Organizational Barriers to Technology Adoption: Evidence from Soccer-Ball Producers in Pakistan

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Appendix B: Theory Appendix

B A Model of Organizational Barriers to Technology Adoption

This appendix develops a model of strategic communication in a principal-agent setting that captures the intra-firm dynamics we have observed and motivates our second experiment, as discussed in Section V of the main text. After describing the theoretical setting in Section B.1, we consider the case in which the principal is not able to offer conditional contracts in Section B.2, and the case in which she is able to offer such contracts in Section B.3. In Section B.4, we present an additional result that a one-time lump-sum transfer can generate an equilibrium similar to the conditional-contracts case.

B.1 Theoretical setting

B.1.1 Basic set-up

Consider a one-period game. There is a principal (she) and an agent (he). The principal can sell any output q at an exogenously given price p. The principal incurs two costs: a constant marginal cost of materials, c, and a wage $w(q)$ that she pays to the agent. Her payoff is $\pi = pq - w(q) - cq$. The agent produces output $q = se$ where s is the speed of the technology (e.g. the cuts per minute), and e is effort, which is not contractible and is costly to the agent to provide. The agent has utility $U = w(q) - \frac{e^2}{2}$ and an outside option of zero. We assume that contracts must be of the linear form $w(q) = \alpha + \beta q$, where $\beta \geq 0$.[1] We further assume that the agent has limited liability, $\alpha \geq 0$ — a reasonable assumption given that no worker in our setting pays an owner to work in the factory. Below we will consider cases which differ in the ability of the principal to condition the piece rate, β, on marginal cost, c, a characteristic of the technology that will in general only be revealed ex post.

Technologies are characterized by speed, s, and materials cost, c. There is an existing technology, θ_0, with (s_0, c_0). There is also a new technology, which can be one of three types:

- Type θ_1, with $c_1 = c_0$ and $s_1 < s_0$. This technology is material-neutral (neither raises nor lowers material costs) and labor-using (is slower). It is dominated by the existing technology; we refer to it as the "bad" technology.

- Type θ_2, with $c_2 < c_0$ and $s_2 < s_0$. This technology is material-saving but labor-using: it lowers material costs but is slower than the existing technology. It is analogous to our offset die.

- Type θ_3, with $c_3 = c_0$ and $s_3 > s_0$. This technology is material-neutral and labor-saving. It dominates the existing technology because it has the same material costs but is faster. It is analogous to the original two-panel non-offset "back-to-back" die (faster than the one-piece die that preceded it) discussed in Section VII.

We assume that both players are aware ex ante of the existence of the technology, but differ in their knowledge about the technology type. The principal has prior ρ_i that the technology is type θ_i, with $\sum_1^3 \rho_i = 1$, and Nature does not reveal the type to her. In contrast, Nature reveals the type with certainty to the agent. While this is clearly an extreme assumption, it captures in an analytically

[1]We restrict attention to a single contract rather than a menu of contracts since there was no evidence such menus were on offer in Sialkot. Also, we rule out by assumption the possibility that the contract can be conditioned on messages sent by the agent, implicitly assuming that the costs of implementing such contracts are prohibitively high. There is an active theoretical literature on optimal contracts in settings similar to ours, in which agents need to be induced to experiment; see e.g. Halac, Kartik, and Liu (2016).

tractable way the observation from our qualitative work that the cutters are better informed about the cutting dies (which they work with all day every day) than are owners. Adopting the new technology requires a fixed cost, F.

In Stage 1 of the game, the principal chooses a wage contract. In Stage 2, Nature reveals the technology type to the agent. In Stage 3, the agent can send a costless message regarding the type of the new technology. In Stage 4, the principal decides whether to adopt the new technology, given the agent's message. In Stage 5, the agent chooses his level of effort. In Stage 6, output is observed, the technology is revealed to the principal, and payoffs are realized. The key feature of the timing is that the wage contract must be chosen before the agent sends his message.[2,3]

Given this setup, the optimal effort choice for the agent, for a given β, is:

$$e = \arg\max_{e} \left(\alpha + \beta s_i e - \frac{e^2}{2} \right) = \beta s_i \tag{B1}$$

In all the cases we consider below, the limited-liability constraint binds and the principal will set $\alpha = 0$. Conditional on technology θ_i being used, the agent's utility is then:

$$U(\beta, \theta_i) = \frac{\beta^2 s_i^2}{2} \tag{B2}$$

which makes it clear that conditional on β the agent prefers faster technologies. Given the agent's optimal effort choice, the principal's profit from adopting technology type θ_i can be written as a function of the piece rate, β. Writing $\pi(\beta, \theta_i)$ as $\pi_i(\beta)$ to reduce clutter, we have:

$$\pi_i(\beta) \quad = \quad s_i^2 \beta \left(p - \beta - c_i\right) - F \cdot \mathbb{1}(i = 1, 2, 3) \tag{B3}$$

where β need not be the optimal choice for technology θ_i.

B.1.2 Benchmarks

As a preliminary step, it is useful to consider the optimal contract under two counterfactual benchmark cases. In the first, suppose that the principal is fully informed about the technology. In this case, in the absence of the limited-liability constraint the principal would make the agent the residual claimant: she would set $\beta = p - c_i$ and bring the agent down to his reservation utility through a negative value of α. With the limited-liability constraint this is not possible. Since the

[2] Since Nature does not reveal the technology type to the principal, it is not crucial for the analysis whether Nature's move, which we can think of as the initial technology drop by our survey team, happens before or after the wage contract is set. (That is, the order of Stages 1 and 2 can be reversed.) Thus the model can also accommodate a scenario in which the principal's priors are set when our survey team does the technology drop and the technology type is revealed to the agent.

[3] One concern with static cheap-talk models is that the principal has no chance to respond to lying, and hence no way to encourage truth telling, if she later discovers that the technology is a good one (for example, from another firm that adopts). A more general version of our model would partially address this concern. Consider the following modification to Stage 2 of the game: if the technology is type θ_3, Nature signals θ_3 to the agent; if the technology is type θ_2, Nature signals θ_2 to the agent with probability $\varphi > \frac{1}{2}$ and θ_1 with probability $1 - \varphi$; and if the technology is type θ_1, Nature signals θ_1 to the agent with probability $\varphi > \frac{1}{2}$ and θ_2 with probability $1 - \varphi$. (Recall that conditional on β the agent's utility is only a function of s_i and not c_i so it is reasonable that he can more easily distinguish slow from fast technologies than low cost from high cost.) This leaves our model essentially unchanged since the agent still wishes to block adoption if he receives either signal θ_1 or θ_2 and to encourage adoption if he receives signal θ_3. Hence, he continues to pool types θ_1 and θ_2, and the principal anticipates this pooling. The key difference is that in this variant of the model, if the principal later discovers the technology is of type θ_2, she cannot be sure that the agent received the signal θ_2.

agent's effort ($e = \beta s_i$, as above) is independent of α (refer to (B1)), the principal sets $\alpha = 0$. The optimal contract for a known technology type θ_i is then:

$$\alpha_i = 0,\ \beta_i = \frac{p - c_i}{2} \tag{B4}$$

Note that the optimal piece rate depends on marginal cost.[4] The optimal piece rate for technology θ_2, β_2, is higher than the optimal piece rate for the existing technology, β_0, since $c_2 < c_0$.[5] In this case, the principal would like to incentivize more effort from the agent because profits per cut are higher.

In the second benchmark, suppose that the principal is imperfectly informed but receives no message from the agent. Define expected profit in this case as:

$$\widetilde{\pi}(\beta) \equiv \rho_1\pi_1(\beta) + \rho_2\pi_2(\beta) + \rho_3\pi_3(\beta) \tag{B5}$$

In this case, it can be shown that the principal, if she were to adopt the technology, would choose the wage contract: $\alpha = 0, \widetilde{\beta} = \sum_{i=1}^{3} \lambda_i\beta_i$, where β_i is as in (B4) and $\lambda_i = \frac{\rho_i s_i^2}{\sum_1^3 \rho_i s_i^2}$. The optimal piece rate would thus be a weighted average of the optimal piece rates in the full-information case. Given this contract, the expected profit from adoption would be:

$$\widetilde{\pi}(\widetilde{\beta}) = \left(\sum_{i=1}^{3} \rho_i s_i^2\right)\left(\widetilde{\beta}\right)^2 - F \tag{B6}$$

B.1.3 Parameter restrictions

As noted above, the aim of the model is to capture the intra-organizational dynamics we have observed, in particular that workers may seek to discourage owners from adopting a technology like ours, and that modifying wage contracts may lead to successful adoption. These features are not present under all possible parameter values. To focus on what we consider to be the interesting case in the model, we impose three parameter restrictions. Using the definitions of $\pi_i(\cdot)$ from (B3), of β_i from (B4), and of $\widetilde{\pi}(\widetilde{\beta})$ from (B6), they are:

$$\pi_2(\beta_0) > \pi_0(\beta_0) \tag{B7a}$$

$$\pi_3(\beta_2) > \pi_0(\beta_2) \tag{B7b}$$

$$\pi_0(\beta_0) > \widetilde{\pi}(\widetilde{\beta}) \tag{B7c}$$

To understand the intuition for these conditions, consider the schematic representation of the

[4] There are alternative scenarios where the piece rate would depend not only on the price and marginal cost, but also on speed. As a first example, if speed and effort were substitutes, $q = s + e$, then agents will optimally provide effort $e = \beta$ and the optimal piece rate for a given technology is $\beta = \frac{p-c_i-s_i}{2}$. However, the agent's utility conditional on β, $U(\beta, \theta_i) = \frac{\beta^2}{2} + \beta s_i$, is still increasing in speed. Since wages are set in Stage 1, the agent will still try to avoid slower technologies like type θ_2 in Stage 3. Whether the principal will want to adopt types θ_2 and θ_3 will again depend on parameters and there will still be a potential misalignment in incentives. Second, suppose that the principal has to produce a target $\bar{q}$ with no time limit and, as in our model, $q = se$ and optimal effort for technology type θ_i is βs_i. Since $q = \beta s_i^2$, the optimal piece rate is to pay $\beta = \frac{\bar{q}}{s_i^2}$ to hit the target (assuming this is less than $\beta = \frac{p-c}{2}$, otherwise she would just hire several cutters at this optimal piece rate). But since the agent's utility conditional on β is still increasing in s, $U(\beta, \theta_i) = \frac{\beta^2 s_i^2}{2}$, the agent will still try to avoid slower technologies like type θ_2 creating a potential misalignment in incentives.

[5] Since $c_3 = c_1 = c_0$, the optimal piece rate for types θ_1 and θ_3 is the same as for the existing technology.

profit functions $\pi_i(\beta)$ in Figure B.1. Each of the profit functions $\pi_i(\beta)$ defines a concave parabola with vertex at β_i.[6] Since $c_0 = c_1 = c_3$, the functions $\pi_0(\beta)$, $\pi_1(\beta)$, and $\pi_3(\beta)$ have maxima at the same value of the piece rate, β_0; $\pi_2(\beta)$ has a maximum at $\beta_2 > \beta_0$.

Condition (B7a) requires that $\pi_2(\beta)$ lie above $\pi_0(\beta)$ even at the optimal piece rate for the existing technology β_0, as in the figure. We believe that this condition is realistic in our empirical setting, where our technology is profitable to adopt for almost all firms even at existing wage rates. The condition in turn implies $\pi_2(\beta_2) > \pi_0(\beta_0)$; that is, a fully informed principal would adopt type θ_2.

