Aggregation in the Presence of Demand and Supply Shocks

Emi Nakamura *
Harvard University

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Abstract

This paper points out that real GDP statisics respond differently to sector-specific demand and supply shocks for the *exactly* same changes in physical quantities. The paper illustrates how this property arises from the theoretical quantity aggregates that real GDP is designed to approximate. The paper also presents an application to US data suggesting that these factors have contributed significantly to the behavior of US aggregate investment growth in the post-WWII period.

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1 Introduction

At an abstract level, macroeconomists have often interpreted real GDP as a measure of the *physical quantity* of output. For example, in a classic paper, Robert Solow (1957) notes that aggregate output is denominated in "physical" units. Yet, actual real GDP statistics have certain features that measures of “physical quantity” typically do not. In particular, the “quantity” concept seems to imply that a given change in disaggregated quantities would always have the *same* effect on the aggregate quantity. This is not, however, the case for current measures of US real GDP.

First, real GDP statistics respond differently to demand and supply shocks for exactly the same change in physical quantities. To make things concrete, consider the following simple example. Suppose the economy produces only apples and oranges. There are two periods, and in the second period, either a demand shock– such as an increase in people’s taste for apples– or a supply shock– such as an improvement in apple-making technology– occurs. Let’s assume, furthermore, that both shocks lead to *exactly the same* changes in the numbers of apples and oranges produced. The demand shock, however, leads to an increase in the relative price of apples while the supply shock leads to a decrease in the relative price of apples. (Think of the standard Economics 101 diagram of downward sloping demand curves and upward sloping supply curve.) Since empirical quantity indexes such as real GDP use prices to weight quantites, the demand shock leads to a greater increase in real GDP for exactly the same change in disaggregated quantities. This is because, in the demand shock case, the quantity increase is weighted by the higher price of apples rather than the lower price that arises from the supply shock. (See Section 3 for the details of this argument.)

Second, real GDP statistics respond differently to a given quantity increase depending on the *initial* level of production of that good. Once again, this effect works through the role of prices in empirical quantity indexes. The nature of the non-linearity depends on the source of the quantity increase. For the apple-orange economy above, the tenth apple contributes *more* to real GDP than the second apple in the demand shock case. In the supply shock case, however, the tenth apple contributes *less* to real GDP than the second apple. In contrast, the "quantity" of shoes in one’s closet always increases by the same amount in response to an additional pair– regardless of how many shoes one already owns.

One might refer to the property that the growth rate of an empirical quantity index depends only on the size of changes in physical quantities as a “quantity axiom”. Real GDP does not have this property, as I discuss above. This paper investigates the theoretical foundations for this feature of real GDP statistics. These issues have not previously been investigated in the index number theory literature because this literature typically views price changes as exogenous from the consumer’s perspective. This framework implicitly assumes that relative price changes arise from supply shocks. Yet, allowing for *both* demand and supply shocks in a model of the economy has important implications for both the interpretation and properties of empirical quantity indexes.

The paper also presents an empirical application to US data on aggregate investment. The analysis shows that the aggregation issues described above have played an important role in the pattern of US investment growth in the post-WWII period. Unlike standard discussions of economic growth, this analysis shows that economic growth depends on factors other than growth in physical quantities alone.

This paper is related to the long-standing literature on aggregation and index number theory, for example, by Fisher (1971), Lau (1979), Diewert (1976, 1978, 1992, 1998) and Caves, Christensen and Diewert (1982). From a methodological perspective, this paper makes use of duality approaches and results on flexible functional forms of cost functions as discussed, for example, in important papers by Diewert (1974) and Bernstein and Nadiri (1989).

The paper proceeds as follows. Section 2 provides a motivating example for the analysis that follows. Section 3 presents the theoretical approaches to aggregation and demonstrates the violation of the “quantity” axiom described above. Section 4 presents the application to US aggregate investment data. Section 5 concludes.

2 A First Example

It is useful to first look directly at the formulas for empirical quantity indexes. For simplicity, let’s consider a simple two-period example. Suppose the economy produces only apples and oranges and there is endogenous labor supply. Assume that the demand curves for apples and oranges are downward sloping and the supply curves are upward sloping all else constant.

