Labor Markets with On the Job-Search and Wage Posting: Equilibrium Wage and Welfare Implications

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Abstract

In this paper we enrich the concept of worker-firm matches, describing them with two parameters, the level of productivity and its growth rate. We present a model of labor market characterized by asymmetry of information among firms and contractual advantage of the incumbent firm, and we compare the results obtained with different assumptions on the level of competition in the labor market. We find that, decreasing the level of competition among firms in the labor market, produce a bias towards high growth firms.

1 Introduction

For the past several decades search-and-matching has been at the forefront of theoretical labor economics. In particular, the idea that each worker-firm pair is characterized by an idiosyncratic productivity level has provided important insights into the determination of wages and turnover. In this paper we present an analysis of equilibrium in the labor market by enriching the concept of a worker-firm match. More specifically, we assume that each worker-firm match is characterized by not only a productivity level but also by an idiosyncratic productivity growth rate – that is, by a entire firm-specific productivity profile

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– while retaining some of the salient features of search-and-matching theories. For example, workers search for better job opportunities while employed because of search frictions. However, because on-the-job productivity increases the division of rents between the worker and the firm is no longer a simple static decision. Rather, its characterization takes on a dimension that is dynamic and inherently more complex. The existence of rents and the division of rents between the worker and firm have of course been extensively studied in the theoretical literature on compensation and turnover. Our analysis explicitly focuses on the ramifications of heterogeneity of productivity growth rates on the division of rents and on the properties of the dynamic assignment of workers across potential jobs in the presence of search frictions. Our theoretical results are consistent with a wide, and often puzzling, array of empirical findings related to wage and turnover dynamics. Moreover, an analysis of the welfare properties of this labor market equilibrium shows that the market outcome is biased toward high growth jobs.

The market for prospective workers are determined by competition between an incumbent firm and an outside firm. The first critical assumption is that outside firms make take-it or leave-it fixed wage offer contracts to prospective workers. The second assumption is that the incumbent firm can match this outside wage offer. The third key assumption is that the outside firm has incomplete information about the productivity profile (and wages) of the incumbent worker-firm match. Hence in our modeling choice we give the incumbent firm an advantage over the outside firm. Thus firms face a twofold problem. First, new firms must find the profit maximizing fixed wage contract to offer prospective workers, and second, incumbent firms must find the highest outside wage offer to match in all future time periods. This market mechanism clearly generates within-job wage and turnover dynamics. The first key set of results of the paper pertain to the derivation of the highest outside wage offer matching function of incumbent firms, and the existence and characterization of the equilibrium wage function of new firms. Next, we detail the social planner solution in order to compare our market outcome to the first best outcome. Here we show that the market turnover rate is inefficiently low owing to the information asymmetry between new and incumbent firms. In our final welfare analysis, we show that the market also over selects high wage growth jobs in the sense that a new firm will offer a higher equilibrium wage to the high growth match even though the low growth match has the same social planner match value as the former.

The properties of the equilibrium wage function and the highest outside wage offers a firm is willing to match generate various model implications that provide an unified explanation for a host of widely documented and often puzzling findings. In particular, the model implications reconcile two seemingly contradictory findings in the literature: evidence that past wage growth on a job reduces turnover, and the lack of evidence of differences in permanent rates of wage growth among jobs. More specifically, we show among other modeling results that: (1) wages increase and turnover rates decrease over the duration of an employment relationship, but this positive tenure effect on wages is weaker than the negative tenure effect on turnover, (2) wages increase and turnover rates decrease over the duration of an employment relationship, but this positive tenure effect on wages is weaker than the negative tenure effect on turnover; (3) wage growth is higher and turnover is lower in high productivity growth jobs than in low productivity growth jobs; and (4) covariance of successive wage increases is negative for a given productivity profile whereas the same covariance, without the conditioning, is indeterminate. These results are consistent with a variety of empirical findings.[1]

In an earlier study (Munasinghe 2003), a theory of wage and turnover dynamics based on a similar technology was proposed as an explanation of these various empirical findings. Although the model presented here is also consistent with this gamut of findings, the primary objective of this paper is different. The focus here is the theoretical analysis of the functioning of labor markets in the presence of search frictions when employment matches are characterized by heterogeneity of productivity growth rates. The model in Munasinghe (2003) rested on two extreme assumptions, namely, competition for prospective workers among a cluster of identical outside job offers and complete firm bargaining power in the wage setting process. The extreme bargaining power of the firm was of course counterbalanced by Bertrand competition among the cluster of outside firms. In this paper, however, we introduce a more realistic set of assumptions despite significantly augmenting the technical hurdles to finding an equilibrium solution to the problem. First, we drop the cluster assumption and presume that the worker only receives a single outside wage offer in every period. In that sense it is similar to Postel-Vinay and Robin (2002) where, because of this assumption in conjunction with full information, prospective or new firms have complete monopsony power. However, in our model we restrict this monopsony power by also assuming that new firms have no information about the productivity profile and wages of the worker in the incumbent firm. Thus new firms make their wage offers based only on their (known) prospective productivity profile. As a result, when workers switch jobs they do collect some of the rents from the new match and not just the part of rents incumbent firms are willing to match.

2 Model Assumptions

The basic assumption of the model is a distribution of productivity profiles across all worker-firm pairs. Each productivity profile is characterized by an initial productivity level and a growth rate that determines future productivity on the job. Productivity increases on the job are firm-specific and this skill accumulation occurs automatically at the match-specific growth rate. The production technology exhibits constant returns to scale and labor is the only factor of production. Hence, firm size is indeterminate and the model can be viewed as a single worker problem – i.e. each worker-firm pair can be treated independently because the underlying match-specific productivity profile is independent

[1] See Munasinghe (2003) for a detailed summary of these findings on wage and turnover dynamics.

of firm size.

Assumption 1 *Workers face an infinite number of potential firms and each worker-firm match is characterized by a two-dimensional [idiosyncratic] vector* $\sigma \equiv (p, g)$*, where* p *is the initial productivity level and* g *is the growth rate of productivity. Also* $\sigma \in \Sigma \subset R^2_+$*, where* Σ *is compact and* ϕ *is a nonatomic probability measure on* Σ.[2] *Workers are infinitely lived and* β *is the common discount factor for both the worker and the firm. Further* $g(\sigma)\beta < 1$ *and* $g(\sigma) > 1\ \forall \sigma \in \Sigma$.

A worker with the same level of initial productivity in two firms can have two different growth rates of productivity. The correlation between initial productivity and growth rate of productivity on a job is exogenously given by ϕ. Moreover, the increase in productivity in any given job is not portable to another firm and hence all workers face a constant Σ.

Assumption 2 *At any time* t *the worker is employed by one firm* $\sigma_t \equiv (g^t p, g)$*. At the end of every period* t *the worker receives an outside job offer* $\widetilde{\sigma}$ *drawn randomly from* Σ *according to* ϕ.

When a worker meets a firm, both the worker and firm know their entire productivity profile – i.e. the initial productivity level and the growth rate of productivity. Therefore, in the search parlance jobs are treated as inspection goods. The fact that a worker receives a single outside job offer every period implies that search, though costly, is exogenous. If a prospective firm is not chosen by the worker then this firm re-enters the population of firms in Σ and remains dormant. If a worker at any time t exits a firm then this ex-incumbent firm re-enters Σ not as $\sigma_t \equiv (g^t p, g)$ but as $\sigma \equiv (p, g)$. That is, the match profile reverts to its original profile whenever it becomes inactive. Therefore a worker faces the identical distribution of outside job offers in every time period.

Assumption 3 *Firm* $\widetilde{\sigma}$ *offers a wage* $\mathbf{w}(\widetilde{\sigma}) : \Sigma \longrightarrow R^+$*. If this outside wage offer is higher than the worker's current wage the incumbent firm can match this offer and keep the worker or not match the offer and allow the worker to costlessly move to* $\widetilde{\sigma}$*. Firm* $\widetilde{\sigma}$ *can commit to a fixed wage and it does not have information about the match* σ_t.

An incumbent firm increases wages if and only if the worker receives a better outside wage offer. Given this wage renegotiation policy the instantaneous payoffs to the worker and firm are given as follows. At time t the worker receives a wage w_t and produces p_t. At time period $t+1$ the worker receives $max\{w_t, \mathbf{w}(\widetilde{\sigma})\}$ and produces either $(1+g)p_t$ if the worker remains with the

[2] For simplicity we assume Σ a convex compact subset of R^2_+. Moreover we assume that Σ is a lattice. These assumptions might be relaxed at the expense of making the presentation more obscure without necessarily gaining any further economic intuition. Also note that we refer to a probability space as a couple (Σ, Φ), i.e., to a set and a probability measure. We do not explicitly mention the sigma-algebra in our definition because we are always employing the Borel sigma-algebra $B(\Sigma)$ over the set Σ.

incumbent firm or $p(\widetilde{\sigma})$ if the worker quits and moves to the new firm $\widetilde{\sigma}$. The profit for firm σ_t at time t is $p_t - w_t$, and the profit at time period $t+1$ is $(1+g)p_t - max\{w_t, \mathbf{w}(\widetilde{\sigma})\}$ if the firm keeps the worker, and 0 if the worker quits. The profit for the $\widetilde{\sigma}$ firm at time period $t+1$ is $p(\widetilde{\sigma}) - \mathbf{w}(\widetilde{\sigma})$ if the worker quits firm σ_t and joins firm $\widetilde{\sigma}$, and 0 otherwise.[3]

The impetus for within-job wage growth is both the receipt of better outside wage offers and the wage renegotiation policy. From the worker's perspective the source of any wage increase, i.e. within-job or between jobs, is the receipt of a better outside wage offer. As a consequence, the wage at any given time is a sufficient statistic of the job value to the worker.

The wage offer $\mathbf{w}(\widetilde{\sigma})$ is only a function of $\widetilde{\sigma}$ because the outside firm has no information about the incumbent firm. That includes information not only about the productivity profile given by σ_t, but also about the current wage, job tenure, and labor market experience. Observe that a new firm can only commit to a fixed wage forever, and cannot, for example, commit to an increasing wage profile. An implication of this assumption is downward wage rigidity. Moreover, although there are no switching costs the fact that a new firm has no information about σ_t confers an advantage to the incumbent firm. This information asymmetry has the same effect of switching costs that advantages the incumbent firm.

3 Social Planner Problem

Before we proceed to the analysis of the labor market, we introduce the so-called Social Planner (SP) problem. The SP problem is to find the optimal assignment of workers to jobs given the matching technology specified in Assumptions 1 and 2 of the previous section. Note that this SP problem is a single agent problem because we assume that the production technology is constant returns to scale and labor is the only factor of production. Hence this single agent is constrained by the same production and matching technology described in Section 2 – i.e. by the same probability space (Σ, Φ).[4] At any given period t the agent is associated with an existing job (or plant or production technology) characterized by a parameter $\sigma_t = (g^t p, g)$ where (p, g) belongs to Σ. At the beginning of the next period $t+1$ the worker encounters another production technology $\widetilde{\sigma}$ drawn from the probability space (Σ, Φ). At the beginning of every period the problem of this single agent (i.e. the SP problem) is to choose whether to keep working with the existing technology or to switch to the new technology. The solution to this SP problem is the benchmark case against

[3]The model excludes mobility costs associated with job switching. Although mobility cost, like specific capital, also creates a wedge between current and outside job values, firm-specific productivity growth generates richer wage and turnover dynamics than any alternative rendition of mobility costs.

[4]Put differently, the assumptions over the probability space (Σ, Φ) are common to the two models, namely, the labor market characterized by Assumption 3 in Section 2 and the SP.

which we will evaluate the market assignment of workers to jobs.[5]

We can express the SP problem through the Bellman equation[6], and thereby define the value function $W(\sigma)$ as the maximum of the present value of expected lifetime productivity if the worker is using technology σ. In order to define this value function, we begin with a definition.

Definition 1 *Given a probability space* (Σ, Φ) *the set* $[\underline{\sigma}, \sigma]$ *is defined as* $\{\widetilde{\sigma} \in \Sigma | \underline{\sigma} \leq \widetilde{\sigma} \leq \sigma\}$.

