Appendix to:

Crises and Recoveries in an Empirical Model of Consumption Disasters

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A Model Estimation

We employ a Bayesian MCMC algorithm to estimate our model. More specifically, we employ a Metropolized Gibbs sampling algorithm to sample from the joint posterior distribution of the unknown parameters and variables conditional on the data. This algorithm takes the following form in the case of our model.

The full probability model we employ may be denoted by

$$f(Y, X, \Theta) = f(Y, X|\Theta)f(\Theta),$$

where $Y \in \{C_{i,t}\}$ is the set of observable variables for which we have data,

$$X \in \{x_{i,t}, z_{i,t}, I_{W,t}, I_{i,t}, \phi_{i,t}, \theta_{i,t}\}$$

is the set of unobservable variables,

$$\Theta \in \{p_W, p_{CbW}, p_{CbI}, p_{Ce}, \rho_z, \theta, \sigma^2_\theta, \phi, \sigma^2_\phi, \mu_i, \sigma^2_{\epsilon,i,t}, \sigma^2_{\eta,i}, \sigma^2_{\nu,i}\}$$

is the set of parameters. From a Bayesian perspective, there is no real importance to the distinction between X and Θ. The only important distinction is between variables that are observed and those that are not. The function $f(Y, X|\Theta)$ is often referred to as the likelihood function of the model, while $f(\Theta)$ is often referred to as the prior distribution. Both $f(Y, X|\Theta)$ and $f(\Theta)$ are fully specified in sections 3 and 4 of the paper. The likelihood function may be constructed by combining equations (1)-(3), the distributional assumptions for the shocks in these equations and the distributional assumptions made about $I_{i,t}$ and $I_{W,t}$ in section 3. The prior distribution is described in detail in section 4.

The object of interest in our study is the distribution $f(X, \Theta|Y)$, i.e., the joint distribution of the unobservables conditional on the observed values of the observables. For expositional simplicity, let $\Phi = (X, \Theta)$. Using this notation, the object of interest is $f(\Phi|Y)$. The Gibbs sampler algorithm produces a sample from the joint distribution by breaking the vector of unknown variables into subsets and sampling each subvector sequentially conditional on the value of all the other unknown variables (see, e.g., Gelman et al., 2004, and Geweke, 2005). In our case we implement the Gibbs sampler as follows.

1. We derive the conditional distribution of each element of Φ conditional on all the other elements and conditional on the observables. For the ith element of Φ, we can denote this

conditional distribution as $f(\Phi_i|\Phi_{-i}, Y)$, where Φ_i denotes the ith element of Φ and Φ_{-i} denotes all but the ith element of Φ. In most cases, $f(\Phi_i|\Phi_{-i}, Y)$ are common distributions such as normal distributions or gamma distributions for which samples can be drawn in a computationally efficient manner. For example, the distribution of potential consumption for a particular country in a particular year, $x_{i,t}$, conditional on all other variables is normal. This makes using the Gibbs sampler particularly efficient in our application. Only in the case of a (ρ_z, $\sigma^2_{\epsilon,i,t}$, $\sigma^2_{\eta,i}$, $\sigma^2_{\nu,i}$, ϕ, σ^2_{ϕ}, σ^2_{θ}) are the conditional distributions not readily recognizable. In these cases, we use the Metropolis algorithm to sample values of $f(\Phi_i|\Phi_{-i}, Y)$.[1]

2. We propose initial values for all the unknown variables Φ. Let Φ^0 denote these initial values.

3. We cycle through Φ sampling Φ_i^t from the distribution $f(\Phi_i|\Phi_{-i}^{t-1}, Y)$ where

$$\Phi_{-i}^{t-1} = (\Phi_1^t, ..., \Phi_{i-1}^t, \Phi_{i+1}^{t-1}, ..., \Phi_d^{t-1})$$

   and d denotes the number of elements in Φ. At the end of each cycle, we have a new draw Φ^t. We repeat this step N times to get a sample of N draws for Φ.

4. It has been shown that samples drawn in this way converge to the distribution $f(\Phi|Y)$ under very general conditions (see, e.g., Geweke, 2005). We assess convergence and throw away an appropriate burn-in sample.

In practice, we run four such "chains" starting two from one set of initial values and two from another set of initial values. We choose starting values that are far apart in the following way: The first set of starting values has $I_{i,t} = 0$ for all i and all t and sets $x_{i,t} = c_{i,t}$ and $z_{i,t} = 0$ for all i and all t. The second set of starting values is constructed as follows. $I_{i,t} = 1$ for all i and all t. We extract a smooth trend (with breaks in 1946 and 1973) from $c_{i,t}$. Denote this trend by $c_{i,t}^t$ and denote the remaining variation in consumption as $c_{i,t}^c = c_{i,t} - c_{i,t}^t$. We set $z_{i,t} = \min(\max(-0.5, c_{i,t}^c), 0)$ and $x_{i,t} = c_{i,t} - z_{i,t}$. The first set of starting values thus attributes all the variation in the data to $x_{i,t}$, while the second attributes the bulk of the variation in the data around a smooth trend to $z_{i,t}$.

[1] The Metropolis algorithm samples a proposal Φ_i^* from a proposal distribution $J_t(\Phi_i^*|\Phi_i^{t-1})$. This proposal distribution must by symmetric, i.e., $J_t(x_a|x_b) = J_t(x_b|x_a)$. The proposal is accepted with probability $\min(r, 1)$ where $r = f(\Phi_i^*|\Phi_{-i}, Y)/f(\Phi_i^{t-1}|\Phi_{-i}, Y)$. If the proposal is accepted, $\Phi_i^t = \Phi_i^*$. Otherwise $\Phi_i^t = \Phi_i^{t-1}$. Using the Metropolis algorithm to sample from $f(\Phi_i|\Phi_{-i}, Y)$ is much less efficient than the standard algorithms used to sample from known distributions such as the normal distribution in most software packages. Intuitively, this is because it is difficult to come up with an efficient proposal distribution. The proposal distribution we use is a normal distribution centered at Φ_i^{t-1}.

Given a sample from the joint distribution $f(\Phi|Y)$ of the unobserved variables conditional on the observed data, we can calculate any statistic of interest that involves Φ. For example, we can calculate the mean of any element of Φ by calculating the sample analogue of the integral

$$\int \Phi_i f(\Phi_i|\Phi_{-i}^{t-1}, Y) d\Phi_i.$$

B Estimation Results for All Countries

Below we report detailed figures with estimates of the key state variables in our model for each country. The following list is a key to these figures:

1. The top-left figures plot consumption (black), the posterior mean of potential consumption (green) and the probability of disaster (red).

2. The top-right figures plot the posterior mean of the disaster gap (black) and 5% and 95% posterior probability bands (green and blue, respectively).

3. The middle-left figures plot the posterior mean of the size of the short run disaster shock (red) as well as consumption and potential consumption. More specifically, the red line is the posterior mean of $I_{i,t}\phi_{i,t}$, i.e., $E[I_{i,t}\phi_{i,t}|T]$.

4. The middle-right figures plot the posterior mean of the size of the long run disaster shock (red) as well as consumption and potential consumption. More specifically, the red line is the posterior mean of $I_{i,t}\theta_{i,t}$, i.e., $E[I_{i,t}\theta_{i,t}|T]$.

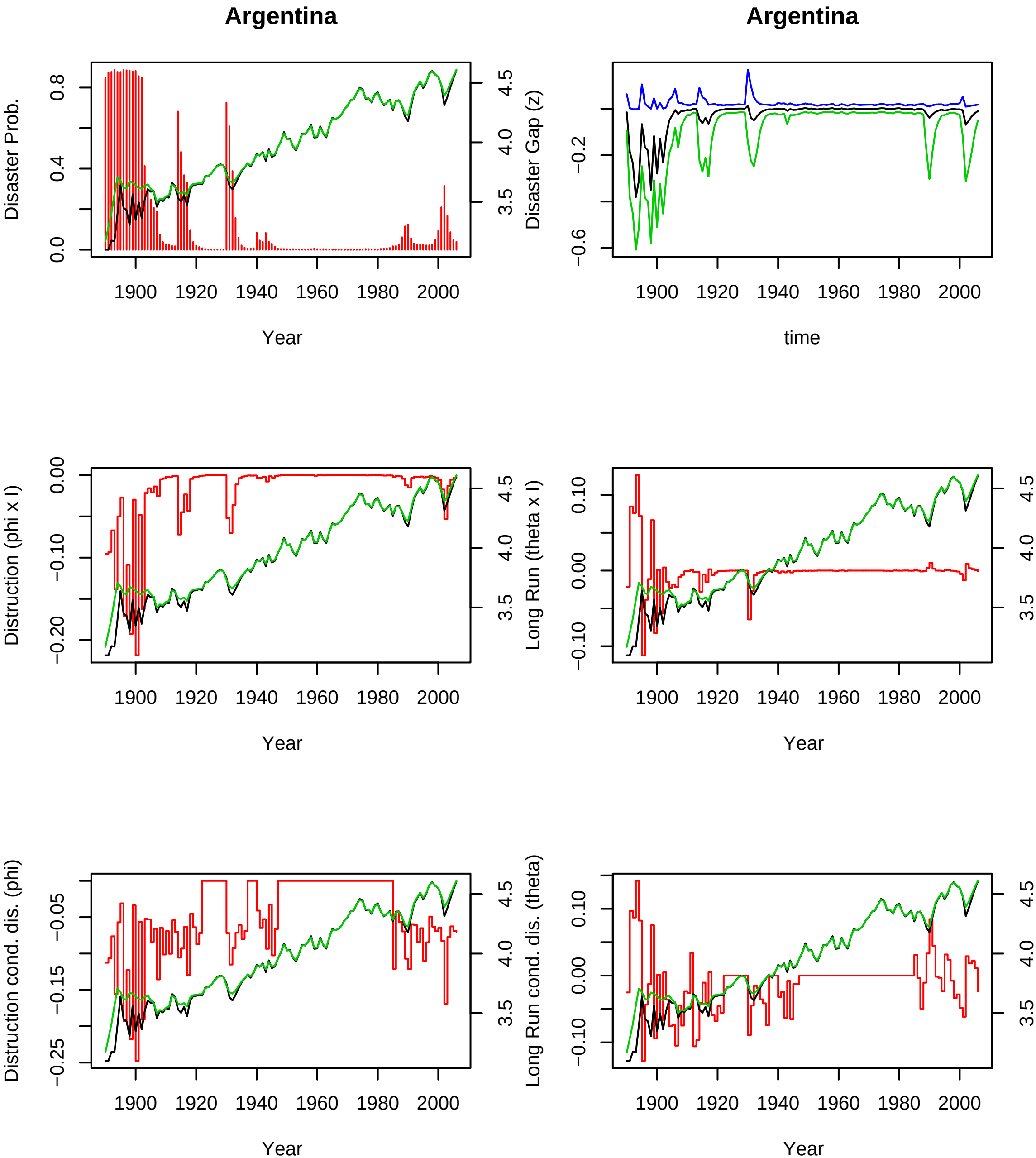
5. The bottom-left figures plot the size of the short run shocks conditional on a disaster, i.e., $E[I_{i,t}\phi_{i,t}|T]/E[I_{i,t}|T]$.

6. The bottom-right figures plot the size of the long run shocks conditional on a disaster, i.e., $E[I_{i,t}\theta_{i,t}|T]/E[I_{i,t}|T]$.