Condition (B7b) requires that technology θ_3 dominates the existing technology θ_0 even at β_2, the optimal piece rate for type θ_2. This condition guarantees that θ_3 dominates θ_0 at all values of the piece rate between β_0 and β_2, which will be the region of primary interest.[7]

Condition (B7c) implies that a principal with no information beyond her priors would choose not to adopt. Intuitively, it requires that the payoff to the bad technology θ_1 is sufficiently low and that the principal's prior on θ_1 is sufficiently high that the $\widetilde{\pi}(\widetilde{\beta})$ curve defined in (B6) (a weighted average of $\pi_1(\beta)$, $\pi_2(\beta)$, and $\pi_3(\beta)$, and itself a concave parabola) lies everywhere below $\pi_0(\beta_0)$.

Remarks 1-3 in Appendix B.5 show formally that conditions (B7a)-(B7c) imply that the relative locations of the $\pi_i(\beta)$ curves are as illustrated in Figure B.1.[8] Remark 4 shows formally that conditions (B7a)-(B7c) are compatible with each other.

B.1.4 Cheap-talk Subgames Conditional on β

For a given choice of the piece rate β by the principal, the agent and principal engage in a cheap-talk subgame similar to the interaction considered by Crawford and Sobel (1982) (hereafter CS) but with a discrete set of possible states of the world. Let Θ be the set of possible new technologies (i.e. $\{\theta_1, \theta_2, \theta_3\}$). We refer to an agent who observes θ_i as being "type θ_i." Let m be a message and M the set of possible messages. Let $q(m|\theta)$ be the agent's probability of sending message m if he is type θ, where $\sum_{m \in M} q(m|\theta) = 1$ for each θ. Let $\rho(\theta)$ be the principal's prior distribution; that is, $\rho(\theta_1) = \rho_1$, $\rho(\theta_2) = \rho_2$, $\rho(\theta_3) = \rho_3$. Let $a(m) \in [0, 1]$ be the probability of adoption by the principal in response to the message m. Let $\widehat{U}(a(m), \beta, \theta_i)$ be the expected utility of the agent of type θ_i, prior to the adoption decision of the principal:

$$\widehat{U}(a(m), \beta, \theta_i) = a(m)U(\beta, \theta_i) + (1 - a(m))U(\beta, \theta_0) \quad \text{(B9)}$$

where $U(\cdot, \cdot)$ is as defined in (B2). Define $\widehat{\pi}(a, \beta, \theta_i)$ as the expected profit of the principal prior to the adoption decision, conditional on θ_i:

$$\widehat{\pi}(a, \beta, \theta_i) \quad = \quad a\pi_i(\beta) + (1 - a)\pi_0(\beta) \quad \text{(B10)}$$

where $\pi_i(\cdot)$ is as defined in in (B3).

[6] To see this, note that (B3) can be rewritten:

$$\pi_i(\beta) = -s_i^2 (\beta - \beta_i)^2 + s_i^2 \beta_i^2 - F \cdot \mathbb{1}(i = 1, 2, 3) \quad \text{(B8)}$$

where β_i is as defined in (B4). The width of each parabola is declining in speed, s_i.

[7] As piece rates rise towards $p - c_0$, operating profits fall to zero for both the existing technology and type θ_3 and so the existing technology, which requires no fixed cost F of adoption, becomes preferred to θ_3. Thus, the condition will be satisfied by some combination of large speed gains from θ_3, low fixed costs of adoption for θ_3, and small marginal cost gains from θ_2.

[8] Note that the conditions do not carry an implication for the relative magnitudes of $\pi_3(\beta_0)$ and $\pi_2(\beta_2)$; it may be that $\pi_3(\beta_0) > \pi_2(\beta_2)$ as in the figure, or that $\pi_3(\beta_0) < \pi_2(\beta_2)$.

By Bayes' rule, the principal's posterior beliefs after receiving any message m sent with positive probability, i.e. an m for which $q(m|\theta_i) > 0$ for some θ_i, are:[9]

$$p(\theta|m) = \frac{q(m|\theta)\rho(\theta)}{\sum_{\theta' \in \Theta} q(m|\theta')\rho(\theta')} \tag{B11}$$

An equilibrium in the cheap-talk subgame is a family of reporting rules $q(m|\theta)$ for the agent (sender) and an action rule $a(m)$ for the principal (receiver) such that the following conditions hold:

1. If $q(m^*|\theta) > 0$ then
$$m^* = \arg\max_{m \in M} \widehat{U}(a(m), \beta, \theta) \tag{B12}$$

2. For each m,
$$a(m) = \arg\max_{a \in [0,1]} \sum_{\theta \in \Theta} \widehat{\pi}(a, \beta, \theta) p(\theta|m) \tag{B13}$$

That is, the rule of each player must be a best response to the rule of the other player.

Following CS, we describe two messages m and m' as *equivalent* in a given subgame equilibrium if they induce the same action, that is $a(m) = a(m')$. Let $M_a = \{m : a(m) = a\}$ be the set of equivalent messages that lead the principal to choose action a. Following CS, we say that an action a is *induced* by an agent of type θ if $\sum_{m \in M_a} q(m|\theta) > 0$. For a given subgame equilibrium, let $m_{min} \equiv \arg\min_{m \in M} a(m)$ and $m_{max} \equiv \arg\max_{m \in M} a(m)$ be messages that induce the lowest and highest probabilities of adoption, respectively. Let $a_{min} \equiv a(m_{min})$ and $a_{max} \equiv a(m_{max})$ be the corresponding lowest and highest induced probabilities of adoption, and $M_{a_{min}}$ and $M_{a_{max}}$ be the sets of equivalent messages that induce them.

To streamline the exposition, we treat each set M_a as a single message and treat subgame equilibria that differ only in which messages from a set M_a are chosen as the *same* subgame equilibrium. We refer to M_1 as "technology is good" and M_0 as "technology is bad."[10]

B.2 No conditional contracts

We first consider the case in which the (imperfectly informed) principal is not able to condition the wage payment on marginal cost, which is only revealed ex post. In this case, there exists an equilibrium in which, if the technology is type θ_2, the agent seeks to discourage the principal from adopting and the principal does not adopt. If we restrict attention to the most informative equilibria in all cheap-talk interactions, then this equilibrium is unique. For conciseness, the following proposition only states the on-equilibrium-path strategies; in the proof below we consider the entire strategy space.

Proposition 1. *Under (B7a)-(B7c), if contracts conditioned on marginal cost are not available, then the following strategies are part of a perfect Bayesian equilibrium.*

1. *In Stage 1, the principal offers wage contract* $(\alpha^* = 0, \beta^* = \beta_0 = \frac{p - c_0}{2})$.

[9] Following Crawford and Sobel (1982), we assume that if the principal receives a message that is off the equilibrium path, i.e. an m for which $q(m|\theta_i) = 0 \; \forall \; \theta_i$, she takes one of the actions induced on the equilibrium path for that β.

[10] If we were to limit the set of messages to have just three elements, $m_1 =$"the techology is type θ_1", $m_2 =$"the techology is type θ_2", and $m_3 =$"the techology is type θ_3", then a natural subgame equilibrium would have $m_1 \in M_0$ and $m_2, m_3 \in M_1$; that is, to discourage adoption the agent would say that the technology is type θ_1 and to encourage it he would say type θ_2 or θ_3. Since formally there is no need to limit the set of messages in this way, we consider the richer set of potential messages in the proofs below.

2. *In Stage 3, the agent:*
   (a) *says "technology is bad" if the technology is type θ_1 or θ_2,*
   (b) *says "technology is good" if the technology is type θ_3.*
3. *In Stage 4, the principal:*
   (a) *adopts if the agent says "technology is good",*
   (b) *does not adopt if the agent says "technology is bad".*

Intuitively, given that the principal has committed in Stage 1 to a piece rate (not conditioned on cost), the agent strictly prefers the existing technology to type θ_2. So if the technology is type θ_2, the agent discourages adoption, and the principal does not adopt.[11] Why does the principal pay any attention to the agent's message, given that she knows that the agent does not want to adopt type θ_2? The intuition is that the players' interests are aligned if the technology is of type θ_1 or θ_3, and the agent's advice is valuable enough in these states of the world that it is worthwhile for the principal to allow herself to be influenced by the agent — and possibly discouraged from using type θ_2 — rather than to ignore the agent's advice altogether.

B.2.1 Proof of Proposition 1

To prove this proposition, we first consider the cheap-talk interaction in the subgame defined by any choice of piece rate, β (Subsection B.2.1.1). We then consider the particular subgame defined by a particular choice of β, namely $\beta = \beta_0$ (Subsection B.2.1.2). We then show that this choice is optimal for the principal (Subsection B.2.1.3).

B.2.1.1 Subgames conditional on β

For the subgame corresponding to any choice of β, we have the following.

Lemma 1. *In any equilibrium with $a_{min} < a_{max}$ the following statements are true:*

$$\sum_{m\in M_{a_{min}}} q(m|\theta_1) = 1 \qquad \sum_{m\in M_{a_{max}}} q(m|\theta_1) = 0 \qquad \sum_{m\notin\{M_{a_{min}}\cup M_{a_{max}}\}} q(m|\theta_1) = 0 \tag{B14a}$$

$$\sum_{m\in M_{a_{min}}} q(m|\theta_2) = 1 \qquad \sum_{m\in M_{a_{max}}} q(m|\theta_2) = 0 \qquad \sum_{m\notin\{M_{a_{min}}\cup M_{a_{max}}\}} q(m|\theta_2) = 0 \tag{B14b}$$

$$\sum_{m\in M_{a_{min}}} q(m|\theta_3) = 0 \qquad \sum_{m\in M_{a_{max}}} q(m|\theta_3) = 1 \qquad \sum_{m\notin\{M_{a_{min}}\cup M_{a_{max}}\}} q(m|\theta_3) = 0 \tag{B14c}$$

For $m \in M_{a_{min}}$, $p(\theta_1|m) = \frac{\rho_1}{\rho_1+\rho_2}$, $p(\theta_2|m) = \frac{\rho_2}{\rho_1+\rho_2}$, and $p(\theta_3|m) = 0$. For $m \in M_{a_{max}}$, $p(\theta_1|m) = 0$, $p(\theta_2|m) = 0$, and $p(\theta_3|m) = 1$.

In any equilibrium with $a_{min} = a_{max}$ (letting $a^* \equiv a_{min} = a_{max}$), the following statements are true:

$$\sum_{m\in M_{a^*}} q(m|\theta_i) = 1 \;\forall\; \theta_i, \qquad \sum_{m\notin M_{a^*}} q(m|\theta_i) = 0 \;\forall\; \theta_i \tag{B15}$$

For $m \in M_{a^*}$, $p(\theta_1|m) = \rho_1$, $p(\theta_2|m) = \rho_2$, and $p(\theta_3|m) = \rho_3$.

[11] In the language of Aghion and Tirole (1997), the principal in this equilibrium retains formal authority over the adoption decision but cedes real authority to the agent.

Proof. Note from (B9) that $\frac{\partial}{\partial a}\widehat{U}(a,\beta,\theta_1) < 0$, $\frac{\partial}{\partial a}\widehat{U}(a,\beta,\theta_2) < 0$ and $\frac{\partial}{\partial a}\widehat{U}(a,\beta,\theta_3) > 0$ for all $a \in [0,1]$, since $s_1 < s_0$, $s_2 < s_0$ and $s_3 > s_0$. Hence if $a_{min} \neq a_{max}$ then in order for (B12) to be satisfied it must be the case that an agent of type θ_1 or θ_2 chooses a message that induces the lowest possible probability of adoption, a_{min}. Similarly, an agent who observes θ_3 must choose a message that induces the highest possible probability of adoption, a_{max}. Given these reporting rules, the updating of the principal's beliefs follow by Bayes' rule.