Since Σ is a subset of R^2, the set $[\underline{\sigma}, \sigma]$ is measurable and therefore $\Phi[\underline{\sigma}, \sigma]$ is a well defined probability of an event belonging to the Borel σ-algebra defined over Σ. Therefore from definition 1 we have:

$$\Phi\{[\underline{\sigma}, \sigma]\} = \Phi\{\widetilde{\sigma} \in \Sigma | \underline{\sigma} \leq \widetilde{\sigma} \leq \sigma\} = \Phi\left\{(p, g) | \underline{p} \leq p \leq p(\sigma), \underline{g} \leq g \leq g(\sigma)\right\}.$$

We can now define the value function $W(\sigma)$ as follows:

$$W(p, g) = p + \beta \int_{\Sigma} \max\{W(gp, g), W(\widetilde{\sigma}) d\Phi(\widetilde{\sigma})\}.$$

Note that $W(\sigma)$ can be viewed as the match value of a production technology (or match) σ. Clearly this match value is a function of initial productivity p and the growth rate g. However, given search frictions – that we have modeled by assuming a single outside offer $\widetilde{\sigma}$ from the distribution Φ – the match value will also depend on the distribution of outside job offers Φ. The Bellman formulation above says match value is current productivity plus the discounted expected match value in the next period. The match value in the next period is equal to the next period match value with the current technology $W(g\sigma)$ if the agent does not switch in the next period. And it will be equal to the match value of the outside technology $\widetilde{\sigma}$ if the agent switches technologies in the next period. Clearly the agent will only switch technologies if the match value of technology $\widetilde{\sigma}$ – i.e. $W(\widetilde{\sigma})$ – is greater than the next period match value of the existing technology – $W(g\sigma)$. Note that by definition of match value, turnover (i.e. the choice of technology over time) is optimal.

We can consider the characterization of the optimal choice of technology in the SP problem as the benchmark against which we evaluate the corresponding market outcome that assign workers to jobs. To this extent, it might be useful to interpret $W(\sigma)$ as a "welfare" function that expresses the preferences of the Social Planner over the existing technologies for a given probability distribution Φ – i.e. for a given matching technology. If we do so, we can compare the preferences of the Social Planner with the ordering of jobs by the labor market. We pursue this approach in Section 7, where we compare the functions $W(\sigma)$ and $\mathbf{w}(\sigma)$ – i.e., the equilibrium wage function we derive in Section 5 below.

We now summarize various properties of the match value function $W(\sigma)$. The following proposition states that the match value function $W(\sigma)$ exists and that it is unique.

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[5] Note that this benchmark case is based on the same matching technology as in Munasinghe (2003).

[6] See the appendix for a proof of this statement.

Proposition 1 *For each SP problem (Σ, Φ, β), there exist a unique $W(\sigma)$ that is continuous, bounded and strictly increasing in p and g.*

PROOF. See appendix.

Since match value is the present value of life-time productivity under a policy of optimal turnover, it is not surprising that it is an increasing function of the productivity level p and the growth rate g. We now turn to a more focussed analysis of the relative valuation of p and g in the determination of $W(\sigma)$.

For a given Φ consider match value in the absence of turnover. Without turnover the match value of any given technology $\sigma = (p, g)$ is simply the present discounted value of all current and future productivities, and is given by:

$$W(p, g) = p\left(\frac{1+\beta}{\beta - g}\right).$$

If we consider a set of technologies that have the same match value (say k) then the trade-off between p and g can be observed by noting that:

$$g = \beta - \left(\frac{1+\beta}{k}\right)p.$$

Hence in the absence of turnover the marginal rate of substitution of p for g is $(1+\beta)/k$, implying that the indifference curves are linear but they flatten as you consider technologies with higher match value.

Of course, in the absence of turnover there is little of interest in our model. The point however is to ask how the trade-off between p and g are likely to change when turnover is reintroduced into the model. Note with turnover p becomes relatively more important than g because the possibility of switching technologies implies that future productivity is no longer completely determined by the g of the current technology. Therefore if we consider the equivalence of a given technology with and without turnover, a given increase in p would imply a relatively lower g with turnover. Put differently, the marginal rate of substitution of p for g will be higher with turnover than in the absence of turnover.

Note further that even with turnover there will be some technologies that would never lead to turnover because after one period they become "best technology." Clearly there will never be turnover for the "best technology." denoted by $\overline{\sigma} = (\overline{p}, \overline{g})$, and hence its match value is always given by:

$$W(\overline{\sigma}) = \overline{p}\left(\frac{1+\beta}{\beta - \overline{g}}\right).$$

If we let $\overline{k} = W(\overline{\sigma})$ then all technologies that have a match value in the next period that exceeds $\overline{k}$ will never encounter a technology to which it will ever switch. Hence all technologies that satisfy the following condition:

$$\frac{p(1+g)(1+\beta)}{\beta - g} \geq \overline{k}$$

will never switch to another technology either. Note, the MRS_{pg} among the technologies that become exactly indifferent to the best technology a period later are no longer constant.

So far we have derived properties of the match value $W(\sigma)$ that are valid for any Φ. The next proposition is a "comparative static" result that compares how match value $W(\sigma)$ changes when the exogenous probability measure Φ changes. In particular, we are interested in comparing the match value $W(\sigma)$ when the distribution Φ "improves". Recall the definition of first order stochastic dominance.

Definition 2 *Φ^* first order stochastically dominates (FOSD) Φ, if for any non-decreasing function $u(p,g)$ we have*
$\int\int_{\Sigma} u(p,g)d\Phi^*(p,g) \geq \int\int_{\Sigma} u(p,g)d\Phi(p,g)$

Proposition 2 *If we have two SP problems (Σ,Φ) and (Σ,Φ^*) such that Φ^* FOSD Φ, then:*

$$W(\sigma;\Phi^*) \geq W(\sigma;\Phi) \text{ for all } \sigma\text{, with the equality holding only when } \sigma \in \Sigma_1 \cup \Sigma_0.$$

The indifference curves of SP^* are steeper than the indifference curves of SP.

Proof. See Appendix. ■

Proposition 2 makes an easy but important point. We have already stated that $W(\sigma)$ is a function of the probability measure Φ over the available technologies because lifetime productivity depends on the productivity of the technologies used in the future. If the probability of finding a better technology increases – e.g. shifts from Φ_2 to Φ_1 – then the gain from a higher productivity growth rate g is lower because the probability to switch to the new technology is now higher. We will see in Section 7 that this property of the match value $W(\sigma)$ is the underlying reason for why the equilibrium wage function $w(\sigma)$ is not a monotone transformation of $W(\sigma)$.

4 Labor Market Participants

The assumptions 2 and 3 in Section 2 describe how we model competition among firms in the labor market. We assume that at any time period the competition to hire a worker is only between two firms – i.e., the "incumbent firm" (where the worker is currently employed) and the "new firm" (where the worker drew an outside job offer). Again, as discussed in Section 2, we assume that firms have all the bargaining power in the contractual relationship with the worker. This assumption is a departure from the classic models of matching where it is typically assumed that the division of rents between worker and firm is the result of some form of bargaining.[7] Notice that the focus on the competition between the two firms for the worker in a single period t makes this

[7] See for example the recent survey on search models of the labor market by Rogerson, Shimer and Wright (1994).

framework very similar to the multi-principal single agent models. This makes the modelling choice about the bargaining power somehow natural.[8] In fact, the assumption of full bargaining power on one side of the market, in common agency models, is so to speak, inherited from the single agency framework.

4.1 The Worker Problem

Even if the "single stage" problem can be viewed as a common agency problem, this model of competition is more complicated because of the dynamic nature of the problem. Nevertheless the problem of the worker is trivial because the worker switches jobs only if the wage offer is not matched by the incumbent firm. This result is an immediate consequence of our assumption 3, namely that the two firms competing for the worker do not have a symmetric relationship with the worker. While we assume that the new firm can only make a take-it or leave-it wage offer, we allowed the incumbent firm to make a counter offer to the new firm's wage offer. In fact offering a wage contract that depends on the wage offered by the other firm is equivalent to our assumption that the incumbent has the option to match the wage and keep the worker.[9]

It is important to stress that the wage here, like in Munasinghe (2003), is a sufficient statistic for job value. As a consequence, various features related to wage and turnover dynamics carry over from that model to the one here. In particular, we generate the result that turnover always results in wage increases even if the worker moves to a high growth job. This result is consistent empirical observations, and sharply contrasts with the result in Postel-Vinay and Robin (2002).

4.2 The Firm Problem

A firm's problem is twofold: first, it must make an instantaneous wage offer to a new worker w, and second, it must determine the highest outside wage offers it will match in all future time periods $\{z_t\}_{t=1,2,..}$. Even though the two problems are clearly interrelated, we consider these two problems sequentially. That is, the problem of the firm as an incumbent – the choice of the sequences $\{z_t\}_{t=1,2,..}$– is nested in the problem of the firm as a "new" firm – the wage offer to make to a prospective worker. Clearly both these problems depend on what the other firms do when they are also bidding for a new worker and when they are trying to retain a currently employed worker. The result of this analysis will be, among others, an equilibrium wage function $\mathbf{w}(\sigma)$ that we study in Section 5.

[8] We are not suggesting that this modelling choice is closer to the real world, only that it is consistent with tradition.

[9] The careful reader will notice that this is not entirely correct. The more accurate statement is that the space of contracts contingent upon the new-firm wage offer includes the space of contracts we endow to the incumbent firm. Nevertheless, it is obvious that nothing can be achieved by the incumbent firm with this more general set of contracts that cannot also be achieved with the matching option.

In the rest of this section we set up the problem of the new firm and study more closely the incumbent firm's problem.

4.2.1 Problem of the New Firm

The expected profits of the new firm σ that offers a wage w are defined as:

$$\Pi_N[\sigma, w] = \Pi_I[\sigma, w]Q(w),$$

where $Q(w)$ is the probability that the new firm will outbid the incumbent firm with a wage w, and $\Pi_I[\sigma, w]$ is the expected discounted value of profits of the firm σ once it is the incumbent firm.

We first derive the profit function of the incumbent firm – i.e. $\Pi_I[\sigma, w]$ – in the next subsection, and then proceed to address the problem of the new firm in Section 5. Note, the probability $Q(w)$ depends in principle on the optimal choice $\{z_t\}_{t=1,2,..}$ of the incumbent firm. Therefore it depends upon all the information available to the new firm about the incumbent firm with whom the new firm is competing for the services of the worker. There are of course many assumptions we can make about this information set available to a new firm σ, which would clearly affect its wage offer to a prospective worker. In this paper we make the simplifying assumption that the distribution function $Q(w)$ is exogenous. This of course leaves our analysis somewhat incomplete and open to criticism. However, it is a gap we hope to fill in the near future.[10]

4.2.2 Problem of the Incumbent Firm

The problem of the incumbent firm depends on the wage offers of the other firms. As a consequence we need to explicitly introduce the outside wage offer function $\mathbf{w} : \Sigma \to R_+$ in defining and solving the incumbent firm's problem. In fact, it is more convenient to deal with a given outside wage offer distribution function $F(w)$ rather than with an outside wage offer function.

The formal relation between $\mathbf{w}(\sigma)$ and $F(w)$ can be stated as follows. Given the probability space (Σ, Φ), the random variable $\mathbf{w} : \Sigma \to R_+$ defines a new probability space (W, F) where $W = [\underline{w}, \overline{w}]$ and $F(w) = \Phi\{\sigma | \mathbf{w}(\sigma) \leq w\}$. We also have $F(\underline{w}) = 0$, $F(\overline{w}) = 1$ for $\underline{w} = \mathbf{w}(\underline{\sigma})$ and $\overline{w} = \mathbf{w}(\overline{\sigma})$.[11]

In the following analysis we assume the existence of a distribution function $F(w)$ that is exogenously given. This distribution function represents the outside offers any employed worker faces in the labor market. Clearly when we consider the profit maximizing wage a firm offers to a prospective worker, the distribution $F(w)$ is endogenous. We deal with this issue in the next section. Here the problem is to characterize the behavior of a single incumbent firm where the initial productivity and growth rate of productivity is given. The incumbent firm's objective is to maximize the total expected profits by choosing the sequence $\{z_t\}_{t=1,2,..}$.

[10] We discuss this issue in more depth in the Section 8.

[11] Note that here we are using the assumption of Σ being a lattice. None of the results depends on this assumption, but the analysis would be less clear.

The following proposition establish the first set of results concerning the value function of the profits of the incumbent firm.

Proposition 3 *For a given distribution function $F(w) \in F_W$ we have the following:*

1) there exists a unique continuous and bounded value function $\Pi_I(p, g, w; F(w))$ satisfying the optimality equation:

$$\Pi_I(p, g, w; F(w)) = \begin{cases} (p-w) & \text{if } \Pi_I(gp, g, w; F(w)) < 0 \\ (p-w) + \beta\Pi(gp, g, w; F(w))F(w) + & \\ \max\limits_{z\in[w,\overline{w}]} \beta \int_w^z \Pi_I(gp, g, \widetilde{w}; F(w)) dF(\widetilde{w}) & \text{if } \Pi_I(gp, g, w; F(w)) \geq 0 \end{cases}$$

2) $\Pi_I(p, g, w; F(w))$ is strictly increasing in p and strictly decreasing in w.