Either a demand or a supply shock occurs in the second period. The demand shock increases people’s taste for apples, while the supply shock improves the farmers’ efficiency at growing apples. As in the introduction, let's assume that both shocks lead to *exactly the same* changes in the numbers of apples and oranges produced. However, the demand shock increases the relative price of apples resulting in a high weight on apples in the quantity index. In contrast, the supply shock lowers the relative price of apples, reducing the weight on apples.

Now let us consider the effect of these shocks on real GDP statistics. The well-known Laspeyres quantity index defined,

$$Q_L = \frac{p_0^T q_1}{p_0^T q_0}, \tag{1}$$

where p_t is a vector of period t prices and q_t is a vector of period t quantities. Notice that the Laspeyres quantity index uses period 0 price weights to aggregate quantities.[1] Similarly, the Paasche index is defined,

$$Q_P = \frac{p_1^T q_1}{p_1^T q_0}, \tag{2}$$

where the only difference is that the quantities are weighted by period 1 rather than period 0 prices. The US BEA currently uses the Fisher quantity index Q_F to measure real GDP, where this index is defined as the geometric mean of the Laspeyres and Paasche quantity indexes,

$$Q_F = (Q_L Q_P)^{0.5}. \tag{3}$$

The Laspeyres index depends on only period 0 prices, implying that Q_L is identical in the demand and supply shock scenarios since the quantity change is assumed to be identical in the two scenarios. In contrast, the Paasche index Q_P depends on period 1 prices, which react differently in the demand shock versus the supply shock scenario. Since the number of apples increases more than the number of oranges, the demand shock implies a higher weight on the more rapidly growing commodity– apples. Thus, Q_F increases more following the demand shock than the supply shock.[2]

Furthermore, the formula for Q_F implies that if subsequent supply shocks yield further declines in apple prices, then increases in apple production contribute non-linearly to Q_F

[1] To a macroeconomist, it may seem a little strange to define the quantity index directly, rather than deflating nominal GDP by a price index to compute real GDP, but since we are interested in the properties of real GDP, it is simpler to deal with the formulae for the quantity indexes directly, rather than viewing real GDP as deflated nominal GDP. A little algebra shows that dividing nominal GDP by a Laspeyres price index gives a Paasche quantity index (defined below); dividing nominal GDP by a Paasche price index gives a Laspeyres quantity index; dividing nominal GDP by a Fisher price index gives a Fisher quantity index. See Diewert and Nakamura (2002b) for a derivation of these results.

[2] I am envisioning an economy with endogenous labor supply so the number of both apples and oranges can be increased by increasing labor (and decreasing leisure). See the numerical example below.

since the weight on apples in Q_F is higher for the initial increases in production than for subsequent increases. The nonlinearity is reversed in the demand shock case since further demand shocks yield further *increases* in apple prices. Thus, the weight on apples in Q_F is lower for the initial increases in production than for subsequent increases.

In order to make these points more concrete, consider the following numerical example. The labor force consists of two yeoman farmers. Farmer 1 produces good 1 and farmer 2 produces good 2. Farmer i's utility is,

$$U_i = C_i - \frac{1}{\gamma} L_i^{\gamma}, \tag{4}$$

where C_i is a consumption aggregate, L_i is the labor supply of farmer i, and γ is a parameter determining the elasticity of labor supply. The consumption aggregate (14) has the CES form,

$$C_i = [z_1^{\frac{1}{\eta}} c_{1,i}^{\frac{\eta-1}{\eta}} + z_2^{\frac{1}{\eta}} c_{2,i}^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}}, \tag{5}$$

where z_j is an exogenous demand shock to good j, $c_{j,i}$ is consumption of good j by farmer i, and η is a parameter describing the substitutibility of the consumption goods. I assume standard values of the parameters, $\gamma = 2$ and $\eta = 3$. To simplify the notation, set $z_1 = 1$. A little algebra shows that the farmers' demands are given by,

$$c_{1,i} = \frac{I}{1+z_2 p_2^{1-\eta}}, \tag{6}$$

$$c_{2,i} = \frac{I}{1+z_2 p_2^{1-\eta}} \frac{z_2}{p_2^{\eta}}, \tag{7}$$

where I is nominal income and p_j is the price of good j. Normalize $p_1 = 1$. Thus, the farmers' utility can be rewritten as,

