

**Estimating Production Functions in Differentiated-Product  
Industries with Quantity Information and External Instruments\***

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Dec. 2020

ONLINE APPENDIX

## A Theory Appendix

### A.1 Construction of CES Price/Quantity Indexes, Output Side

#### A.1.1 Consumer's Minimization Problem

The Lagrangian corresponding to the first stage of the consumer's problem is given by:

$$\mathcal{L}^y = \sum_{j \in \Omega_{it}^y} Y_{ijt} P_{ijt} - \lambda \left( \left[ \sum_{j \in \Omega_{it}^y} (\varphi_{ijt} Y_{ijt})^{\frac{\sigma_i^y - 1}{\sigma_i^y}} \right]^{\frac{\sigma_i^y}{\sigma_i^y - 1}} - \tilde{Y}_{it} \right)$$

The first order condition with respect to product  $j$ ,  $\frac{\partial \mathcal{L}^y}{\partial Y_{ijt}} = 0$ , implies:

$$\frac{P_{ijt}}{\varphi_{ijt}} = \lambda (\varphi_{ijt} Y_{ijt})^{-\frac{1}{\sigma_i^y}} \tilde{Y}_{it}^{\frac{1}{\sigma_i^y}} \quad (\text{A1})$$

Raising both sides of this equation to the power  $1 - \sigma_i^y$ , summing over the  $j \in \Omega_{it}^y$ , using the definition of  $\tilde{P}_{it}$  in (2), and rearranging, we have:

$$\lambda = \tilde{P}_{it} \quad (\text{A2})$$

The second-order conditions for minimization are satisfied without further assumptions if and only if  $\sigma_i^y \in (0, 1) \cup (1, \infty)$ .<sup>1</sup> To see this, note that a necessary and sufficient condition for  $\mathcal{L}^y$  to be convex is that all principal minors of order  $r$  of the Hessian matrix of  $\mathcal{L}^y$  are non-negative, for  $r = 1, \dots, J$ , where  $J$  is the number of products in  $\Omega_{it}^y$ . (See e.g. Theorem 2.3.3 in Sydsaeter et al. (2005).) Chen (2012) shows (Theorem 5.1) that the determinant of the Hessian matrix of a CES function is always zero. This implies that the principal minor of order  $J$  of the Hessian for  $\mathcal{L}^y$  is zero. Furthermore, every principal minor of degree  $1 \leq r < J - 1$  corresponds to the determinant of the Hessian matrix of a CES aggregator with  $J - r$  varieties and hence is also zero.<sup>2</sup> We are left only with the principal minors of order one, which correspond to the elements of the diagonal of the Hessian matrix of  $\mathcal{L}^y$ , which are the second derivatives:

$$\frac{\partial^2 \mathcal{L}^y}{\partial^2 Y_{ijt}} = \frac{-\lambda}{\sigma_i^y} \left[ \tilde{Y}_{it}^{\frac{1}{\sigma_i^y}} \varphi_{ijt}^{\frac{\sigma_i^y - 1}{\sigma_i^y}} Y_{ijt}^{\frac{-1 - \sigma_i^y}{\sigma_i^y}} \right] \left[ \left( \frac{\varphi_{ijt} Y_{ijt}}{\tilde{Y}_{it}} \right)^{\frac{\sigma_i^y - 1}{\sigma_i^y}} - 1 \right]$$

Given that the second term in brackets is always negative and  $\lambda > 0$  (see (A2)), all principal minors of order one are greater than zero if and only if  $\sigma_i^y \in (0, 1) \cup (1, \infty)$ . Hence  $\mathcal{L}^y$  is convex and the second-order conditions for minimization are satisfied for a critical point satisfying the Lagrangian first order conditions if and only if  $\sigma_i^y \in (0, 1) \cup (1, \infty)$ . In the terminology of Sun and Yang (2006), goods in the bundle  $\Omega_{it}^y$  are *gross substitutes* when  $\sigma_i^y > 1$  and *gross complements* when  $0 < \sigma_i^y < 1$ . That is, the demand for product  $j$  increases in response to an increase in the price of any other variety  $k$ , holding everything else constant, if and only if  $\sigma_i^y > 1$ ; it decreases

<sup>1</sup>For the limiting case  $\sigma_i^y \rightarrow 1$ ,  $\tilde{Y}_{it}$  tends to a Cobb-Douglas function with exponents  $\varphi_{ijt}$ . In this case,  $\tilde{Y}_{it}$  is concave if and only if  $\sum_{j \in \Omega_{it}^y} \varphi_{ijt} \leq 1$ .