Proof.

For existence and uniqueness we can use contraction mapping theorem. We take a function $\Pi_I(p, g, w) \in C'(\Sigma \times W)$, where $C'(\Sigma \times W)$ is a set of continuous bounded functions in R defined over $\Sigma \times W$ non-increasing in w. Define the operator $(T\Pi_I)(p, g, w)$ as follows:

$$(T\Pi_I)(p, g, w) = \begin{cases} (p-w) & \text{if } \Pi_I(gp, g, w; F(w)) < 0 \\ (p-w) + \beta\Pi(gp, g, w; F(w))F(w) + & \\ \max\limits_{z\in[w,\overline{w}]} \beta \int_w^z \Pi_I(gp, g, \widetilde{w}; F(w)) dF(\widetilde{w}) & \text{if } \Pi_I(gp, g, w; F(w)) \geq 0 \end{cases}$$

We first prove that the operator T brings elements of $C'(\Sigma \times W)$ into itself.

1) Consider the case where $\Pi_I(gp, g, w) > 0$. Then
$T\Pi_I(p, g, w) = \max_{z\in[w,\overline{w}]} \left[p - w + \beta\Pi_I(gp, g, w)F(w) + \beta \int_w^z \Pi_I(gp, g, \widetilde{w})dF(\widetilde{w})\right]$.
Since $\Pi_I(gp, g, \widetilde{w})$ is non-increasing in $\widetilde{w}$ and $\Pi_I(gp, g, \widetilde{w}) > 0$ we have
$T\Pi_I(p, g, w) = p - w + \beta\Pi_I(gp, g, w)F(w) + \beta \int_w^{z^*} \Pi_I(gp, g, \widetilde{w})dF(\widetilde{w})$, where z^* is defined from the following equation $\Pi_I(p, g, z^*) = 0$.

Define

$$\widehat{\Pi}_I(p, g, w, \widetilde{w}) = \begin{cases} \Pi_I(gp, g, w) & \text{if } \widetilde{w} \leqslant w \\ \Pi_I(gp, g, \widetilde{w}) & \text{if } \widetilde{w} > w \end{cases}$$

By construction since $\Pi_I(gp, g, w)$ is continuous, bounded and non-increasing in w, $\widehat{\Pi}_I(p, g, w, \widetilde{w})$ is also continuous and non-increasing in $\widetilde{w}$ and w. We have $T\Pi_I(p, g, w) = p - w + \beta \int_{\underline{w}}^{z^*} \widehat{\Pi}_I(gp, g, w, \widetilde{w})dF(\widetilde{w})$. $\widehat{\Pi}_I(p, g, w, \widetilde{w})$ is continuous and bounded function, non-increasing in w, therefore $\int_{\underline{w}}^{z^*} \widehat{\Pi}_I(gp, g, w, \widetilde{w})dF(\widetilde{w})$ is continuous, bounded and non-increasing in w since integral with fixed extreme points is a continuous, bounded, increasing operator.

2) Consider the case where $\Pi_I(gp, g, w) < 0$. Then

$T\Pi_I(p,g,w) = p - w$. $T\Pi_I(p,g,w)$ is continuous and bounded function decreasing in w.

3) The last case to consider is $\Pi_I(gp,g,w^*) = 0$. Since $\lim_{w\to w^*-} T\Pi_I(p,g,w) = \lim_{w\to w^*+} T\Pi_I(p,g,w) = T\Pi_I(p,g,w^*) = p - w^*$, $T\Pi_I(p,g,w)$ is continuous and bounded at point $w = w^*$.

We have shown that the operator T brings elements of $C'(\Sigma\times W)$ into itself. Also $C'(\Sigma\times W)$ is a complete metric space. Moreover T is a contraction since: (a) it is immediate to show that it is (weakly) monotone, and (b) it satisfies the discounting property because for any $a > 0$

if $\Pi_I(gp,g,w) \geqslant 0$ we have $(T(\Pi_I + a))(p,g,w) =$

$= \sup_{z\in[w,\overline{w}]}(p - w + \beta\Pi_I(gp,g,w)F(w) + \beta aF(w) + \beta\int_w^z(\Pi_I(gp,g,\widetilde{w}) + a)dF(\widetilde{w}) =$

$p - w + \beta\Pi_I(gp,g,w)F(w) + \beta aF(w) + \beta(\sup_{z\in[w,\overline{w}]}\int_w^z(\Pi_I(gp,g,\widetilde{w})dF(\widetilde{w}) + aF(z) - aF(w)) <$

$p - w + \beta\Pi_I(gp,g,w)F(w) + \beta(T(\Pi_I + a))(p,g,w) = \sup_{z\in[w,\overline{w}]}(p - w + \beta\Pi_I(gp,g,w)F(w) + \beta aF(w) + \beta\int_w^z(\Pi_I(gp,g,\widetilde{w}) + a)dF(\widetilde{w})\int_w^z(\Pi_I(gp,g,\widetilde{w})dF(\widetilde{w}) + \beta a = T\Pi_I(p,g,w) + \beta a$;

if $\Pi_I(gp,g,w) < 0$, $\Pi_I(p,g,w) + a \geqslant 0$ we have $(T(\Pi_I + a))(p,g,w) =$

$= \sup_{z\in[w,\overline{w}]}(p - w + \beta\int_w^z(\widehat{\Pi}_I(p,g,w,\widetilde{w}) + a)dF(\widetilde{w})) < p - w + \beta aF(z) - \beta aF(w) < p - w + \beta a = T\Pi_I(p,g,w) + \beta a$;

if $\Pi_I(gp,g,w) + a < 0$ we have $(T(\Pi_I + a))(p,g,w) = p - w < p - w + \beta a = T\Pi_I(p,g,w) + \beta a$.

Therefore from the contraction mapping theorem there exists a unique, continuous and bounded function $\Pi_I(p,g,w)$ that is non-increasing in w, satisfying the optimality equation.

To prove that $\Pi_I(p,g,w)$ is strictly decreasing in w, it is enough to show that for all $\Pi_I \in C'(\Sigma\times W)$ we obtain $T\Pi_I \in C''(\Sigma\times W)$, where $C''(\Sigma\times W)$ is the subset of $C'(\Sigma\times W)$ composed of functions strictly decreasing in w. To show this we notice that $p - w$ is strictly decreasing in w, and $\int_{\underline{w}}^{z^*}\widehat{\Pi}_I(gp,g,w,\widetilde{w})dF(\widetilde{w})$ is weakly decreasing in w. The same reasoning applies to prove that $\Pi_I(p,g,w)$ is strictly increasing in p.

■

Corollary 1 *1. If $\Pi_I[g\sigma,z] \geq 0$ for $z = w$, and $\Pi_I[g\sigma,z] \leq 0$ for $z = \overline{w}$, then $G(p,g,w)$ is such that $\Pi_I[g\sigma, G(p,g,w)] = 0$. If $\Pi_I[g\sigma,w] < 0$ for $z = w$, then $G(p,g,w) = w$. If $\Pi_I[g\sigma,w] \geq 0$ for $z = \overline{w}$, then $G(p,g,w) = \overline{w}$.*

2. $G(p,g,w)$ is a continuous function in (p,g,w).

3. $G(p,g,w)$ is weakly increasing in (p,g,w).

4. If $\Pi[g\sigma,z] \geq 0$ for $z = w$ and $\Pi[g\sigma,z] < 0$ for $z = \overline{w}$ then $G(p,g,w)$ is strictly increasing in (p,g). Otherwise $G(p,g,w)$ is constant (p,g)

5. If $\Pi_I[g\sigma,w] \leq 0$ for $z = w$, then $G(p,g,w)$ is strictly increasing in w. Otherwise $G(p,g,w)$ is constant in w.

Proof.

The proof of the corollary is immediate. (1) is proved noticing that $\Pi_I[p, g, w; F(w)]$ is strictly decreasing in w. (2) follows given (1) and noticing that the feasibility correspondence is $Z(\sigma, w) = [w, \overline{w}]$. (3) follows (1) and (2) noticing that $\Pi_I[p, g, w; F(w)]$ is strictly increasing in (p, g) .(4) and (5) follow trivially from (1) and (3).

■

Since $G(p, g, w)$ is a function, we sometime refers to it as the **z** function, i.e. the highest outside wage offer the firm is willing to match. Therefore the corollary tells us that apart possible corner solutions, the incumbent firms will match outside wage offers up to a zero profit condition. The intuition is simple: when the incumbent firm looses a worker, it means that the new firm offered a wage that the incumbent firm is not willing to match. When the worker leaves the ex-incumbent firm "disappears" making zero profits from that moment on. Therefore as long the worker is offered a wage by the new firm that, if paid by the incumbent firm, gives an expected profit greater than zero, the incumbent firm should match the offer. We will further discuss this result in section 6.

We can further characterize the profits of the incumbent firm, as illustrated by the following proposition and corollary.

5 Existence of Equilibrium Wage Function

From the previous section it is clear that, for any distribution function $F(w)$, the value of expected profits of the incumbent firm σ is dependent on the wage w paid to the worker – i.e. $\Pi_I[\sigma, w; F]$. The distribution function $F(w)$ of course cannot be considered as given exogenously because it is the result of the wage offers of all new firms. In this section we derive the equilibrium distribution function $F(w)$, or equivalently, the equilibrium wage function $\mathbf{w}(\sigma)$.[12] We begin by turning to the problem faced by the new firm – i.e., the determination of the profit maximizing fixed wage to offer to the prospective worker. As stated in Section 4 the expected profits of a new firm σ that offers a wage w are equal to the expected profits of the incumbent firm σ multiplied by the probability of getting the worker by offering a wage w. Therefore, we can write the maximization problem of the new firm as:

$$\max_{w \in R_+} \Pi_I[p, g, w; F] Q(w) .$$

In the previous section we focussed on the first component of the new firm's profit function $\Pi_I[p, g, w; F]$. Here we treat the probability of getting the worker with a wage offer w as exogenous given, and we make the following assumption about $Q(w)$.

Assumption 4 $Q(w)$ *is a distribution function that is weakly increasing, continuous from below, and* $Q(0) = 0$ *and* $\lim_{w \to \infty} Q(w) = 1$.

[12] We indicate the equilibrium wage function with the bold character $\mathbf{w}(\cdot)$ to avoid confusion with the wage paid by the firm. Clearly in equilibrium we have $\mathbf{w}(\cdot) = w$.

Given the exogeneity of $Q(w)$, assumption 4 is barely restrictive.

The trade-off faced by the new firm is clear: the higher the wage offer the more likely is the firm to get the worker, but the profits will be lower if the worker accepts – i.e., if the incumbent firm does not match the offer. This is the same trade-off faced by a bidder in a first-price auction.[13]

The solution to the problem for the new firm σ is an optimal wage offer w. In order to ensure the existence of such a solution, we introduce a second assumption on the probability $Q(w)$.

Assumption 5 *$Q(w)$ is continuous.*

Since the profits of the incumbent firm depend on the wage distribution F and the optimal wage offer w determines the actual wage distribution F, we have to ensure the existence of an equilibrium wage distribution or equivalently, an equilibrium wage function $\mathbf{w}(\sigma)$. We first introduce formal definitions of the equilibrium wage distribution and equilibrium wage function, respectively.

Definition 3 *A wage function $w : \Sigma \to R_+$ that maps the probability space (Σ, Φ) into the probability space (W, F) is an equilibrium wage function of the labor market if for any $\sigma \in \Sigma$:*

$$\mathbf{w}(\sigma) = \arg \max_{\xi \in [\underline{w}, \overline{w}]} \left(\Pi_I[p, g, \xi; F] Q(\xi) \right).$$

In order to prove the existence of an equilibrium wage function $\mathbf{w}(\sigma)$, we first translate this problem to the equivalent problem of finding an equilibrium distribution function $F(w)$.

Definition 4 *A wage distribution $F(w)$ is an equilibrium for the labor market if*

$$F(w) = \Phi\left(\sigma : \arg \max_{\xi \in [\underline{w}, \overline{w}]} \left(\Pi_I[p, g, \xi; F] Q(\xi) \right) \leq w\right).$$

We can now state the main proposition of this section.

Proposition 4 *If assumptions 1-5 hold then there exists an equilibrium wage function $\mathbf{w}(\sigma)$, and an associated equilibrium distribution function $F(w)$.*

Proof.