If $a_{min} = a_{max}$ then trivially all messages sent by the agent induce the same action and hence are equivalent ($M_{a_{min}} = M_{a_{max}} = M_{a^*}$). Given this, the principal does not update. □

Intuitively, conditional on a piece rate, β, types θ_1 and θ_2 strictly prefer non-adoption, and type θ_3 strictly prefers adoption; the types report accordingly, with the most discouraging and most encouraging messages. So in any equilibrium in which the principal's action is influenced by the agent's message (i.e. $a_{min} < a_{max}$), if an agent sends a discouraging message the principal infers that he is type θ_1 or θ_2; if the message is encouraging, she infers that he is type θ_3. She then updates by Bayes' rule.

The Lemma holds that for a given β, only two subgame equilibria (modulo treating messages that induce the same action as equivalent) may exist: one in which $a_{min} \neq a_{max}$ and agent types θ_1 and θ_2 are indistinguishable and another in which the principal ignores the message from the agent (a "babbling" equilibrium). In the terminology of Sobel (2013), an equilibrium is "informative" if the message from the sender shifts the receiver's beliefs and is "influential" if different messages induce the receiver to take different actions. By these definitions, the equilibrium with $a_{min} \neq a_{max}$ is both informative and influential.

It will be convenient below to define the principal's ex-ante expected profit in a given subgame equilibrium. Given Lemma 1, we have:

$$\begin{aligned}\pi^*(\beta) &= \rho_1\widehat{\pi}\left[a\left(m|m \in M_{a_{min}}\right),\beta,\theta_1\right] + \rho_2\widehat{\pi}\left[a\left(m|m \in M_{a_{min}}\right),\beta,\theta_2\right] \\ &\quad +\rho_3\widehat{\pi}\left[a\left(m|m \in M_{a_{max}}\right),\beta,\theta_3\right] \qquad \text{(B16)}\end{aligned}$$

where $a(m)$ represents the principal's best response, given by (B13), and we may or may not have $a_{min} = a_{max}$.

B.2.1.2 Subgame with $\beta = \beta_0$

Now consider the particular subgame with $\beta = \beta_0$.

Lemma 2. *If $\beta = \beta_0$, there exists a perfect Bayesian subgame equilibrium with the strategies outlined in Proposition 1.*

Proof. As outlined by Proposition 1, the agent's reporting rules are given by (B14a)-(B14c), where $a_{min} = 0$ and $a_{max} = 1$, and the principal's action rule is $a(m) = 0 \;\forall\; m \in M_0$, $a(m) = 1 \;\forall\; m \in M_1$.[12] To prove that this is a subgame equilibrium, it suffices to show that neither agent nor principal wants to deviate. That the agent does not want to deviate follows from the fact (from (B9)) that $\frac{\partial}{\partial a}\widehat{U}(a,\beta,\theta_1) < 0$, $\frac{\partial}{\partial a}\widehat{U}(a,\beta,\theta_2) < 0$ and $\frac{\partial}{\partial a}\widehat{U}(a,\beta,\theta_3) > 0$ for all $a \in [0,1]$. Now consider the principal's decision. Given Lemma 1, if the agent says "technology is bad" (i.e. any $m \in M_0$) then the condition for the principal not to deviate is:

$$\pi_0(\beta) \geq \frac{\rho_1}{\rho_1+\rho_2}\pi_1(\beta) + \frac{\rho_2}{\rho_1+\rho_2}\pi_2(\beta)$$

[12]Note that in the Proposition-1 equilibrium all messages fall into one of the following sets: M_0, M_1, or the complement to $M_0 \cup M_1$, no element of which is used in equilibrium.

where the left-hand side is the profit from the existing technology and the right-hand side the expected profit to adoption. That this condition holds for $\beta = \beta_0$ is demonstrated by Remark 5 below.

If the agent says "technology is good" (i.e. any $m \in M_1$), then the condition for the principal not to deviate is:

$$\pi_0(\beta) \leq \pi_3(\beta)$$

which by Remark 2 is satisfied for $\beta = \beta_0$. □

In this subgame, the principal's ex-ante expected profit (refer to (B16)) is:

$$\begin{aligned} \pi^*(\beta_0) &= \rho_1 \widehat{\pi}(0, \beta_0, \theta_1) + \rho_2 \widehat{\pi}(0, \beta_0, \theta_2) + \rho_3 \widehat{\pi}(1, \beta_0, \theta_3) \\ &= (\rho_1 + \rho_2)\pi_0(\beta_0) + \rho_3 \pi_3(\beta_0) \end{aligned} \tag{B17}$$

B.2.1.3 Principal's choice of β

We now turn to the supergame where the principal selects the optimal β given the subgame payoffs. We can show that the principal has no incentive to deviate from β_0 in the supergame.

Lemma 3. *The principal's expected payoff in the informative equilibrium of the subgame with piece rate β_0 is strictly greater than the payoffs in the subgames for all other possible values of β.*

Proof. From Lemma 1, types θ_1 and θ_2 can never be distinguished. Therefore, the highest possible payoff to the principal from any subgame equals the payoff that the principal would obtain if she observed whether $\theta = \theta_3$ or $\theta \in \{\theta_1, \theta_2\}$.[13] Consider the maximal payoffs in all subgames under the assumption that the principal is able to extract this information. There are only four cases to consider, which may exist for different values of β.[14]

1. It is profitable for the principal to adopt if $\theta = \theta_3$ but not if $\theta \in \{\theta_1, \theta_2\}$:

$$\begin{aligned} \widehat{\pi}(1, \beta, \theta_3) &\geq \widehat{\pi}(0, \beta, \theta_3) \\ \frac{\rho_1}{\rho_1 + \rho_2}\widehat{\pi}(1, \beta, \theta_1) + \frac{\rho_2}{\rho_1 + \rho_2}\widehat{\pi}(1, \beta, \theta_2) &< \frac{\rho_1}{\rho_1 + \rho_2}\widehat{\pi}(0, \beta, \theta_1) + \frac{\rho_2}{\rho_1 + \rho_2}\widehat{\pi}(0, \beta, \theta_2) \end{aligned}$$

   In this case, the principal's maximal ex-ante expected profit (refer to (B16)) is:

$$\begin{aligned} \pi^*(\beta) &= \rho_1 \widehat{\pi}(0, \beta, \theta_1) + \rho_2 \widehat{\pi}(0, \beta, \theta_2) + \rho_3 \widehat{\pi}(1, \beta, \theta_3) \\ &= (\rho_1 + \rho_2)\pi_0(\beta) + \rho_3 \pi_3(\beta) \end{aligned} \tag{B18}$$

   By Remarks 1-3, this case holds when $\beta = \beta_0$. From the definition of $\pi_i(\cdot)$ in (B3) it follows immediately that the expected payoff is maximized at $\beta^* = \beta_0 \equiv \frac{p - c_0}{2}$. That is, $\pi^*(\beta_0) > \pi^*(\beta)\ \forall\ \beta \neq \beta_0$. Hence for all subgames that fall into this case, the principal prefers β_0 to any other β.

[13] This is a simple application of Blackwell's ordering, i.e. that the decision maker's payoff must be weakly higher with more information (see Blackwell (1953).)

[14] The sets of values of β for which the different cases hold depend on the values of the parameters $\hat{\hat{\beta}}_3$ and $\overline{\beta}$ defined in Remarks 2 and 5. There are two possibilities: (1) $\overline{\beta} \geq \hat{\hat{\beta}}_3$. Here the region $(0, \hat{\beta}_3)$ corresponds to Case 2 below, $[\hat{\beta}_3, \hat{\hat{\beta}}_3]$ to Case 1, $[\hat{\hat{\beta}}_3, \overline{\beta}]$ to Case 2, and $(\overline{\beta}, \infty)$ to Case 4. (2) $\overline{\beta} < \hat{\hat{\beta}}_3$. Here the region $(0, \hat{\beta}_3)$ corresponds to Case 2 below, $[\hat{\beta}_3, \overline{\beta})$ to Case 1, $[\overline{\beta}, \hat{\hat{\beta}}_3]$ to Case 3, and $(\hat{\hat{\beta}}_3, \infty)$ to Case 4.

2. It is profitable for the principal to adopt neither if $\theta = \theta_3$ nor if $\theta \in \{\theta_1, \theta_2\}$:

$$\begin{aligned} \widehat{\pi}(1,\beta,\theta_3) &< \widehat{\pi}(0,\beta,\theta_3) \\ \frac{\rho_1}{\rho_1+\rho_2}\widehat{\pi}(1,\beta,\theta_1) + \frac{\rho_2}{\rho_1+\rho_2}\widehat{\pi}(1,\beta,\theta_2) &< \frac{\rho_1}{\rho_1+\rho_2}\widehat{\pi}(0,\beta,\theta_1) + \frac{\rho_2}{\rho_1+\rho_2}\widehat{\pi}(0,\beta,\theta_2) \end{aligned}$$

In this case, the principal's ex-ante expected profit (refer to (B16)) is:

$$\pi^*(\beta) = \rho_1\widehat{\pi}(0,\beta,\theta_1) + \rho_2\widehat{\pi}(0,\beta,\theta_2) + \rho_3\widehat{\pi}(0,\beta,\theta_3) = \pi_0(\beta) \tag{B19}$$

From (B3), β_0 maximizes $\pi_0(\beta)$, i.e. $\pi_0(\beta_0) \geq \pi_0(\beta)\ \forall\ \beta$. But by Remark 2, $\pi_3(\beta_0) > \pi_0(\beta_0)$ and hence $\pi^*(\beta_0) > \pi_0(\beta_0) \geq \pi_0(\beta)$ (where $\pi^*(\beta_0)$ is from (B17)). Hence the principal prefers the subgame with β_0 to any subgame that falls under this case.

3. It is profitable for the principal to adopt either if $\theta = \theta_3$ and or if $\theta \in \{\theta_1, \theta_2\}$:

$$\begin{aligned} \widehat{\pi}(1,\beta,\theta_3) &\geq \widehat{\pi}(0,\beta,\theta_3) \\ \frac{\rho_1}{\rho_1+\rho_2}\widehat{\pi}(1,\beta,\theta_1) + \frac{\rho_2}{\rho_1+\rho_2}\widehat{\pi}(1,\beta,\theta_2) &\geq \frac{\rho_1}{\rho_1+\rho_2}\widehat{\pi}(0,\beta,\theta_1) + \frac{\rho_2}{\rho_1+\rho_2}\widehat{\pi}(0,\beta,\theta_2) \end{aligned}$$

In this case, the principal's ex-ante expected profit is:

$$\pi^*(\beta) = \rho_1\pi_1(\beta) + \rho_2\pi_2(\beta) + \rho_3\pi_3(\beta) = \widetilde{\pi}(\beta) \tag{B20}$$

where $\widetilde{\pi}(\cdot)$ is defined in (B5). Since $\widetilde{\beta}$ is the optimal choice if the principal bases her decision only on her priors and adopts (refer to (B6)), it must be the case that $\widetilde{\pi}(\widetilde{\beta}) \geq \widetilde{\pi}(\beta)$ for all β. But by condition (B7c), $\pi_0(\beta_0) > \widetilde{\pi}(\widetilde{\beta})$. As above, Remark 2 implies $\pi^*(\beta_0) > \pi_0(\beta_0)$ (where $\pi^*(\beta_0)$ is from (B17)). Hence $\pi^*(\beta_0) > \pi_0(\beta_0) > \widetilde{\pi}(\widetilde{\beta}) \geq \widetilde{\pi}(\beta)\ \forall\ \beta$; the principal prefers the subgame with β_0 to any subgame that falls under this case.