<sup>2</sup>Note that the theorem still applies when replacing  $p(x)$  by  $p(x) + c$ , where the additional constant arises due to the excluded varieties that now enter as constant terms within the sum.

if and only if  $0 < \sigma_i^y < 1$ . Although our methodology can accommodate either case, we believe that given the sectors we consider in our empirical application, it is reasonable to assume  $\sigma_i^y > 1$ . In other settings, it may be plausible to allow  $0 < \sigma_i^y < 1$ .

Plugging (A2) into (A1) and rearranging, we can express the output quantity for product  $j$  in terms of its price, its quality, and the firm-level aggregate output and price index:

$$Y_{ijt} = \tilde{Y}_{it} \left( \frac{\tilde{P}_{it}}{P_{ijt}} \right)^{\sigma_i^y} \varphi_{ijt}^{\sigma_i^y - 1} \quad (\text{A3})$$

Note that:

$$R_{it} = \sum_{j \in \Omega_{it}^y} R_{ijt} = \sum_{j \in \Omega_{it}^y} P_{ijt} Y_{ijt} = \tilde{P}_{it} \tilde{Y}_{it} \left( \tilde{P}_{it} \right)^{\sigma_i^y - 1} \underbrace{\sum_{j \in \Omega_{it}^y} \left( \frac{P_{ijt}}{\varphi_{ijt}} \right)^{1 - \sigma_i^y}}_{= \tilde{P}_{it}^{1 - \sigma_i^y}} = \tilde{P}_{it} \tilde{Y}_{it} \quad (\text{A4})$$

That is,  $\tilde{P}_{it}$  is indeed the price index that sets  $R_{it} = \tilde{P}_{it} \tilde{Y}_{it}$ .

### A.1.2 Price Index Log Change

Using (A3),

$$S_{ijt}^y = \frac{P_{ijt} Y_{ijt}}{R_{it}} = \frac{P_{ijt} Y_{ijt}}{\tilde{P}_{it} \tilde{Y}_{it}} = \left( \frac{\left( \frac{P_{ijt}}{\varphi_{ijt}} \right)}{\tilde{P}_{it}} \right)^{1 - \sigma_i^y} \quad (\text{A5})$$

Hence from the definitions in (5) in the main text:

$$\chi_{it,t-1}^y = \frac{\sum_{j \in \Omega_{it}^{y*}} S_{ijt}^y}{\sum_{j \in \Omega_{it}^y} S_{ijt}^y} = \frac{\sum_{j \in \Omega_{it}^{y*}} \left( \frac{P_{ijt}}{\varphi_{ijt}} \right)^{1 - \sigma_i^y}}{\sum_{j \in \Omega_{it}^y} \left( \frac{P_{ijt}}{\varphi_{ijt}} \right)^{1 - \sigma_i^y}}, \quad \chi_{it-1,t}^y = \frac{\sum_{j \in \Omega_{it}^{y*}} S_{ijt-1}^y}{\sum_{j \in \Omega_{it-1}^y} S_{ijt-1}^y} = \frac{\sum_{j \in \Omega_{it}^{y*}} \left( \frac{P_{ijt-1}}{\varphi_{ijt-1}} \right)^{1 - \sigma_i^y}}{\sum_{j \in \Omega_{it-1}^y} \left( \frac{P_{ijt-1}}{\varphi_{ijt-1}} \right)^{1 - \sigma_i^y}}$$

