, \ldots, n\}$ is a *blocking type* if

$$u_2\left(a_1, b_2\left(a_1, \theta^i\right), \theta^i\right) - u_2\left(a_1, b_2^*\left(a_1, \theta^i\right), \theta^i\right) \leq \sum_{j=m(b_2)}^{i-1} \left[\vec{U}_2\left(a_1, b_2^*\right)_{2j-1} - \vec{U}_2\left(a_1, b_2\right)_{2j-1}\right] \tag{10}$$

or

$$u_2\left(a_1, b_2\left(a_1, \theta^i\right), \theta^i\right) - u_2\left(a_1, b_2^*\left(a_1, \theta^i\right), \theta^i\right) \leq \sum_{j=i+1}^{m(b_2)} \left[\vec{U}_2\left(a_1, b_2^*\right)_{2(j-1)} - \vec{U}_2\left(a_1, b_2\right)_{2(j-1)}\right] \tag{11}$$

We first state a corollary of Lemma 1:

**Corollary 5** *A strategy $b_2^* \in A_2^{A_1 \times \Theta}$ is renegotiation-proof only if for any $a_1 \in A_1$, there exists an $i \in \{1, 2, \ldots, n\}$ such that $b_2^*(a_1, \theta^i) \in BR_2(a_1, \theta^i)$.*

*Proof of Corollary 5* Since $b_2^*$ is renegotiation-proof, there is an $f^*$ such that $(f^*, b_2^*)$ is renegotiation-proof. Fix $f^*$, an arbitrary $a_1 \in A_1$, and $i = 1, \ldots, n$ such that $b_2^*(a_1, \theta^i) \notin BR_2(a_1, \theta^i)$. If such an $i$ does not exist, we are done. If it does, pick a selection $br_2(a_1, \theta^i) \in BR_2(a_1, \theta^i)$ and note that $br_2(a_1, \theta^i) \in \mathfrak{B}(a_1, i, b_2^*)$. Lemma 1 and $\vec{U}_2(a_1, br_2) \geq 0$ imply that there exists a $j \neq i$ such that $f(a_1)_j - f(a_1)_i > 0$. If $b_2^*(a_1, \theta^j) \in BR_2(a_1, \theta^j)$, we are done. If not, apply Lemma 1 again to conclude that there exists a $k \neq j$ such that $f(a_1)_k - f(a_1)_j > 0$. Since $\Theta$ is finite, by continuing with this process we end up either with a $\theta$ such that $b_2^*(a_1, \theta) \in BR_2(a_1, \theta)$ or in a cycle $\theta^i, \theta^j, \ldots, \theta^i$. If it is the former, we are done; if it is the latter we reach a contradiction because we have $f(a_1)_j > f(a_1)_i, \ldots > f(a_1)_j$. □

We can also show that a strategy is renegotiation-proof only if for any alternative strategy that increases total surplus, there is a blocking type.

**Lemma 2** *A strategy $b_2^* \in A_2^{A_1 \times \Theta}$ is renegotiation-proof only if for any $a_1 \in A_1$, $i \in \{1, 2, \ldots, n\}$, and $b_2 \in \mathfrak{B}(a_1, i, b_2^*)$ there is a blocking type.*

*Proof* Similar to the proof of Theorem 2 in Gerratana and Koçkesen (2012). For details see Gerratana and Koçkesen (2015). □

The above condition becomes also sufficient for renegotiation-proofness with an additional requirement about the relationship between blocking types for different renegotiation opportunities.

**Lemma 3** *A strategy $b_2^* \in A_2^{A_1 \times \Theta}$ is renegotiation-proof if for any $a_1 \in A_1$, $i \in \{1, 2, \ldots, n\}$, and $b_2^i \in \mathfrak{B}(a_1, i, b_2^*)$ there is a blocking type $m(b_2^i)$ such that $k < l$, $m(b_2^k) > k$, and $m(b_2^l) < l$ imply $m(b_2^k) \leq m(b_2^l)$.*

*Proof* Similar to the proof of Theorem 2 in Gerratana and Koçkesen (2012). For details see Gerratana and Koçkesen (2015). □

We should note that Lemma 2 identifies necessary and *sufficient* conditions when there are only two types.