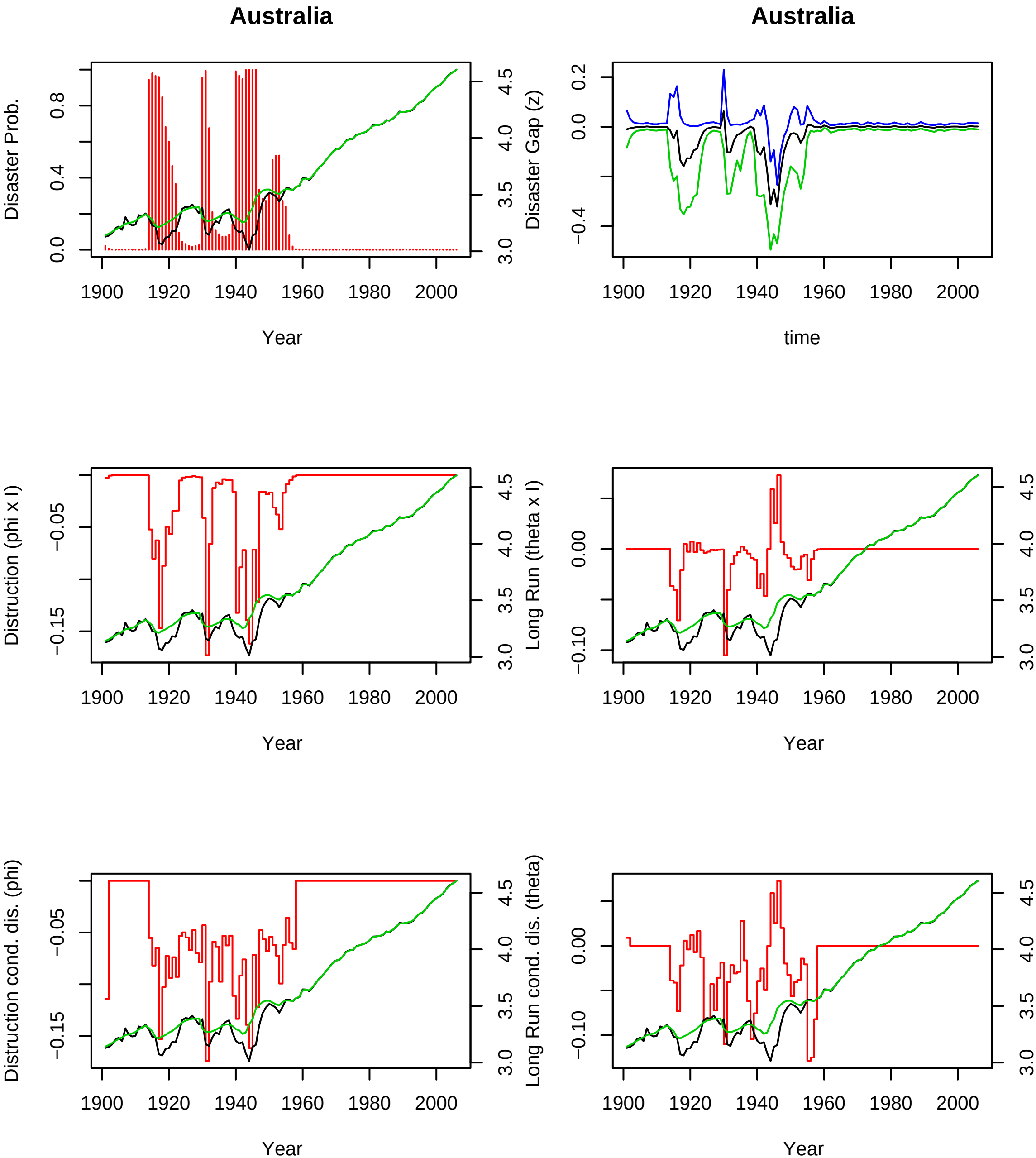
References

Gelman, A., J. B. Carlin, H. S. Stern, and D. B. Rubin (2004): *Bayesian Data Analysis.* John Wiley and Sons, Hoboken, New Jersey.

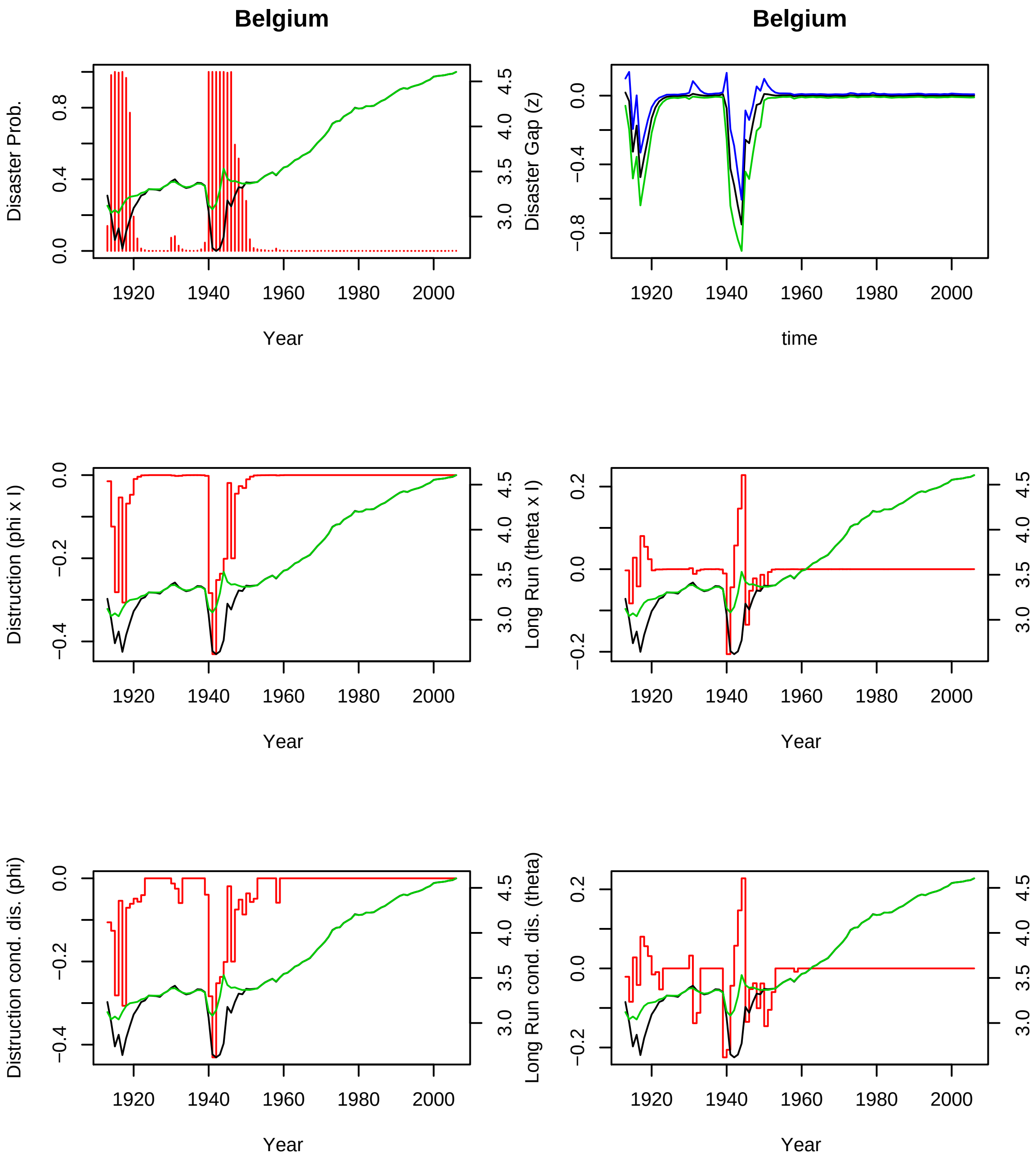
Geweke, J. (2005): *Contemporary Bayesian Econometrics and Statistics.* Chapman & Hall/CRC, Boca Raton, Florida.



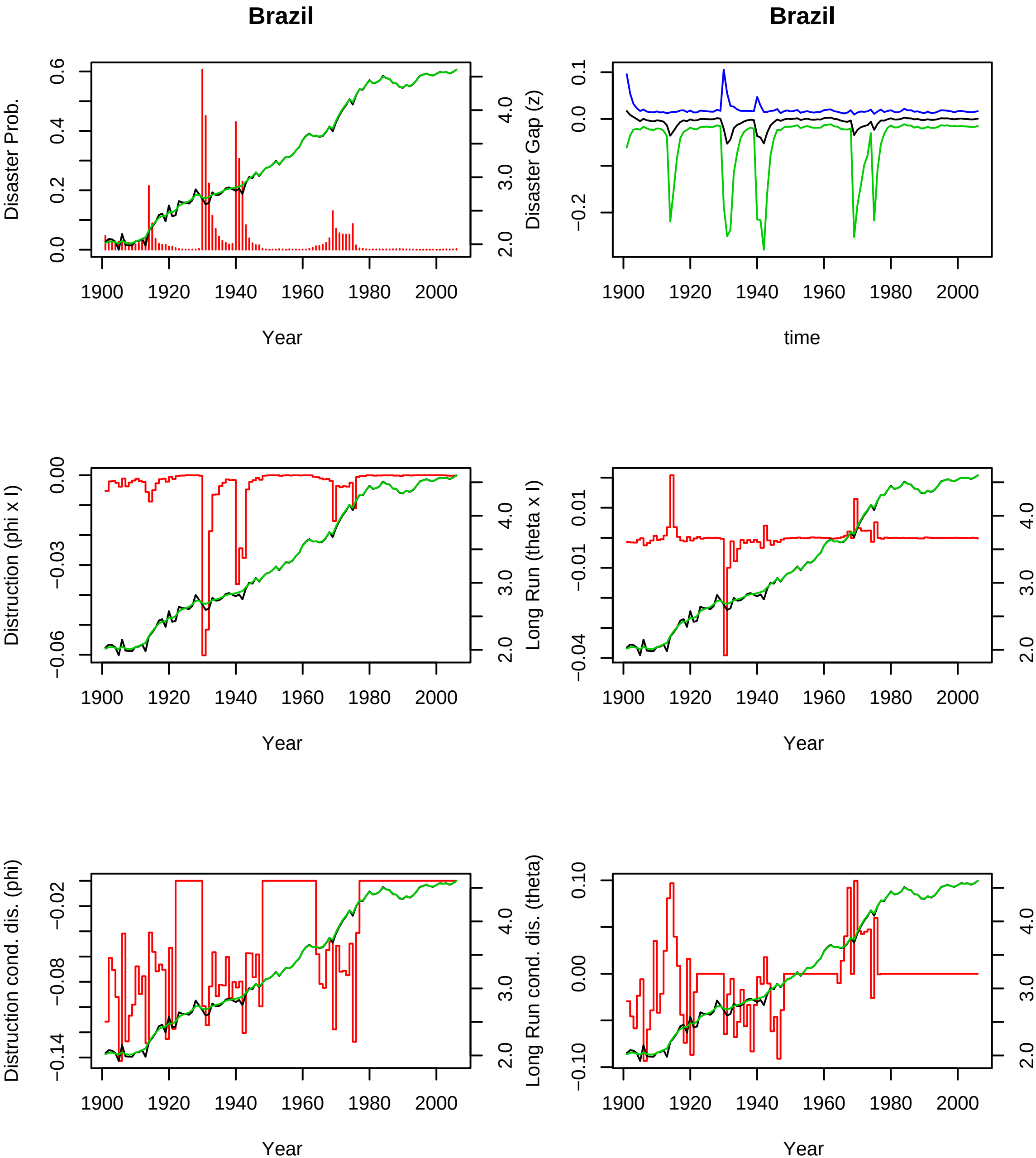
Argentina
Argentina
Disaster Prob.
Disaster Gap (z)
Year
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year



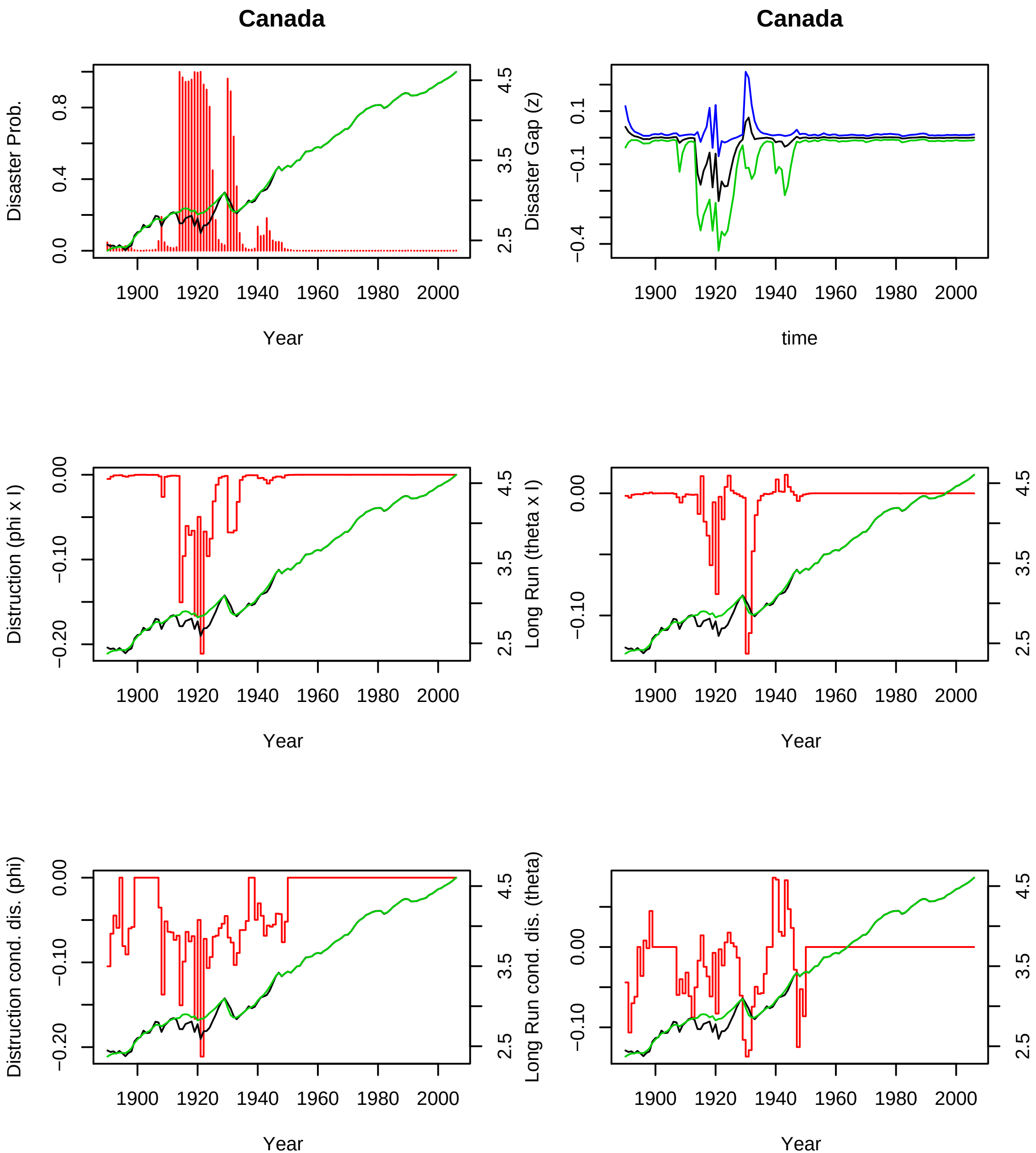
Australia
Disaster Prob.
Disaster Gap (z)
Year
Australia
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year