Then using the definition of  $\tilde{P}_{it}$ , (2),

$$\begin{aligned} \frac{\tilde{P}_{it}}{\tilde{P}_{it-1}} &= \frac{\left[ \sum_{j \in \Omega_{it}^y} \left( \frac{P_{ijt}}{\varphi_{ijt}} \right)^{1 - \sigma_i^y} \right]^{\frac{1}{1 - \sigma_i^y}}}{\left[ \sum_{j \in \Omega_{it-1}^y} \left( \frac{P_{ijt-1}}{\varphi_{ijt-1}} \right)^{1 - \sigma_i^y} \right]^{\frac{1}{1 - \sigma_i^y}}} \\ &= \left( \frac{\chi_{it-1,t}^y}{\chi_{it,t-1}^y} \right)^{\frac{1}{1 - \sigma_i^y}} \frac{\left( \sum_{j \in \Omega_{it}^{y*}} \left( \frac{P_{ijt}}{\varphi_{ijt}} \right)^{1 - \sigma_i^y} \right)^{\frac{1}{1 - \sigma_i^y}}}{\left( \sum_{j \in \Omega_{it}^{y*}} \left( \frac{P_{ijt-1}}{\varphi_{ijt-1}} \right)^{1 - \sigma_i^y} \right)^{\frac{1}{1 - \sigma_i^y}}} = \left( \frac{\chi_{it-1,t}^y}{\chi_{it,t-1}^y} \right)^{\frac{1}{1 - \sigma_i^y}} \frac{\tilde{P}_{it}^*}{\tilde{P}_{it-1}^*} \quad (\text{A6}) \end{aligned}$$

where  $\tilde{P}_{it}^*$  is the common-goods price index defined in the main text (footnote 15).

To derive an expression for  $\frac{\tilde{P}_{it}^*}{\tilde{P}_{it-1}^*}$ , note that (A5) implies a similar expression for the expendi-

ture share of common goods:

$$S_{ijt}^{y*} = \frac{P_{ijt}Y_{ijt}}{\tilde{P}_{it}^* \tilde{Y}_{it}^*} = \frac{P_{ijt}Y_{ijt}}{\tilde{P}_{it} \tilde{Y}_{it}} \cdot \frac{\tilde{P}_{it} \tilde{Y}_{it}}{\tilde{P}_{it}^* \tilde{Y}_{it}^*} = \left( \frac{\left( \frac{P_{ijt}}{\varphi_{ijt}} \right)}{\tilde{P}_{it}} \right)^{1-\sigma_i^y} \frac{\tilde{P}_{it} \tilde{Y}_{it}}{\tilde{P}_{it}^* \tilde{Y}_{it}^*}$$

Using (A3),

$$\frac{\tilde{P}_{it} \tilde{Y}_{it}}{\tilde{P}_{it}^* \tilde{Y}_{it}^*} = \frac{\tilde{P}_{it} \tilde{Y}_{it}}{\sum_{j \in \Omega_{it}^{y*}} P_{ijt} Y_{ijt}} = \left( \frac{\tilde{P}_{it}}{\tilde{P}_{it}^*} \right)^{1-\sigma_i^y}$$

Hence:

$$S_{ijt}^{y*} = \left( \frac{\left( \frac{P_{ijt}}{\varphi_{ijt}} \right)}{\tilde{P}_{it}^*} \right)^{1-\sigma_i^y} \quad (\text{A7})$$

Divide (A7) by the same equation for the previous year, take logs, and re-arrange:

$$\frac{\ln \left( \frac{\tilde{P}_{it}^*}{\tilde{P}_{it-1}^*} \right) - \ln \left( \frac{\frac{P_{ijt}}{\varphi_{ijt}}}{\frac{P_{ijt-1}}{\varphi_{ijt-1}}} \right)}{\ln \left( \frac{S_{ijt}^{y*}}{S_{ijt-1}^{y*}} \right)} = \frac{1}{\sigma_i^y - 1}$$

Multiply both sides by  $S_{ijt}^{y*} - S_{ijt-1}^{y*}$  and sum over the common goods:

$$\sum_{j \in \Omega_{it}^{y*}} \left( S_{ijt}^{y*} - S_{ijt-1}^{y*} \right) \frac{\ln \left( \frac{\tilde{P}_{it}^*}{\tilde{P}_{it-1}^*} \right) - \ln \left( \frac{\frac{P_{ijt}}{\varphi_{ijt}}}{\frac{P_{ijt-1}}{\varphi_{ijt-1}}} \right)}{\ln \left( \frac{S_{ijt}^{y*}}{S_{ijt-1}^{y*}} \right)} = \left( \frac{1}{\sigma_i^y - 1} \right) \sum_{j \in \Omega_{it}^{y*}} \left( S_{ijt}^{y*} - S_{ijt-1}^{y*} \right) = 0$$