The proof is based on establishing the existence of a fixed point of the operator applied over distribution function $F(w)$, defined as follows:

$(\widetilde{T}F)(w) = \Phi(\sigma : \arg\max_{\xi \in [\underline{w}, \overline{w}]} (\Pi_I(p, g, \xi; F(w)) Q(\xi)) \leqslant w)$.

The proof strategy is to show that the operator $\widetilde{T}$ (defined over the compact set $F_{[\underline{w}, \overline{w}]}$) maps elements $F_{[\underline{w}, \overline{w}]}$ into itself, and that it is continuous. Since $\Pi_I(p, g, \xi; F(w))$ is continuous in p, g, ξ, $Q(\xi)$ is also continuous in ξ, and ξ is defined over a compact $[\underline{w}, \overline{w}]$ we conclude that $\arg\max_{\xi \in [\underline{w}, \overline{w}]} (\Pi_I(p, g, \xi; F(w)) Q(\xi))$

[13] In Section 9 we further discuss this linkage.

is non-empty, closed-valued, bounded correspondence, upper-hemicontinuous in p, g (by Maximum Theorem).

By Measurable Selection Theorem the function

$$\widetilde{w}(\sigma) = \arg \max_{\xi \in [\underline{w}, \overline{w}]} (\Pi_I(p, g, \xi; F(w)) Q(\xi))$$

is measurable. Therefore, we may conclude that the operator $\widetilde{T}$ brings elements of $F_{[\underline{w}, \overline{w}]}$ into itself.

To prove continuity of the operator $\widetilde{T}$ we first prove that the operator T is continuous in function $F(w)$. Assume that there is a sequence of distribution functions $F_n(w)$ such that $F_n(w) \longrightarrow F(w)$, i.e. $\sup_{w \in [\underline{w}, \overline{w}]} |F_n(w) - F(w)| < \varepsilon_n$ and $\lim_{n \longrightarrow \infty} \varepsilon_n = 0$. Then we can write that $F_n(w) = F(w) + f(w)$, where $|f(w)| < \varepsilon_n$. For the case where $\Pi_I(gp, g, w) \geqslant 0$ we have

$$T\Pi_I(p, g, w; F(w)) = p - w + \beta \int_{\underline{w}}^{z^*} \widehat{\Pi}_I(gp, g, w, \widetilde{w}) dF(\widetilde{w})$$

and

$$\begin{aligned} T\Pi_I(p, g, w; F_n(w)) &= p - w + \beta \int_{\underline{w}}^{z^*} \widehat{\Pi}_I(gp, g, w, \widetilde{w}) dF_n(\widetilde{w}) \\ &= p - w + \beta \int_{\underline{w}}^{z^*} \widehat{\Pi}_I(gp, g, w, \widetilde{w}) d(F(\widetilde{w}) + f(\widetilde{w})) \\ &= p - w + \beta \int_{\underline{w}}^{z^*} \widehat{\Pi}_I(gp, g, w, \widetilde{w}) dF(\widetilde{w}) \\ &\quad + \beta \int_{\underline{w}}^{z^*} \widehat{\Pi}_I(gp, g, w, \widetilde{w}) df(\widetilde{w}) \end{aligned}$$

Therefore,

$$\begin{aligned} &T\Pi_I(p, g, w; F_n(w)) - T\Pi_I(p, g, w; F(w)) = \\ &\beta \int_{\underline{w}}^{z^*} \widehat{\Pi}_I(gp, g, w, \widetilde{w}) df(\widetilde{w}) < \\ &\beta\varepsilon_n \int_{\underline{w}}^{z^*} \widehat{\Pi}_I(gp, g, w, \widetilde{w}) d\widetilde{w} \leqslant \\ &\beta\varepsilon_n \Pi_I(gp, g, \underline{w})(z^* - \underline{w}) \end{aligned}$$

$\lim_{n \longrightarrow \infty} \beta\varepsilon_n \Pi_I(gp, g, \underline{w})(z^* - \underline{w}) = 0$,
thus $\lim_{n \longrightarrow \infty} (T\Pi_I(p, g, w; F_n(w)) - T\Pi_I(p, g, w; F(w))) = 0$,
which means that the operator T is continuous in $F(\cdot)$.
The case, where $\Pi_I(gp, g, w) < 0$ is trivial.

Since T is continuous in $F(w)$ its fixed point is also continuous in $F(w)$.

$\Pi_I(p, g, \xi; F(w))Q(\xi)$ is continuous in $p, g, w, F(w)$ and bounded. Therefore, $\arg\max_{\xi \in [\underline{w}, \overline{w}]}(\Pi_I(p, g, \xi; F(w))Q(\xi))$ is upper-hemicontinuous in $F(w)$ and $\widetilde{T}$ is continuous in $F(w)$. ■

This equilibrium wage function differs from the equilibrium wage function in Munasinghe (2003) and the wage offers in Postel-Vinay and Robin (2002) although the wage setting in all three models are based on offer matching. In Munasinghe the equilibrium wage function generated zero profit wages because of the cluster assumption. As a consequence all rents went to the workers. In sharp contrast, the wage offers in Postel-Vinay and Robin shift rents to the firm because both only a single outside firm competes with the incumbent firm and the new firm has full information about the highest outside offer the incumbent firm is willing to match. As a consequence, the new firm offers a wage just sufficient to lure the worker. Therefore, for reasons that are not surprising, our equilibrium wage function generates wage offers that fall somewhere between these two extremes. Recall that in our model formulation we drop the cluster assumption of the former and the full information assumption of the latter.

Our equilibrium wage function has two distinct advantages over these two closely related wage offer functions. First, the restriction on competition and the limited information prospective new firms have about incumbent firms seem more realistic than the extreme assumptions that characterize Munasinghe, and Postel-Vinay and Robin. Second, our equilibrium wage offers are still consistent with a well know empirical puzzles in the literature. Namely, that wages increase and turnover rates decrease over the duration of an employment relationship, but this positive tenure effect on wages is weaker than the negative tenure effect on turnover. Note that turnover is generated by the highest outside wage offer the incumbent firm is willing to match, which increases with job tenure because of productivity growth. The mean wage increase however is attenuated by the initial equilibrium wage. If the initial wage is relatively high then the scope for subsequent wage growth is of course reduced. In Munasinghe the zero-profit wage clearly generated this attenuation so that the modelling results were consistent with this fact. In contrast, Postel-Vinay and Robin's model allowed for large wage increases on the job because firms pay the minimum initial wage to get their workers. Our model, by placing the equilibrium wage somewhere between the zero-profit wage and a minimum wage of the Postel-Vinay and Robin type can therefore still reconcile the empirical regularity of a weak tenure effect on wages.

6 Model Implications

We now turn to a more careful analysis of the $\mathbf{z}$ function that specifies the highest outside wage offer the incumbent firm is willing to match. The key result that we want to derive here is that if two jobs offer the same equilibrium wage then the highest outside wage offers the firm is willing to match will be higher for the high growth job. This result leads to a variety of implications that are consistent with empirical findings on wage and turnover dynamics. First, it implies that mean within-job wage growth is higher in high productivity growth jobs. Second, turnover is lower in high productivity growth jobs. The joint implication is that past wage growth on a job is negatively related to turnover.

We begin by noting that incumbent firms will match outside wage offers up to a zero profit condition (see Corollary 1). We have already shown that the function $\mathbf{z}(.)$ is increasing in p and g.

Here we derive a first set of results.

Proposition 5 *The profits of the incumbent firm is given by:*

$$\Pi_I[\sigma, \mathbf{w}(\sigma)] = \frac{1}{1-\beta F(\mathbf{w}(\sigma))} \frac{Q(\mathbf{w}(\sigma))}{q(\mathbf{w}(\sigma))},$$

Proof. See Appendix

■

The expression of proposition 6 above can be interpreted as the indirect profit function, and it is increasing in p and g. The expression for the profits of the new firm is therefore given by:

$$\Pi_N[\sigma, \mathbf{w}(\sigma)] = \frac{1}{1-\beta F(\mathbf{w}(\sigma))} \frac{Q^2(\mathbf{w}(\sigma))}{q(\mathbf{w}(\sigma))}.$$

Using these expressions we derive the following lemmata.

Lemma 1 *If σ and σ' are such that $\mathbf{w}(\sigma) = \mathbf{w}(\sigma')$ then $\Pi_j[\sigma, \mathbf{w}(\sigma)] = \Pi_j[\sigma', \mathbf{w}(\sigma')]$, with $j = I, N$.*

Proof. This is immediate from the expressions obtained above since we express profits only as a function of the equilibrium wage $\mathbf{w}(\sigma)$. ■

The Lemma below is used for the proof of Lemma 6.

Lemma 2 *If $\Pi_I[g'\sigma', \mathbf{w}(\sigma)] > \Pi_I[g\sigma, \mathbf{w}(\sigma)]$, then $\Pi_I[g'\sigma', w] > \Pi_I[g\sigma, w]$, for any w.*

Proof.

In the appendix we show (see Lemma ???) that

$$\frac{d\Pi_I[\sigma, \mathbf{w}(\sigma)]}{dw} = -\frac{1}{1-\beta F(\mathbf{w}(\sigma))}$$

for any σ – i.e., the equilibrium wage is a sufficient statistic for profits. Hence

$$\frac{d\Pi_I[g'\sigma', \mathbf{w}(\sigma)]}{dw} = \frac{d\Pi_I[\sigma, \mathbf{w}(\sigma)]}{dw},$$

because the equilibrium wage is the same. Now define

$$\Pi(w) = \Pi_I[g'\sigma', w] - \Pi_I[g\sigma, w].$$

We have $\Pi(\mathbf{w}(\sigma)) > 0$ and $\frac{d\Pi_I[w]}{dw} = 0$. Therefore $\Pi_I[g'\sigma', w] - \Pi_I[g\sigma, w] > 0$ for any w. ■

Lemma 3 *Let σ and σ' such that $p(\sigma) > p(\sigma')$ and $g(\sigma') > g(\sigma)$. If $\mathbf{w}(\sigma) = \mathbf{w}(\sigma')$, then*

$$\mathbf{z}\left(g'^t\sigma'\right) > \mathbf{z}\left(g^t\sigma\right) \text{ for all } t > 0,$$

and

$$\Pi_j[g'^t\sigma', \mathbf{w}(.)] > \Pi_j[g^t\sigma, \mathbf{w}(.)], \text{ for all } t > 0, \text{ and } j = I, N.$$

Proof.

Since $\mathbf{w}(\sigma) = \mathbf{w}(\sigma')$, from Lemma 4 it follows that $\Pi_j[\sigma, \mathbf{w}(\sigma)] = \Pi_j[\sigma', \mathbf{w}(\sigma')]$. Expanding these profit terms implies:

$$\begin{aligned} & [p(\sigma) - \mathbf{w}(\sigma)] + \beta\left\{\Pi_I[g\sigma, \mathbf{w}(\sigma)]F[\mathbf{w}(\sigma)] + \int_{\mathbf{w}(\sigma)}^{\mathbf{z}(g\sigma)} \Pi_I[g\sigma, \widetilde{w}]dF[\widetilde{w}]\right\} \\ = \; & [p(\sigma') - \mathbf{w}(\sigma')] + \beta\left\{\Pi_I[g'\sigma', \mathbf{w}(\sigma')]F[\mathbf{w}(\sigma')] + \int_{\mathbf{w}(\sigma')}^{\mathbf{z}(g\sigma')} \Pi_I[g'\sigma', \widetilde{w}]dF[\widetilde{w}]\right\} \end{aligned}$$

Since $p(\sigma) > p(\sigma')$ and $\mathbf{w}(\sigma) = \mathbf{w}(\sigma')$ we have

$$\begin{aligned} & \Pi_I[g'\sigma', \mathbf{w}(\sigma)]F[\mathbf{w}(\sigma)] + \int_{\mathbf{w}(\sigma)}^{\mathbf{z}(g'\sigma')} \Pi_I[g'\sigma', \widetilde{w}]dF[\widetilde{w}] \\ > \; & \Pi_I[g\sigma, \mathbf{w}(\sigma)]F[\mathbf{w}(\sigma)] + \int_{\mathbf{w}(\sigma)}^{\mathbf{z}(g\sigma)} \Pi_I[g\sigma, \widetilde{w}]dF[\widetilde{w}]. \end{aligned}$$

Now assume that $\Pi_I[g'\sigma', \mathbf{w}(\sigma)] > \Pi_I[g\sigma, \mathbf{w}(\sigma)]$. Then $\Pi_I[g'\sigma', w] > \Pi_I[g\sigma, w]$ from Lemma 5 for any w. Therefore if we take the zero-profit wage $w = \mathbf{z}(g\sigma)$ – i.e., $\Pi_I[g\sigma, w] = 0$ – then $\Pi_I[g'\sigma', w] > \Pi_I[g\sigma, w] = 0$. Since profits decrease with wages, the w' that satisfies $\Pi_I[g'\sigma', w'] = 0$ implies that $z(g'\sigma') = w' > w = z(g\sigma)$.