4. It is not profitable for the principal to adopt if $\theta = \theta_3$ but it is profitable if $\theta \in \{\theta_1, \theta_2\}$:[15]

$$\begin{aligned} \widehat{\pi}(1,\beta,\theta_3) &< \widehat{\pi}(0,\beta,\theta_3) \\ \frac{\rho_1}{\rho_1+\rho_2}\widehat{\pi}(1,\beta,\theta_1) + \frac{\rho_2}{\rho_1+\rho_2}\widehat{\pi}(1,\beta,\theta_2) &\geq \frac{\rho_1}{\rho_1+\rho_2}\widehat{\pi}(0,\beta,\theta_1) + \frac{\rho_2}{\rho_1+\rho_2}\widehat{\pi}(0,\beta,\theta_2) \end{aligned}$$

In this case, the principal's ex-ante expected profit (refer to (B16)) is:

$$\pi^*(\beta) = \rho_1\pi_1(\beta) + \rho_2\pi_2(\beta) + \rho_3\pi_0(\beta) \tag{B21}$$

Consider the function $\widetilde{\widetilde{\pi}}(\beta) = \rho_1\pi_1(\beta) + \rho_2\pi_2(\beta) + \rho_3\pi_0(\beta)$ defined over all possible values of β. (That is, $\widetilde{\widetilde{\pi}}(\cdot)$ and $\pi^*(\cdot)$ coincide for values of β that yield Case 4, but for values of β that yield the other cases $\pi^*(\cdot) \geq \widetilde{\widetilde{\pi}}(\cdot)$ since $\pi^*(\cdot)$ reflects optimal adoption decisions.) Maximizing $\widetilde{\widetilde{\pi}}(\beta)$ over β yields:

$$\widetilde{\widetilde{\beta}} = \left(\frac{\rho_1 s_1^2 + \rho_3 s_0^2}{\rho_1 s_1^2 + \rho_2 s_2^2 + \rho_3 s_0^2}\right)\beta_0 + \left(\frac{\rho_2 s_2^2}{\rho_1 s_1^2 + \rho_2 s_2^2 + \rho_3 s_0^2}\right)\beta_2$$

Because $\widetilde{\widetilde{\beta}}$ maximizes $\widetilde{\widetilde{\pi}}(\beta)$ and $\widetilde{\widetilde{\pi}}(\cdot)$ is strictly convex, $\widetilde{\widetilde{\pi}}(\widetilde{\widetilde{\beta}}) > \widetilde{\widetilde{\pi}}(\beta)\ \forall\ \beta \neq \widetilde{\widetilde{\beta}}$. Because $\widetilde{\widetilde{\beta}}$ is

[15] We include this case for completeness, although no informative equilibria will exist in this case as interests are completely misaligned.

a weighted average of β_0 and β_2, we know $\tilde{\tilde{\beta}} \in (\beta_0, \beta_2)$. By Remark 2, $\pi_3(\beta) > \pi_0(\beta)$ for all $\beta \in (\beta_0, \beta_2)$ and hence $\pi^*(\tilde{\tilde{\beta}}) > \tilde{\tilde{\pi}}(\tilde{\tilde{\beta}})$ in this region. Since it is profitable to adopt θ_3 for $\beta = \tilde{\tilde{\beta}}$, Case 1 or 3 must hold. By the argument in one or the other case, $\pi^*(\beta_0) > \tilde{\tilde{\pi}}(\tilde{\tilde{\beta}})$ (where $\pi^*(\beta_0)$ is from (B17)). Hence $\pi^*(\beta_0) > \tilde{\tilde{\pi}}(\tilde{\tilde{\beta}}) > \tilde{\tilde{\pi}}(\beta)$ for values of β for which this case holds. Once again, the principal prefers the subgame with β_0 to any subgame that falls under this case.

Considering the four cases together, we can conclude the maximum possible payoff to the principal for $\beta \neq \beta_0$ is less than $\pi^*(\beta_0)$ from (B17). □

Intuitively, from Lemma 1, types θ_1 and θ_2 can never be distinguished. Therefore, the highest possible payoff to the principal from any subgame equals the payoff that the principal would obtain if she observed whether $\theta = \theta_3$ or $\theta \in \{\theta_1, \theta_2\}$.[16] It is possible to show that the payoff when $\beta = \beta_0$ is greater than the maximal payoffs in all other subgames.

Given Lemma 3, the principal does not have an incentive to deviate from $\beta^* = \beta_0$, her chosen wage in Proposition 1. By Lemma 2, the strategies outlined in Proposition 1 form a perfect Bayesian equilibrium of the subgame with $\beta^* = \beta_0$. Hence neither player has an incentive to deviate at any stage. This completes the proof of the existence of the equilibrium described in Proposition 1.

B.2.2 Discussion

Crawford and Sobel (1982) and others have argued that it is reasonable to assume that players coordinate on the most informative equilibrium in cheap-talk interactions. If one is willing to assume this, then the equilibrium described by Proposition 1 is unique. Recall that Lemma 1 implies that there are at most two possible equilibria in each subgame for each β. The restriction that we focus on the most informative equilibrium in each subgame implies that there is at most one equilibrium in each subgame. Lemma 3 implies that the principal prefers the subgame with $\beta = \beta_0$ to all other subgames. Hence the only equilibrium of the supergame is the one characterized by the Proposition-1 strategies on the equilibrium path.

One other result is worth highlighting. Lemma 1 implies that there does not exist an equilibrium in which the agent always truthfully reveals the technology type. That is, information about the technology is necessarily lost in some states of the world. Intuitively, if the agent were to reveal the technology type truthfully, then the principal would want to adopt type θ_2 and not type θ_1. But given this strategy of the principal, and the fact that the wage contract is fixed ex ante, the agent would be better off misreporting type θ_2 to be type θ_1.

B.3 Conditional contracts

Now suppose that in Stage 0 the principal can pay a transaction cost G and gain access to a larger set of wage contracts — in particular to contracts that condition the piece rate on marginal cost, c. This larger set of possible contracts includes contracts that offer a per-sheet incentive to reduce waste of laminated rexine, as these can be interpreted as an increase in the piece rate conditional on using the lower-marginal-cost technology.[17] The fixed cost G can be interpreted as the cost of a commitment device to pay a piece rate above the one that would be paid for the existing technology

[16] This is a simple application of Blackwell's ordering, i.e. that the decision maker's payoff must be weakly higher with more information (see Blackwell (1953).)

[17] In our framework, the only way to reduce waste is to use the low-marginal-cost technology; exerting additional effort would raise output but not reduce waste per sheet.

(β_0), which otherwise the principal would not be able to credibly commit to. (We discuss other possible interpretations in Section V.B.3 of the main text.)

The optimal contracts under the existing technology and types θ_1 and θ_3 are identical (since $c_3 = c_1 = c_0$ and hence $\beta_3 = \beta_1 = \beta_0$). The ability to condition on marginal cost matters only if the technology is type θ_2. Allowing for conditioning, the principal can offer contracts of the form:

$$w(q) = \alpha + (\beta + \gamma_2)q \qquad \text{if } c = c_2 \tag{B22}$$

$$w(q) = \alpha + \beta q \qquad \text{if } c \neq c_2$$

If G is sufficiently small, then there exists an equilibrium in which the agent reports truthfully, in the sense that he encourages adoption of profitable technologies and discourages adoption of unprofitable ones. Again, for conciseness, the proposition only states the on-equilibrium-path strategies, but the proof below covers the entire strategy space.

Proposition 2. *Under (B7a)-(B7c), if contracts conditioned on marginal cost are available at fixed cost G, then the following strategies are part of a perfect Bayesian equilibrium.*

1. *In Stages 0 and 1,*
   (a) *For*
   $$\rho_2 > \frac{G}{\pi_2(\beta_2) - \pi_0(\beta_0)}, \tag{B23}$$
   the principal pays G and offers wage contract $\left(\alpha^{**} = 0, \beta^{**} = \frac{p-c_0}{2}, \gamma_2^{**} = \frac{c_0-c_2}{2}\right)$.
   (b) *For* $\rho_2 \leq \frac{G}{\pi_2(\beta_2)-\pi_0(\beta_0)}$ *the principal does not pay G and offers wage contract* $\left(\alpha^{**} = 0, \beta^{**} = \frac{p-c_0}{2}\right)$.
2. *In Stage 3,*
   (a) *given* $\left(\alpha^{**} = 0, \beta^{**} = \frac{p-c_0}{2}, \gamma_2^{**} = \frac{c_0-c_2}{2}\right)$, *the agent:*
      i. *says "technology is bad" if the technology is type* θ_1,
      ii. *says "technology is good" if the technology is type* θ_2 *or* θ_3.
   (b) *given* $\left(\alpha^{**} = 0, \beta^{**} = \frac{p-c_0}{2}\right)$, *the agent:*
      i. *says "technology is bad" if the technology is type* θ_1 *or* θ_2,
      ii. *says "technology is good" if the technology is type* θ_3.
3. *In Stage 4, the principal:*
   (a) *adopts if the agent says "technology is good",*
   (b) *does not adopt if the agent says "technology is bad".*

Intuitively, if the principal offers the conditional contract, the higher piece rate if $c = c_2$ is enough to induce the agent to prefer adoption if the technology is of type θ_2.[18] Paying the transaction cost, G, will be in the interest of the principal if (B23) is satisfied, which is to say that the expected additional profit from adopting type θ_2 (with the optimal piece rate for type θ_2) is greater than the fixed cost of offering the new contract. In this case, the availability of the conditional contract solves the misinformation problem, in that type θ_2 will be adopted in equilibrium. At the same time, if (B23) is not satisfied, for instance because the principal has a low prior, ρ_2, then there again exists the equilibrium of Proposition 1, in which type θ_2 is not adopted.

[18] Note that, using the notation of (B4), $\beta^{**} = \beta_0$ and $\beta^{**} + \gamma_2^{**} = \beta_2$, the optimal piece rate for type θ_2 in the full-information case.

B.3.1 Proof of Proposition 2

To prove this proposition, in Subsection B.3.1.1 we consider the subgame conditional on paying G and offering β^{**} and γ_2^{**} and show first that the strategies in 2(a) and 3(a) in the proposition are part of a perfect Bayesian subgame equilibrium and second that the equilibrium replicates the payoff to the principal if the technology type were fully revealed to the principal at the beginning of Stage 1 (before setting the contract). This implies that, conditional on paying G, the principal cannot do better by choosing a subgame with a different β and γ_2. In Subsection B.3.1.2, we consider the principal's decision about whether to pay G and show that the strategies outlined in the proposition form an equilibrium of the full game.