where the second equality follows because  $\sum_{j \in \Omega_{it}^{y*}} S_{ijt}^{y*} = \sum_{j \in \Omega_{it}^{y*}} S_{ijt-1}^{y*} = 1$ . This implies:

$$\sum_{j \in \Omega_{it}^{y*}} \left( \frac{S_{ijt}^{y*} - S_{ijt-1}^{y*}}{\ln S_{ijt}^{y*} - \ln S_{ijt-1}^{y*}} \right) \ln \left( \frac{\tilde{P}_{it}^*}{\tilde{P}_{it-1}^*} \right) = \sum_{j \in \Omega_{it}^{y*}} \left( \frac{S_{ijt}^{y*} - S_{ijt-1}^{y*}}{\ln S_{ijt}^{y*} - \ln S_{ijt-1}^{y*}} \right) \ln \left( \frac{\frac{P_{ijt}}{\varphi_{ijt}}}{\frac{P_{ijt-1}}{\varphi_{ijt-1}}} \right).$$

Since  $\ln \left( \frac{\tilde{P}_{it}^*}{\tilde{P}_{it-1}^*} \right)$  does not vary with  $j$ , this can be re-written as:

$$\ln \left( \frac{\tilde{P}_{it}^*}{\tilde{P}_{it-1}^*} \right) = \sum_{j \in \Omega_{it}^{y*}} \delta_{ijt} \ln \left( \frac{P_{ijt}}{P_{ijt-1}} \right) - \sum_{j \in \Omega_{it}^{y*}} \delta_{ijt} \ln \left( \frac{\varphi_{ijt}}{\varphi_{ijt-1}} \right), \quad (\text{A8})$$

where  $\delta_{ijt}$  is as defined in (5) above. Combining (A6) and (A8), we have:

$$\ln \left( \frac{\tilde{P}_{it}}{\tilde{P}_{it-1}} \right) = \sum_{j \in \Omega_{it}^{y*}} \delta_{ijt} \ln \left( \frac{P_{ijt}}{P_{ijt-1}} \right) - \sum_{j \in \Omega_{it}^{y*}} \delta_{ijt} \ln \left( \frac{\varphi_{ijt}}{\varphi_{ijt-1}} \right) - \frac{1}{\sigma_i^y - 1} \ln \left( \frac{\chi_{it-1,t}^y}{\chi_{it,t-1}^y} \right) \quad (\text{A9})$$

which is (4).

### A.1.3 Quantity Index Log Change

To derive the log change in the quantity index, start by noting that (A3) implies,

$$P_{ijt} = \tilde{P}_{it} \left( \frac{\tilde{Y}_{it}}{Y_{ijt}} \right)^{\frac{1}{\sigma_i^y} \frac{\sigma_i^y - 1}{\sigma_i^y}} \varphi_{ijt}$$

Therefore,

$$\ln \left( \frac{P_{ijt}}{P_{ijt-1}} \right) = \ln \left( \frac{\tilde{P}_{it}}{\tilde{P}_{it-1}} \right) + \frac{1}{\sigma_i^y} \ln \left( \frac{\tilde{Y}_{it}}{\tilde{Y}_{it-1}} \right) - \frac{1}{\sigma_i^y} \ln \left( \frac{Y_{ijt}}{Y_{ijt-1}} \right) + \frac{\sigma_i^y}{\sigma_i^y - 1} \ln \left( \frac{\varphi_{ijt}}{\varphi_{ijt-1}} \right)$$

Plugging this into (A9), re-arranging, and using the fact that  $\sum_{j \in \Omega_{it}^{y*}} \delta_{ijt} = 1$  gives:

$$\ln \left( \frac{\tilde{Y}_{it}}{\tilde{Y}_{it-1}} \right) = \sum_{j \in \Omega_{it}^{y*}} \delta_{ijt} \ln \left( \frac{Y_{ijt}}{Y_{ijt-1}} \right) + \sum_{j \in \Omega_{it}^{y*}} \delta_{ijt} \ln \left( \frac{\varphi_{ijt}}{\varphi_{ijt-1}} \right) + \frac{\sigma_i^y}{\sigma_i^y - 1} \ln \left( \frac{\chi_{it-1,t}^y}{\chi_{it,t-1}^y} \right)$$

which is (6). The fact that  $\tilde{P}_{it}^* \tilde{Y}_{it}^* = R_{it}^*$  can be shown as in (A4), using just common goods.