*Proof of Proposition 3* Follows from Lemma 2. Proof is similar to the proof of Lemma 7 in Gerratana and Koçkesen (2012). See Gerratana and Koçkesen (2015) for more details. □

*Proof of Proposition 4* Follows from Lemma 3. Proof is similar to the proof of Lemma 6 in Gerratana and Koçkesen (2012). See Gerratana and Koçkesen (2015) for more details. □

*Proof of Proposition 5* **Part (1)**: Let $b_2^* \in \text{argmax}_{b_2 \in B_2^+} \max_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)$ and $b_1^* \in \text{argmax}_{b_1 \in BR_1(b_2^*)} U_2(b_1, b_2^*)$. Note that $\bar{U}_2^B = U_2(b_1^*, b_2^*)$. Since $b_2^*$ is increasing by construction, there exists a contract $f^*$ such that $(f^*, b_2^*)$ is incentive compatible and $F(b_1^*, b_2^*) = \delta$. For any $f \in \mathscr{C}, a_1 \in A_1, \theta \in \Theta$ choose $b_{2,f} \in \text{argmax}_{a_2 \in A_2} u_2(a_1, a_2, \theta) - f(a_1, a_2)$ and $b_{1,f} \in \text{argmax}_{a_1' \in A_1} U_1(a_1', b_{2,f})$.

Consider the following assessment $(\beta, \mu)$ of $\Gamma(G)$: $\beta_2(\emptyset) = f^*$, $\beta_3(f^*) = y$, $\beta_3(f) = y$ iff $F(b_{1,f}, b_{2,f}) \geq \delta$, $\beta_1(f^*) = b_1^*$, $\beta_1(f) = b_{1,f}$, for $f \neq f^*$, $\beta_2(f^*, \theta, a_1) = b_2^*(a_1, \theta)$, $\beta_2(f, \theta, a_1) = b_{2,f}(a_1, \theta)$ for all $f \neq f^*$, $a_1 \in A_1$, and $\theta \in \Theta$.

If player 2 offers any contract $f \neq f^*$, the continuation play will be $(b_{1,f}, b_{2,f})$. If $F(b_{1,f}, b_{2,f}) < \delta$ it will be rejected and hence it cannot be a profitable deviation. If $F(b_{1,f}, b_{2,f}) \geq \delta$, then

$$U_2\left(b_1^*, b_2^*\right) - F\left(b_1^*, b_2^*\right) = U_2\left(b_1^*, b_2^*\right) - \delta \geq U_2\left(b_{1,f}, b_{2,f}\right) - F\left(b_{1,f}, b_{2,f}\right)$$

by construction. Therefore, it is optimal for player 2 to offer $f^*$. Sequential rationality at other information sets are easily checked and we conclude that this assessment is a perfect Bayesian equilibrium of the game with observable contracts.

**Part (2)**: Let $b_1^*(b_2) \in \text{argmin}_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)$, $b_2^* \in \text{argmax}_{b_2 \in B_2^+} U_2(b_1^*(b_2), b_2)$, and $a_1^* = b_1^*(b_2^*)$. Note that $U_2(a_1^*, b_1^*) = \bar{U}_2^W$ and suppose, for contradiction, that player 2 gets a payoff $\tilde{U}_2 < \bar{U}_2^W - \delta$. We will show that player 2 can offer a contract that supports $(a_1^*, b_2^*)$ and yields a higher payoff.

For any $a_1$ choose $\hat{b}_{2,a_1} \in \text{argmin}_{b_2 \in B_2^+} U_1(a_1, b_2)$. By construction $\hat{b}_{2,a_1}$ is increasing and hence there exists a contract that makes it optimal to play. We will further show that there exists a contract that makes it the unique optimal strategy after $a_1$. Assume without loss of generality that $\hat{b}_{2,a_1}(a_1, \theta) \neq \hat{b}_{2,a_1}(a_1, \theta')$ whenever $\theta \neq \theta'$ and hence $\hat{b}_{2,a_1}(a_1, \theta^i) \succ_\theta \hat{b}_{2,a_1}(a_1, \theta^{i-1})$ for all $i = 1, \ldots, n$.[19] Define $\vec{U}_2(a_1, \hat{b}_{2,a_1})$ as usual and note that strictly increasing differences and $\hat{b}_{2,a_1}(a_1, \theta^i) \succ_\theta \hat{b}_{2,a_1}(a_1, \theta^{i-1})$ imply

[19] If there exist $i$ such that $\hat{b}_{2,a_1}(a_1, \theta^i) = \hat{b}_{2,a_1}(a_1, \theta^{i-1})$ simply eliminate the incentive compatibility constraint between them and set $\hat{f}_{a_1}(a_1, \hat{b}_{2,a_1}(a_1, \theta^i)) = \hat{f}_{a_1}(a_1, \hat{b}_{2,a_1}(a_1, \theta^{i-1}))$.

that

$$\vec{U}_2\left(a_1,\hat{b}_{2,a_1}\right)_{2i-1}+\vec{U}_2\left(a_1,\hat{b}_{2,a_1}\right)_{2i}>0,\quad \forall i=1,\ldots,n-1.$$