$$U_i = \frac{I_i}{P} - \frac{1}{\gamma} L_i^{\gamma}. \tag{8}$$

The yeoman farmer's nominal income equals his sales revenue,

$$I_i = p_i y_i. \tag{9}$$

The production function is linear,

$$Y_j = A_j L_j, \tag{10}$$

where A_j reflects the labor productivity of farmer j. Substituting the expression for nominal income (9) and the production function (10) into the utility function (8), we have,

$$U_i = \frac{p_i A_i L_i}{P} - \frac{1}{\gamma} L_i^{\gamma}. \tag{11}$$

The farmer's utility-maximizing labor supply can be found by maximizing equation (11) with respect to his labor supply, which gives,

$$L_i = (\frac{A_i p_i}{P})^{\frac{1}{\gamma-1}}. \tag{12}$$

Finally, by setting output equal to the combined demands of the two farmers, one can easily show that the market-clearing price is,

$$p_2 = \left[\frac{z_2}{(\frac{A_2}{A_1})^{\frac{\gamma}{\gamma-1}}}\right]^{\frac{1}{\frac{1}{\gamma-1}+\eta}}. \tag{13}$$

The expression for p_2 shows that if farmer 2 switches from a hand plow to a tractor (A_2 increases), the price of good 2 falls, all else constant. However, if the farmers' taste for good 2 suddenly increases (z_2 increases), then the price of good 2 rises, all else constant. These effects are quite mundane– supply shocks cause price declines, while demand shocks cause price rises.

Now for the first point– the asymmetric effects of demand and supply shocks. Suppose there are two periods. In the first period, $A_1 = A_2 = 1$ and $z_1 = z_2 = 1$. By symmetry, equal amounts of the two goods are produced: $y_1 = y_2 = 1.41$. In addition, prices are equal at 1. In period 2, a demand or supply shock occurs. The shocks have the following characteristics: the demand shock increases z_2 from 1 to 2, and the supply shock increases A_1 by 3.5% and A_2 by 16%. Following the demand shock, output increases to $y_1 = 1.554$ and $y_2 = 1.848$, while the price of good 2 rises to 1.19. Following the supply shock, output increases to $y_1 = 1.560$ and $y_2 = 1.851$, while the price of good 2 falls to 0.94. Notice that both outputs increase more following the supply shock than the demand shock.

What happens to the quantity aggregates? Following the demand shock, the CES consumption index rises by 45.75% and the Fisher index rises by 21%; whereas, following the supply shock, the CES consumption index rises by 20.25%, and the Fisher index rises by 20%. Thus, the CES consumption index and the Fisher indexes increase by more following the demand shock than the supply shock– even though *both* outputs increase by less.

Moreover, larger demand shocks have larger effects on measured output: a demand shock that causes a 17% increase in output of good 1 and a 54% increase in output of good 2 yields a 38% increase in average labor productivity, while a larger demand shock that causes a 45% increase in good 1 and a 157% increase in good 2 yields a 6% increase in average labor productivity. The apparent increasing returns to scale, despite the linearity of the production function, result from the fundamental non-linearity of the macroeconomic aggregates. These issues are investigated from a theoretical perspective in the next section.

3 Aggregation Theory

3.1 Aggregation on the Demand side

The main approach to motivating quantity aggregates in the theoretical macroeconomics literature is to assume that utility depends on a linearly homogenous consumption aggregate. For example, it is often assumed that utility depends on the CES consumption index,

$$C(q_t) = \left[\int_0^1 q_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \tag{14}$$

where $q_t(j)$ is output of good j, and θ is a parameter destermining the substitutability of the consumption goods.

Suppose the household's utility $u(C)$ depends on a consumption aggregate C, where C is a linearly homogenous, concave function of the quantities consumed q_t. The household's expenditure minimization problem can then be written,

$$E(C_t, p_t) = \min_{q_t} \left\{ p_t.q_t : C(q_t) \geq C_t \right\}, \tag{15}$$

where p_t is the vector of period t prices. The homotheticity assumption implies that,

$$q_t = C_t f(p_t), \tag{16}$$

where q_t is the vector of consumption goods. In other words, the household's consumption is simply a scaled up version of its consumption for $C_t = 1$. Thus, a natural choice for the consumption aggregate is

$$Q_D^* = \frac{C_{t+1}}{C_t}. \tag{17}$$

We can now use properties of the expenditure function to derive a well-known relationship between Q_D^* and observable prices and quantities. The expression (16) and the definition of the expenditure function imply that the expenditure function is separable in prices,