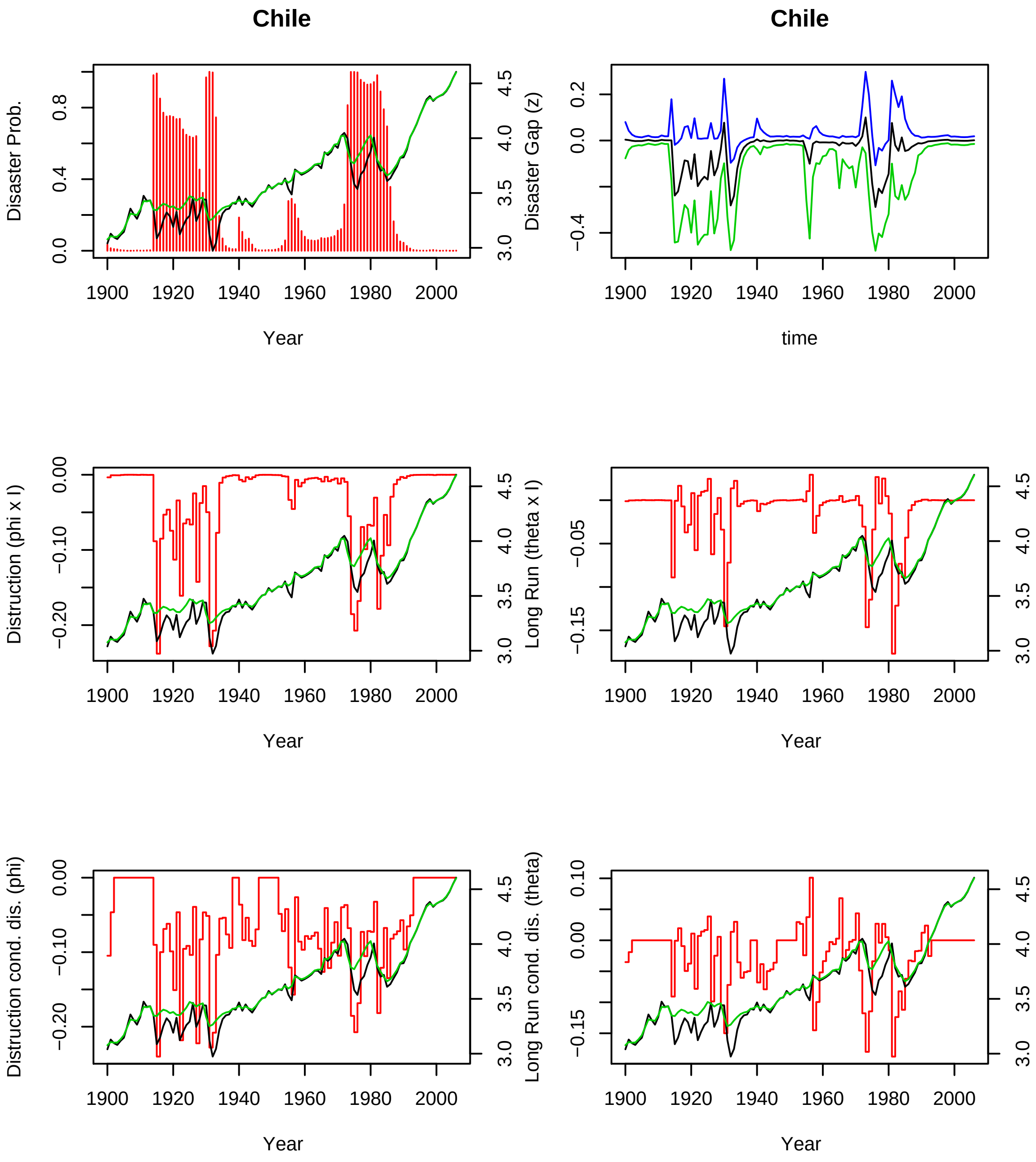


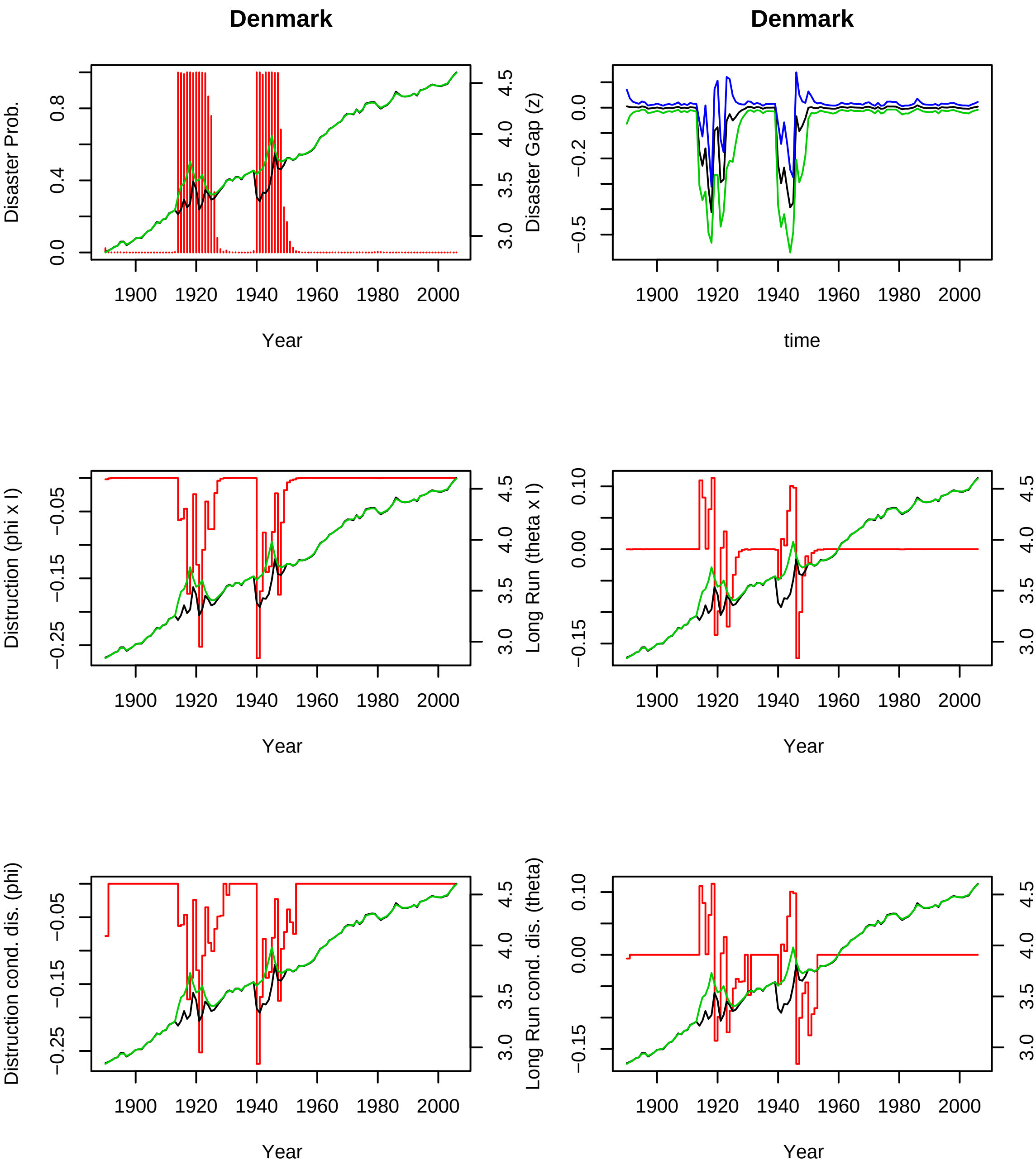
Belgium
Belgium
Disaster Prob.
Disaster Gap (z)
Year
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year
1920
1940
1960
1980
2000
0.0
0.4
0.8
3.0
3.5
4.0
4.5
−0.8
−0.4
−0.2
0.2