We will show that there exists $f^{a_1}$ such that $Df^{a_1} \ll \vec{U}_2(a_1,\hat{b}_{2,a_1})$. Define

$$A=\begin{pmatrix}\vec{U}_2\left(a_1,\hat{b}_2\right) & -D\\ 1 & 0\end{pmatrix}$$

and note that there exists $f^{a_1}$ such that $Df^{a_1} \ll \vec{U}_2(a_1,\hat{b}_{2,a_1})$ iff there exists $x$ such that $Ax \gg 0$.[20] By Gordan's Theorem (Mangasarian 1994), this is true iff $A'y=0$ implies $y\leq 0$. It is easy to show that $A'y=0$ implies $y_1=y_2, y_3=y_4,\ldots,y_{2(n-1)-1}=y_{2(n-1)}$. Therefore,

$$A'y=y_{2(n-1)+1}+\sum_{i=1}^{n-1}\left(\vec{U}_2\left(a_1,\hat{b}_{2,a_1}\right)_{2i-1}+\vec{U}_2\left(a_1,\hat{b}_{2,a_1}\right)_{2i}\right)y_{2i-1}$$

$\vec{U}_2(a_1,\hat{b}_{2,a_1})_{2i-1}+\vec{U}_2(a_1,\hat{b}_{2,a_1})_{2i}>0, \forall i=1,\ldots,n-1$, and $A'y=0$ imply $y\leq 0$.

Let $\varepsilon>0$ be small and define $f(b_1^*,a_2)=\delta+\varepsilon$ for all $a_2$. For any $a_1\neq b_1^*$ define

$$f\left(a_1,a_2\right)=\begin{cases} f_i^{a_1}, & a_2=\hat{b}_{2,a_1}\left(a_1,\theta^i\right)\\ \infty, & \text{otherwise}\end{cases}$$

Under this contract, player 2 plays a best response to $a_1^*$ and according to $\hat{b}_{2,a_1}$ after any $a_1\neq a_1^*$. Player 1, on the other hand, must play a best response to $b_2^*$. This is because for any $a_1\notin BR_1(b_2^*)$ and $a_1'\in BR_1(b_2^*)$, we have $U_1(a_1',b_2^*)>U_1(a_1,b_2^*)\geq U_1(a_1,\hat{b}_{2,a_1})$. Therefore, deviation to such a contract yields a payoff of $U_2(b_1^*,b_2^*)-\delta-\varepsilon>\tilde{U}_2$, for small enough $\varepsilon$. In other words, player 2 has a profitable deviation, contradicting that $\tilde{U}_2$ is an equilibrium payoff. □

Before we proceed to the proof of Proposition 6 we need to introduce a definition and prove a supplementary lemma.

**Definition 9** We say that a perfect Bayesian equilibrium $(\beta,\mu)$ of the game with renegotiable contracts has conservative beliefs if

$$\begin{aligned}\beta_2\left(f,a_1,\theta\right)=g\in\mathscr{C},\ \beta_2\left(f,a_1,\theta,g,y\right)=b_g\left(a_1,\theta\right),\ \beta_2\left(f,a_1,\theta,g,n\right)\\ =b_f\left(a_1,\theta\right),\ \beta_3\left(I_3\left(f,a_1,\theta\right)\right)=y\end{aligned}$$

imply $g(b_g(a_1,\theta))\geq f(b_f(a_1,\theta))$.

[20] To see this let $x=\begin{pmatrix}\zeta\\ f^{a_1}\end{pmatrix}$.

In other words, whenever, in equilibrium, type $\theta$ renegotiates the contract from $f$ to $g$, the third party should not expect a decrease in the transfer from that type.

**Lemma 4** *Take any a perfect Bayesian equilibrium of the game with renegotiable contracts and assume that it has conservative beliefs. Suppose that a contract $f$ is renegotiated after some $a_1$ and $\theta$. Then, there exists a contract strategy pair that is incentive compatible, renegotiation-proof, and induces the same outcome as $f$ after $a_1$.*

*Proof of Lemma 4* Fix a a perfect Bayesian equilibrium with conservative beliefs and suppose that contract $f$ is renegotiated after some $a_1$ and $\theta$. Let the set of types after which $f$ is renegotiated be $\Theta^R$ and $\Theta^{NR} = \Theta \backslash \Theta^R$. For any $\theta \in \Theta^R$, let $g_\theta$ be the new contract and $b_{g_\theta}(a_1, \theta)$ be the new strategy of player 2 after $a_1$ and $\theta$. Also let $b_f(a_1, \theta)$ be the equilibrium strategy of player 2 after $a_1$ and $\theta$ if he does not renegotiate $f$. In other words, we have $\beta_2(f, \theta, a_1) = b_f(a_1, \theta),\ \forall \theta \in \Theta^{NR}$, $\beta_2(f, \theta, a_1) = g_\theta,\ \forall \theta \in \Theta^R$, $\beta_2(f, \theta, a_1, g_\theta, y) = b_{g_\theta}(a_1, \theta)$, $\beta_2(f, \theta, a_1, g_\theta, n) = b_f(a_1, \theta)$, and $\beta_3(I_3(f, a_1, g_\theta)) = y$. For ease of exposition we will omit the reference to $a_1$ in the following. Consider the following mixture menu:

$$\left\{(g_\theta(b_{g_\theta}(\theta)), b_{g_\theta}(\theta))_{\theta \in \Theta^R}\right\} \cup \left\{(f(b_f(\theta)), b_f(\theta))_{\theta \in \Theta^{NR}}\right\}$$