$$E(C_t, p_t) = C_t e(p_t), \tag{18}$$

where $e(p_t)$ is the expenditure for $C_t = 1$ at a given level of prices. The concavity of the expenditure function implies,

$$E(C_1, p_0) \leq E(C_1, p_1) + \nabla_p E(C_1, p_1)(p_0 - p_1) \tag{19}$$

$$= E(C_1, p_1) + q_1(p_0 - p_1) \tag{20}$$

$$= p_0 q_1, \tag{21}$$

where the second equality follows from the envelope theorem. Dividing expression (21) by $E(C_0, p_0)$ and using the definition $E(C_0, p_0) = p_0 q_0$ we have,

$$\frac{p_0 q_1}{p_0 q_0} \geq \frac{E(C_1, p_0)}{E(C_0, p_0)} \tag{22}$$

$$= \frac{C_1 e(p_0)}{C_0 e(p_0)} \tag{23}$$

$$= \frac{C_1}{C_0}, \tag{24}$$

where the third line follows from the separability of the expenditure function in prices (18).

This expression can be rewritten using the definition of the demand-side aggregator Q_D^* as,

$$Q_L \geq Q_D^*, \tag{25}$$

where Q_L is the well-known Laspeyres quantity index defined above. A very similar line of reasoning shows that,

$$Q_P \leq Q_D^*, \tag{26}$$

where Q_P is the Paasche index defined above. Combining the two inequalities (25) and (26), we have the expression,

$$Q_L \geq Q_D^* \geq Q_P. \tag{27}$$

Expression (25) is the economic justification for the idea that there is an upward bias in the fixed-weighted Laspeyres measure of real GDP relative to the theoretical ideal. Thus, the discrepency between Q_L and Q_D^* is often referred to as “consumer substitution bias”.

These result can be extended directly to the case of the aggregate investment component of real GDP under the assumption that the firm’s production function, $G(L_t, K_t)$ depends on a capital aggregate K_t and $K_{t+1} = K_t(1 - \delta) + I_t$, where I_t is a linearly homogenous investment aggregate and δ is the depreciation rate. Notice that this is a rather unusual assumption on the form of the capital aggregate. Although the term “consumer substitution bias” is often used to describe discrepencies among alternative measures of real GDP as a whole, it is perhaps more appropriate to refer to this discrepancy as "final demander substitution bias" for aggregate investment since in this case, the firm is the "final demander" of investment goods.

An important feature of the consumer substitution bias result is that relative price changes are taken as given. In this sense, the standard index number theory arguments proceed in a partial equilibrium framework. The theoretical macroeconomics literature also often abstracts from the issue of relative price change, and thus does not generally allow for factors generating fluctuations in relative prices and quantities. Let us now consider how the analysis of index numbers changes in a general equilibrium framework.

3.2 Aggregation on the Supply side

For simplicity, let us make the following assumptions about the supply side of the economy. Suppose that the production possibilities set for the economy can be represented as,

$$S_t = \{(y, x) : g(y_t) = f(x_t)\}, \tag{28}$$

where y is a vector of outputs and x is a vector of inputs. Let us assume, furthermore, that $g(y)$ is a linearly homogenous, convex function of the output vector y.[3] The convexity of $g(y)$ is an implication of the classical condition of increasing marginal rates of substitution between products, as discussed in Hicks (1946) pg. 87.

The firm's profit maximization problem can then be solved in two stages: in the first stage, the firm chooses revenue-maximizing output ratios given a particular level of $g(y)$; and in the second stage, the firm chooses a level of aggregate output. The problem of optimal factor demands can then be solved entirely separately from the problem of choosing output ratios by minimizing costs subject to the constraint $g(y) = f(x)$.

For a particular level of $g(y)$, the firm's optimization problem can be written,

$$R(p_t, f(x_t)) = \max_{y_t} \{p_t.y_t : g(y_t) = f(x_t)\}. \tag{29}$$

The resulting optimal outputs can be written,

$$y_t = f(x_t)h(p_t), \tag{30}$$

where h is a function of prices alone. In other words, the outputs at any given price vector are simply a scaled up version of outputs for the case where $g(y_t) = 1$. This expression is

[3]These assumptions on technology provide a simple justification for the existence of theoretical output aggregate, as I discuss below.

a direct analogy to the expression for consumption goods (16). In the case of the firm's outputs, the scaling property follows from the linear homogeneity of $g(y_t)$. Given this set-up, a natural concept for the theoretical output index Q_S^* is the ratio,

$$Q_S^* = \frac{g(y_1)}{g(y_0)}. \tag{31}$$

Moreover, the scaling property implies that the revenue function can be written,

$$R(p_t, f(x_t)) = r(p_t)g(y_t), \tag{32}$$

where $r(p_t)$ is the revenue function for $g(y_t) = 1$.