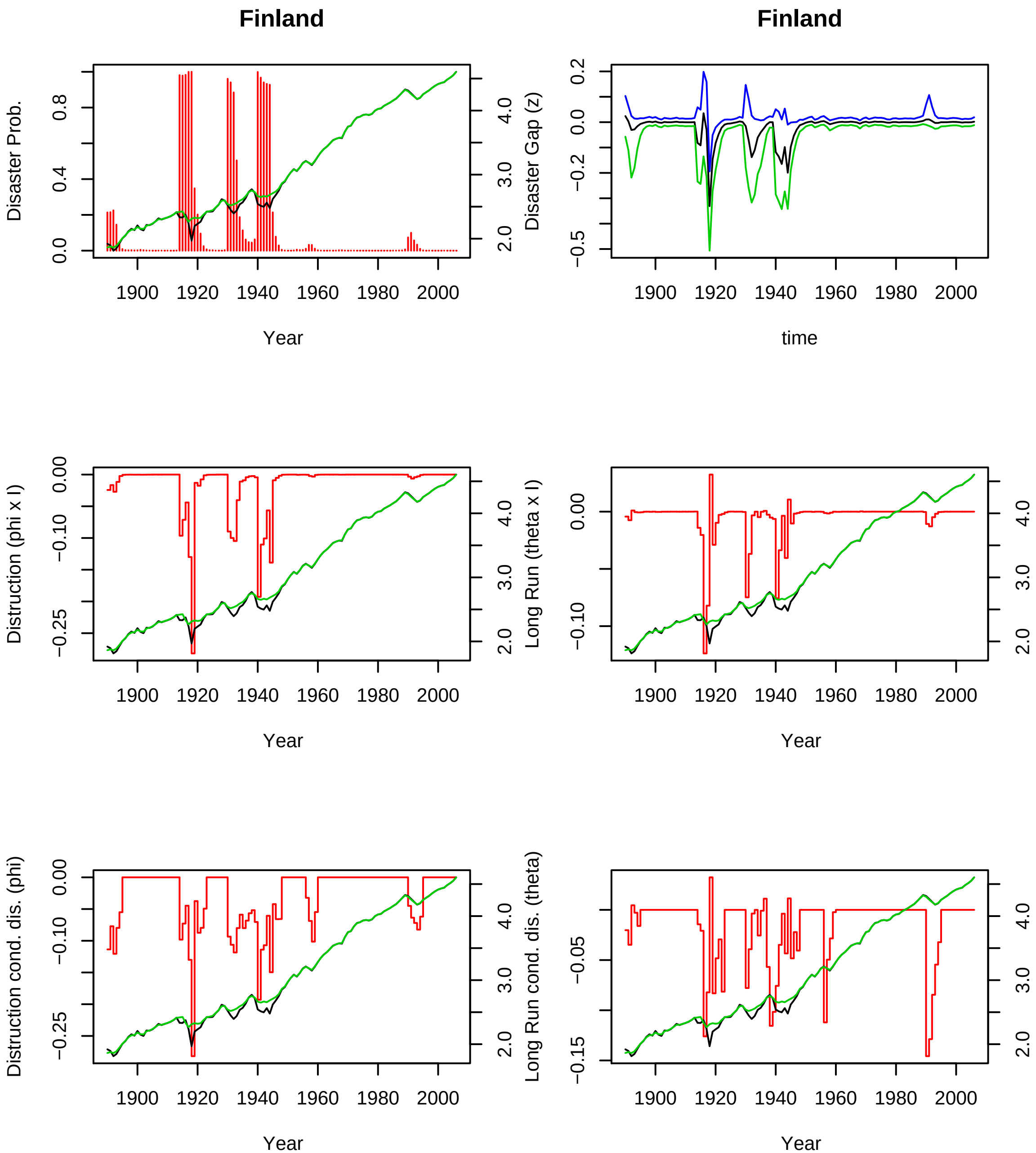


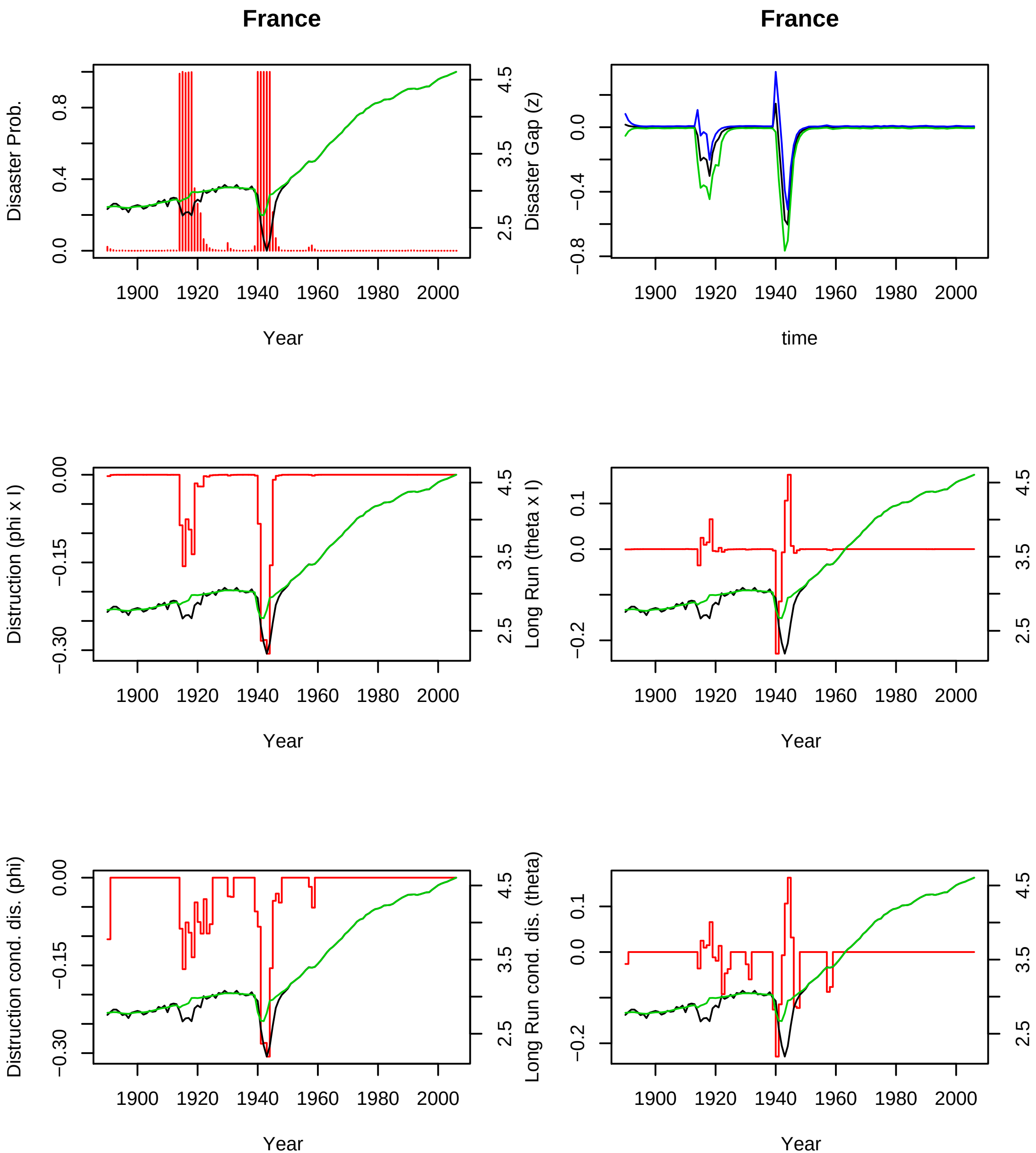
Brazil
Brazil
Disaster Prob.
Disaster Gap (z)
Year
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year
0.0
0.2
0.4
0.6
2.0
3.0
4.0
0.1
0.0
−0.2
0.00
−0.03
−0.06
0.01
−0.01
−0.04
−0.02
−0.08
−0.14
0.10
0.00
−0.10
1900
1920
1940
1960
1980
2000

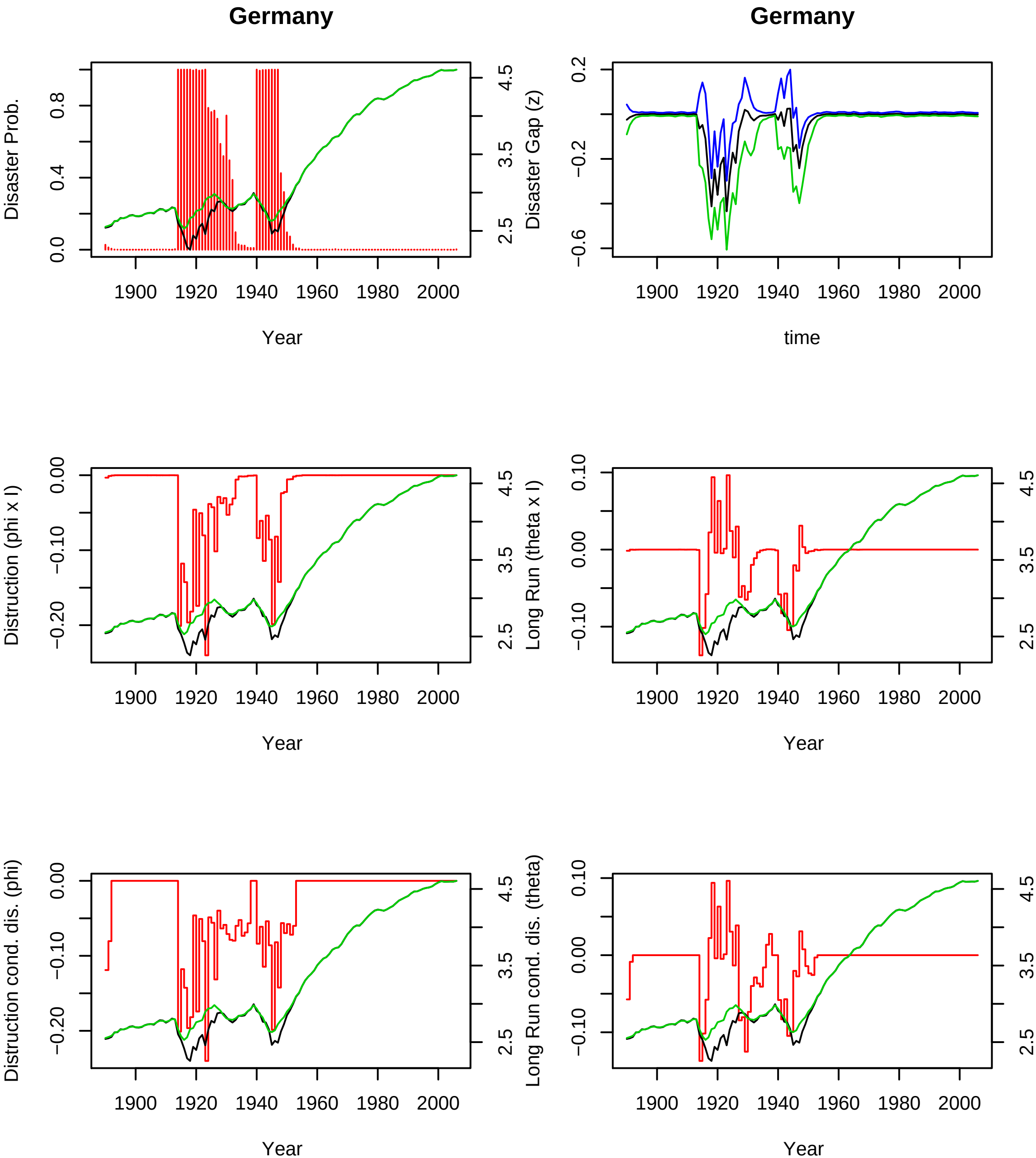

Canada
Disaster Prob.
0.0
0.4
0.8
Disaster Gap (z)
2.5
3.5
4.5
1900
1920
1940
1960
1980
2000
Year
Canada
0.1
−0.1
−0.4
time
Distruction (phi x l)
0.00
−0.10
−0.20
Long Run (theta x l)
0.00
−0.10
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year


Chile
Chile
Disaster Prob.
Disaster Gap (z)
Year
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year
1900
1920
1940
1960
1980
2000
0.0
0.4
0.8
3.0
3.5
4.0
4.5
−0.4
0.2
0.00
−0.10
−0.20
−0.05
−0.15
0.10

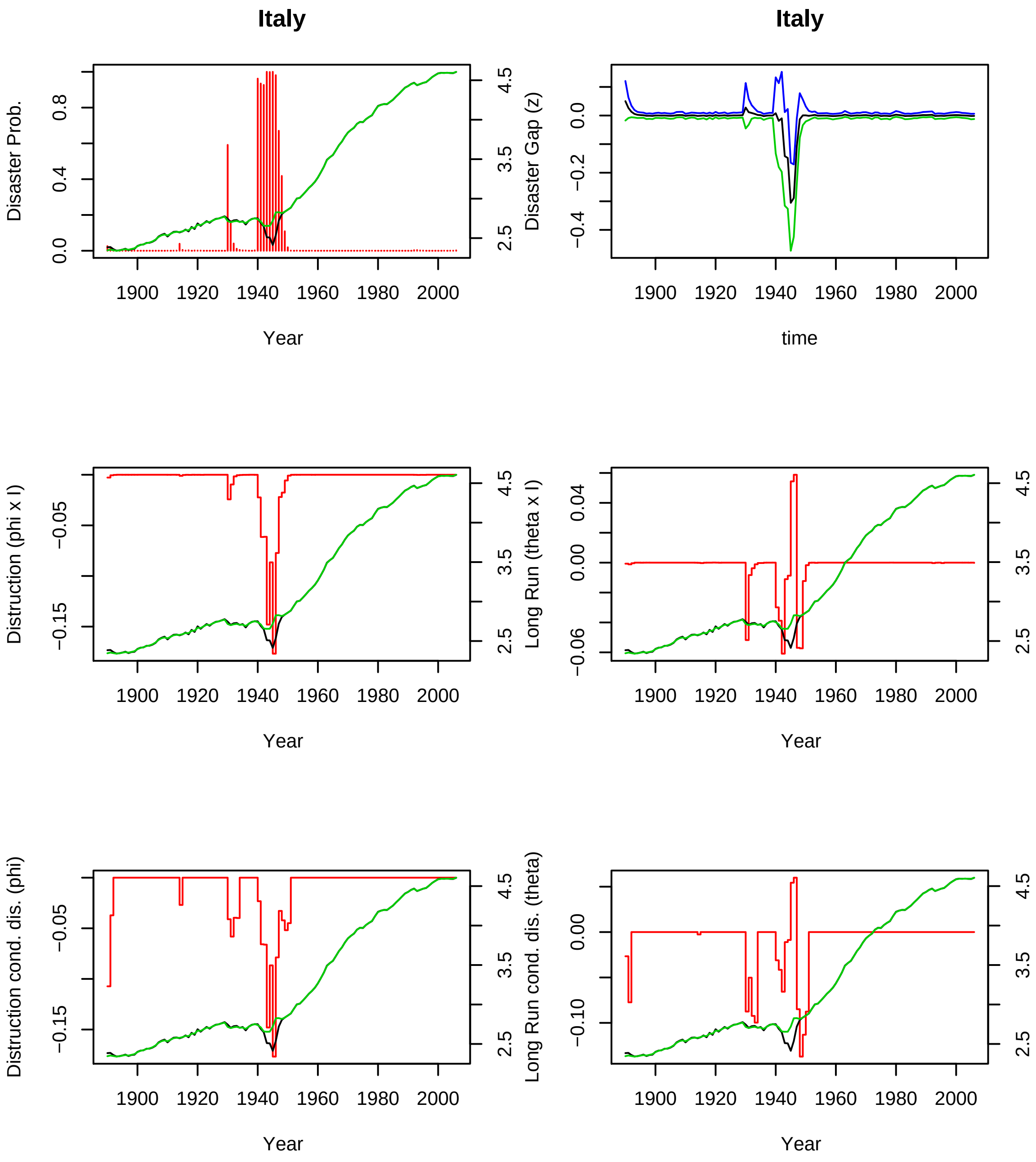

Denmark
Disaster Prob.
Disaster Gap (z)
Year
Denmark
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year