It is clear that this menu replicates the outcome induced by $f$ after $a_1$. We also claim that this menu is incentive compatible and renegotiation proof after $a_1$.

Incentive compatibility of $(f, b_f)$ implies that no two types in $\Theta^{NR}$ has an incentive to mimic each other. Consider $\theta, \theta' \in \Theta^R$ and suppose, for contradiction, that

$$u_2\left(b_{g_\theta}(\theta), \theta\right) - g_\theta\left(b_{g_\theta}(\theta)\right) < u_2\left(b_{g_{\theta'}}(\theta'), \theta\right) - g_{\theta'}\left(b_{g_{\theta'}}(\theta')\right)$$

But then type $\theta$ could increase her payoff after $(f, a_1)$ by offering $g_{\theta'}$ and playing $b_{g_{\theta'}}(\theta')$ rather than offering $g_\theta$ and playing $b_{g_\theta}(\theta)$.[21]

Now let $\theta' \in \Theta^{NR}$ and $\theta \in \Theta^R$, and suppose for contradiction that

$$u_2\left(b_f\left(\theta'\right), \theta'\right) - f\left(b_f\left(\theta'\right)\right) < u_2\left(b_{g_\theta}(\theta), \theta'\right) - g_\theta\left(b_{g_\theta}(\theta)\right)$$

This implies that after $(f, a_1)$ offering $g_\theta$, which is accepted in equilibrium, and playing $b_{g_\theta}(\theta)$ is a profitable deviation for type $\theta'$.

Finally, let $\theta' \in \Theta^{NR}$ and $\theta \in \Theta^R$ and suppose, for contradiction, that

$$u_2\left(b_{g_\theta}(\theta), \theta\right) - g_\theta\left(b_{g_\theta}(\theta)\right) < u_2\left(b_f\left(\theta'\right), \theta\right) - f\left(b_f\left(\theta'\right)\right)$$

But then type $\theta$ could play $b_f(\theta')$ after $(f, a_1)$ and receive a higher payoff rather than offering $g_\theta$, which is accepted, and playing $b_{g_\theta}(\theta)$. This proves that the mixture menu is incentive compatible.

[21] Note that $g_{\theta'}$ is accepted after $(f, a_1)$ in equilibrium by assumption.

Suppose now, for contradiction, that the mixture menu is not renegotiation-proof after $a_1$. Then, there exists $\theta$ and an incentive compatible contract strategy pair $(h, b_h)$ such that if $\theta \in \Theta^{NR}$, then,

$$u_2\left(b_h\left(\theta\right), \theta\right) - h\left(b_h\left(\theta\right)\right) > u_2\left(b_f\left(\theta\right), \theta\right) - f\left(b_f\left(\theta\right)\right) \tag{12}$$

if $\theta \in \Theta^R$, then

$$u_2\left(b_h\left(\theta\right), \theta\right) - h\left(b_h\left(\theta\right)\right) > u_2\left(b_{g_\theta}\left(\theta\right), \theta\right) - g_\theta\left(b_{g_\theta}\left(\theta\right)\right) \tag{13}$$

and

$$h\left(b_h\left(\hat{\theta}\right)\right) > f\left(b_f\left(\hat{\theta}\right)\right), \forall\hat{\theta} \in \Theta^{NR} \tag{14}$$

$$h\left(b_h\left(\hat{\theta}\right)\right) > g_{\hat{\theta}}\left(b_{g_{\hat{\theta}}}\left(\hat{\theta}\right)\right), \forall\hat{\theta} \in \Theta^{R} \tag{15}$$

Since $g_{\hat{\theta}}$ is accepted for all $\hat{\theta} \in \Theta^R$ and the equilibrium has conservative beliefs,

$$g_{\hat{\theta}}\left(b_{g_{\hat{\theta}}}\left(\hat{\theta}\right)\right) \geq f\left(b_f\left(\hat{\theta}\right)\right), \quad \forall\hat{\theta} \in \Theta^{R} \tag{16}$$

which, together with (14) and (15), implies that

$$h\left(b_h\left(\hat{\theta}\right)\right) > f\left(b_f\left(\hat{\theta}\right)\right), \forall\hat{\theta} \in \Theta \tag{17}$$

Suppose first that $\theta \in \Theta^{NR}$. Inequalities (12) and (17) imply that after $(f, a_1)$ type $\theta$ could offer $h$, which would be accepted, and increase her payoff, a contradiction that in equilibrium she plays $b_f(\theta)$ after $(f, a_1)$.