Applying Hotelling's lemma to this expression for the revenue function implies that,

$$y_t = f(x_t)\nabla r(p_t). \tag{33}$$

Furthermore, if we assume that the households minimize expenditure as descibed by equation (15), then consumption may be written,

$$q_t = C_t \nabla e(p_t), \tag{34}$$

from Roy's identity. For simplicity, we assume that all output is consumed. Equilibrium in the goods market then requires,

$$f(x_t)\nabla r(p_t) = C_t \nabla e(p_t). \tag{35}$$

Since the level of p_t is indeterminate without some further assumption, let us normalize the level of prices to, $p_t(1) = 1$. This system of $N + 1$ equations determines p_t and C_t, given a pre-determined level of $f(x_t)$. In order to simplify notation, we have so far suppressed the time subscripts on the unit revenue and expenditure functions. At this point, in order to explicitly account for shifts in preferences and technology in this economy, let us rewrite the unit revenue and expenditure functions as, $r_t(p_t, \epsilon_t) = A_t \tilde{r}(p_t, \epsilon_t)$ and $e_t(p_t, \sigma_t) = B_t \tilde{e}(p_t, \sigma_t)$, where variation in A_t or B_t represent aggregate supply and demand shocks respectively, while ϵ_t and σ_t are sector-specific demand and supply shocks that enter non-multiplicatively into the functions $\tilde{r}_t$ and $\tilde{e}_t$. We can then rewrite the equilibrium condition as,

$$g(y_t)A_t \nabla \tilde{r}(p_t, \epsilon_t) = C(y_t)B_t \nabla \tilde{e}(p_t, \sigma_t). \tag{36}$$

This vector equality implies that for any two goods j and k,

$$\frac{d\tilde{r}(p_t, \epsilon_t)/dp_t(j)}{d\tilde{r}(p_t, \epsilon_t)/dp_t(k)} = \frac{d\tilde{e}(p_t, \sigma_t)/dp_t(j)}{d\tilde{e}(p_t, \sigma_t)/dp_t(k)}. \tag{37}$$

This system of equations and the normalization $p_t(1) = 1$ can be solved for the vector of prices p_t. Moreover, the expressions (16) and (30) show that relative quantities depend only on prices. The punchline of this analysis is that if the revenue and expenditure functions R_t and E_t are stable over time, or if they vary only according to the multiplacative constants A_t and B_t then *relative prices and quantities are fixed in this economy.* Relative price and quantity changes *only* occur when $\tilde{r}_t$ and $\tilde{e}_t$ shift due to fluctuations in the non-multiplicative shocks ϵ_t and σ_t, or when the linear homogeneity assumptions on either the demand or supply side of the economy fail. Thus, the discussion that follows of the effects of demand and supply shocks on the economy refers exclusively to the effects of the sector specific shocks ϵ_t and σ_t.

The standard index number theory framework assumes that preferences are stable i.e. $e_t = e$ for all t. However, relative prices and quantities are assumed to vary over time– indeed, it is this variation that makes the index number problem interesting. The implicit assumption in the literature is, thus, that variation in relative prices is caused by sector-specific supply shocks– shifts in $\tilde{r}_t$– or non-homotheticity on the supply side of the economy.