B.3.1.1 Subgame with G paid, $\beta = \beta^{**}$, $\gamma_2 = \gamma_2^{**}$

As before, for types θ_1 and θ_3, $\frac{\partial}{\partial a}\widehat{U}(a,\beta,\theta_1) < 0$ and $\frac{\partial}{\partial a}\widehat{U}(a,\beta,\theta_3) > 0$ for all $a \in [0,1]$, since $s_1 < s_0$ and $s_3 > s_0$. (Refer to (B9).) For type θ_2, expected utility is now given by:

$$\widehat{U}(a,\beta,\gamma_2,\theta_2) = a\left(\frac{(\beta+\gamma_2)^2 s_2^2}{2}\right) + (1-a)\left(\frac{\beta^2 s_0^2}{2}\right) \tag{B24}$$

Here condition (B7a) implies that $\frac{\partial}{\partial a}\widehat{U}(a,\beta,\gamma_2,\theta_2) > 0$ for $\beta = \beta^{**}$, $\gamma_2 = \gamma_2^{**}$ and hence that agent type θ_2 wants to *encourage* adoption.[19] In this subgame, a result analogous to Lemma 1 holds.[20]

Lemma 4. *In any equilibrium with $a_{min} < a_{max}$ the following statements are true:*

$$\sum_{m\in M_{a_{min}}} q(m|\theta_1) = 1 \qquad \sum_{m\in M_{a_{max}}} q(m|\theta_1) = 0 \qquad \sum_{m\notin\{M_{a_{min}}\cup M_{a_{max}}\}} q(m|\theta_1) = 0 \tag{B25a}$$

$$\sum_{m\in M_{a_{min}}} q(m|\theta_2) = 0 \qquad \sum_{m\in M_{a_{max}}} q(m|\theta_2) = 1 \qquad \sum_{m\notin\{M_{a_{min}}\cup M_{a_{max}}\}} q(m|\theta_2) = 0 \tag{B25b}$$

$$\sum_{m\in M_{a_{min}}} q(m|\theta_3) = 0 \qquad \sum_{m\in M_{a_{max}}} q(m|\theta_3) = 1 \qquad \sum_{m\notin\{M_{a_{min}}\cup M_{a_{max}}\}} q(m|\theta_3) = 0 \tag{B25c}$$

For $m \in M_{a_{min}}$, $p(\theta_1|m) = 1$, $p(\theta_2|m) = 0$, and $p(\theta_3|m) = 0$. For $m \in M_{a_{max}}$, $p(\theta_1|m) = 0$, $p(\theta_2|m) = \frac{\rho_2}{\rho_2+\rho_3}$, and $p(\theta_3|m) = \frac{\rho_3}{\rho_2+\rho_3}$.

In any equilibrium with $a_{min} = a_{max}$ (letting $a^* \equiv a_{min} = a_{max}$), the following statements are true:

$$\sum_{m\in M_{a^*}} q(m|\theta_i) = 1 \;\forall\; \theta_i, \qquad \sum_{m\notin M_{a^*}} q(m|\theta_i) = 0 \;\forall\; \theta_i \tag{B26}$$

For $m \in M_{a^*}$, $p(\theta_1|m) = \rho_1$, $p(\theta_2|m) = \rho_2$, and $p(\theta_3|m) = \rho_3$.

Proof. Given that $\frac{\partial}{\partial a}\widehat{U}(a,\beta,\theta_1) < 0$, $\frac{\partial}{\partial a}\widehat{U}(a,\beta,\theta_2) > 0$ and $\frac{\partial}{\partial a}\widehat{U}(a,\beta,\theta_3) > 0$ for all $a \in [0,1]$, if $a_{min} \neq a_{max}$ then in order for (B12) to be satisfied it must be the case that an agent of type θ_1 chooses messages that induce the lowest possible probability of adoption, a_{min}, and agents of type θ_2 or θ_3 choose a message that induces the highest possible probability of adoption, a_{max}. Given these reporting rules, the updating of the principal's beliefs follow by Bayes' rule. If $a_{min} = a_{max}$

[19] To see this, note that condition (B7a) implies $\pi_2(\beta_2) > \pi_0(\beta_0)$, since $\pi_2(\beta)$ is increasing for $\beta \in [\beta_0, \beta_2)$. Using (B3) and (B4), we have that $\frac{((\beta^{**}+\gamma_2^{**})s_2)^2}{2} > \frac{(\beta^{**}s_0)^2}{2}$, which in turn implies $\frac{\partial}{\partial a}\widehat{U}(a,\beta,\gamma_2,\theta_2) > 0$.

[20] This lemma holds in any subgame in which $\gamma_2 > \beta\left(\frac{s_0-s_2}{s_2}\right)$ and hence $\frac{\partial}{\partial a}\widehat{U}(a,\beta,\gamma_2,\theta_2) > 0$.

then trivially all messages sent by the agent induce the same action and hence are essentially equivalent ($M_{a_{min}} = M_{a_{max}} = M_{a^*}$). Given this, the principal does not update. □

As in Lemma 1, we have shown that there exist at most two equilibria in this subgame. We can also show existence of the informative subgame equilibrium.

Lemma 5. *If G is paid, $\beta^{**} = \frac{p-c_0}{2}$ and $\gamma_2^{**} = \frac{c_0-c_2}{2}$, there exists a perfect Bayesian subgame equilibrium with the strategies outlined in Proposition 2.*

Proof. In the subgame equilibrium outlined by Proposition 2 (after paying G), the agent's reporting rules are given by (B25a)-(B25c), where $a_{min} = 0$, $a_{max} = 1$, and the principal's action rule is $a(m) = 0 \ \forall \ m \in \ M_0$, $a(m) = 1 \ \forall \ m \in \ M_1$.

To prove that this is a subgame equilibrium, it suffices to show that neither agent nor principal wants to deviate. Since $\frac{\partial}{\partial a}\widehat{U}(a, \beta, \theta_1) < 0$, $\frac{\partial}{\partial a}\widehat{U}(a, \beta, \theta_2) > 0$ and $\frac{\partial}{\partial a}\widehat{U}(a, \beta, \theta_3) > 0$ for all $a \in [0, 1]$, the agent has no incentive to deviate. Now consider the principal's decision. Given Lemma 4, if the agent says $m \in M_0$ ("technology is bad"), the condition for the principal not to deviate is:

$$\pi_0(\beta^{**}) > \pi_1(\beta^{**})$$

which is satisfied for $\beta = \beta^{**} = \beta_0$ by Remark 3. If the agent says $m \in M_1$ ("technology is good"), the condition for the principal not to deviate is:

$$\pi_0(\beta_0) \leq \left(\frac{\rho_2}{\rho_2 + \rho_3}\right) \pi_2(\beta_2) + \left(\frac{\rho_3}{\rho_2 + \rho_3}\right) \pi_3(\beta_0) \tag{B27}$$

since $\beta^{**} = \beta_0$ and $\beta^{**} + \gamma_2^{**} = \beta_2$. Since $\pi_2(\beta)$ is increasing for $\beta \in [\beta_0, \beta_2)$, condition (B7a) implies $\pi_0(\beta_0) < \pi_2(\beta_2)$ as noted in Remark 1. Similarly $\pi_0(\beta_0) < \pi_3(\beta_0)$ follows from condition (B7b) as noted in Remark 2. Hence the right-hand-side of (B27) is a weighted average of two quantities greater than $\pi_0(\beta_0)$ and (B27) is satisfied. □

In this subgame, the principal's ex-ante expected profit (refer to (B16)) is:

$$\begin{aligned} \pi^{**}(\beta^{**}, \gamma_2^{**}) &= \rho_1\widehat{\pi}(0, \beta_0, \theta_1) + \rho_2\widehat{\pi}(1, \beta_2, \theta_2) + \rho_3\widehat{\pi}(1, \beta_0, \theta_3) - G \\ &= \rho_1\pi_0(\beta_0) + \rho_2\pi_2(\beta_2) + \rho_3\pi_3(\beta_0) - G \end{aligned} \tag{B28}$$

Compare this payoff to what would obtain in the conditional contracts case if the technology were fully revealed to the principal at the beginning of Stage 1. Recalling (B4), the principal would offer β_0 under the existing technology and types θ_1 and θ_3 and β_2 under type θ_2. She would adopt types θ_2 and θ_3, and not adopt type θ_1.[21] Her ex-ante expected profit would be equal to (B28). That is, when G is paid, the conditional contract with β^{**} and γ_2^{**} exactly replicates the payoff to the principal if she observed the technology type herself at the beginning of Stage 1. The insight of Blackwell (1953), mentioned above, is that the principal cannot do better than she would do with full information. Hence conditional on paying G, no subgame can offer the principal a better payoff than the one she receives from offering $(\beta^{**}, \gamma_2^{**})$. Conditional on paying G, the principal has no incentive to deviate to offer a different β or γ_2.

[21] To see this, note that $\pi_0(\beta_0) > \pi_1(\beta_1)$ and $\pi_0(\beta_0) < \pi_3(\beta_0)$ from Remarks 2-3, and $\pi_0(\beta_0) < \pi_2(\beta_2)$ from Remark 1 (and the fact that $\pi_2(\beta_2) \geq \pi_2(\beta_0)$ since β_2 is the optimal wage under type θ_2).

B.3.1.2 Principal's choice whether to pay G

We now consider whether the principal has an incentive to deviate from the strategies outlined in Proposition 2 at Stage 0, when choosing whether to pay G. Suppose $\rho_2 > \frac{G}{\pi_2(\beta_2)-\pi_0(\beta_0)}$ (i.e. that (B23) is satisfied). If the principal pays G, her payoff is given by $\pi^{**}(\beta^{**}, \gamma_2^{**})$ in (B28) above. If she were to deviate and not pay G, the resulting subgame would identical to the interaction analyzed in Proposition 1. In this case, the maximal payoff she could obtain would be $\pi^*(\beta_0)$ from (B17). If (B23) holds, then $\pi^{**}(\beta^{**}, \gamma_2^{**}) > \pi^*(\beta_0)$ and she does not have an incentive to deviate to receive the maximal payoff from not paying G. If this is true for the maximal payoff from deviating, it is also true for all other payoffs from deviating.

Similarly, suppose $\rho_2 \leq \frac{G}{\pi_2(\beta_2)-\pi_0(\beta_0)}$ (i.e. that (B23) is not satisfied).[22] If the principal does not pay G and offers $\beta = \beta_0 = \frac{p-c_0}{2}$, her payoff is given by $\pi^*(\beta_0)$ from (B17). If she were to deviate and pay G, the maximal payoff she could obtain is $\pi^{**}(\beta^{**}, \gamma_2^{**})$ in (B28). If (B23) holds, then $\pi^{**}(\beta^{**}, \gamma_2^{**}) < \pi^*(\beta_0)$ and she does not have an incentive to deviate to receive the maximal payoff from paying G. If this is true for the maximal payoff from deviating, it is also true for all other payoffs from deviating.

We have already shown that neither player has an incentive to deviate in the resulting subgames. Hence the strategies described in Proposition 2 form a perfect Bayesian equilibrium.

B.3.2 Uniqueness of the equilibrium in Proposition 2

As with Proposition 1, if we are willing to assume that players coordinate on the most informative equilibrium in cheap-talk interactions, the equilibrium described by Proposition 2 is unique. Recall that in the case where $\rho_2 \leq \frac{G}{\pi_2(\beta_2)-\pi_0(\beta_0)}$, the equilibrium is identical to that in Proposition 1 which was unique if players coordinated on the most informative equilibrium of each subgame. Thus, to prove that the equilibrium in Proposition 2 is unique, we only need to show that the equilibrium is unique in the case where $\rho_2 > \frac{G}{\pi_2(\beta_2)-\pi_0(\beta_0)}$. We do this in two steps. First, Lemma 6 shows that in any subgame conditional on a wage contract, there are at most two equilibria and these can be strictly ordered in terms of which is most informative. Second, Lemma 7 shows that the principal's ex-ante expected profits under the contract $(\beta^{**}, \gamma_2^{**})$ are strictly greater than under any other possible wage contract, and so the principal prefers that subgame to any other. Hence, the only equilibrium of the supergame conditional on paying G is the one characterized by the Proposition-2 strategies on the equilibrium path.