Similarly, if $\theta \in \Theta^R$, then (13) and (17) imply that after $(f, a_1)$ type $\theta$ could offer $h$, which would be accepted, and increase her payoff, rather than offering $g_\theta$, a contradiction. Therefore, the mixture menu is renegotiation-proof.

Since the mixture is incentive compatible we can easily extend it to a contract whose domain is the entire $A_2$ rather than just the range of $b_f$ and $b_{g_\theta}$. Define the new contract as

$$h(a_2) = \begin{cases} f(a_2), & \exists\theta : a_2 = b_f(\theta) \\ g_\theta(a_2), & \exists\theta : a_2 = b_{g_\theta}(\theta) \\ \infty, & \text{otherwise} \end{cases}$$

and note that $h$ is well-defined since incentive compatibility of the mixture menu implies that whenever $b_f(\theta') = b_{g_\theta}(\theta) = a_2$ for some $\theta \in \Theta^R$ and $\theta' \in \Theta^{NR}$ we must also have $f(a_2) = g_\theta(a_2)$. □

This lemma tells us that in any equilibrium with conservative beliefs, one can achieve any outcome that is achieved via renegotiation after $a_1$ by using a renegotiation-proof contract.

*Proof of Proposition 6* **Part (1)**: Let $b_2^* \in \text{argmax}_{b_2 \in B_2^R} \max_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)$ and $b_1^* = \text{argmax}_{b_1 \in BR_1(b_2^*)} U_2(b_1, b_2^*)$. Note that $\bar{U}_2^{BR} = U_2(b_1^*, b_2^*)$. Since $b_2^*$ is increasing and renegotiation-proof, there exists $f^* \in \mathscr{C}$ such that $(f^*, b_2^*)$ is incentive compatible and renegotiation-proof with $F^*(b_1^*, b_2^*) = \delta$. For any $f \in \mathscr{C}$, $a_1$, and $\theta$, let $b_{2,f}(a_1, \theta) \in \text{argmax}_{a_2} u_2(a_1, a_2, \theta) - f(a_1, a_2)$ and $g_{(f,\theta,a_1)} \in \text{argmax}_g u_2(a_1, b_{2,g}(a_1, \theta), \theta) - g(a_1, b_{2,g}(a_1, \theta))$ subject to $g(a_1, b_{2,g}(a_1, \theta')) \geq f(a_1, b_{2,f}(a_1, \theta'))$ for all $\theta'$.

Consider the following assessment $(\beta, \mu)$: $\beta_2(\emptyset) = f^*$; $\beta_1(f^*) = b_1^*, \beta_3(f^*) = y$, $\beta_2(f^*, \theta, a_1) = b_2^*(a_1, \theta)$ for all $(a_1, \theta)$;

$$\beta_2(f, \theta, a_1) = \begin{cases} g_{(f,\theta,a_1)}, & \text{if } \begin{array}{r} u_2\left(a_1, b_{2,g_{(f,\theta,a_1)}}(a_1, \theta), \theta\right) - g_{(f,\theta,a_1)}\left(a_1, b_{2,g_{(f,\theta,a_1)}}(a_1, \theta)\right) \\ > u_2\left(a_1, b_{2,f}(a_1, \theta), \theta\right) - f\left(a_1, b_{2,f}(a_1, \theta)\right) \end{array} \\ b_{2,f}(a_1, \theta), & \text{otherwise} \end{cases}$$

for any $f \neq f^*$ and $(\theta, a_1)$; $\beta_2(f, \theta, a_1, g, y) = b_{2,g}(a_1, \theta)$ and $\beta_2(f, \theta, a_1, g, n) = b_{2,f}(a_1, \theta)$ for all $f \neq f^*$ and $(a_1, \theta, g)$; $\beta_2(f^*, \theta, a_1, g, n) = b_2^*(a_1, \theta)$ for all $(a_1, \theta, g)$;

$$\beta_3\left(I_3\left(f^*, a_1, g\right)\right) = \begin{cases} y, & g\left(a_1, b_{2,g}(a_1, \theta)\right) > f^*\left(a_1, b_2^*(a_1, \theta)\right) \quad \forall\theta \\ n, & \text{otherwise} \end{cases}$$