Alternatively, if we suppose that $\Pi_I[g'\sigma', \mathbf{w}(\sigma)] \leq \Pi_I[g\sigma, \mathbf{w}(\sigma)]$ then $\Pi_I[g'\sigma', w] \leq \Pi_I[g\sigma, w]$ for any w, and $z(g'\sigma') \leq z(g\sigma)$. But that implies

$$\begin{aligned} & \Pi_I[g'\sigma', \mathbf{w}(\sigma)]F[\mathbf{w}(\sigma)] + \int_{\mathbf{w}(\sigma)}^{\mathbf{z}(g'\sigma')} \Pi_I[g'\sigma', \widetilde{w}]dF[\widetilde{w}] \\ < \; & \Pi_I[g\sigma, \mathbf{w}(\sigma)]F[\mathbf{w}(\sigma)] + \int_{\mathbf{w}(\sigma)}^{\mathbf{z}(g\sigma)} \Pi_I[g\sigma, \widetilde{w}]dF[\widetilde{w}], \end{aligned}$$

a contradiction. ■

The fact that the highest outside wage offer an incumbent firm is willing to match is bigger for a high growth job than for a low growth job holding the equilibrium wages constant across both jobs is the key theoretical result that underpins many of the model implications consistent with a variety of empirical findings on wage and turnover dynamics. First, it leads to the implication that mean wage growth is higher in high growth jobs than in low growth jobs. The obvious corollary is that turnover will be lower in the high growth job. Second, the stochastic nature of within-job wage increases implies that, conditional on a productivity profile, the covariance of wage increases in adjacent time periods is negative, whereas the unconditional covariance is indeterminate. These results collectively explain the empirical puzzle mentioned in the introduction of the paper: evidence that past wage growth on a job reduces turnover, and the lack of evidence of differences in permanent rates of wage growth among jobs.

7 Welfare Analysis

In this section we focus on the welfare implications of our model. The key result is that the asymmetry between the incumbent and new firm creates a bias such that the market favors high productivity growth matches. More specifically, we analyze the market outcomes of two jobs, denoted σ and σ', such that job σ has a relatively higher initial productivity ($p > p'$) and a lower productivity growth rate ($g < g'$) than the job σ'.[14] Recall that in Section 5 we showed the existence of an equilibrium wage function $\mathbf{w} : \Sigma \rightarrow R_+$, and we begin this section with several observations related to this equilibrium wage function. First, for any $\sigma \in \Sigma$ the profits of the new firm – $\Pi_N(\sigma)$ – are greater than zero, and hence also the profits of the incumbent firm – $\Pi_I(\sigma, \mathbf{w}(\sigma))$. Second, for any $\sigma \in \Sigma$ the equilibrium wage is less than the highest outside wage offer an incumbent firm will match – i.e., $\mathbf{w}(\sigma) < z(\sigma)$. Hence if two jobs are equivalent – have the same σ – at any moment in time then our market mechanism will always select the incumbent firm over the new firm. In fact, the incumbent firm will be selected even when the new firm is superior – i.e. $\sigma_N > \sigma_I$ – due to continuity. The implication of this result is of course that our model generates inefficiently low turnover. Note the result $\mathbf{w}(\sigma) < z(\sigma)$ is a direct consequence of the information asymmetry between the incumbent and new firm (see Assumption 2 above).

In order to formally assess the welfare implications of our model we first need to introduce an additional assumption. Our previous set of assumptions (1-5) are insufficient in scope to make sharp welfare predictions. Recall in Section 5 we drew attention to the similarity between the problem faced by the new firm and the problem of a bidder in a first price auction. In the first price auction we expect a higher valuation for the bidder to lead to a higher submitted bid. Analogously, in our model we expect a higher profit for the incumbent firm – i.e. conditional on getting the worker – to lead to a higher equilibrium wage offer to a prospective worker. Unfortunately, for an arbitrary distribution function $Q(w)$, we do not know whether this might in fact be the case. A sufficient condition that is routinely used to ensure the monotonicity between the valuation – $\Pi_I(\sigma, w)$ – and the bid – $\mathbf{w}(\sigma)$ – is given in the following assumption.[15]

[14] Throughout this section we refer to σ' as the high productivity growth job and σ as the high initial productivity job.

[15] As a word of caution, note that although Assumption 6 is very familiar to the reader of auction theory, we are aware that the more assumptions we make on the exogenous distribution

Assumption 6 *The distribution function $Q(w)$ has density $q(w)$ and the ratio $\frac{Q(w)}{q(w)}$ is increasing in w.*

With this additional assumption we can now state the following proposition.

Proposition 6 *Given Assumption 1-6, the equilibrium wage function $\mathbf{w}:\Sigma\to R_+$ is (strictly) increasing in p and g.*

Proof. By definition $\Pi_N(\sigma)=\max_{w\in[\underline{w},\overline{w}]}\Pi_I(\sigma,w)\,Q(w)$. From the first order condition we obtain:

$$\Pi_I[\sigma,\mathbf{w}(\sigma)]=-\frac{d\Pi_I[\sigma,\mathbf{w}(\sigma)]}{dw}\frac{Q(\mathbf{w}(\sigma))}{q(\mathbf{w}(\sigma))}$$

Recall from the previous section that $\Pi_I[\sigma,w]|_{w=\mathbf{w}(\sigma)}$ is differentiable at w if $F(w)$ is continuous. In the discontinuity points of $F(w)$ there exist both the left and right derivative and they are finite.[16] Also recall from the previous section that:

$$\frac{\Pi_I[\sigma,w]}{dw}|_{w=\mathbf{w}(\sigma)}=-\frac{dV(w)}{dw}|_{w=\mathbf{w}(\sigma)}=-\frac{1}{1-\beta F(w)}|_{w=\mathbf{w}(\sigma)}.$$

By substituting from above we obtain:

$$\Pi_I[\sigma,w]|_{w=\mathbf{w}(\sigma)}=[\frac{1}{1-\beta F(w)}\frac{Q(w)}{q(w)}]|_{w=\mathbf{w}(\sigma)}.$$

In Proposition 3 (see Section 4) we established that $\Pi_I[\sigma,w]$ is increasing in (p,g) for any w. Therefore, to guarantee the FOC the right hand side must also increase and this implies w increasing in (p,g). ■

It is clear that with Assumption 6 we can make sharper model predictions. For example, all the results of the previous section (without Assumption 6) still hold. Moreover, we can now further strengthen these results in ways that are useful for the welfare analysis. First, we can compare the labor market outcome of this model with the Social Planner outcome obtained in Section 3. Second, we can compare the outcomes of our model with Munasinghe (2003) where the modelling details are different.

We know from Munasinghe (2003), that when we impose a zero-profit condition on the new firm – i.e. if the job σ drawn by the worker is from a cluster of identical firms in a Bertrand competition – we obtain the following set of results: (1) All match rents go to the worker. This result is a trivial consequence of the fact that each firm makes zero-profits. (2) The turnover from an incumbent firm

$Q(w)$, the more difficult it will be to obtain an equilibrium $Q(w)$ satisfying those assumptions. Since we intend to extend our model so that we obtain the distribution $Q(w)$ endogenously we are certainly concerned about not making such assumptions arbitrarily. However, given that we are currently working with an exogenous $Q(w)$, we are unable to evaluate the extent to which Assumption 6 is restrictive.

[16] In this case, the proof needs to be amended but none of the substantive results are altered as a result.

to a new firm is efficient. In other words, there are no possible "local" switches between the incumbent and a new firm that give a Pareto improvement. (3) The labor market is efficient overall in the sense that the equilibrium wage function is a monotone transformation of $W(\sigma)$. Put differently, if we denote the equilibrium wage function in Munasinghe as $\omega(.)$ then $\omega(\sigma) \geq \omega(\sigma')$ if and only if the Social Planner prefers σ to σ'.

Given Assumptions 1-6, in the following proposition we state various welfare results that draw sharp contrasts with Munasinghe (2003) and the outcomes of the Social Planner.

Proposition 7 *(1)* $\Pi_N(\sigma) > 0$ *and* $V(\mathbf{w}(\sigma)) < V(\omega(\sigma))$. *(2)* $\forall \sigma \in int\Sigma$ $\exists\ \varepsilon$ *such that* $z(\sigma) > w(\sigma + \varepsilon)$. *(3) If* σ, σ' *such that* $W(\sigma) = W(\sigma')$ *then* $w(\sigma) \leq w(\sigma')$, *with equality only if* $\sigma, \sigma' \in \Sigma_1$.

PROOF. See appendix.

The first item of the proposition says that the rents are divided between the worker and the firm. Since firms make positive profits clearly the value of any job σ to the worker – i.e. $V(\mathbf{w}(\sigma))$ – is less than the value of a job if competition among firms shifted all rents to the worker – i.e $V(\omega(\sigma))$. The second item of the proposition claims that the turnover from the incumbent to a new firm is not always efficient. For example, if σ_I and σ_N are "close enough" with $W(\sigma_N) > W(\sigma_I)$ then under our market mechanism the incumbent firm will keep the worker even if allowing the worker to switch jobs for a transfer payment from the new firm would lead to a Pareto improvement. In other words, there are Pareto improving "local" switches between the incumbent and a new firm. The claims in both items (1) and (2) are immediate and not surprising.

The third item of Proposition 8, namely that the labor market is not ex-ante efficient, is far from obvious and leads to a variety of interesting observations regarding the welfare implications of the model. What this result says is that when we compare two new firms, the equilibrium wage function $\mathbf{w}(\sigma)$ is no longer a monotone transformation of $W(\sigma)$ for all $\sigma \in \Sigma$. Since the intuition for this result is not transparent, we begin by revisiting the problem of the Social Planner and the relative weights placed on initial productivity and the growth rate of productivity in the computation of match value.

In particular, recall from Section 3 the role of the outside offer distribution in determining the indifference map of jobs. We argued earlier that if we consider an alternate offer distribution that is say stochastically dominant then the Social Planner would value the initial productivity level relatively more than the growth rate, and vice versa. The intuition for this result is relatively straightforward. If the outside offer distribution improves it implies that the likelihood of the worker finding a better match increases. Since the worker is now more likely to switch jobs in the future current productivity is relatively more valuable than a high growth rate that would lead to high productivity only in the future. The question is what does this result have to do with why market mechanism over selects high growth jobs? Note from item (2) above that turnover is inefficiently low in our market mechanism. Hence it is as if the worker is less likely to receive a better outside offer under the market mechanism than under the direction of the Social Planner. Put otherwise, the shift from the Social Planner to the market mechanism can be interpreted as a change in the outside offer distribution where the likelihood of a better match for any given σ is lower. As a consequence, the market vis-a-vis the first best of the Social Planner will place a higher relative value on the growth rate than on the initial productivity level of a job.

The final part of the intuition for item (3) is that if the market places a relatively higher value on growth then the high growth job will lead to higher profits than the low growth job even of both jobs have the same match value. Higher profits would of course imply that the firm is willing to pay a higher wage to lure workers into a high growth job than to low growth job. That completes, in no doubt in a circuitous route, the intuition behind the claim in item (3). One other observation is that if any job becomes a best firm in the next period then this market bias vanishes. The reason of course is because in such jobs there is no turnover. In the absence of turnover the scope for this bias is thwarted. This point highlights the fact that the effects of the advantage given to the incumbent firm in terms of information asymmetry works precisely through the turnover mechanism of the model.

This result also tells us something about the distribution of high growth jobs in a more complicated environment that explicitly takes into account the entry of new workers into the labor market. Once again if we suppose that the Social Planner problem leads to a specific distribution of high growth jobs once matches become "best firm" then we can ask how this distribution of growth rates would compare under market conditions. Given the earlier discussion, the answer is of course fairly straightforward. The market would over select high growth jobs.

Finally we make a few other observations. First, the market bias for high growth jobs can be interpreted as if the market mechanism displays a lower discount rate than the Social Planner. Second, competition in Munasinghe (2003) clearly leads to efficient turnover and thus mimics the outcome of the Social Planner. However, our model by restraining the extent of competition and by introducing information asymmetry gives rise to not only to inefficiently low turnover but also to a bias for high growth jobs that is somewhat unexpected. Note that restraints on competition appears to lead to a bias for high growth.