Lemma 6. *In any subgame conditional on a wage contract, there are at most two equilibria, one of which is strictly more informative that the other.*

Proof. Under conditional contracts, the principal can offer contracts of the form:

$$\begin{aligned} w(q) &= \alpha + (\beta + \gamma_1)q \qquad \text{if } c = c_1 \\ w(q) &= \alpha + (\beta + \gamma_2)q \qquad \text{if } c = c_2 \\ w(q) &= \alpha + (\beta + \gamma_3)q \qquad \text{if } c = c_3 \\ w(q) &= \alpha + \beta q \qquad \text{if } c = c_0 \end{aligned} \tag{B29}$$

As in Proposition 1, $\alpha > 0$ is costly and does not induce effort and so the principal will always set $\alpha = 0$. We denote a set of piece-rate contracts by $(\beta, \gamma_1, \gamma_2, \gamma_3)$. (Note that the contract in

[22] We implicitly assume that the principal prefers the simpler option of not paying G if $G = \rho_2(\pi_2(\beta_2) - \pi_0(\beta_0))$.

Proposition 2 corresponds to $(\frac{p-c_0}{2}, 0, \frac{c_0-c_2}{2}, 0)$ in this notation.) Define the expected utility of the agent of type θ, prior to the adoption decision of the principal:

$$\widehat{\widehat{U}}(a,\beta,\gamma_i,\theta_i) = aU(\beta+\gamma_i,\theta_i) + (1-a)U(\beta,\theta_0) \tag{B30}$$

where $U(\cdot,\cdot)$ is as defined in (B2) and where $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_i,\theta_i) = U(\beta+\gamma_i,\theta_i) - U(\beta,\theta_0)$ is independent of a. We assume a tie breaking rule where the principal and agent prefer the status quo technology if $U(\beta+\gamma_i,\theta_i) = U(\beta,\theta_0)$.

Define $\widehat{\widehat{\pi}}(a,\beta,\gamma_i,\theta_i)$ as the expected profit of the principal prior to the adoption decision, conditional on θ_i:

$$\widehat{\widehat{\pi}}(a,\beta,\gamma_i,\theta_i) \quad = \quad a\pi_i(\beta+\gamma_i) + (1-a)\pi_0(\beta) \tag{B31}$$

where $\pi_i(\cdot)$ is as defined in in (B3). As before, the principal's posterior beliefs after receiving message m, by Bayes' rule, are given by (B11). An equilibrium in the cheap-talk subgame is a family of reporting rules $q(m|\theta)$ for the agent (sender) and an action rule $a(m)$ for the principal (receiver) such that the following conditions hold:

1. If $q(m^*|\theta) > 0$ then
$$m^* = \arg\max_{m\in M} \widehat{\widehat{U}}(a(m),\beta,\gamma_i,\theta) \tag{B32}$$

2. For each m,
$$a(m) = \arg\max_{a\in[0,1]} \sum_{\theta\in\Theta} \widehat{\widehat{\pi}}(a,\beta,\gamma_i,\theta)p(\theta|m) \tag{B33}$$

That is, the rule of each player must be a best response to the rule of the other player.

For there to be more than one equilibrium, it must be the case that $a_{min} < a_{max}$. In any equilibrium with $a_{min} < a_{max}$, the following statements are true:

$$\sum_{m\in M_{a_{min}}} q(m|\theta_1) = 1 \quad \sum_{m\in M_{a_{max}}} q(m|\theta_1) = 0 \quad \sum_{m\notin\{M_{a_{min}}\cup M_{a_{max}}\}} q(m|\theta_1) = 0 \text{ if } \frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_1,\theta_1) \le 0 \tag{B34}$$

$$\sum_{m\in M_{a_{min}}} q(m|\theta_1) = 0 \quad \sum_{m\in M_{a_{max}}} q(m|\theta_1) = 1 \quad \sum_{m\notin\{M_{a_{min}}\cup M_{a_{max}}\}} q(m|\theta_1) = 0 \text{ if } \frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_1,\theta_1) > 0 \tag{B35}$$

$$\sum_{m\in M_{a_{min}}} q(m|\theta_2) = 1 \quad \sum_{m\in M_{a_{max}}} q(m|\theta_2) = 0 \quad \sum_{m\notin\{M_{a_{min}}\cup M_{a_{max}}\}} q(m|\theta_2) = 0 \text{ if } \frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_2,\theta_2) \le 0 \tag{B36}$$

$$\sum_{m\in M_{a_{min}}} q(m|\theta_2) = 0 \quad \sum_{m\in M_{a_{max}}} q(m|\theta_2) = 1 \quad \sum_{m\notin\{M_{a_{min}}\cup M_{a_{max}}\}} q(m|\theta_2) = 0 \text{ if } \frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_2,\theta_2) > 0 \tag{B37}$$

$$\sum_{m\in M_{a_{min}}} q(m|\theta_3)=1 \quad \sum_{m\in M_{a_{max}}} q(m|\theta_3)=0 \qquad \sum_{m\notin\{M_{a_{min}}\cup M_{a_{max}}\}} q(m|\theta_3)=0 \text{ if } \frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_3,\theta_3)\leq 0 \tag{B38}$$

$$\sum_{m\in M_{a_{min}}} q(m|\theta_3)=0 \quad \sum_{m\in M_{a_{max}}} q(m|\theta_3)=1 \qquad \sum_{m\notin\{M_{a_{min}}\cup M_{a_{max}}\}} q(m|\theta_3)=0 \text{ if } \frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_3,\theta_3)> 0 \tag{B39}$$

Given the signs of $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_1,\theta_1,$ $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_2,\theta_2)$ and $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_3,\theta_3)$ for all $a\in[0,1]$, if $a_{min}\neq a_{max}$ then in order for (B32) to be satisfied it must be the case that an agent of type θ_i chooses messages that induce the lowest possible probability of adoption, a_{min}, if $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_i,\theta_i)\leq 0$, and chooses messages that induce the highest possible probability of adoption, a_{max}, if $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_i,\theta_i)> 0$. For $a_{min}<a_{max}$, both messages must be used. Given these reporting rules, the updating of the principal's beliefs follow by Bayes' rule.

There are eight possible cases to consider:

1. If $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_1,\theta_1)\leq 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_2,\theta_2)\leq 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_3,\theta_3)\leq 0$: For $m\in M_{a_{min}}$, $p(\theta_1|m)=\rho_1$, $p(\theta_2|m)=\rho_2$, and $p(\theta_3|m)=\rho_3$. For $m\in M_{a_{max}}$, $p(\theta_1|m)=0$, $p(\theta_2|m)=0$, and $p(\theta_3|m)=0$. As only a_{min} is used there is no equilibrium with $a_{min}<a_{max}$. There is a single equilibrium with $a_{min}=a_{max}$.

2. If $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_1,\theta_1)\leq 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_2,\theta_2)\leq 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_3,\theta_3)> 0$: For $m\in M_{a_{min}}$, $p(\theta_1|m)=\frac{\rho_1}{\rho_1+\rho_2}$, $p(\theta_2|m)=\frac{\rho_2}{\rho_1+\rho_2}$, and $p(\theta_3|m)=0$. For $m\in M_{a_{max}}$, $p(\theta_1|m)=0$, $p(\theta_2|m)=0$, and $p(\theta_3|m)=1$. As both a_{min} and a_{max} are used, there are at most two equilibria, an informative one with $a_{min}<a_{max}$ and potentially a babbling one with $a_{min}=a_{max}$.

3. If $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_1,\theta_1)\leq 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_2,\theta_2)> 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_3,\theta_3)\leq 0$: For $m\in M_{a_{min}}$, $p(\theta_1|m)=\frac{\rho_1}{\rho_1+\rho_3}$, $p(\theta_2|m)=0$, and $p(\theta_3|m)=\frac{\rho_3}{\rho_1+\rho_3}$. For $m\in M_{a_{max}}$, $p(\theta_1|m)=0$, $p(\theta_2|m)=1$, and $p(\theta_3|m)=0$. As both a_{min} and a_{max} are used, there are at most two equilibria, an informative one with $a_{min}<a_{max}$ and potentially a babbling one with $a_{min}=a_{max}$.

4. If $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_1,\theta_1)\leq 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_2,\theta_2)> 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_3,\theta_3)> 0$: For $m\in M_{a_{min}}$, $p(\theta_1|m)=1$, $p(\theta_2|m)=0$, and $p(\theta_3|m)=0$. For $m\in M_{a_{max}}$, $p(\theta_1|m)=0$, $p(\theta_2|m)=\frac{\rho_2}{\rho_2+\rho_3}$, and $p(\theta_3|m)=\frac{\rho_3}{\rho_2+\rho_3}$. As both a_{min} and a_{max} are used, there are at most two equilibria, an informative one with $a_{min}<a_{max}$ and potentially a babbling one with $a_{min}=a_{max}$.

5. If $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_1,\theta_1)>$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_2,\theta_2)> 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_3,\theta_3)> 0$: For $m\in M_{a_{min}}$, $p(\theta_1|m)=0$, $p(\theta_2|m)=0$, and $p(\theta_3|m)=0$. For $m\in M_{a_{max}}$, $p(\theta_1|m)=\rho_1$, $p(\theta_2|m)=\rho_2$, and $p(\theta_3|m)=\rho_3$. As only a_{min} is used there is no equilibrium with $a_{min}<a_{max}$. There is a single equilibrium with $a_{min}=a_{max}$..

6. If $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_1,\theta_1)> 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_2,\theta_2)\leq 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a,\beta,\gamma_3,\theta_3)\leq 0$: For $m\in M_{a_{min}}$, $p(\theta_1|m)=0$, $p(\theta_2|m)=\frac{\rho_2}{\rho_2+\rho_3}$, and $p(\theta_3|m)=\frac{\rho_3}{\rho_2+\rho_3}$. For $m\in M_{a_{max}}$, $p(\theta_1|m)=1$, $p(\theta_2|m)=0$, and $p(\theta_3|m)=0$. As both a_{min} and a_{max} are used, there are at most two equilibria, an informative one with $a_{min}<a_{max}$ and potentially a babbling one with $a_{min}=a_{max}$.

7. If $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a, \beta, \gamma_1, \theta_1) > 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a, \beta, \gamma_2, \theta_2) \leq 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a, \beta, \gamma_3, \theta_3) > 0$: For $m \in M_{a_{min}}$, $p(\theta_1|m) = 0$, $p(\theta_2|m) = 1$, and $p(\theta_3|m) = 0$. For $m \in M_{a_{max}}$, $p(\theta_1|m) = \frac{\rho_1}{\rho_1+\rho_3}$, $p(\theta_2|m) = 0$, and $p(\theta_3|m) = \frac{\rho_3}{\rho_1+\rho_3}$. As both a_{min} and a_{max} are used, there are at most two equilibria, an informative one with $a_{min} < a_{max}$ and potentially a babbling one with $a_{min} = a_{max}$.