and

$$\beta_3\left(I_3\left(f, a_1, g\right)\right) = \begin{cases} y, & \text{if } g\left(a_1, b_{2,g}(a_1, \theta)\right) \geq f\left(a_1, b_{2,f}(a_1, \theta)\right) \quad \forall\theta \\ n, & \text{otherwise} \end{cases}$$

for any $a_1$, $g$ and $f \neq f^*$. Obviously, any $f \neq f^*$ induces a continuation strategy $b_2^f$ for player 2, which may involve renegotiation after some $\theta$. Let player 1 play the same best response to the continuation play irrespective of the contract that induces it. Let the third party accept $f$ iff continuation play yields expected transfers at least equal to $\delta$. Specify beliefs as follows: $\mu(I_3(f^*, a_1, g))(\theta) = p(\theta)$ if $g(a_1, b_{2,g}(a_1, \theta)) > f^*(a_1, b_2^*(a_1, \theta))$ for all $\theta$ and $\mu(I_3(f^*, a_1, g))(\theta') = 1$ if there exists $\theta'$ such that $f^*(a_1, b_2^*(a_1, \theta')) \geq g(a_1, b_{2,g}(a_1, \theta'))$; For any $f \neq f^*$ and $(a_1, g)$, $\mu(I_3(f, a_1, g))(\theta) = p(\theta)$ if $g(a_1, b_{2,g}(a_1, \theta)) \geq f(a_1, b_{2,f}(a_1, \theta))$ for all $\theta$ and $\mu^*(I_3(f, a_1, g))(\theta') = 1$ if there exists $\theta'$ such that $f(a_1, b_{2,f}(a_1, \theta')) > g(a_1, b_{2,g}(a_1, \theta'))$.

Now consider any contract $f \neq f^*$. If $(f, b_{2,f})$ is renegotiation-proof, then $b_{2,f} \in B_2^R$ and hence $f$ cannot yield a higher payoff than $f^*$. Therefore, suppose that $(f, b_{2,f})$ is not renegotiation-proof and let $b_2^f$ be the induced strategy, which includes renegotiation after some $a_1$ and $\theta$. Since $\beta_3(I_3(f, a_1, g)) = y$ iff $g(a_1, b_{2,g}(a_1, \theta)) \geq f(a_1, b_{2,f}(a_1, \theta))$ for all $\theta \in \Theta$, the equilibrium constructed above has conservative beliefs. Lemma 4 therefore implies that there exists $(h, b_{2,h})$

which is incentive compatible and renegotiation-proof and induces the same outcome as $(f, b_2^f)$. But no renegotiation-proof strategy can yield a payoff that is higher than $\bar{U}_2^{BR}$ and hence deviation to $f$ cannot be profitable.

Sequential rationality at other information sets and consistency of beliefs can be checked easily to show that the above assessment is a perfect Bayesian equilibrium.

**Part (2)**: Similar to Part (2) of Proposition 5. □

*Proof of Proposition 7* **(Only if)** Suppose that $(b_1^*, b_2^*)$ can be supported. Then, there exists a perfect Bayesian equilibrium $(\beta^*, \mu^*)$ that induces $(b_1^*, b_2^*)$, i.e., $\beta_2^*(\emptyset) = f^*$, $\beta_3(f^*) = y$, $\beta_1^*(\mathscr{C}) = b_1^*$, $\beta_2^*(f^*, \theta, a_1) = b_2^*(a_1, \theta)$ for all $a_1 \in A_1$ and $\theta \in \Theta$. The fact that $(b_1^*, b_2^*)$ is a Bayesian Nash equilibrium of $G$ is a direct consequence of sequential rationality of players 1 and 2. It must also be the case that it is optimal to play according to $b_2^*$ under $f^*$. Increasing differences and Proposition 2 implies that $b_2^*$ is increasing.

**[If]** Let $(b_1^*, b_2^*)$ be a Bayesian Nash equilibrium of $G$ such that $b_2^*$ is increasing. Proposition 2 implies that there exists a contract $f'$ such that $(f', b_2^*)$ is incentive compatible. It is not difficult to show that we can find such a contract whose expected value under $(b_1^*, b_2^*)$ is equal to $\delta$. So assume $F'(b_1^*, b_2^*) = \delta$. For any $b_2 \in A_2^{A_1 \times \Theta}$ and $a_1 \in A_1$, let $b_2(a_1, \Theta)$ be the image of $\Theta$ under $b_2(a_1, .)$. Define

$$f^*(a_1, a_2) = \begin{cases} f'(a_1, a_2), & \text{if } a_2 \in b_2^*(a_1, \Theta) \\ \infty, & \text{otherwise} \end{cases}$$