3.3 Aggregation in the Presence of Demand and Supply Shocks

Let us now consider how the standard index number theory results change if relative price and quantity fluctuations are assumed to arise from sector-specific demand rather than supply shocks. The argument is very similar to that for the supply shock case except that we are now dealing with a stable *convex* revenue function rather than a stable *concave* expenditure function. (The time subscript on the revenue function is suppressed in this case since the revenue function is assumed to be stable: the relative price fluctuations are assumed to arise from variation in σ_t rather than ϵ_t.) By the convexity of the revenue function,

$$R(p_0, f(x_1)) \geq R(p_1, f(x_1)) + \nabla_p R(p_1, f(x_1))(p_0 - p_1) \tag{38}$$

$$= R(p_1, f(x_1)) + y_1(p_0 - p_1) \tag{39}$$

$$= p_0 y_1, \tag{40}$$

where the second equality follows from the envelope theorem. Dividing expression (40) by $R(p_0, f(x_0))$ and using the separability assumption (32) we have the inequality,

$$\frac{p_0 q_1}{p_0 q_0} \leq \frac{R(p_0, f(x_1))}{R(p_0, f(x_0))} \tag{41}$$

$$= \frac{g(y_1)r(p_0)}{g(y_0)r(p_0)} \quad (42)$$

$$= \frac{g(y_1)}{g(y_0)}. \quad (43)$$

This expression can be rewritten simply as,

$$Q_L \leq Q_S^*, \quad (44)$$

where Q_L is the Laspeyres quantity index defined in Section 2. A very similar line of reasoning shows that,

$$Q_P \geq Q_S^*. \quad (45)$$

Combining the two inequalities (44) and (45), we have the expression,

$$Q_P \geq Q_S^* \geq Q_L. \quad (46)$$

Notice that these are exactly the *opposite* set of inequalities from those derived for the supply shock scenario discussed at the beginning of this section.

Let us now discuss the issue presented in the introduction– that is, the effect of sector-specific demand versus supply shocks on empirical quantity indexes when both shocks have identical effects on disaggregated quantities. Let us denote by $Q_D^*(S)$ the value of Q_D^* in the supply shock scenario; and $Q_S^*(D)$ the value of Q_S^* in the demand shock scenario. Notice that the notation distinguishes *both* between theoretical aggregators on the demand versus the supply side of the economy, and between the value of these aggregators in the demand shock versus the supply shock scenarios. In this case, combining expressions (27) and (46) yields,

$$Q_D^*(S) \leq Q_L \leq Q_S^*(D). \quad (47)$$

This expression follows from noting that Q_L is identical in both scenarios.[4]

What remains to show is how these theore *theoretical* aggregators relate to empirical quantity indexes used in practice. Let us begin with the standard case where relative price fluctuations are caused by sector-specific supply shocks. In this case, a well-known result in the index number theory literature shows that the Fisher index corresponds exactly to the demand-side aggregator Q_D^* assuming that the demand-side aggregator $C(q)$ has the following "flexible functional form",

$$(q^T Aq)^{0.5}, \quad (48)$$

[4]To see this, note that Q_L depends only on q_0, q_1 and p_0, which are assumed to be the same in both scenarios.

where A is a symmetric NxN matrix of constants.[5]

In the supply shock case, an exactly analagous argument can be made for the supply-side aggregator Q_S^* as I show in the following proposition.

Proposition 1: The Fisher index Q_F defined above corresponds exactly to the theoretical output aggregate (31) given a a stable, linearly homogenous $g_t(y)$ with the flexible functional form described by expression (48) above.

Proof: The proof is in Appendix A.

Thus, expression (47) can be rewritten as,

$$Q_F(S) = Q_D^*(S) \leq Q_S^*(D) = Q_F(D). \tag{49}$$

The inequality (49) demonstrates the violation of the "quantity axiom" discussed in the introduction: the Fisher index responds differently to demand and supply shocks for the same changes in disaggregated quantities. Moreover, the expression shows that the asymmetry between demand and supply shocks follows directly from properties of the theoretical aggregator functions Q_D^* or Q_S^*.

In particular, since Q_D^* reflects growth in $C(q)$ while or Q_S^* reflects growth in $g(y)$, the aggregators have opposite curvature properties: the aggregator on the supply side is concave in current quantities, whereas the aggregator on the demand side is convex in current quantities. The inequality (49) shows that the Fisher index reflects *either* Q_D^* or Q_S^*, depending on the source of the economic fluctuation. Although either Q_S^* *or* Q_D^* individually satisfies the "quantity axiom", real GDP does not satisfy the axiom since it may alternately reflect either Q_D^* or Q_S^* in the presence of sector-specific demand and supply shocks.

The curvature properties of $C(q)$ and $g(y)$ have clear economic intuitions. On the one hand, the concavity of $C(q)$ follows from diminishing marginal utility: an additional apple contributes less to C_t than the previous apple due to diminishing marginal utility. On the other hand, the convexity of $g(y)$ follows from the increasing martinal rate of substitution between products: an additional apple requires more resources to produce than the previous apple, and thus contributes more to the output aggregate $g(y)$.