## A.2 Construction of CES Price/Quantity Indexes, Input Side

The derivations for the price and quantity indexes for the input side are analogous to the ones from the output side. We include them for the sake of completeness.

### A.2.1 Firm's Minimization Problem

The Lagrangian corresponding to the first stage of the firm's problem is given by:

$$\mathcal{L}^m = \sum_{h \in \Omega_{it}^m} M_{iht} W_{iht} - \lambda \left( \left[ \sum_{h \in \Omega_{it}^m} (\alpha_{iht} M_{iht})^{\frac{\sigma_i^m - 1}{\sigma_i^m}} \right]^{\frac{\sigma_i^m}{\sigma_i^m - 1}} - \tilde{M}_{it} \right)$$

The first order condition with respect to input  $h$ ,  $\frac{\partial \mathcal{L}^m}{\partial M_{iht}} = 0$ , implies:

$$\frac{W_{iht}}{\alpha_{iht}} = \lambda (\alpha_{iht} M_{iht})^{-\frac{1}{\sigma_i^m}} \tilde{M}_{it}^{\frac{1}{\sigma_i^m}} \quad (\text{A10})$$

Raising both sides of this equation to the power  $1 - \sigma_i^m$ , summing over the  $h \in \Omega_{it}^m$ , using the definition of  $\widetilde{W}_{it}$  in (9) in the main text, and rearranging, we have:

$$\lambda = \widetilde{W}_{it} \quad (\text{A11})$$

Analogously to the output case, it can be shown that (without further assumptions) any point satisfying the first order conditions constitutes an global minimum if and only if  $\sigma_i^m \in (0, 1) \cup (1, \infty)$ . Therefore, our method allows material inputs to be *gross complements*,  $\sigma_i^m < 1$ , or to be *gross substitutes*,  $\sigma_i^m > 1$ . Nevertheless, given the type of sectors we consider in our empirical analysis, we assume material inputs to be *gross substitutes*, that is, we assume  $\sigma_i^m > 1$ .

Plugging (A11) into (A10) and rearranging:

$$M_{iht} = \widetilde{M}_{it} \left( \frac{\widetilde{W}_{it}}{W_{iht}} \right)^{\sigma_i^m} \alpha_{iht}^{\sigma_i^m - 1} \quad (\text{A12})$$

As for revenues,

$$E_{it} = \sum_{h \in \Omega_{it}^m} E_{iht} = \sum_{h \in \Omega_{it}^m} W_{iht} M_{iht} = \widetilde{W}_{it} \widetilde{M}_{it} \left( \widetilde{W}_{it} \right)^{\sigma_i^m - 1} \underbrace{\sum_{h \in \Omega_{it}^m} \left( \frac{W_{iht}}{\alpha_{iht}} \right)^{1 - \sigma_i^m}}_{= \widetilde{W}_{it}^{1 - \sigma_i^m}} = \widetilde{W}_{it} \widetilde{M}_{it} \quad (\text{A13})$$

## A.2.2 Price Index Log Change

Using (A12),

$$S_{iht}^m = \frac{W_{iht} M_{iht}}{E_{it}} = \frac{W_{iht} M_{iht}}{\widetilde{W}_{it} \widetilde{M}_{it}} = \left( \frac{\left( \frac{W_{iht}}{\alpha_{iht}} \right)}{\widetilde{W}_{it}} \right)^{1 - \sigma_i^m} \quad (\text{A14})$$