8. If $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a, \beta, \gamma_1, \theta_1) > 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a, \beta, \gamma_2, \theta_2) > 0$, $\frac{\partial}{\partial a}\widehat{\widehat{U}}(a, \beta, \gamma_3, \theta_3) \leq 0$: For $m \in M_{a_{min}}$, $p(\theta_1|m) = 0$, $p(\theta_2|m) = 0$, and $p(\theta_3|m) = 1$. For $m \in M_{a_{max}}$, $p(\theta_1|m) = \frac{\rho_1}{\rho_1+\rho_2}$, $p(\theta_2|m) = \frac{\rho_2}{\rho_1+\rho_2}$, and $p(\theta_3|m) = 0$. As both a_{min} and a_{max} are used, there are at most two equilibria, an informative one with $a_{min} < a_{max}$ and potentially a babbling one with $a_{min} = a_{max}$.

Given $(\beta, \gamma_1, \gamma_2, \gamma_3)$, in cases 2-4 and 6-8 there can exist at most two equilibria in each subgame: one more-informative equilibrium in which $a_{min} \neq a_{max}$ and two agent types are indistinguishable from each other but are distinguishable from the third type; and one less-informative equilibrium in which $a_{min} = a_{max}$ and the principal ignores the message from the agent (a "babbling" type). In cases 1 and 5, only a single equilibrium exists in which $a_{min} = a_{max}$ and the principal ignores the message from the agent.

□

Lemma 7. *The principal's ex-ante expected profits under the contract* $(\beta^{**}, \gamma_2^{**})$ — $(\beta^{**}, 0, \gamma_2^{**}, 0)$ *using the notation of Lemma 6 — are strictly greater than under any other possible wage contract.*

Proof. From (B4), the maximal ex-ante expected profits are equal to:

$$\begin{aligned} \pi^{max} &= \rho_1 max(\pi_0(\beta_0), \pi_1(\beta_1)) + \rho_2 max(\pi_0(\beta_0), \pi_2(\beta_2)) + \rho_3 max(\pi_0(\beta_0), \pi_3(\beta_3)) - G \\ &= \rho_1\pi_0(\beta_0) + \rho_2\pi_2(\beta_2) + \rho_3\pi_3(\beta_3) - G \end{aligned} \tag{B40}$$

where the second line follows from the fact $\pi_0(\beta_0) > \pi_1(\beta_1)$ and $\pi_0(\beta_0) < \pi_3(\beta_0)$ from Remarks 2-3 (and the fact that $\beta_0 = \beta_3$ since $c_0 = c_3$), and $\pi_0(\beta_0) < \pi_2(\beta_2)$ from Remark 1 (and the fact that $\pi_2(\beta_2) \geq \pi_2(\beta_0)$ since β_2 is the optimal wage under type θ_2). This is the expected profit both under full information and under the contract $(\beta^{**}, 0, \gamma_2^{**}, 0)$. From Blackwell (1953), we know that the principal cannot do better than this.

It remains to show that other wage contracts and/or adoption patterns cannot provide equally large ex-ante expected profits. First, if the principal pays optimal piece rates, β_i for type θ_i, profits will be strictly smaller under any adoption patterns other than adopt θ_2 and θ_3, do not adopt θ_1: $\pi_0(\beta_0) > \pi_1(\beta_1)$, $\pi_0(\beta_0) < \pi_2(\beta_2)$, and $\pi_0(\beta_0) < \pi_3(\beta_3)$ are all strict inequalities. Second, profits at optimal piece rates, $\pi_i(\beta_i)$ for type θ_i, are also strictly higher under piece rate β_i than under any other piece rate: $\pi_i(\beta)$ is twice continuously differentiable and strictly concave as $\frac{\partial^2\pi_i(\beta)}{\partial\beta^2} = -2s_i^2 < 0$; hence, the stationary point β_i (the value of β for which $\frac{\partial\pi_i(\beta)}{\partial\beta} = 0$) is a global maximizer and any deviations from these optimal piece-rates will strictly reduce profits.

□

B.4 Theoretical prediction for incentive intervention

Here we prove that a lump-sum payment offered by a third-party experimenter conditional on the technology being revealed to be type θ_2, if sufficiently large, can induce the agent to reveal truthfully and lead to adoption of the type θ_2 technology. Suppose that the players have coordinated on the equilibrium described in Proposition 1 (or, equivalently, on the equilibrium in Proposition 2 where G is large relative to the expected benefits of adopting type θ_2 and hence (B23) is not satisfied and the conditional contracts are not offered.) In Stage 1, the principal offers wage contract ($\alpha^* = 0, \beta^* = \frac{p-c_0}{2}$). Now suppose that in Stage 2 a third-party experimenter, without forewarning, offers a conditional lump-sum payment, L, conditional on the marginal cost being c_2. Suppose that players place zero prior on this event in Stages 0 and 1. For the subgame that follows this intervention, we have the following.

Proposition 3. *In the subgame described above, if*

$$L > \frac{(p-c_0)^2(s_0^2 - s_2^2)}{8} \tag{B41}$$

then the following strategies are part of a perfect Bayesian subgame equilibrium.

1. *In Stage 3, the agent:*

   (a) *says "technology is bad" if the technology is type θ_1,*

   (b) *says "technology is good" if the technology is type θ_2 or θ_3.*

2. *In Stage 4, the principal:*

   (a) *adopts if the agent says "technology is good",*

   (b) *does not adopt if the agent says "technology is bad".*

Proof. It again suffices to show that there is no profitable deviation for either principal or agent. In the subgame equilibrium outlined by Proposition 3, the agent's reporting rules are given by (B25a)-(B25c), where $a_{min} = 0$, $a_{max} = 1$, and the principal's action rule is $a(m) = 0 \ \forall m \in \ M_0$, $a(m) = 1 \ \forall m \in \ M_1$. That the agent does not want to deviate if he is of type θ_1 or θ_3 follows from the fact (from (B9)) that $\frac{\partial}{\partial a}\widehat{U}(a, \beta, \theta_1) < 0$ and $\frac{\partial}{\partial a}\widehat{U}(a, \beta, \theta_3) > 0$ for all $a \in [0, 1]$. That the agent does not want to deviate if he is of type θ_2 follows from the fact that

$$\widehat{U}(a, \beta^*, L, \theta_2) = a\left(\frac{(\beta^*)^2 s_2^2}{2} + L\right) + (1-a)\left(\frac{(\beta^*)^2 s_0^2}{2}\right) \tag{B42}$$

and (B41) implies $\frac{\partial}{\partial a}\widehat{U}(a, \beta^*, L, \theta_2) > 0$ for all $a \in [0, 1]$.

Now consider the principal's decision. If $m \in M_0$ ("technology is bad"), then the condition for the principal not to deviate can be written:

$$\pi_0(\beta_0) \geq \pi_1(\beta_0)$$

which is true by Remark 3. If $m \in M_1$ ("technology is good"), then the condition for the principal not to deviate can be written:

$$\left(\frac{\rho_2}{\rho_2+\rho_3}\right)\pi_2(\beta_0) + \left(\frac{\rho_3}{\rho_2+\rho_3}\right)\pi_3(\beta_0) \geq \pi_0(\beta_0) \tag{B43}$$

where there is no L on the left hand side since we, rather than the principal, pay the lump-sum bonus. Since $\pi_2(\beta_0) > \pi_0(\beta_0)$ by Remark 1 and $\pi_3(\beta_0) > \pi_0(\beta_0)$ is true by Remark 2, the left-hand side is a weighted average of two quantities greater than $\pi_0(\beta_0)$ and (B43) holds. Hence neither the agent nor the principal has an incentive to deviate. □

B.5 Miscellaneous Proofs

The following remarks establish several properties of the profit functions defined in (B3).

Remark 1. *Given the definition of $\pi_i(\cdot)$ in (B3), of β_i in (B4), and condition (B7a), we have:*

$$\pi_2(\beta) - \pi_0(\beta) \begin{cases} < 0 & \text{if } 0 < \beta < \hat{\beta}_2 \\ = 0 & \text{if } \beta = \hat{\beta}_2 \\ > 0 & \text{if } \beta > \hat{\beta}_2 \end{cases}$$

where $0 < \hat{\beta}_2 < \beta_0$.

Proof. From (B3), we can write:

$$\pi_2(\beta) - \pi_0(\beta) = \left({s_0}^2 - {s_2}^2\right)(\beta - \Omega)^2 - \Omega^2\left({s_0}^2 - {s_2}^2\right) - F \tag{B44}$$

where $\Omega = \frac{s_0^2\beta_0 - s_2^2\beta_2}{s_0^2 - s_2^2}$. This defines a convex parabola with vertex at $\left(\Omega, -\Omega^2\left({s_0}^2 - {s_2}^2\right) - F\right)$. Setting $\pi_2(\beta) - \pi_0(\beta) = 0$ gives two critical values of β. Since we are requiring $\beta > 0$, we ignore the negative root. The positive root defines the value of $\hat{\beta}_2$: $\hat{\beta}_2 = \Omega + \sqrt{\Omega^2 + \frac{F}{{s_0}^2 - {s_2}^2}}$. Condition (B7a) requires that $\pi_2(\beta_0) - \pi_0(\beta_0) > 0$ and hence that $\beta_0 > \hat{\beta}_2$. □

Remark 2. *Given the definition of $\pi_i(\cdot)$ in (B3), of β_i in (B4), and condition (B7b), we have:*

$$\pi_3(\beta) - \pi_0(\beta) \begin{cases} < 0 & \text{if } \beta < \hat{\beta}_3 \\ = 0 & \text{if } \beta = \hat{\beta}_3 \\ > 0 & \text{if } \hat{\beta}_3 < \beta < \hat{\hat{\beta}}_3 \\ = 0 & \text{if } \beta = \hat{\hat{\beta}}_3 \\ < 0 & \text{if } \beta > \hat{\hat{\beta}}_3 \end{cases}$$

where $0 < \hat{\beta}_3 < \beta_0 < \beta_2 < \hat{\hat{\beta}}_3 < 2\beta_0$.

Proof. From (B3), we can write:

$$\pi_3(\beta) - \pi_0(\beta) = -\left({s_3}^2 - {s_0}^2\right)(\beta - \beta_0)^2 + \beta_0^2\left({s_3}^2 - {s_0}^2\right) - F \tag{B45}$$

This defines a concave parabola with vertex at $\left(\beta_0, \beta_0^2\left({s_3}^2 - {s_0}^2\right) - F\right)$. Condition (B7b) implies that $\pi_3(\beta_0) > \pi_0(\beta_0)$, since $\pi_3(\beta) - \pi_0(\beta)$ is decreasing over $(\beta_0, \beta_2]$, and this in turn implies $\beta_0^2\left({s_3}^2 - {s_0}^2\right) - F > 0$. Setting $\pi_3(\beta) - \pi_0(\beta) = 0$ defines the values of the roots: $\hat{\beta}_3 = \beta_0 - \sqrt{\beta_0^2 - \frac{F}{{s_3}^2 - {s_0}^2}}$, $\hat{\hat{\beta}}_3 = \beta_0 + \sqrt{\beta_0^2 - \frac{F}{{s_3}^2 - {s_0}^2}}$. The facts that $0 < \hat{\beta}_3 < \beta_0 < \hat{\hat{\beta}}_3 < 2\beta_0$ follow directly from the expressions for $\hat{\beta}_3$ and $\hat{\hat{\beta}}_3$. Condition (B7b) requires that $\pi_3(\beta_2) - \pi_0(\beta_2) > 0$ and hence that $\beta_2 < \hat{\hat{\beta}}_3$. □

Remark 3. *Given the definition of $\pi_i(\cdot)$ in (B3) and of β_i in (B4), we have:*

$$\pi_1(\beta) - \pi_0(\beta) \begin{cases} < 0 & \text{if } 0 < \beta < \hat{\hat{\beta}}_1 \\ = 0 & \text{if } \beta = \hat{\hat{\beta}}_1 \\ > 0 & \text{if } \beta > \hat{\hat{\beta}}_1 \end{cases}$$

where $\hat{\hat{\beta}}_1 > 2\beta_0 > \hat{\hat{\beta}}_3$.