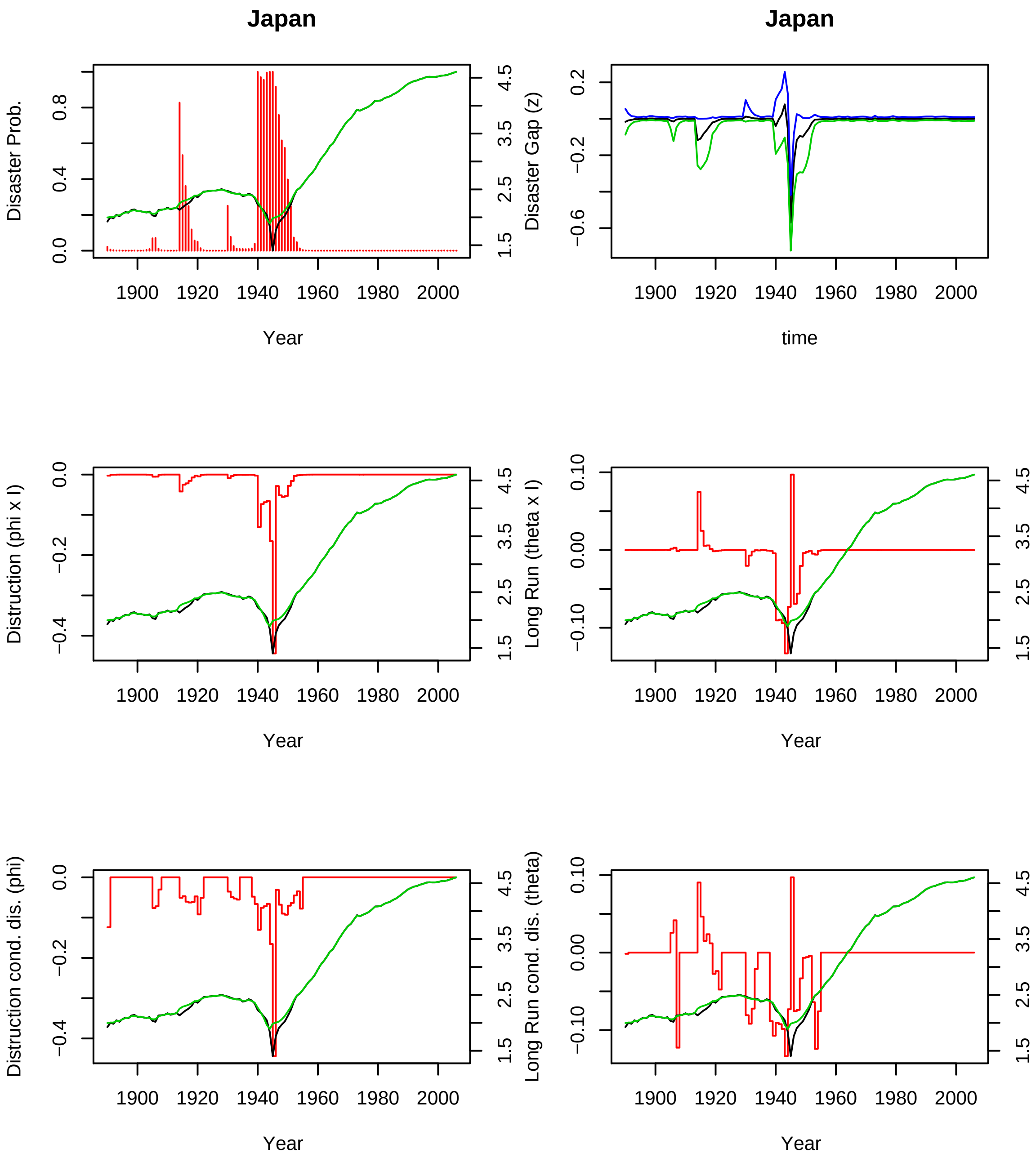

Finland
Disaster Prob.
Disaster Gap (z)
Year
Finland
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year
0.0
0.4
0.8
2.0
3.0
4.0
1900
1920
1940
1960
1980
2000
−0.5
−0.2
0.0
0.2
−0.25
−0.10
0.00
−0.10
0.00
−0.15
−0.05


France
Disaster Prob.
Disaster Gap (z)
Year
France
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year

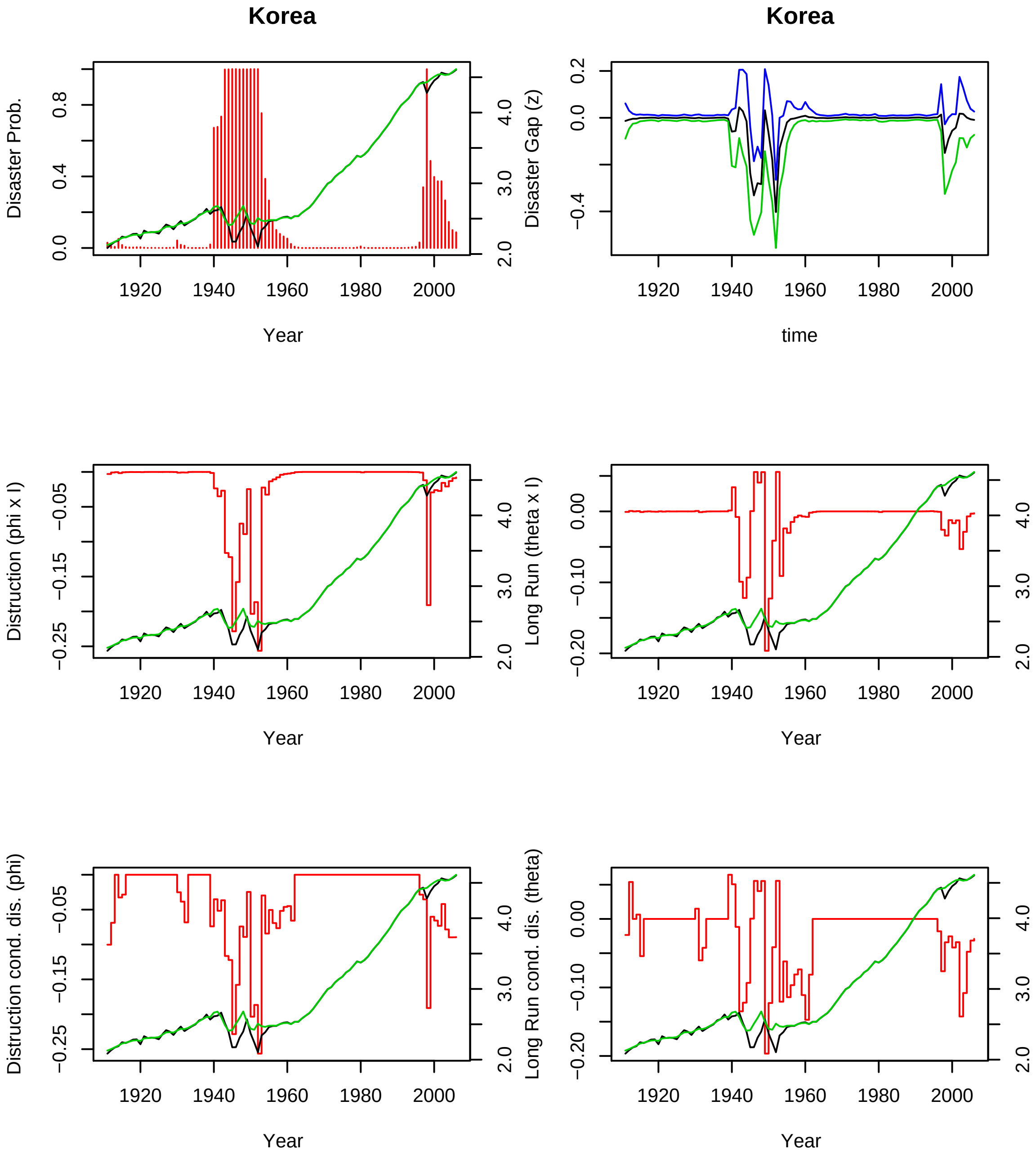


Germany
Disaster Prob.
Disaster Gap (z)
Year
Germany
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year


Italy
Italy
Disaster Prob.
0.0
0.4
0.8
Disaster Gap (z)
2.5
3.5
4.5
1900
1920
1940
1960
1980
2000
Year
0.0
−0.2
−0.4
time
Distruction (phi x l)
−0.05
−0.15
Long Run (theta x l)
0.04
0.00
−0.06
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
0.00
−0.10



Japan
Disaster Prob.
0.0
0.4
0.8
Disaster Gap (z)
1.5
2.5
3.5
4.5
1900
1920
1940
1960
1980
2000
Year
Japan
0.2
−0.2
−0.6
time
Distruction (phi x l)
0.0
−0.2
−0.4
Long Run (theta x l)
0.10
0.00
−0.10
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)