for any $(a_1, a_2) \in A_1 \times A_2$, and

$$b_{2,f}^*(a_1, \theta) = \begin{cases} b_2^*(a_1, \theta), & f = f^* \\ \in \operatorname{argmax}_{a_2} u_2(a_1, a_2, \theta) - f(a_1, a_2), & f \neq f^* \end{cases}$$

for any $f \in \mathscr{C}$, $a_1 \in A_1$, and $\theta \in \Theta$. Consider the assessment $(\beta^*, \mu^*)$ of $\Gamma(G)$, where $\beta_2^*[\emptyset] = f^*$, $\beta_3[f] = y$ iff $F(b_1^*, b_{2,f}^*) \geq \delta$, $\beta_1^*[\mathscr{C}] = b_1^*$, $\beta_2^*[f, \theta, a_1] = b_{2,f}^*(a_1, \theta)$ for all $f \in \mathscr{C}$, $a_1 \in A_1$, and $\theta \in \Theta$, and $\mu^*[\mathscr{C}](f^*) = 1$. It is easy to check that this assessment induces $(b_1^*, b_2^*)$ and is a perfect Bayesian equilibrium of $\Gamma(G)$. □

*Proof of Proposition 8* **[If]** Let $(b_1^*, b_2^*)$ be a Bayesian Nash equilibrium of $G$ such that $b_2^*$ is increasing and renegotiation-proof. This implies that there exists $f' \in \mathscr{C}$ such that $(f', b_2^*)$ is incentive compatible and renegotiation-proof. Let $f^*(a_1, a_2) = f'(a_1, a_2) - F'(b_1^*, b_2^*) + \delta$ for all $(a_1, a_2)$ and note that $F^*(b_1^*, b_2^*) = \delta$. Furthermore, using Lemma 1, it can be easily checked that $(f^*, b_2^*)$ is incentive compatible and renegotiation-proof. For any $f \in \mathscr{C}$, $a_1$, and $\theta$, let $b_{2,f}(a_1, \theta) \in \operatorname{argmax}_{a_2} u_2(a_1, a_2, \theta) - f(a_1, a_2)$ and $g_{(f,\theta,a_1)} \in \operatorname{argmax}_g u_2(a_1, b_{2,g}(a_1, \theta), \theta) - g(a_1, b_{2,g}(a_1, \theta))$ subject to $g(a_1, b_{2,g}(a_1, \theta')) \geq f(a_1, b_{2,f}(a_1, \theta'))$ for all $\theta'$.

Consider the following assessment $(\beta^*, \mu^*)$ of $\Gamma_R(G)$: $\beta_2^*(\emptyset) = f^*$; $\beta_3(f) = y$ iff continuation play yields an expected transfer of at least $\delta$, $\beta_1^*(\mathscr{C}) = b_1^*$, $\beta_2^*(f^*, \theta, a_1) = b_2^*(a_1, \theta)$ for all $(a_1, \theta)$;

$$\beta_2^*(f,\theta,a_1) = \begin{cases} g_{(f,\theta,a_1)}, & \text{if} \quad u_2\left(a_1, b_{2,g_{(f,\theta,a_1)}}(a_1,\theta),\theta\right) - g_{(f,\theta,a_1)}\left(a_1, b_{2,g_{(f,\theta,a_1)}}(a_1,\theta)\right) \\ & \qquad > u_2\left(a_1, b_{2,f}(a_1,\theta),\theta\right) - f\left(a_1, b_{2,f}(a_1,\theta)\right) \\ b_{2,f}(a_1,\theta), & \text{otherwise} \end{cases}$$

for any $f \neq f^*$ and $(\theta, a_1)$; $\beta_2^*(f,\theta,a_1,g,y) = b_{2,g}(a_1,\theta)$ and $\beta_2(f,\theta,a_1,g,n) = b_{2,f}(a_1,\theta)$ for all $f \neq f^*$ and $(a_1,\theta,g)$; $\beta_2(f^*,\theta,a_1,g,n) = b_2^*(a_1,\theta)$ for all $(a_1,\theta,g)$;

$$\beta_3^*(I_3(f^*,a_1,g)) = \begin{cases} y, & g(a_1, b_{2,g}(a_1,\theta)) > f^*(a_1, b_2^*(a_1,\theta)) \quad \forall\theta \\ n, & \text{otherwise} \end{cases}$$

and

$$\beta_3^*(I_3(f,a_1,g)) = \begin{cases} y, & \text{if } g(a_1, b_{2,g}(a_1,\theta)) \geq f(a_1, b_{2,f}(a_1,\theta)) \quad \forall\theta \\ n, & \text{otherwise} \end{cases}$$