8 Model Limitations and Extensions

In this section we discuss some of the restrictions of our modelling assumptions and also suggest possible extensions for a future research agenda.

We mentioned earlier that the assumption that the probability distribution of getting a worker if an outside firm offers a wage w – $Q(w)$ – is exogenously given is unpalatable since ideally we should consider this distribution as an equilibrium characteristic of the model. There clearly a variety of ways to try and tackle this issue. First, we can assume that the new firm knows the age and thus the labor market experience of the worker.[17] This would also imply that the incumbent firm would face a different F for workers of different age. Clearly this a complicated problem since incumbent firms will need to calculate the probability that a prospective worker is at a job σ conditional on age. One advantage that still remains is that we can still treat any worker as independent of all other workers, as we do here, because of the constant returns to scale assumption.

An alternatively strategy is to make assumptions about the existence of a steady-state distribution of ages in the workforce. On the basis of this the new firm can calculate the prior on the set Sigma. Note, the calculation of this prior is more complicated than the calculation suggested above. Moreover there is a single F for any age like in our model, but this F now also depends on the steady-state distribution of ages. Clearly the task of endogenizing $Q(w)$ is complicated. We hope to tackle this problem in a future research project.

Another restriction of our model is that it does not give the worker a major role. In particular, since the model assumes a *constant* arrival rate of outside job offers, search effort is not endogenously determined in the model. But of course workers are likely to influence the arrival rate of outside job offers by searching more or less intensely, and their optimal effort level will be determined by a benefit-cost analysis of search effort. Under standard assumptions – an increasing marginal cost function – search effort will be a function of the wage level since the current wage is a sufficient statistic for job value. Since lower wages imply higher marginal gains to search, optimal search effort will be a negative function of current wages. If search effort is a direct function of wages then endogeneity of search will not alter the qualitative results of the paper because the wage level still remains a sufficient statistic for job value. As a consequence, the model implications that hold current wages constant remain robust with endogenously determined search effort.

Although endogenous search effort is unlikely to change the qualitative results of the paper, it will affect some of the welfare properties of the wage renegotiation policy. In particular, as Mortensen (1978) observed, a wage policy of matching outside offers will lead to inefficiently high levels of search intensity even though the turnover rule remains optimal. Since intensity of a worker's search effort is a function only of the wage the worker receives, optimal search effort is inefficiently high because the worker does not take into consideration the capital loss incurred by the firm. This leads to an interesting conjecture in the context of our model. As we noted earlier the information asymmetry of the model implies an inefficiently low turnover rate. Hence the question is whether and to what extent the inefficiency of endogenous search might ameliorate this first inefficiency in a model that also incorporated endogenous search effort on the part of workers. The formal incorporation of search effort into the current framework adds considerable complexity to the modelling details, and hence this challenging extension is left to a future research project.

[17] If the new firm knew the tenure of the worker in the current job then the wage offer will clearly depend on the σ and job tenure in the incumbent firm.

As a final observation, note that in Section 5 we mentioned a strong connection between the problem of the new firm and the problem of a bidder in a first price auction. The trade-off faced by the firm is clear: a higher wage offer increases the probability of getting a prospective worker, but if the worker accepts the higher wage reduces expected profits. This trade-off is the same one faced by a bidder in a first-price auction. The new firm can be interpreted as a bidder in an auction where $Q(w)$ is the probability to get the object that has a net private value for the bidder equal to $\Pi_I[p, g, w; F]$. This interpretation is suggestive because it links our model of the labor market to a rich and well developed literature on auction pricing. Nevertheless the analogy is not as clear cut. First, in our model the value of the good for the bidder is not independent from the bid. In other words, the bidder has a non-linear utility in money. More importantly, if we try to interpret the incumbent firm as the other bidder, we have to consider an asymmetric auction (first-price/second-price) where the highest bid of the two bidders gets the object, but one bidder (the incumbent firm) pays the second price while the new firm pays the first price. To push this reasoning further, the incumbent firm pays a price to participate in a second-price auction while the new firm participates in a first-price auction. This price can be interpreted as a reservation price: the worker – who can be thought as the auctioneer – has a reservation price w for the object that he is auctioning, and the incumbent firm makes an enforceable commitment to pay this reservation price. These considerations suggest a future research agenda that analyzes and interprets labor market institutions through auction theory.

9 Conclusions

In this paper we present an analysis of equilibrium in the labor market by explicitly assuming that a worker-firm match is characterized by two idiosyncratic parameters – an initial level of productivity and a growth rate of productivity. Within an environment that is further characterized by search frictions, limited competition between the incumbent and a new firm, and information asymmetry that gives an advantage to the incumbent firm, we solve for the equilibrium of this labor market. The modelling results are not only consistent with a wide, and often puzzling, array of empirical findings related to wage and turnover dynamics, but our welfare analysis shows that the market outcome is biased toward high growth jobs. These results contrast sharply with the first best outcomes of the Social Planner and other models of labor markets.

10 Appendix

The appendix contains the proofs not in the text.

Proof of Proposition 1

Proof.

The proof uses standard techniques of dynamic programming as in Stokey and Lucas (1989).

Let $\underline{\sigma}, \overline{\sigma} \in \Sigma$ be such that for all $\sigma \in \Sigma$ we have $\underline{\sigma} \leq \sigma \leq \overline{\sigma}$. Let $W(\sigma) = \frac{p(\sigma)}{1-\beta g(\sigma)}$, so we have $W(\sigma) \leq W(\overline{\sigma})$ for all $\sigma \in \Sigma$. Notice that $W(\sigma) < \infty$ since $\beta g(\sigma) < 1$.

Let $\Sigma^e \equiv \{\sigma \in R^2_+ | \frac{p(\underline{\sigma})}{1-\beta g(\underline{\sigma})} \leq \frac{p(\sigma)}{1-\beta g(\sigma)} \leq \frac{p(\overline{\sigma})}{1-\beta g(\overline{\sigma})}$ and $g(\underline{\sigma}) \leq g(\sigma) \leq g(\overline{\sigma})\}$.

Define a probability space $(\Sigma^e, B(\Sigma^e), \Phi^e)$ where $B(\Sigma^e)$ is the Borel sigma-algebra over Σ^e and $\Phi^e \equiv \Phi(B \cap \Sigma)$. Notice that Σ^e is compact, convex and that $\Sigma \subset \Sigma^e$. For any σ such that $g(\sigma)\sigma \notin \Sigma$, let $W(g\sigma) = \frac{p(\sigma)g(\sigma)}{1-\beta g(\sigma)}$.

Let $C(\Sigma^e)$ be the space of bounded and continuous functions $W : \Sigma^e \rightarrow R$ with the sup norm and define the operator T^e on $C(\Sigma^e)$ by

$$(T^e W)(\sigma) = p(\sigma) + \beta \int_{\Sigma^e} \max\{W(g\sigma), W(\widetilde{\sigma})\} \, d\Phi^e(\widetilde{\sigma})$$

The return function $p(\sigma)$ is continuous and since Σ^e is compact it is also bounded.

Let $f : \Sigma^e \times \Sigma^e \rightarrow R$ defined by $f(\sigma, \widetilde{\sigma}) = \max\{W(\sigma), W(\widetilde{\sigma})\}$. The function f is trivially bounded and continuous because so is W. Since $\Phi^e(\widetilde{\sigma})$ is a probability measure we have that $\left\| \int_{\Sigma^e} f(\cdot, \widetilde{\sigma}) \, d\Phi^e(\widetilde{\sigma}) \right\| \leq \|f\|$.

Hence $\int_{\Sigma^e} f(\cdot, \widetilde{\sigma}) \, d\Phi^e(\widetilde{\sigma})$ is bounded and so it is $(T^e W)$. Since f is a continuous function, $\int_{\Sigma^e} f(\cdot, \widetilde{\sigma}) \, d\Phi^e(\widetilde{\sigma})$ is continuous. Thus $T^e : C(\Sigma^e) \longrightarrow C(\Sigma^e)$ is continuous and bounded.

Next, we show that the operator T^e is a contraction. In fact T^e satisfies the Blackwell's sufficient conditions for a contraction:

(monotonicity) Let $W_1, W_2 \in C(\Sigma^e)$ with $W_1(\sigma) \leq W_2(\sigma)$ for all $\sigma \in \Sigma^e$; then $\max[W_1(g\sigma), W_1(\widetilde{\sigma})] \leq \max[W_2(g\sigma), W_2(\widetilde{\sigma})]$. It follows that $(T^e W_2)(\sigma) - (T^e W_1)(\sigma)$
$= \beta \int_{\Sigma^e} \{\max[W_2(g\sigma), W_2(\widetilde{\sigma})] - \max[W_1(g\sigma), W_1(\widetilde{\sigma})]\} \, d\Phi^e(\widetilde{\sigma}) \geq 0$ for all $\sigma \in \Sigma^e$; that is $(T^e W_1)(\sigma) \leq (T^e W_2)(\sigma)$.

(discounting) Let $a \geq 0$, then for any $\sigma \in \Sigma^e$ and $W \in C(\Sigma^e)$, we have $[T^e(W + a)](\sigma) = p(\sigma) + \beta \int_{\Sigma^e} \max[W(g\sigma) + a, W(\widetilde{\sigma}) + a] d\Phi^e(\widetilde{\sigma}) = p(\sigma) + \beta \int_{\Sigma^e} \{a + \max[W(g\sigma), W(\widetilde{\sigma})]\} \, d\Phi^e(\widetilde{\sigma}) =$
$= p(\sigma) + \beta \int_{\Sigma^e} \max[W(g\sigma), W(\widetilde{\sigma})] d\Phi^e(\widetilde{\sigma}) + \beta a = (T^e W)(\sigma) + \beta a.$

Therefore, by the Contraction Mapping Theorem, the operator T^e defined above has a unique fixed point $W \in C(\Sigma^e)$.

We still need to prove that $W(\sigma)$ is increasing in (p, g)

Define $C'(\Sigma^e) \subset C(\Sigma^e)$ as the subset of bounded continuous functions weakly increasing in (p, g). Since $C'(\Sigma^e)$ is a closed subset of a complete metric space, then it is also a complete metric space. Let $C''(\Sigma^e) \subset C'(\Sigma^e)$ the set of strictly increasing functions. Therefore it is sufficient to show that $T^e[C'(\Sigma^e)] \subseteq C''(\Sigma^e)$.

Consider a value function $W \in C'(\Sigma^e)$. If $W(\sigma)$ is (weakly) increasing in p and g then, for $\sigma_2 = (p_2, g)$ and $\sigma_1 = (p_1, g)$ with $p_2 > p_1$ we have:

$$
\begin{aligned}
&(T^e W)(p_2, g) = p_2 + \beta \int_{\Sigma^e} \max\{W(gp_2, g), W(\widetilde{\sigma})\} \, d\Phi^e(\widetilde{\sigma}) \\
&> p_1 + \beta \int_{\Sigma^e} \max\{W(gp_1, g), W(\widetilde{\sigma})\} \, d\Phi^e(\widetilde{\sigma}) \\
&= (T^e W)(p_1, g)
\end{aligned}
$$

that implies that $W(p, g)$ is strictly increasing in p.

Now for a $W(p, g)$ is strictly increasing in p and weakly increasing in g, and for $\sigma_2 = (p, g_2)$ and $\sigma_1 = (p, g_1)$ with $g_2 > g_1$ we have:

$$
\begin{aligned}
&(T^e W)(p, g_2) - (T^e W)(p, g_1) = \\
&= \beta \int_{\Sigma^e} [\max\{W(g_2 p, g_2), W(\widetilde{\sigma})\} - \max\{W(g_1 p, g_1), W(\widetilde{\sigma})\}] d\Phi^e(\widetilde{\sigma}) \\
&> 0
\end{aligned}
$$

The last inequality holds with the strict sign because (i) W is strictly increasing in p and (ii) $g_2\sigma_2 = (g_2 p, g_2) > (g_1 p, g_1) = g_1\sigma_1$ therefore implies $W(g_2\sigma_2) > W(g_1\sigma_2)$. Moreover by assumption 1, $\Phi^e(g_2\sigma_2) > \Phi^e(g_1\sigma_1)$, where (with abuse of notation) we denote

$\Phi^e(\sigma) = \Phi(\widetilde{\sigma} \mid W(\widetilde{\sigma}) < W(\sigma))$.