Korea
Korea
Disaster Prob.
0.0
0.4
0.8
Disaster Gap (z)
2.0
3.0
4.0
1920
1940
1960
1980
2000
Year
−0.4
0.0
0.2
time
Distruction (phi x l)
−0.25
−0.15
−0.05
Long Run (theta x l)
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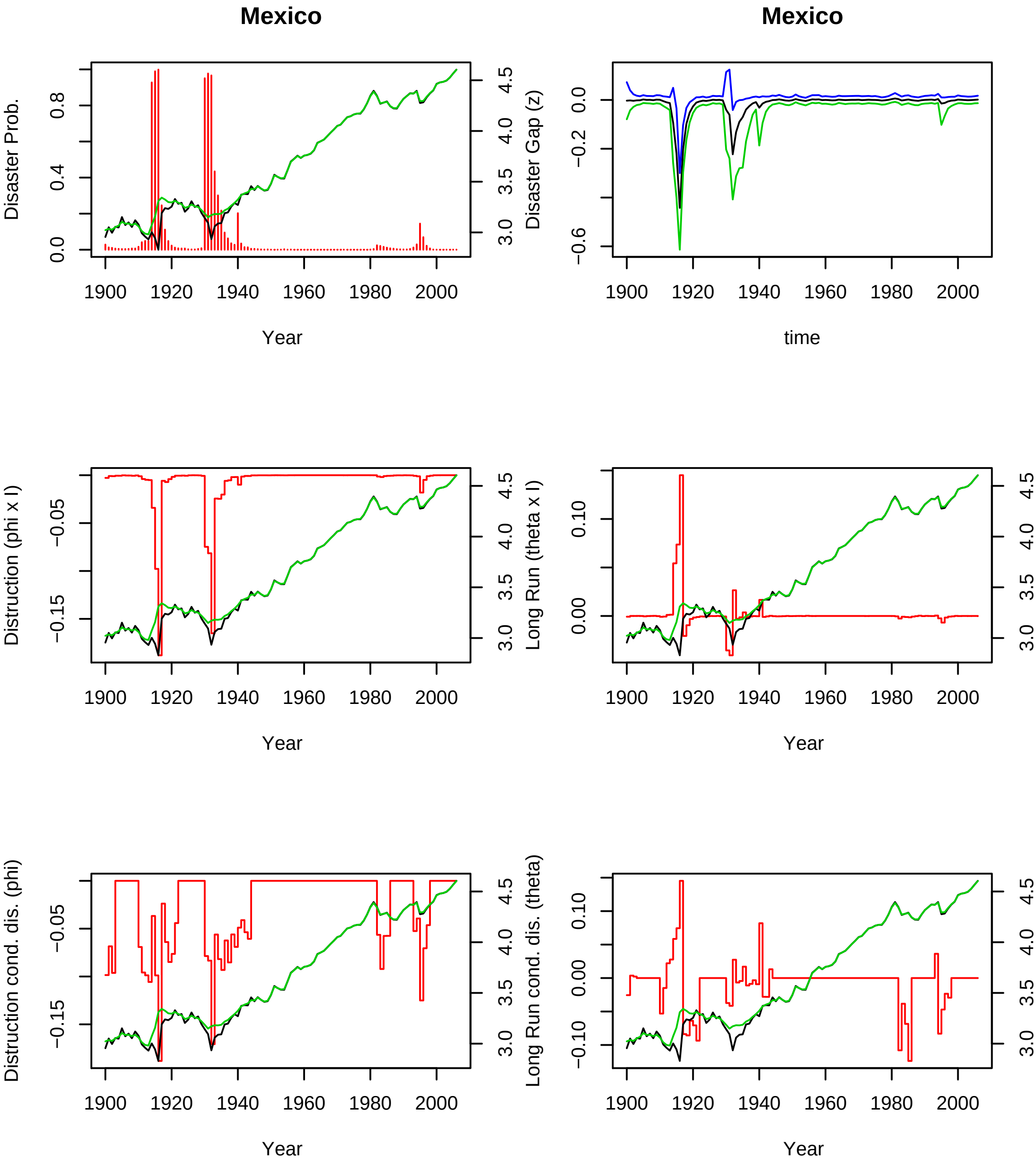
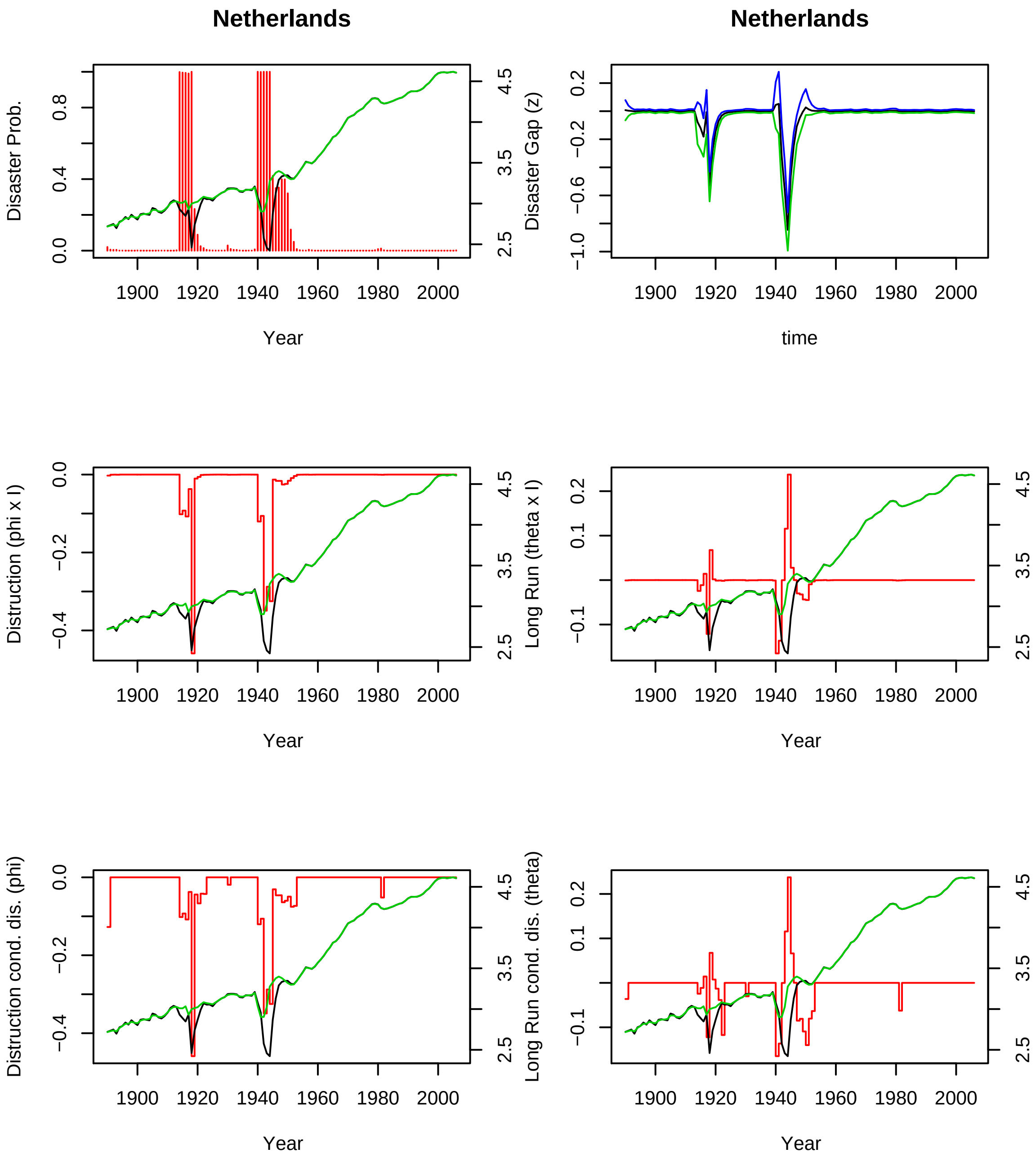
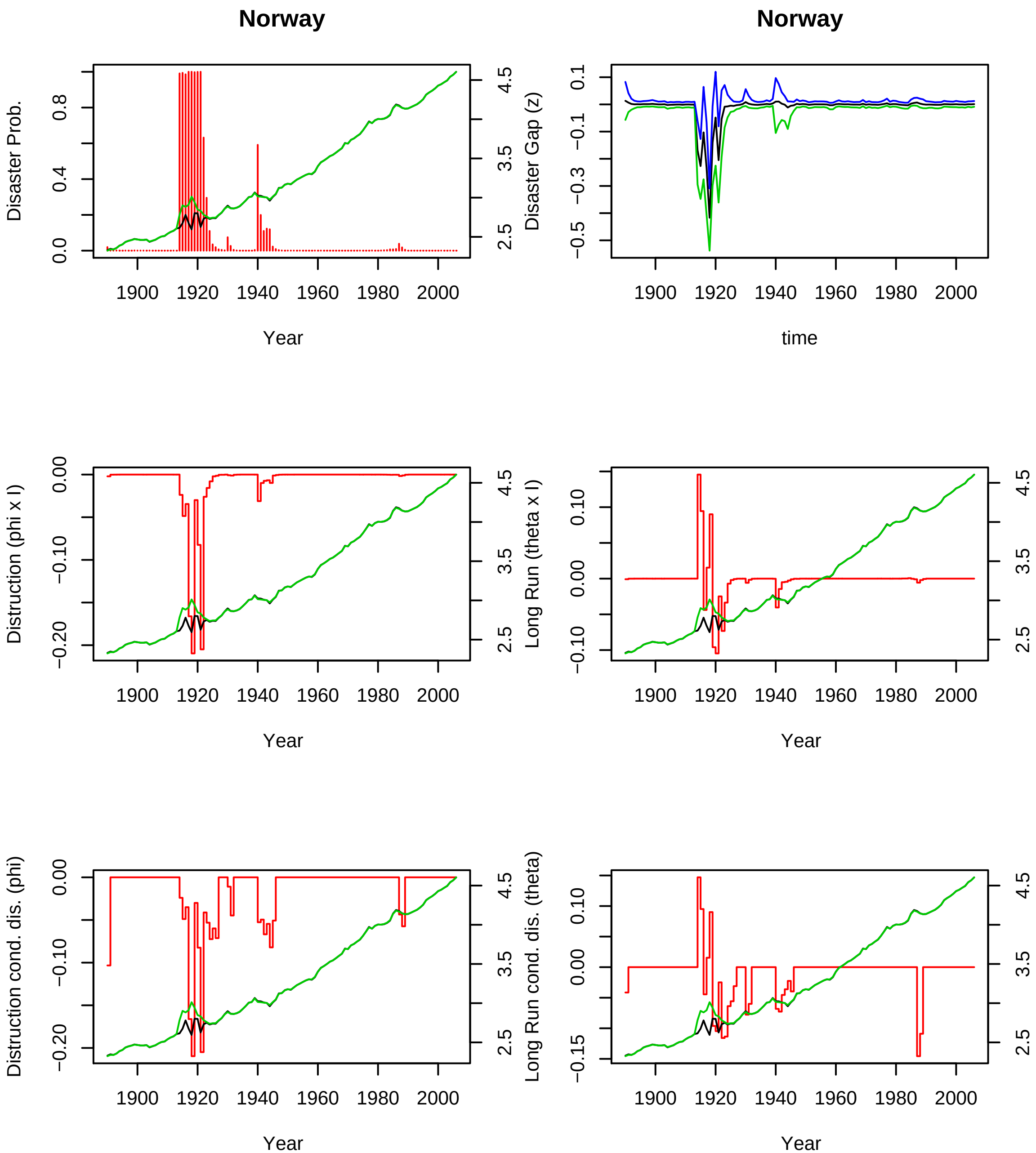
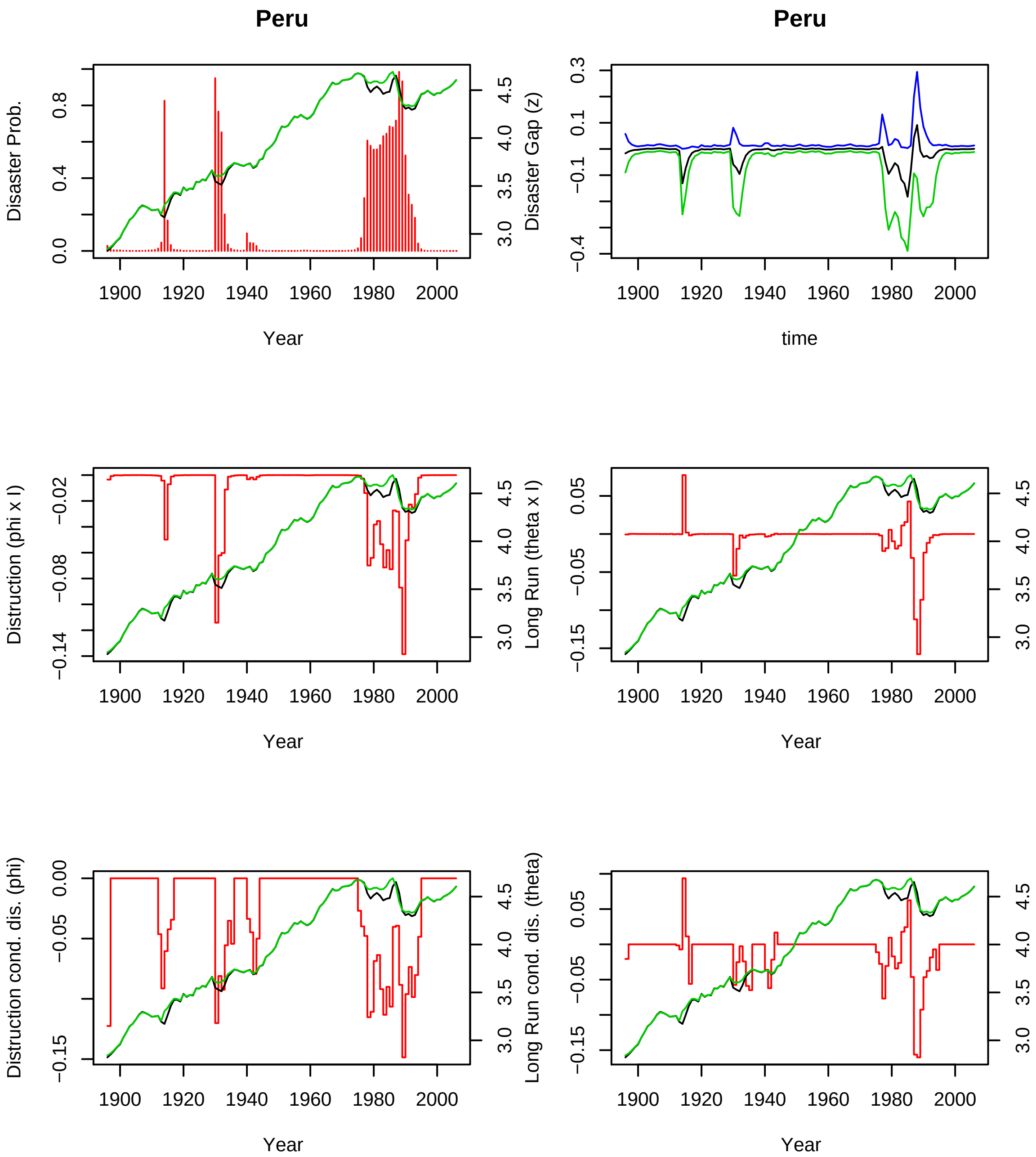
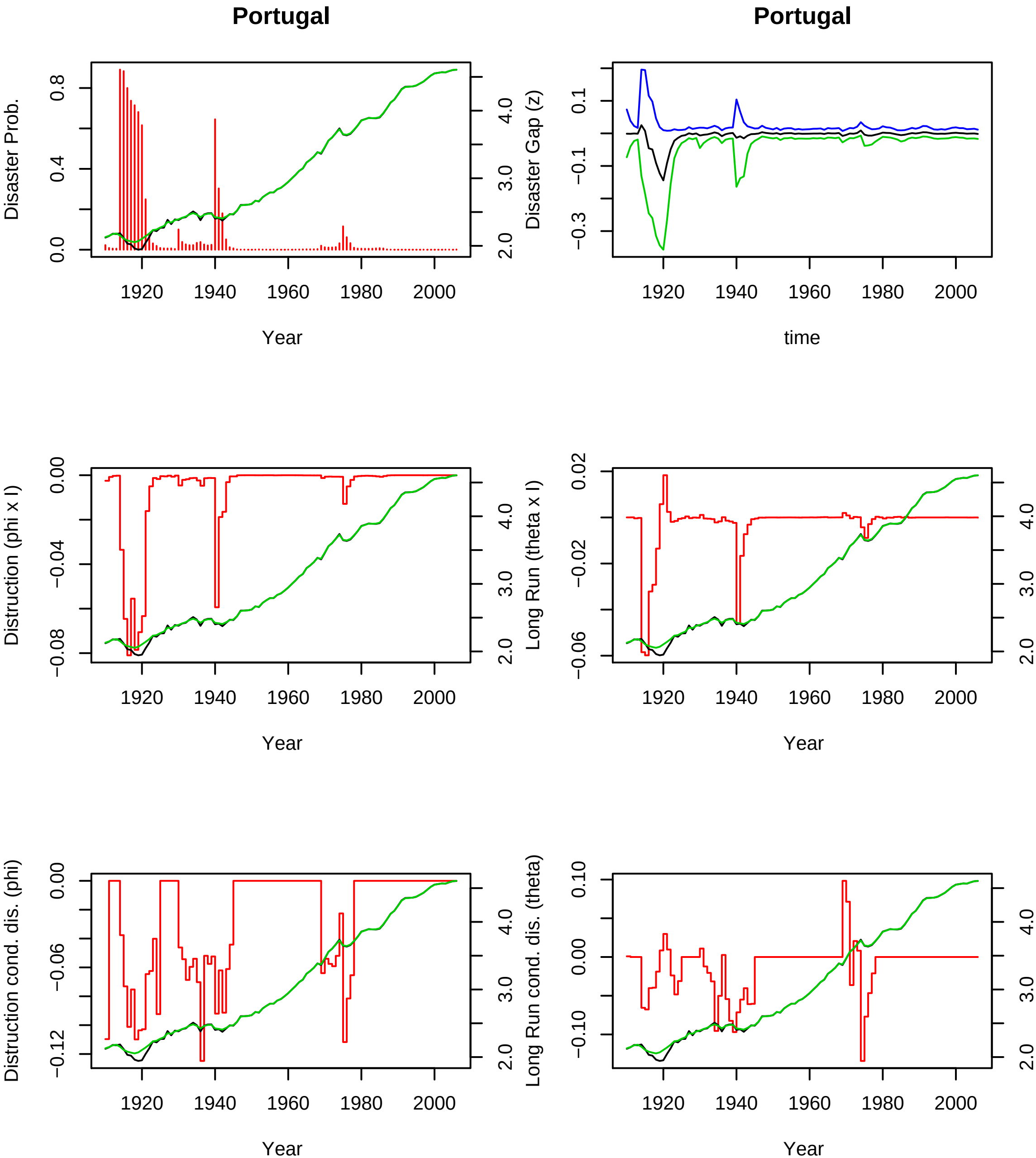
−0.10
0.00
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)