It is useful to note that even if we do not assume that Q_S^* and Q_S^* take the flexible

[5] See Diewert (1976) for a proof of this result.

functional form (48) the inequalities (27), (46), and the definition of the Fisher index (3) imply that,

$$Q_P(S) \leq Q_F(S) \leq Q_L \leq Q_F(D) \leq Q_P(D). \tag{50}$$

Expression (50) demonstrates that the Fisher index responds asymmetrically to sector-specific demand and supply shocks even if the cost and revenue functions do not take the form (48), though in this case the empirical quantity indexes only approximate the theoretical aggregator functions Q_D^* and Q_S^*.

4 Empirical Analysis

Table 1 compares the Laspeyres and Paasche measures of annualized aggregate investment growth for the time intervals 1958q1-1972q4, 1972q1-1982q4, and 1982q1-1987q4. The data are obtained from a "real-time" dataset developed by by Dean Crushore and Tom Stark (Crushore and Stark, 1999).[6] I focus on the aggregate investment series because differences among the alternative index number formulae tend to be particularly large for aggregate investment.

Table 1 shows that both the usual "consumer substitution bias" relationship $Q_L > Q_P$, derived in the supply shock case (27) and the alternative relationship $Q_P > Q_L$ derived in the demand shock case (46) are present in the data. Nakamura (2004) shows that if $Q_L > Q_P$, the data are consistent with the existence of a stable, linearly homogenous aggregator $C(q)$ on the demand side of the economy but not a stable, linearly homogenous output aggregator– indicating the presence of supply shocks. On the other hand, if $Q_P > Q_L$ the data are consistent with the a stable linearly homogenous aggregator $g(y)$ on the supply side, but not a stable, linearly homogenous aggregator on the demand side– suggesting the presence of demand shocks. Note that in the case of aggregate investment, demand shocks are shocks to the "preferences" of the firms *buying* investment goods, while supply shocks are shocks to the technology of the firms *producing* investment goods.

Thus, the evidence suggests that over the period considered in the table the Fisher

[6] In particular, the Laspeyres and Paasche indexes reflect different "vintages" of real GDP released by the BEA. I rely on comparisons between the Laspeyres and Paasche indexes rather than comparisons involving the chain-weighted Fisher index in order to avoid incorporating the effects of chaining. The different vintages of data are calculated using different "base periods", and can therefore be viewed as Laspeyres or Paasche indexes for particular time periods. See Nakamura (2004) for more details on the data. The main caveat to this type of data is that the different vintages of data incorporate some definitional changes in real GDP. I discuss these issues in detail in Nakamura (2004).

index cannot be interpreted as reflecting a stable, linearly homogenous aggregator on the demand side of the economy. However, the evidence is consistent with the interpretation of aggregate investment as reflecting Q_D^* during the period 1958q1-1972q4, and Q_S^* during the later periods.

Beyond the sign of the difference between the Laspeyres and Paasche indexes, Table 1 shows that the magnitude of the difference has changed dramatically over time. The difference between the Laspeyres and Paasche indexes falls from 0.21 for the 1958q1-1972q4 period to -0.91 for the 1982q1-1987q4 period. The generally acknowledged story for the large negative difference in the 1980's was the extremely rapid growth in quantities (and rapid decline in prices) of the "computers and associated products" category of investment during this period. Table 1 shows that the relationship between the prices and quantities of investment goods during the 1980's was, indeed, quite unusual during this period relative to historical experience.

Moreover, the large change in the nature of shocks generating long-term investment growth significantly exacerbated existing trends in aggregate investment growth. According to the chain-weighted Fisher measure of aggregate investment, aggregate investment growth declined from 5.12 for the period 1958q1-1972q4 to 4.06 for the period 1972q1-1982q4, to 1.61 for the period 1982q1-1987q4. The fact that the investment growth during the 1980's was associated to such a great extent with supply shocks– and therefore led to movement along the concave demand-side aggregator $C(q)$– contributed significantly to the low aggregate investment growth during the 1982q1-1987q4 period. Indeed, Table 1 suggests that the growth rate would have been at least 0.46 percentage points higher during this period if *exactly* the same changes in quantity had been motivated by demand rather than supply shocks.[7]

Finally, the large differences between the Laspeyres and Paasche indexes during the 1982q1-1987q4 period indicate that the non-linearity of the demand-side aggregator function $C(q)$ played an important role in aggregate investment growth over this period. Assuming that the movements in price and quantity occurred incrementally over the 1982q1-1987q4 period, the initial increases in computer production contributed significantly more to aggregate investment growth than subsequent growth in this sector of the economy.