Proof. From (B3), we can write

$$\pi_1(\beta) - \pi_0(\beta) = \left({s_0}^2 - {s_1}^2\right)(\beta - \beta_0)^2 - \left({s_0}^2 - {s_1}^2\right)\beta_0^2 - F \tag{B46}$$

which is a convex parabola with roots $\hat{\beta}_1 = \beta_0 - \sqrt{\beta_0^2 + \frac{F}{{s_0}^2 - {s_1}^2}} < 0$ and $\hat{\hat{\beta}}_1 = \beta_0 + \sqrt{\beta_0^2 + \frac{F}{{s_0}^2 - {s_1}^2}}$. Note that $\hat{\beta}_1 < 0$. The fact that $\hat{\hat{\beta}}_1 > 2\beta_0$ follows immediately from the expression for $\hat{\hat{\beta}}_1$. The fact that $2\beta_0 > \hat{\hat{\beta}}_3$ is from Remark 2. □

Remark 4. *Conditions (B7a), (B7b) and (B7c) are compatible with each other.*

Intuitively, it is straightforward to see that condition (B7a) can be satisfied as long as F is not too large and the slower speed s_2 is sufficiently compensated by the lower costs c_2 (compared to s_0 and c_0). Condition (B7b) is satisfied as long as F is not too large and s_3 is sufficiently faster than s_0. Condition (B7c) is satisfied as long as the bad technology is sufficiently likely, i.e. ρ_1 is large, and the bad technology is sufficiently bad relative to the existing technology, i.e. s_1 is sufficiently low.

Proof. There are three conditions:

Condition (B7a)

$$\pi_2(\beta_0) > \pi_0(\beta_0)$$

which can be rewritten in terms of primitives as:

$$s_2^2\left(\frac{p - c_0}{2}\right)\left(p - \left(\frac{p - c_0}{2}\right) - c_2\right) - F > s_0^2\left(\frac{p - c_0}{2}\right)\left(p - \left(\frac{p - c_0}{2}\right) - c_0\right)$$

Condition (B7b)

$$\pi_3(\beta_2) > \pi_0(\beta_2)$$

which can be rewritten in terms of primitives as:

$$s_3^2\left(\frac{p - c_2}{2}\right)\left(p - \left(\frac{p - c_2}{2}\right) - c_0\right) - F > s_0^2\left(\frac{p - c_2}{2}\right)\left(p - \left(\frac{p - c_2}{2}\right) - c_0\right)$$

Condition (B7c)

$$\pi_0(\beta_0) > \widetilde{\pi}(\widetilde{\beta})$$

which can be rewritten in terms of primitives as:

$$s_0^2\left(\frac{p - c_0}{2}\right)\left(p - \left(\frac{p - c_0}{2}\right) - c_0\right) > \left(\sum_{i=1}^{3} \rho_i s_i^2\right)\left(\sum_{i=1}^{3} \frac{\rho_i s_i^2}{\sum_1^3 \rho_i s_i^2}\left(\frac{p - c_i}{2}\right)\right)^2 - F$$

To see that the three can be satisfied simultaneously, consider the following chain:

First, pick a s_0, c_0 and p such that $p - \left(\frac{p-c_0}{2}\right) - c_0$ is positive (i.e. so that the original technology is profitable at β_0).

Second, manipulate s_2 , c_2, s_3 and F to ensure both (B7a) and (B7b) are simultaneously satisfied (i.e. lower s_2 below s_0, lower c_2 below c_0, and raise s_3 above s_0). This is always possible as in the limit when $F \to 0$ and $s_2 \to s_0$ (B7a) will be strictly satisfied, and in the limit when $F \to 0$ (B7b) will be strictly satisfied.

Third, still holding fixed s_0, c_0 and p at the values from step 1, and holding fixed s_2 , c_2, s_3 and F at a combination that satisfies (B7a) and (B7b) from step 2, we can raise ρ_1 (and correspondingly lower ρ_2 and ρ_3 since $\sum_{i=1}^{3} \rho_i = 1$) and lower s_1 below s_0 to ensure that (B7c) holds. This will always be possible since in the limit when $\rho_1 \to 1$ and $s_1 \to 0$ (B7c) will be strictly satisfied.

□

Remark 5. *Given the definition of $\pi_i(\cdot)$ in (B3) and conditions (B7b) and (B7c), we have:*

$$Z(\beta) \equiv \left[\frac{\rho_1}{\rho_1+\rho_2}\pi_1(\beta) + \frac{\rho_2}{\rho_1+\rho_2}\pi_2(\beta)\right] - \pi_0(\beta) \quad \begin{cases} < 0 & \text{if } \hat{\beta}_2 \leq \beta < \overline{\beta} \\ = 0 & \text{if } \beta = \overline{\beta} \\ > 0 & \text{if } \beta > \overline{\beta} \end{cases}$$

for some $\overline{\beta} > \beta_0$.

Proof. We first consider $\beta = \beta_0$. By condition (B7c), $\pi_0(\beta_0) > \widetilde{\pi}(\widetilde{\beta})$. Since $\widetilde{\beta}$ is the optimal choice if the principal bases her decision only on her priors, it must be the case that $\widetilde{\pi}(\widetilde{\beta}) \geq \widetilde{\pi}(\beta_0)$. Hence $\pi_0(\beta_0) > \widetilde{\pi}(\beta_0)$. This in turn implies:

$$\begin{aligned} \pi_0(\beta_0) &> \rho_1\pi_1(\beta_0) + \rho_2\pi_2(\beta_0) + \rho_3\pi_3(\beta_0) \\ \pi_0(\beta_0) - \rho_3\pi_3(\beta_0) &> \rho_1\pi_1(\beta_0) + \rho_2\pi_2(\beta_0) \\ \pi_0(\beta_0) - \rho_3\pi_0(\beta_0) &> \rho_1\pi_1(\beta_0) + \rho_2\pi_2(\beta_0) \\ (\rho_1+\rho_2)\pi_0(\beta_0) &> \rho_1\pi_1(\beta_0) + \rho_2\pi_2(\beta_0) \\ 0 &> \left[\frac{\rho_1}{\rho_1+\rho_2}\pi_1(\beta_0) + \frac{\rho_2}{\rho_1+\rho_2}\pi_2(\beta_0)\right] - \pi_0(\beta_0) = Z(\beta_0) \end{aligned} \tag{B47}$$

where the third inequality follows from the fact that $\pi_3(\beta_0) > \pi_0(\beta_0)$ (from condition (B7b)).

Now consider $\beta \in [\hat{\beta}_2, \beta_0)$. Note that:

$$Z(\beta) = \frac{\rho_1[\pi_1(\beta) - \pi_0(\beta)] + \rho_2[\pi_2(\beta) - \pi_0(\beta)]}{\rho_1+\rho_2} \tag{B48}$$

By Remark 3, $\pi_1(\beta) - \pi_0(\beta) < 0$ in this region. From (B44), $\pi_2(\beta) - \pi_0(\beta)$ is strictly increasing over this region. Hence if $Z(\beta_0) < 0$ then must be the case that $Z(\beta) < 0$ for all $\beta \in [\hat{\beta}_2, \beta_0)$.

Now consider $\beta > \beta_0$. Using (B44), (B46) and (B48), we have:

$$\frac{\partial Z(\beta)}{\partial \beta} = \frac{1}{\rho_1+\rho_2}\left[2\rho_1\left({s_0}^2 - {s_2}^2\right)(\beta - \Omega) + 2\rho_2\left({s_0}^2 - {s_1}^2\right)(\beta - \beta_0)\right]$$

which is strictly positive and increasing in β for $\beta > \beta_0$ (noting that $\Omega < \hat{\beta}_2 < \beta_0$). Since $Z(\beta)$ is negative at β_0 and has strictly positive and increasing slope for $\beta > \beta_0$, it takes the value zero at a single point, call it $\overline{\beta}$, and is negative for $\beta \in (\beta_0, \overline{\beta})$ and positive for $\beta \in (\overline{\beta}, \infty)$.[23] □

[23] To see this, note that $Z(\beta) = \int_c^{\beta} Z'(\beta')d\beta' + C$ for some finite constants c and C. Hence, $\lim_{\beta \longrightarrow \infty} Z(\beta) = \int_c^{\infty} Z'(\beta')d\beta' + C$. Since $Z'(\beta')$ does not go to zero as β' goes to infinity, $Z(\beta) \longrightarrow \infty$ and so must cross zero.

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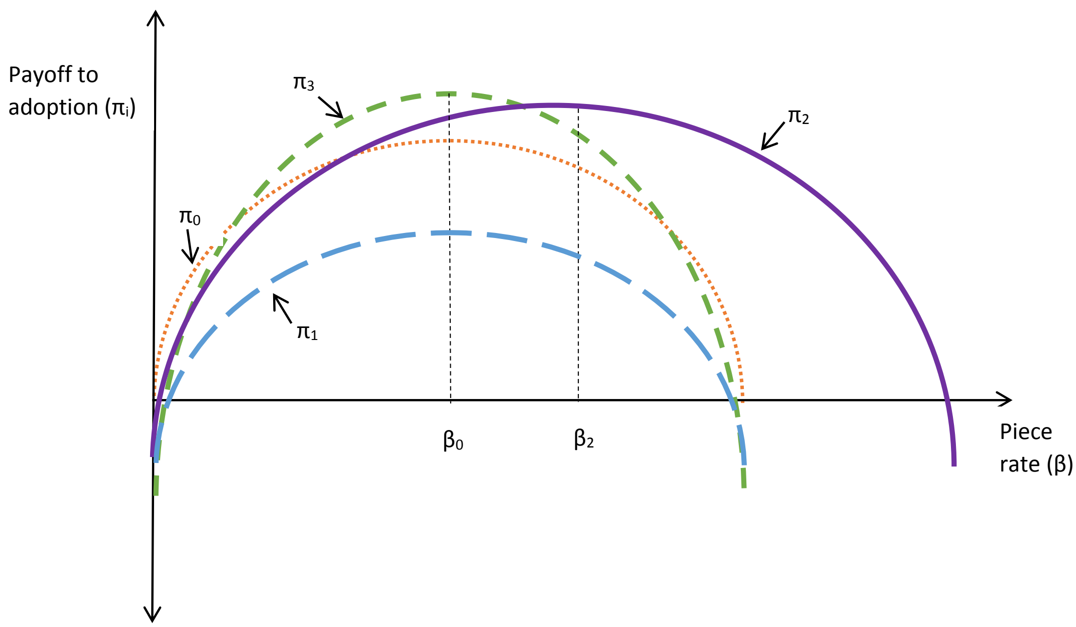
Figure B.1: Profit functions for different technology types



Notes: Figure illustrates the relative positions of the $\pi_i(\beta)$ functions (defined by (B3) in Subsection B.1.1) implied by the parameter restrictions (B7a)-(B7c).