■

Proof of Proposition 2

Proof. Define $W_t(p, g)$ as value function for $\sigma \in \Sigma_t \cup \Sigma_{t-1} \cup ... \cup \Sigma_0$. Lets take two points $\sigma = (p, g)$ and $\sigma' = (p', g')$, $p' < p$, which belong to the same indifference curve if $\sigma \sim \Phi(\sigma)$. We will prove that for $\sigma \sim \Phi^*(\sigma)$, such that $\Phi^*(\sigma) \leqslant \Phi(\sigma)$, $W_t^*(\sigma') \leqslant W_t^*(\sigma)$ for all t, where $W_t^*(\sigma)$ is job value function if $\sigma \sim \Phi^*(\sigma)$. Since $W(p, g)$ is non-decreasing in both arguments it will follow that for $\sigma \sim \Phi^*(\sigma)$ the indifference curve should be no flatter than for $\sigma \sim \Phi(\sigma)$.

$W_0(p, g) = \frac{p}{1-\beta g}$. W_0 does not depend on distribution function Φ and $W_0^*(p', g') - W_0^*(p, g) = W_0(p', g') - W_0(p, g) = 0$ - proposition holds.

$W_1(p, g) = p + \beta \widehat{W}_0 F_0(\widehat{W}_0) + \beta \int_{\widehat{W}_0}^{\overline{W}} \widetilde{W}_0 dF_0(\widetilde{W}_0)$,

where $\widetilde{W}_0 \equiv W_0(\widetilde{p}, \widetilde{g})$, $(\widetilde{p}, \widetilde{g}) \sim \Phi(\cdot, \cdot)$, $\widetilde{W}_0 \sim F_0(\cdot)$, $F_0(x) \equiv \Pr\left\{\widetilde{W}_0 = \frac{\widetilde{p}}{1-\beta\widetilde{g}} \leqslant x\right\} |_{\widetilde{p}, \widetilde{g} \sim \Phi(\cdot, \cdot)}$

$\widehat{W}_0 \equiv W_0(pg, g)$, $\overline{W} = \max_{(p,g)\in\Sigma} \frac{p}{1-pg}$.

$W_1(p', g') = p' + \beta \widehat{W}_0' F_0(\widehat{W}_0') + \beta \int_{\widehat{W}_0'}^{\overline{W}} \widetilde{W}_0 dF_0(\widetilde{W}_0)$,

where $\widehat{W}_0' \equiv W_0(p'g', g')$.

Let's suppose that initially for $(\widetilde{p},\widetilde{g}) \sim \Phi(\cdot,\cdot)$, $\widetilde{W_0} \sim F_0(\cdot)$ (p,g) and (p',g') belong to the same indifference curve, i.e.

$W_1(p,g) = W_1(p',g')$. Since $p' < p$, we have $\widehat{W_0}' > \widehat{W_0}$.

$W_1(p',g') - W_1(p,g) = (p'-p) + \beta \int_{\underline{W}}^{\overline{W}} S_0(\widetilde{W_0}) dF_0(\widetilde{W_0})$,

where

$$S_0(\widetilde{W_0}) \equiv \begin{cases} \widehat{W}_0' - \widehat{W_0} & \text{if } \widetilde{W_0} \leqslant \widehat{W_0} \\ \widehat{W}_0' - \widetilde{W_0} & \text{if } \widehat{W_0} < \widetilde{W_0} \leqslant \widehat{W}_0' \\ 0 & \text{if } \widetilde{W_0} > \widehat{W}_0' \end{cases}$$

$S_0(\widetilde{W_0})$ is non-increasing function.

Now we consider the case $(\widetilde{p},\widetilde{g}) \sim \Phi^*(\cdot,\cdot)$ such that Φ^* FOSD Φ. The corresponding distribution of $\widetilde{W_0}$ is defined as $F_0^*(\cdot)$, $F_0^*(x) \equiv \Pr\left\{\widetilde{W_0} = \frac{\widetilde{p}}{1-\beta\widetilde{g}} \leqslant x\right\} |_{\widetilde{p},\widetilde{g} \sim \Phi^*(\cdot,\cdot)}$.

Define set $A \in \Sigma$ as non-increasing if the corresponding indicator function $I_A(p,g)$ is non-increasing in p,g.

Lemma 4 *If Φ^* FOSD Φ then $\Phi^*(A) \leqslant \Phi(A)$ for any non-increasing set $A \in \Sigma$.*

Proof.

Let $u(p,g) \equiv 1 - I_A(p,g)$. Thus $u(p,g)$ is non-decreasing and therefore $\int\int_\Sigma u(p,g) d\Phi^*(p,g) \geqslant \int\int_\Sigma u(p,g) d\Phi(p,g) \Longrightarrow 1 - \Phi^*(A) \geqslant 1 - \Phi(A)$, that is $\Phi^*(A) \leqslant \Phi(A)$

■

Let $A \equiv \left\{\widetilde{p},\widetilde{g} \in \Sigma \mid \frac{\widetilde{p}}{1-\beta\widetilde{g}} \leqslant x\right\}$. A is a non-increasing set since $\frac{\widetilde{p}}{1-\beta\widetilde{g}}$ is increasing in $\widetilde{p},\widetilde{g} \in \Sigma$. Therefore $\Pr\left\{\frac{\widetilde{p}}{1-\beta\widetilde{g}} \leqslant x\right\} |_{\widetilde{p},\widetilde{g} \sim \Phi^*(\cdot,\cdot)} \equiv F_0^*(x) \leqslant F_0(x) \equiv \Pr\left\{\frac{\widetilde{p}}{1-\beta\widetilde{g}} \leqslant x\right\} |_{\widetilde{p},\widetilde{g} \sim \Phi(\cdot,\cdot)}$ for all x.

$F_0^*(\widetilde{W_0}) \leqslant F_0(\widetilde{W_0})$ for all $\widetilde{W_0}$.

Lemma 5 $S^* \equiv \int_{\underline{W}}^{\overline{W}} S_0(\widetilde{W_0}) dF_0^*(\widetilde{W_0}) \leqslant \int_{\underline{W}}^{\overline{W}} S_0(\widetilde{W_0}) dF_0(\widetilde{W_0}) \equiv S$.

Proof. Define $F_0^*(\widetilde{W_0}) - F_0(\widetilde{W_0}) \equiv H_0(\widetilde{W_0})$, $H_0(\widetilde{W_0}) \leqslant 0$.

$$S_0'(\widetilde{W_0}) \equiv \begin{cases} 0 & \text{if } \widetilde{W_0} \leqslant \widehat{W_0} \\ -1 & \text{if } \widehat{W_0} < \widetilde{W_0} \leqslant \widehat{W}_0' \\ 0 & \text{if } \widetilde{W_0} > \widehat{W}_0' \end{cases}$$

The function $S_0'(\widetilde{W_0})$ is equal to the derivative of $S_0(\widetilde{W_0})$ except points $\widetilde{W_0} = \widehat{W_0}$ and $\widetilde{W_0} = \widehat{W_0}'$, where the derivative of $S_0(\widetilde{W_0})$ is not defined. This subset has measure 0. Since $\Phi(\cdot,\cdot)$ and $\Phi^*(\cdot,\cdot)$ are non-atomic and $\frac{\widetilde{p}}{1-\beta\widetilde{g}}$ is increasing in both $\widetilde{p}$ and $\widetilde{g}$, $F_0(\widetilde{W_0})$ and $F_0^*(\widetilde{W_0})$ are also non-atomic. Therefore, making operation similar to integrating by parts substituting $S_0'(\widetilde{W_0})$ for the derivative of $S_0(\widetilde{W_0})$, we get the following

$S^* - S = \int_{\underline{W}}^{\overline{W}} S_0(\widetilde{W}_0) dH_0(\widetilde{W}_0) =$

$= S_0(\widetilde{W}_0) H_0(\widetilde{W}_0) \mid_{\underline{W}}^{\overline{W}} - \int_{\underline{W}}^{\overline{W}} H_0(\widetilde{W}_0) S_0'(\widetilde{W}_0) d\widetilde{W}_0.$

$S_0(\overline{W}) = 0, \ \ H_0(\underline{W}) = 0 \implies \ \ S_0(\widetilde{W}_0) H_0(\widetilde{W}_0) \mid_{\underline{W}}^{\overline{W}} = 0;$

$H_0(\widetilde{W}_0) \leqslant 0, \ S_0'(\widetilde{W}_0) \leqslant 0 \implies \int_{\underline{W}}^{\overline{W}} H_0(\widetilde{W}_0) S_0'(\widetilde{W}_0) d\widetilde{W}_0 \geqslant 0.$

So, $S^* - S \leqslant 0$

■

Since $S^* \leqslant S$, $W_1^*(p', g') - W_1^*(p, g) \leqslant W_1(p', g') - W_1(p, g) = 0$ and proposition 2 holds for $\sigma \in \Sigma_1 \cup \Sigma_0$.

Define $F_t(\cdot)$ as distribution function of $\widetilde{W}_t$, $\widetilde{W}_t = W_t\ (\widetilde{p}, \widetilde{g})$.

Lemma 6 *If $F_0^*(\widetilde{W}_0) \leqslant F_0(\widetilde{W}_0)$ for all $\widetilde{W}_0$, then $F_t^*(\widetilde{W}_t) \leqslant F_t(\widetilde{W}_t)$ for all $\widetilde{W}_t$.*

Proof.

By induction.

$W_1(p, g) = p + \beta \int_{\underline{W}}^{\overline{W}} u(\widetilde{W}_0) dF_0(\widetilde{W}_0)$, where

$$u(\widetilde{W}_0) = \begin{cases} \widehat{W}_0 & \text{if } \widetilde{W}_0 \leq \widehat{W}_0 \\ \widetilde{W}_0 & \text{if } \widetilde{W}_0 > \widehat{W}_0 \end{cases}$$

$W_1^*(p, g) = p + \beta \int_{\underline{W}}^{\overline{W}} u(\widetilde{W}_0) dF_0^*(\widetilde{W}_0).$

Since $u(\widetilde{W}_0)$ is non-decreasing function and F_0^* first-order stochastically dominates F_0, $W_1^*(p, g) \geqslant W_1(p, g)$ for all p, g and consequently $F_1^*(\widehat{W}_1) = \Pr(W_1\ (\widetilde{p}, \widetilde{g}) \leqslant \widehat{W}_1) \mid_{\widetilde{p}, \widetilde{g} \sim \Phi^*(\cdot,\cdot)} \leqslant \Pr(W_1\ (\widetilde{p}, \widetilde{g}) \leqslant \widehat{W}_1) = F_1(\widehat{W}_1) \mid_{\widetilde{p}, \widetilde{g} \sim \Phi(\cdot,\cdot)}$. (proof is analogous to that of lemma 5).

Let's assume that the lemma holds for t, i.e. $F_t^*(\widetilde{W}_t) \leqslant F_t(\widetilde{W}_t)$. Then

$W_{t+1}(p, g) = p + \beta \int_{\underline{W}}^{\overline{W}} u(\widetilde{W}_t) dF_t(\widetilde{W}_t)$,

where $\widehat{W}_t \equiv W_t(pg, g)$;

$$u(\widetilde{W}_t) = \begin{cases} \widehat{W}_t & \text{if } \widetilde{W}_t \leq \widehat{W}_t \\ \widetilde{W}_t & \text{if } \widetilde{W}_t > \widehat{W}_t \end{cases}$$

Since $F_t^*(\cdot)$ first-order stochastically dominates $F_t(\cdot)$ and $u(\widetilde{W}_t)$ is non-decreasing $W_{t+1}^*(p, g) \geqslant W_{t+1}(p, g)$ and $F_{t+1}^*(W_1) \leqslant F_{t+1}(W_1)$.

■

Point (i) follows from the above lemma. For point (ii), we have:

For any t if

$W_t(p', g') - W_t(p, g) = (p' - p) + \beta \int_{\underline{W}}^{\overline{W}} S_{t-1}(\widetilde{W}_{t-1}) dF_{t-1}(\widetilde{W}_{t-1}) = 0,$

$W_t^*(p', g') - W_t^*(p, g) = (p' - p) + \beta \int_{\underline{W}}^{\overline{W}} S_{t-1}(\widetilde{W}_{t-1}) dF_{t-1}^*(\widetilde{W}_{t-1}).$

Since $F_{t-1}^*(\widetilde{W}_{t-1}) \leqslant F_{t-1}(\widetilde{W}_{t-1})$ and $S_{t-1}(\widetilde{W}_{t-1})$ is non-increasing function $W_t^*(p', g') - W_t^*(p, g) \leqslant W_t(p', g') - W_t(p, g) = 0.$

■

Proof of Proposition 6

Proof.