Mexico
Disaster Prob.
Disaster Gap (z)
Year
Mexico
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year


Netherlands
Netherlands
Disaster Prob.
Disaster Gap (z)
Year
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year
0.0
0.4
0.8
2.5
3.5
4.5
0.2
−0.2
−0.6
−1.0
−0.4
0.1
−0.1
1900
1920
1940
1960
1980
2000


Norway
Norway
Disaster Prob.
Disaster Gap (z)
Year
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year


Peru
Disaster Prob.
0.0
0.4
0.8
Disaster Gap (z)
3.0
3.5
4.0
4.5
Year
1900
1920
1940
1960
1980
2000
Peru
−0.4
−0.1
0.1
0.3
time
Distruction (phi x I)
−0.14
−0.08
−0.02
Long Run (theta x I)
−0.15
−0.05
0.05
Distruction cond. dis. (phi)
−0.15
−0.05
0.00
Long Run cond. dis. (theta)


Portugal
Portugal
Disaster Prob.
Disaster Gap (z)
Year
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year
0.0
0.4
0.8
2.0
3.0
4.0
−0.3
−0.1
0.1
0.00
−0.04
−0.08
0.02
−0.02
−0.06
−0.12
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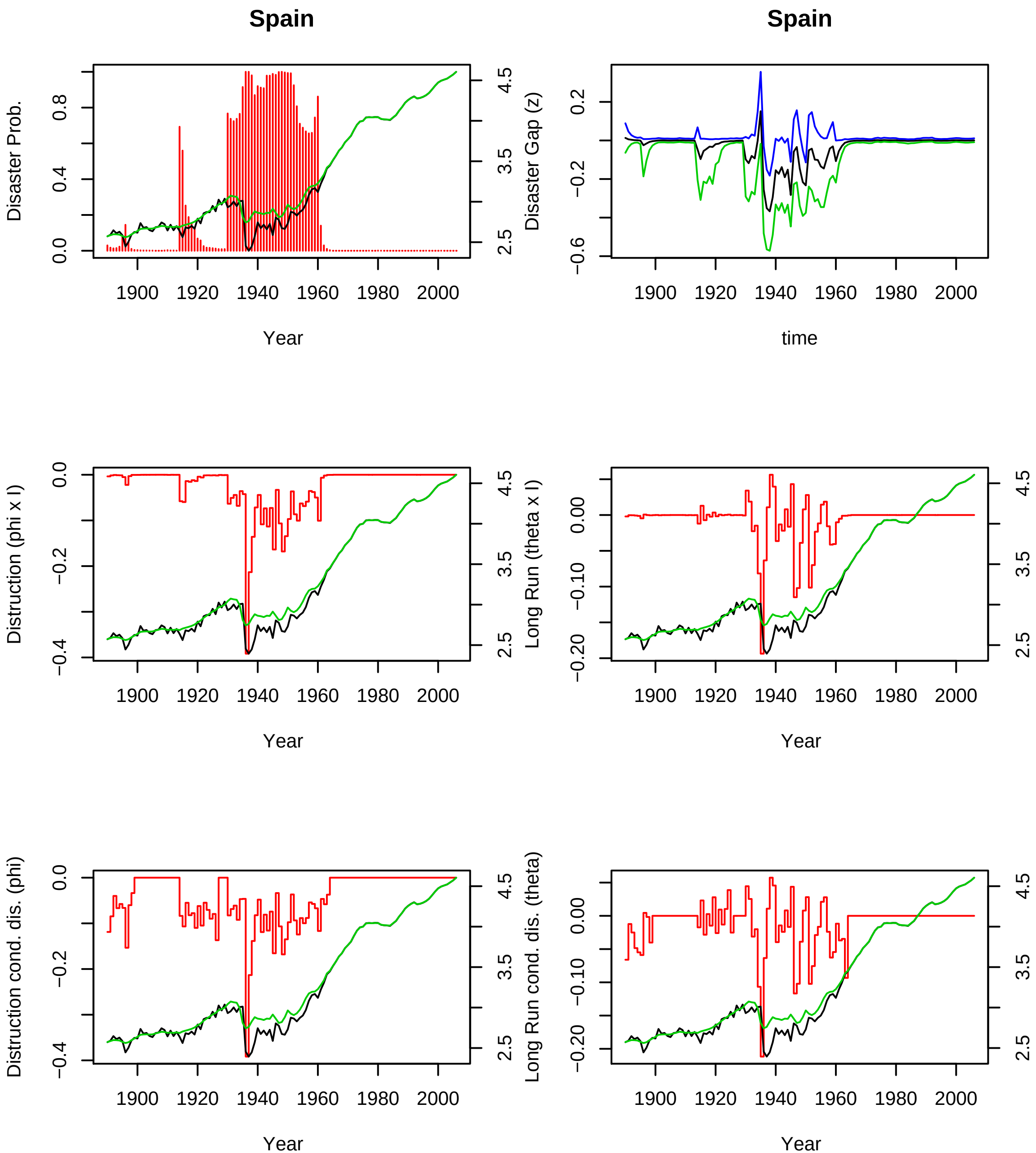
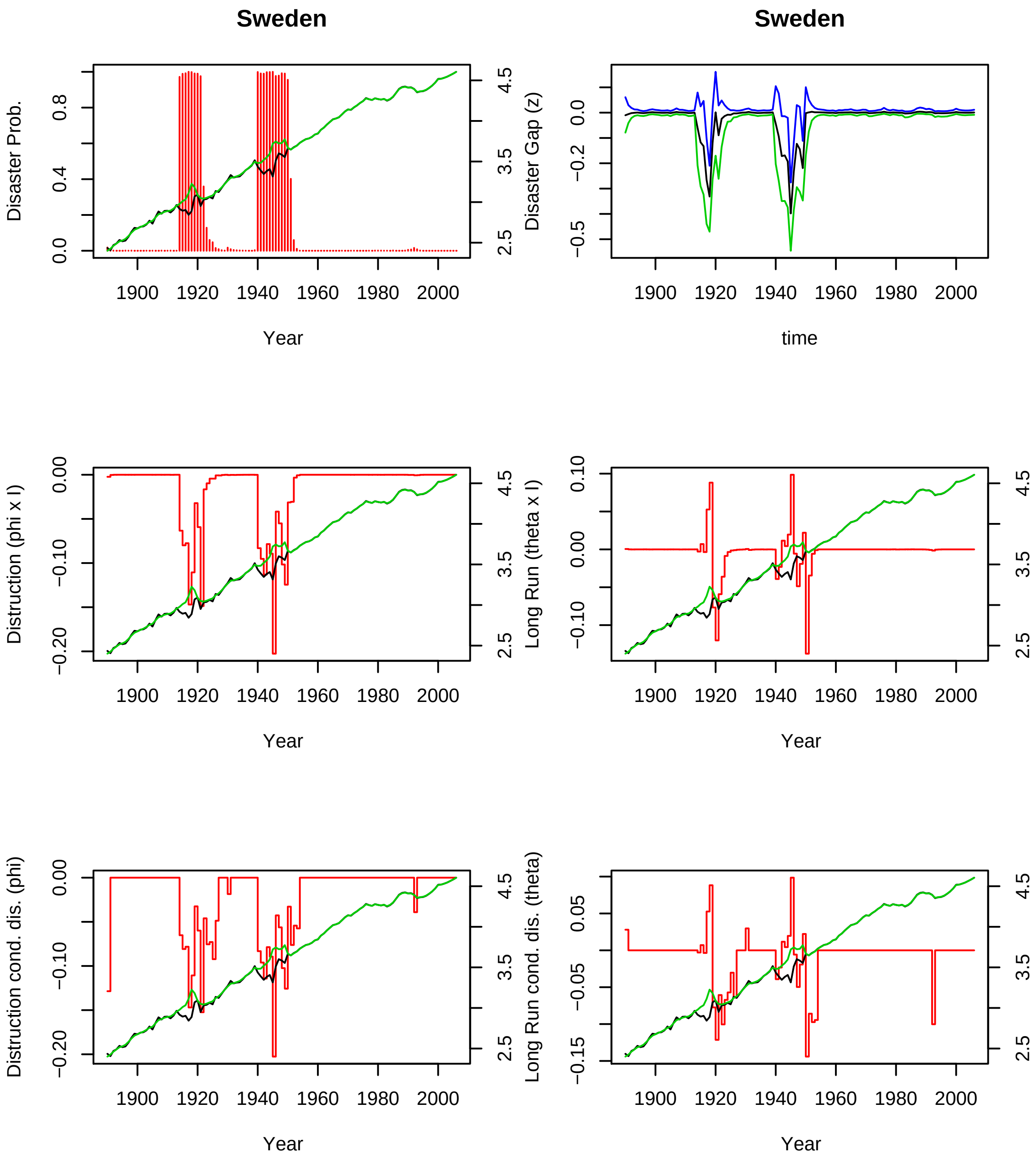
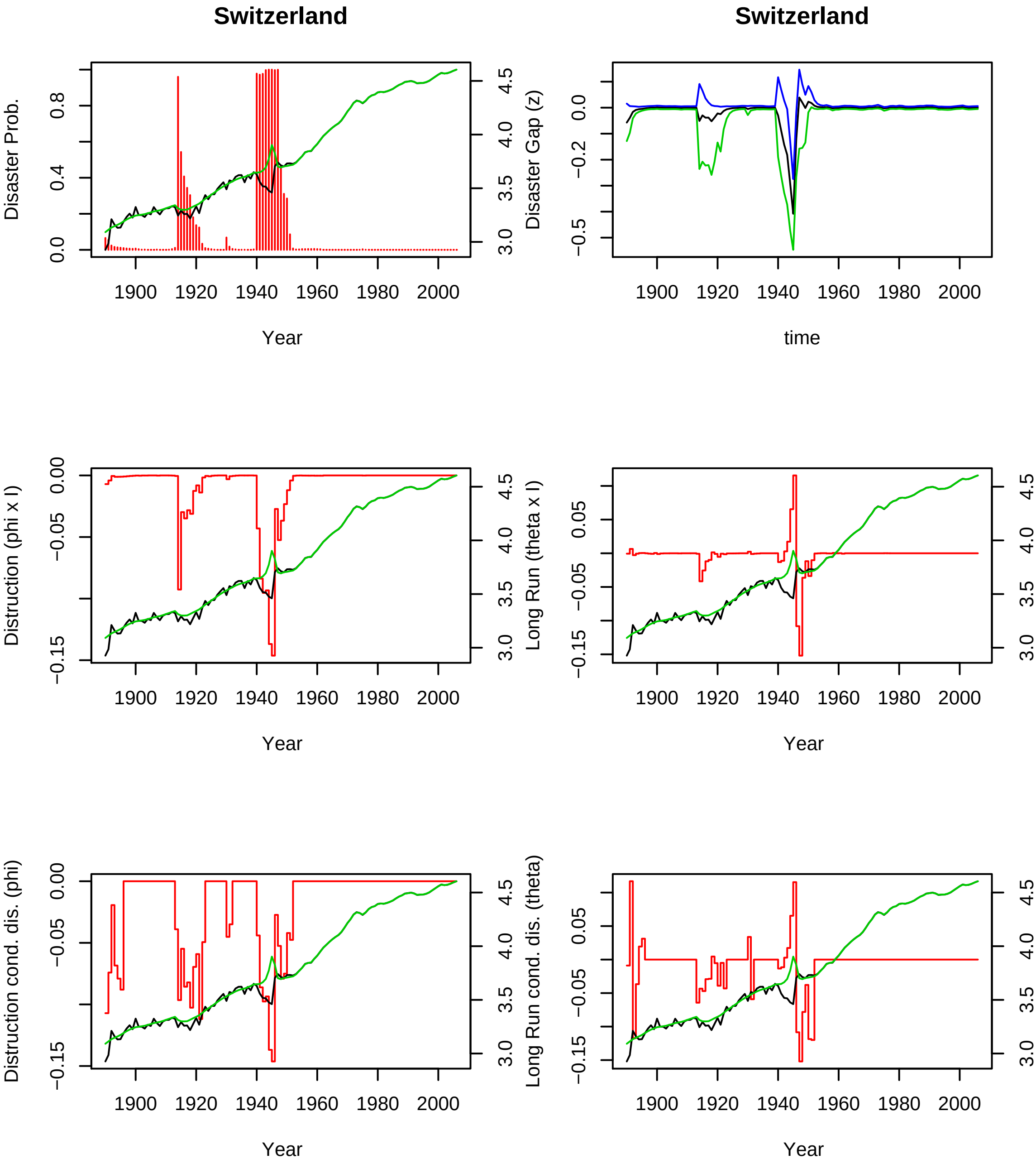
0.10
0.00
−0.10
1920
1940
1960
1980
2000