for any $a_1$, $g$ and $f \neq f^*$; $\mu^*(\mathscr{C})(f^*) = 1$; $\mu^*(I_3(f^*,a_1,g))(\theta) = p(\theta)$ if $g(a_1, b_{2,g}(a_1,\theta)) > f^*(a_1, b_2^*(a_1,\theta))$ for all $\theta$ and $\mu^*(I_3(f^*,a_1,g))(\theta') = 1$ if there exists $\theta'$ such that $f^*(a_1, b_2^*(a_1,\theta')) \geq g(a_1, b_{2,g}(a_1,\theta'))$; For any $f \neq f^*$ and $(a_1, g)$, $\mu^*(I_3(f,a_1,g))(\theta) = p(\theta)$ if $g(a_1, b_{2,g}(a_1,\theta)) \geq f(a_1, b_{2,f}(a_1,\theta))$ for all $\theta$ and $\mu^*(I_3(f,a_1,g))(\theta') = 1$ if there exists $\theta'$ such that $f(a_1, b_{2,f}(a_1,\theta')) > g(a_1, b_{2,g}(a_1,\theta'))$. This assessment induces $(b_1^*, b_2^*)$ and is a renegotiation-proof perfect Bayesian equilibrium.

**[Only if]** Suppose that $\Gamma_R(G)$ has a renegotiation-proof perfect Bayesian equilibrium $(\beta^*, \mu^*)$ that induces $(b_1^*, b_2^*)$. Letting $\beta_2^*(\emptyset) = f^*$, we have $\beta_1^*(\mathscr{C}) = b_1^*$, $\beta_2(f^*,\theta,a_1) = b_2^*(a_1,\theta)$ for all $(a_1,\theta)$, and $\mu^*(\mathscr{C})(f^*) = 1$. Sequential rationality of player 1 implies that

$$b_1^* \in \operatorname{argmax}_{a_1} U_1\left(a_1, b_2^*\right) \tag{18}$$

whereas that of player 2 implies $u_2(a_1, b_2^*(a_1,\theta),\theta) - f^*(a_1, b_2^*(a_1,\theta)) \geq u_2(a_1, b_2^*(a_1,\theta'),\theta) - f^*(a_1, b_2^*(a_1,\theta'))$ for all $a_1$ and $\theta, \theta'$, which, together with increasing differences, implies that $b_2^*$ is increasing.

We also claim that

$$b_2^*\left(b_1^*,\theta\right) \in \operatorname{argmax}_{a_2} u_2\left(b_1^*, a_2, \theta\right) \quad \forall\theta. \tag{19}$$

Suppose, for contradiction, that this is not the case for $\theta'$ and let $\hat{a}_2 \in \operatorname{argmax}_{a_2} u_2(b_1^*, a_2, \theta')$ and define $\varepsilon = u_2(b_1^*, \hat{a}_2, \theta') - u_2(b_1^*, b_2^*(b_1^*,\theta'),\theta') > 0$. Define $f'(a_1,a_2) = F^*(b_1^*, b_2^*) + \varepsilon/2$ and note that the third party accepts $f'$. Assume first that $f'$ is not renegotiated after $b_1^*$ and note that sequential rationality of player 2 implies that $\beta_2^*(f',\theta,b_1^*) \in \operatorname{argmax}_{a_2} u_2(b_1^*, a_2, \theta)$. Let $b_{2,f'}(a_1,\theta) = \beta_2^*(f',\theta,a_1)$. Player 2's expected payoff under $f'$ is

$$U_2\left(b_1^*, b_{2,f'}\right) - F^*\left(b_1^*, b_2^*\right) - \varepsilon/2 > U_2\left(b_1^*, b_2^*\right) - F^*\left(b_1^*, b_2^*\right)$$

contradicting that $(\beta^*, \mu^*)$ is a PBE. A similar argument goes through if $f'$ is renegotiated after $b_1^*$.

Therefore, by (18) and (19), $(b_1^*, b_2^*)$ is a Bayesian Nash equilibrium of $G$ and $b_2^*$ is increasing. Finally, suppose that $b_2^*$ is not renegotiation-proof. This implies that for any contract $f$ such that $(f, b_2^*)$ is incentive compatible, there exist $a_1'$, $\theta'$, and an incentive compatible $(g, b_2)$ such that $u_2(a_1', b_2(a_1', \theta'), \theta') - g(a_1', b_2(a_1', \theta')) > u_2(a_1', b_2^*(a_1', \theta'), \theta') - f(a_1', b_2^*(a_1', \theta'))$ and $g(a_1', b_2(a_1', \theta)) > f(a_1', b_2^*(a_1', \theta))$ for all $\theta$. This implies that, in any perfect Bayesian equilibrium, after history $(f, \theta', a_1')$ player 2 strictly prefers to renegotiate and offer $g$ and the third party accepts it. In other words, there exists no renegotiation-proof perfect Bayesian equilibrium which induces $(b_1^*, b_2^*)$, completing the proof. □

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