Recall that $\Pi_N(\sigma) = \max_{w\in[\underline{w},\overline{w}]} \Pi_I(\sigma, w) Q(w)$. From the first order condition we obtain:

$$\Pi_I[\sigma, \mathbf{w}(\sigma)] = -\frac{d\Pi_I[\sigma, \mathbf{w}(\sigma)]}{dw} \frac{Q(\mathbf{w}(\sigma))}{q(\mathbf{w}(\sigma))}$$

Also recall that

$$\frac{d\Pi_I[\sigma, w]}{dw}|_{w=\mathbf{w}(\sigma)} = -\frac{dV(w)}{dw}|_{w=\mathbf{w}(\sigma)} = -\frac{1}{1-\beta F(w)}|_{w=\mathbf{w}(\sigma)}.$$

By substituting from above we obtain:

$$\Pi_I[\sigma, w]|_{w=\mathbf{w}(\sigma)} = [\frac{1}{1-\beta F(w)} \frac{Q(w)}{q(w)}]|_{w=\mathbf{w}(\sigma)}.$$

■

Proof of Proposition 8

Proof. The proofs of (1) and (2) are trivial (and omitted.) The proof of (3) uses several lemmas. We begin by introducing some new notation and definitions.

Define Σ_0^W as $\Sigma_0^W = \{\widetilde{\sigma} \in \Sigma \text{ s.t. } W(\widetilde{\sigma}) \geq W(\sigma), \forall \sigma \in \Sigma\}$ and Σ_t^W as $\Sigma_t^W = \{\widetilde{\sigma} \in \Sigma \text{ s.t. } W(g^t\widetilde{\sigma}) \geq W(\overline{\sigma}), \text{ and } W(g^{t-1}\widetilde{\sigma}) < W(\overline{\sigma})\}$. Note that $\Sigma_t^W = \{\widetilde{\sigma} \in \Sigma \text{ s.t. } g(\widetilde{\sigma})\widetilde{\sigma} \in \Sigma_{t-1}^W\}$. Moreover the $(\Sigma_t^W)_{t=1,..}$ is a partition of Σ and it is independent on Φ. That is given any Σ (and β) we have a unique partition of Σ, generated by $W(\overline{\sigma}) = \max_{\sigma\in\Sigma} \frac{p(\sigma)}{1-\beta g(\sigma)}$. An element σ of Σ_t^W is such that, if the worker have been using technology σ for t periods, we is stuck using it forever.

Lemma 7 *The restriction of $w(\sigma)$ over $\Sigma_0^W \cup \Sigma_1^W$ is a monotone transformation of the restriction of $W(\sigma)$ over $\Sigma_0^W \cup \Sigma_1^W$. That is $w(\sigma_1) \geq w(\sigma_2)$ if and only if $W(\sigma_1) \geq W(\sigma_2)$ for any $\sigma_1, \sigma_2 \in \Sigma_0^W \cup \Sigma_1^W$.*

Proof. First, if $\sigma \in \Sigma_0^W \cup \Sigma_1^W$ and $\sigma \in \Sigma_0^z \cup \Sigma_1^z$ then $z(\sigma) = \overline{w}$ (no turnover), and hence $\Pi_I(\sigma, w) = W(\sigma) - V(w; F)$ and $\Pi_N(\sigma, w) = [W(\sigma) - V(w; F)]Q(w)$. Therefore if $W(\sigma') \geq W(\sigma)$ then $\Pi_I(\sigma', w) \geq \Pi_I(\sigma, w)$. From Assumption 6 ($\frac{Q(w)}{q(w)}$ weakly increasing in w) we obtain that $\text{argmax}_w \Pi_N(\sigma, w)$ is increasing in $\Pi_I(\sigma, w)$ and therefore a monotone transformation of $W(\sigma)$. Notice that we use the superscript W and z, to indicate that the partition over Σ is given by the social planner function $W(\sigma)$ and the zero profit wage $z(\sigma)$ respectively. That is $\Sigma_t^z = \{\widetilde{\sigma} \in \Sigma \text{ s.t. } z(g^t\widetilde{\sigma}) \geq \overline{w}, \text{ and } z(g^{t-1}\widetilde{\sigma}) < \overline{w}\}$. So the Lemma 10 is proved noting that $\Sigma_0^W \cup \Sigma_1^W \subset \Sigma_0^z \cup \Sigma_1^z$.

■

The rest of the proof is by induction. We proved that $\Pi_I(\sigma', w) - \Pi_I(\sigma, w) > W(\sigma') - W(\sigma)$ for $\sigma, \sigma' \in \Sigma_1$.

Assume that the proposition of the lemma holds for $\sigma, \sigma' \in \Sigma_t$. For $\sigma, \sigma' \in \Sigma_{t+1}$ we can write down the following:

$\Pi_I(\sigma, w) = p(\sigma) - w + \beta \Pi_I(g\sigma, w)F(w) + \beta \int_w^{w(g\sigma)} \Pi_I(g\sigma, \widetilde{w})dF(\widetilde{w});$

$\Pi_I(\sigma', w) = p(\sigma') - w + \beta \Pi_I(g'\sigma', w)F(w) + \beta \int_w^{w(g'\sigma')} \Pi_I(g'\sigma', \widetilde{w})dF(\widetilde{w}).$

$\Pi_I(\sigma, w) - \Pi_I(\sigma', w) = \Delta\Pi =$

$= p(\sigma) - p(\sigma') + \beta(\Pi_I(g\sigma, w) - \Pi_I(g'\sigma', w))F(w(g\sigma)) + \beta \int_{w(g\sigma)}^{z(g\sigma)} (\Pi_I(g\sigma, \widetilde{w}) - \Pi_I(g'\sigma', \widetilde{w}))dF(\widetilde{w}) - \beta \int_{z(g\sigma)}^{z(g'\sigma')} \Pi_I(g'\sigma', \widetilde{w})dF(\widetilde{w}).$

We have to distinguish between two cases:

Case (1): $w(g\sigma) < z(g\sigma) < w(g'\sigma') < z(g'\sigma')$.

$\frac{\Delta W - \Delta\Pi}{\beta} = ((W(g\sigma) - W(g'\sigma')) - (\Pi_I(g\sigma, w) - \Pi_I(g'\sigma', w)))F(w(g\sigma))+$

$+ \int_{w(g\sigma)}^{z(g\sigma)} ((W(\widetilde{\sigma}) - W(g'\sigma')) - (\Pi_I(g\sigma, \widetilde{w}) - \Pi_I(g'\sigma', \widetilde{w})))d\phi(\widetilde{\sigma}) + \int_{z(g\sigma)}^{w(g'\sigma')} ((W(\widetilde{\sigma}) - W(g'\sigma')) + \Pi_I(g'\sigma', \widetilde{w})))d\phi(\widetilde{\sigma})+$

$+ \int_{w(g'\sigma')}^{z(g'\sigma')} \Pi_I(g'\sigma', \widetilde{w})d\phi(\widetilde{\sigma}).$

$(W(g\sigma) - W(g'\sigma')) - (\Pi_I(g\sigma, w) - \Pi_I(g'\sigma', w)) > 0$ because $g\sigma, g\sigma' \in \Sigma_t$;

$W(\widetilde{\sigma}) > W(g\sigma)$ since $w(\widetilde{\sigma}) \geqslant w(g\sigma)$ and $w(\cdot)$ is a monotone transformation of $W(\cdot)$, therefore

$(W(\widetilde{\sigma}) - W(g'\sigma')) - (\Pi_I(g\sigma, \widetilde{w}) - \Pi_I(g'\sigma', \widetilde{w})) > (W(g\sigma) - W(g'\sigma')) - (\Pi_I(g\sigma, w) - \Pi_I(g'\sigma', w)) > 0;$

$(W(\widetilde{\sigma}) - W(g'\sigma')) + \Pi_I(g'\sigma', \widetilde{w})) > (W(\widetilde{\sigma}) - W(g'\sigma')) - (\Pi_I(g\sigma, \widetilde{w}) - \Pi_I(g'\sigma', \widetilde{w})) > 0;$

$\Pi_I(g'\sigma', \widetilde{w}) > 0.$

Thus $\frac{\Delta W - \Delta\Pi}{\beta} > 0$ and $K > 0$.

Case (2): $w(g\sigma) < w(g'\sigma') < z(g\sigma) < z(g'\sigma')$.

$\frac{\Delta W - \Delta\Pi}{\beta} = ((W(g\sigma) - W(g'\sigma')) - (\Pi_I(g\sigma, w) - \Pi_I(g'\sigma', w)))F(w(g\sigma))+$

$+ \int_{w(g\sigma)}^{w(g'\sigma')} ((W(\widetilde{\sigma}) - W(g'\sigma')) - (\Pi_I(g\sigma, \widetilde{w}) - \Pi_I(g'\sigma', \widetilde{w})))d\phi(\widetilde{\sigma}) + \int_{w(g'\sigma')}^{z(g\sigma)} -(\Pi_I(g\sigma, \widetilde{w}) - \Pi_I(g'\sigma', \widetilde{w}))d\phi(\widetilde{\sigma})+$

$+ \int_{z(g\sigma)}^{z(g'\sigma')} \Pi_I(g'\sigma', \widetilde{w})))d\phi(\widetilde{\sigma}).$

■

References

[1] George Baker, Michael Gibbs, and Bengt Holmstrom. The wage policy of a firm. *Quarterly Journal of Economics*, pages 921–956, November 1994.

[2] Gary Becker. Investment in human capital: A theoretical analysis. *Journal of Political Economy*, 70(5, Supplement):9–49, October 1962.

[3] Kenneth Burdett. Theory of employee search: Quit rates. *American Economic Review*, 68:212–220, March 1978.

[4] Kenneth Burdett and Melvyn Coles. Equilibrium wage-tenure contracts. *Econometrica*, 71(5):1377–1404, 2003.

[5] Kenneth Burdett and Dale Mortensen. Wage differentials, employer size, and unemployment. *International Economic Review*, 39(2):257–273, 1998.

[6] Pierre-Andre Chiappori, Bernard Salanie, and Julie Valentin. Early starters versus late beginners. *Journal of Political Economy*, 102:731–760, August 1999.

[7] Henry Farber and Robert Gibbons. Learning and wage dynamics. *Quarterly Journal of Economics*, 111(4):1007–1047, November 1996.

[8] Leonardo Felli and Christopher Harris. Learning, wage dynamics, and firm-specific human capital. *Journal of Political Economy*, 104:838–868, August 1996.

[9] Robert Gibbons and Michael Waldman. A theory of wage and promotion dynamics in internal labor markets. *Quarterly Journal of Economics*, pages 1321–1358, November 1999.

[10] Milton Harris and Bengt Holmstrom. A theory of wage dynamics. *Review of Economic Studies*, 49(3):315–333, July 1982.

[11] Boyan Jovanovic. Firm-specific capital and turnover. *Journal of Political Economy*, 87:1246–60, December 1979.

[12] Boyan Jovanovic. Job matching and the theory of turnover. *Journal of Political Economy*, 87:972–990, October 1979.

[13] W. Bentley MacLeod and James M. Malcomson. Investments, holdup, and the form of market contracts. *American Economic Review*, 83(4):811–837, September 1993.

[14] James M. Malcomson. Contracts, hold-up, and labor markets. *Journal of Economic Literature*, 35:1916–1957, December 1997.

[15] Dale Mortensen. Specific capital and labor turnover. *Bell Journal of Economics*, 9:572–586, Autumn 1978.

[16] Dale Mortensen. Wages, separations, and job tenure: On-the-job specific training or matching? *Journal of Labor Economics*, 6(4):445–471, October 1988.

[17] Lalith Munasinghe. Wage growth and the theory of turnover. *Journal of Labor Economics*, 18:204–220, 2000.

[18] Lalith Munasinghe. A theory of wage and turnover dynamics. Working paper, Barnard College, Columbia University, 2003.

[19] Lalith Munasinghe and Brendan O'Flaherty. Specific training sometimes cuts wages but always cuts turnover. *Journal of Labor Economics*, Forthcoming.

[20] Fabien Postel-Vinay and Jean-Marc Robin. The distribution of earnings in an equilibrium search model with state-dependent offers and counteroffer. *International Economic Review*, 43(4):989–1016, 2002.

[21] Nancy Stokey and Robert Lucas. *Recursive Methods in Economic Dynamics*. Harvard University Press, 1989.

[22] Robert Topel. Specific capital, mobility, and wages: Wages rise with job seniority. *Journal of Political Economy*, 99(1):145–176, February 1991.

[23] Robert Topel and Michael P. Ward. Job mobility and the careers of young men. *Quarterly Journal of Economics*, 107(2):439–479, May 1992.