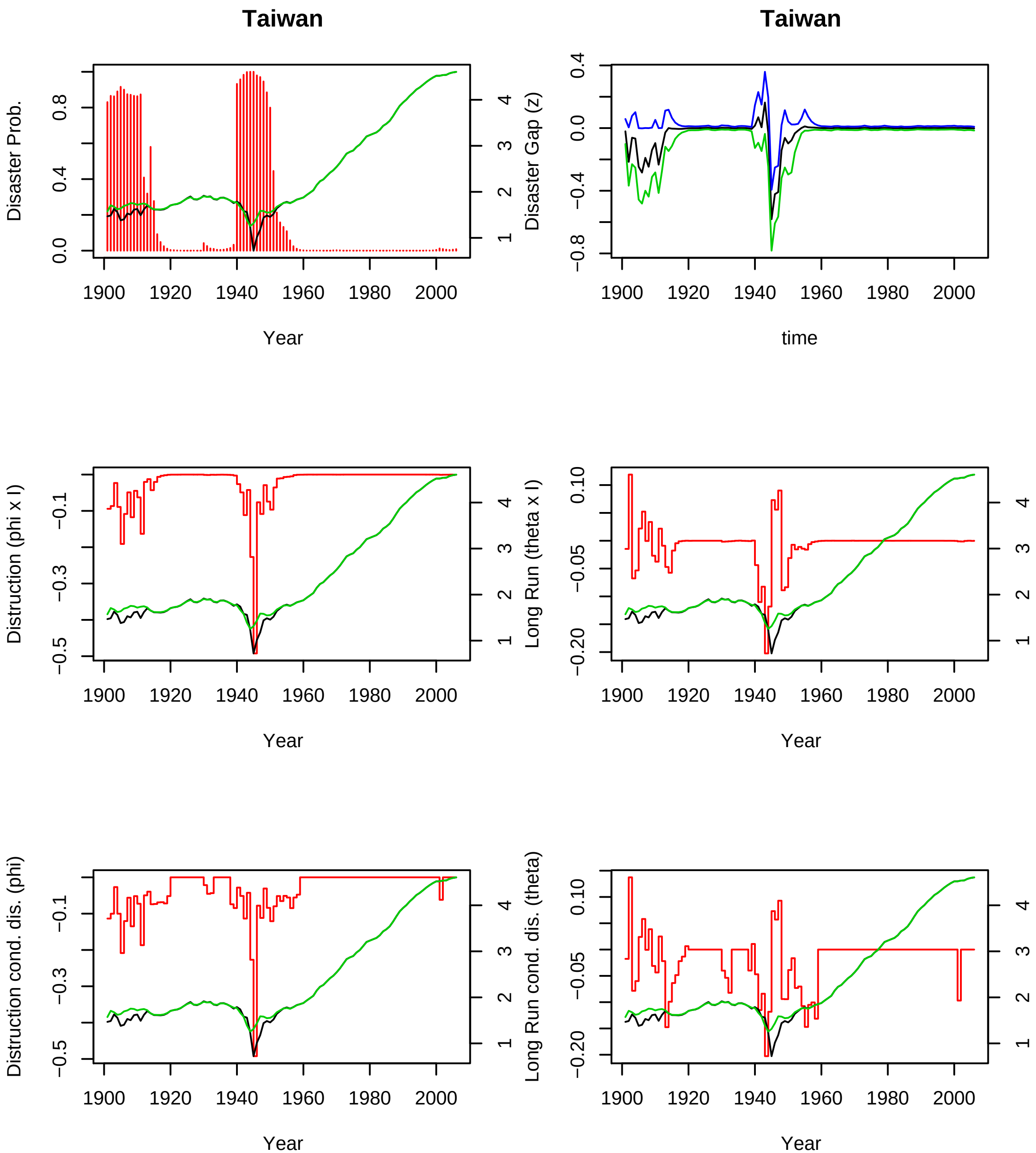

Spain
Disaster Prob.
Disaster Gap (z)
Year
Spain
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year



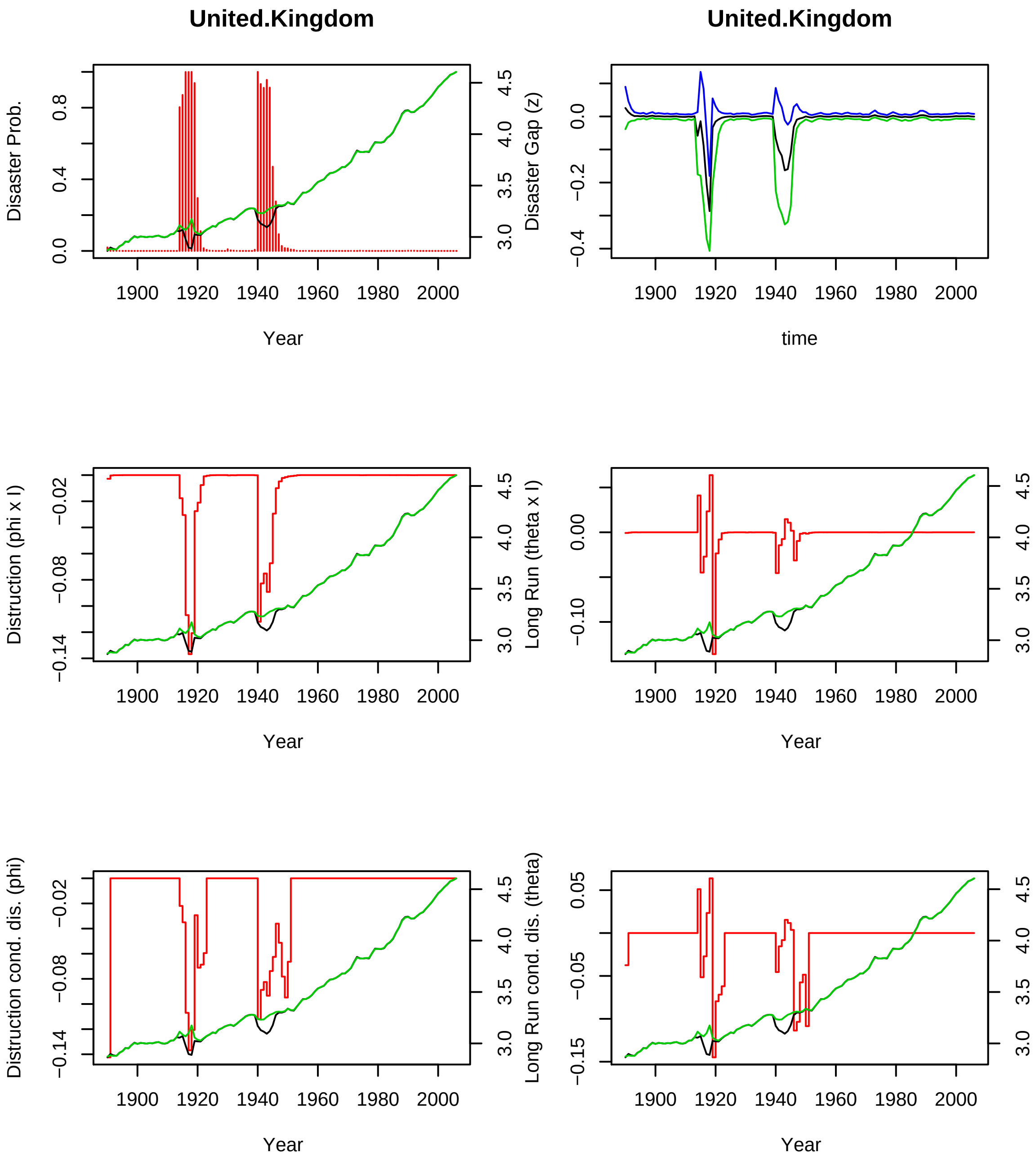
Sweden
Disaster Prob.
0.0
0.4
0.8
Disaster Gap (z)
2.5
3.5
4.5
1900
1920
1940
1960
1980
2000
Year
Sweden
0.0
−0.2
−0.5
time
Distruction (phi x I)
0.00
−0.10
−0.20
Long Run (theta x I)
0.10
0.00
−0.10
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
0.05
−0.05
−0.15
Year

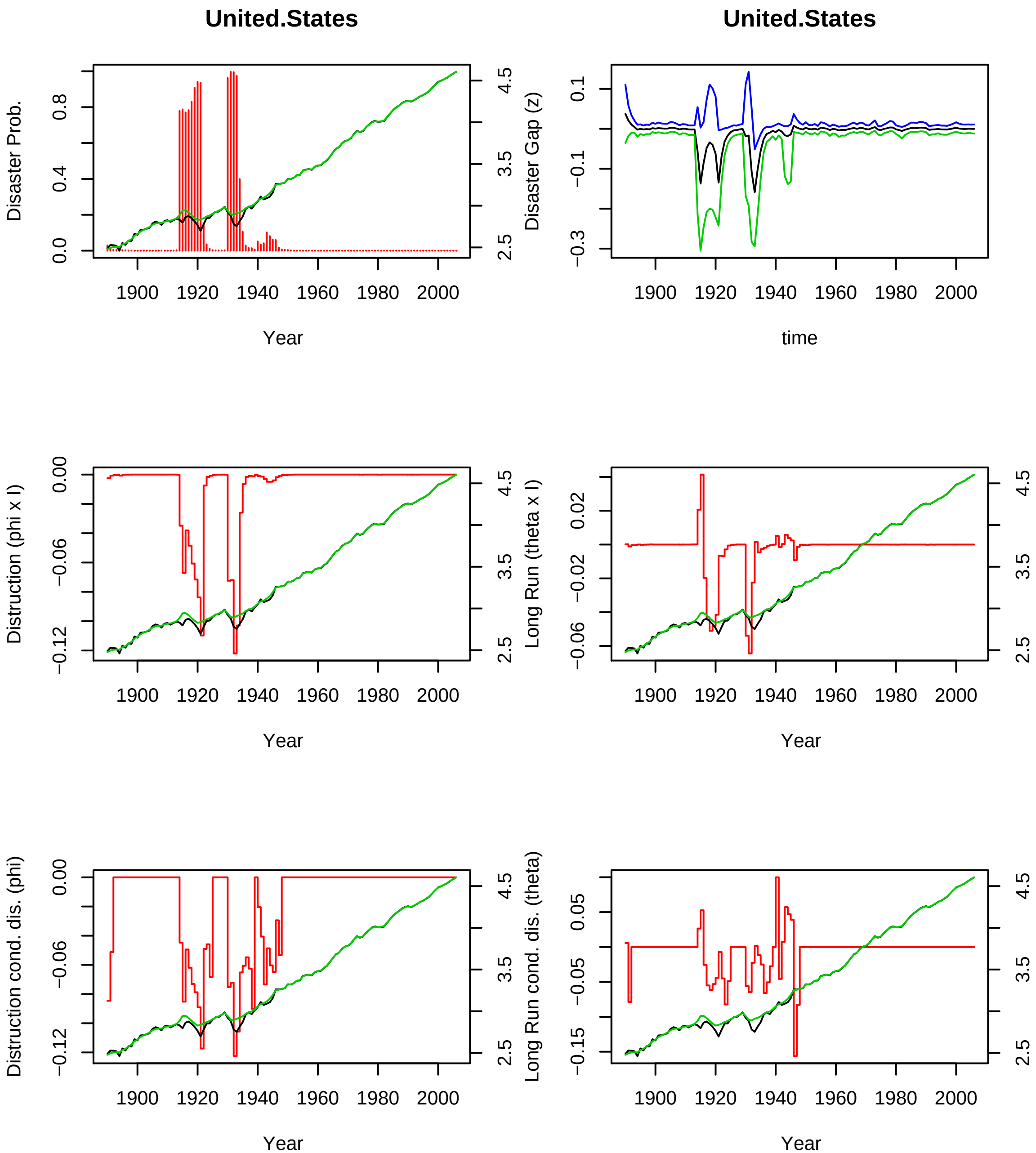


Switzerland
Disaster Prob.
Disaster Gap (z)
Year
Switzerland
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year



Taiwan
Taiwan
Disaster Prob.
Disaster Gap (z)
Year
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year


United.Kingdom
Disaster Prob.
Disaster Gap (z)
Year
United.Kingdom
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year



United.States
United.States
Disaster Prob.
Disaster Gap (z)
Year
time
Distruction (phi x l)
Long Run (theta x l)
Year
Year
Distruction cond. dis. (phi)
Long Run cond. dis. (theta)
Year
Year
0.0
0.4
0.8
2.5
3.5
4.5
0.1
−0.1
−0.3
0.00
−0.06
−0.12
0.02
−0.02
−0.06
0.05
−0.05
−0.15
1900
1920
1940
1960
1980
2000