[7]This follows since in the demand shock case, expression (46) implies that $Q_F \geq Q_L$ and Q_L would be unchanged given the same growth in disaggregated quantities.

5 Conclusion

A great deal of confusion about real GDP statistics persists in the economics literature despite their obvious importance.[8] One reason for the confusion is probably the traditional analogy between real GDP and "physical quantities". As I discuss in this paper, the analogy between real GDP and the concept of quantity is problematic in a number of ways for the recently adopted Fisher chain-weighted measure of real GDP.

This paper shows that the Fisher index cannot be interpreted as reflecting growth in a stable consumption aggregate in the presence of sector-specific demand shocks. Rather, in the presence of both demand and supply shocks, the Fisher index reflects *either* a "demand-side aggregator" or a "supply-side aggregator" depending on the underlying cause of price and quantity fluctuations at a given point in time. Since the demand-side aggregator and the supply-side aggregator have fundamentally different curvature properties, the Fisher index responds differently to demand and supply shocks for identical changes in disaggregated quantities.

This result shows that analyses of aggregate growth that focus only on the evolution of growth in disaggregated quantities are incomplete. The growth in chain-weighted Fisher measures of real GDP statistics is affected by the *source* of economic flucutations– demand or supply shocks– independently from the effects of these shocks on growth in disaggregated quantities.

[8]For example, Paul Krugman (1996) comments in an article that the BEA's 1996 reforms to real GDP reduced growth by correcting for the fact that "while the computer on my desk may be 50 times as powerful as the one I had in 1985, it is nowhere nearly 50 times as useful." However, quality adjustments, such as those undertaken in 1999, tend to increase, not decrease, the growth rate of real GDP. (See Section **??**). The switch to Fisher chain-weighting, on the other hand, reduced growth by recognizing that a computer produced today is much less valuable than one produced twenty years ago, even if the modern computer is of far higher quality.

A Proof of Proposition 1

Consider the revenue maximization problem given by,

$$R(p, f(x)) = \max_{y} \{p.y : g(y) = f(x)\}, \tag{51}$$

where the output aggregator $g(y)$ has the flexible functional form,

$$g(y) = (y^T A y)^{0.5}, \tag{52}$$

where A is a symmetric NxN matrix of constants. The first order conditions for this maximization problem are,

$$p = \lambda \nabla g(y), \tag{53}$$

where λ is the Lagrange multiplier and ∇ denotes the vector of partial derivatives. Moreover, the linear homogeneity of $g(y)$ implies that,

$$p^T y = \lambda \nabla g(y)^T y \tag{54}$$

$$= \lambda g(y), \tag{55}$$

where the second equality follows from Euler's theorem. Dividing the expression (53) by the expression (55) implies that,

$$p/(p^T y) = \nabla g(y)/g(y) \tag{56}$$

$$= Ay/y'Ay, \tag{57}$$

We can now use the relationship between prices and the parameters in A, given by equation (57) to rewrite the Fisher index as follows.

$$Q_F \equiv \left[\frac{p_0' q_1}{p_0' q_0}\right]^{0.5} \left[\frac{p_1' q_1}{p_1' q_0}\right]^{0.5} \tag{58}$$

$$= \left\{\frac{y_1' \left[p_0/p_0' y_0\right]}{y_0' \left[p_1/p_1' y_1\right]}\right\}^{0.5} \tag{59}$$

$$= \left\{\frac{y_1^T A y_1}{y_0^T A y_0}\right\}^{0.5} \tag{60}$$

$$= \frac{g(y_1)}{g(y_0)}, \tag{61}$$

where the third line follows by substituting in expression (57) and the last line follows from the definition (52).

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Table 1

A Comparison of Laspeyres and Paasche Indexes

	Aggregate Investment	
Period	Laspeyres	Paasche
1958q1-1972q4	4.30	4.50
1972q1-1982q4	3.25	2.81
1982q1-1987q4	2.57	1.66