Hence from the definitions in (11) in the main text:

$$\chi_{it,t-1}^m = \frac{\sum_{h \in \Omega_{it}^{m*}} S_{iht}^m}{\sum_{h \in \Omega_{it}^m} S_{iht}^m} = \frac{\sum_{h \in \Omega_{it}^{m*}} \left( \frac{W_{iht}}{\alpha_{iht}} \right)^{1 - \sigma_i^m}}{\sum_{h \in \Omega_{it}^m} \left( \frac{W_{iht}}{\alpha_{iht}} \right)^{1 - \sigma_i^m}}, \quad \chi_{it-1,t}^m = \frac{\sum_{h \in \Omega_{it}^{m*}} S_{iht-1}^m}{\sum_{h \in \Omega_{it-1}^m} S_{iht-1}^m} = \frac{\sum_{h \in \Omega_{it}^{m*}} \left( \frac{W_{iht-1}}{\alpha_{iht-1}} \right)^{1 - \sigma_i^m}}{\sum_{h \in \Omega_{it-1}^m} \left( \frac{W_{iht-1}}{\alpha_{iht-1}} \right)^{1 - \sigma_i^m}}$$

Then using the definition of  $\widetilde{W}_{it}$ , (9),

$$\begin{aligned} \frac{\widetilde{W}_{it}}{\widetilde{W}_{it-1}} &= \frac{\left[ \sum_{h \in \Omega_{it}^m} \left( \frac{W_{iht}}{\alpha_{iht}} \right)^{1 - \sigma_i^m} \right]^{\frac{1}{1 - \sigma_i^m}}}{\left[ \sum_{h \in \Omega_{it-1}^m} \left( \frac{W_{iht-1}}{\alpha_{iht-1}} \right)^{1 - \sigma_i^m} \right]^{\frac{1}{1 - \sigma_i^m}}} \\ &= \left( \frac{\chi_{it-1,t}^m}{\chi_{it,t-1}^m} \right)^{\frac{1}{1 - \sigma_i^m}} \frac{\left( \sum_{h \in \Omega_{it}^{m*}} \left( \frac{W_{iht}}{\alpha_{iht}} \right)^{1 - \sigma_i^m} \right)^{\frac{1}{1 - \sigma_i^m}}}{\left( \sum_{h \in \Omega_{it}^{m*}} \left( \frac{W_{iht-1}}{\alpha_{iht-1}} \right)^{1 - \sigma_i^m} \right)^{\frac{1}{1 - \sigma_i^m}}} = \left( \frac{\chi_{it-1,t}^m}{\chi_{it,t-1}^m} \right)^{\frac{1}{1 - \sigma_i^m}} \frac{\widetilde{W}_{it}^*}{\widetilde{W}_{it-1}^*} \quad (\text{A15}) \end{aligned}$$

where  $\widetilde{W}_{it}^*$  is the common-goods price index defined in the main text (footnote 22).

To derive an expression for  $\frac{\widetilde{W}_{it}^*}{\widetilde{W}_{it-1}^*}$ , note that (A14) implies a similar expression for the expenditure share of common goods:

$$S_{iht}^{m*} = \frac{W_{iht}M_{iht}}{\widetilde{W}_{it}^*\widetilde{M}_{it}^*} = \frac{W_{iht}M_{iht}}{\widetilde{W}_{it}\widetilde{M}_{it}} \cdot \frac{\widetilde{W}_{it}\widetilde{M}_{it}}{\widetilde{W}_{it}^*\widetilde{M}_{it}^*} = \left( \frac{\left(\frac{W_{iht}}{\alpha_{iht}}\right)}{\widetilde{W}_{it}} \right)^{1-\sigma_i^m} \frac{\widetilde{W}_{it}\widetilde{M}_{it}}{\widetilde{W}_{it}^*\widetilde{M}_{it}^*}$$

Using (A3),

$$\frac{\widetilde{W}_{it}\widetilde{M}_{it}}{\widetilde{W}_{it}^*\widetilde{M}_{it}^*} = \frac{\widetilde{W}_{it}\widetilde{M}_{it}}{\sum_{h \in \Omega_{it}^{m*}} W_{iht}M_{iht}} = \left( \frac{\widetilde{W}_{it}}{\widetilde{W}_{it}^*} \right)^{1-\sigma_i^m}$$

Hence:

$$S_{iht}^{m*} = \left( \frac{\left(\frac{W_{iht}}{\alpha_{iht}}\right)}{\widetilde{W}_{it}^*} \right)^{1-\sigma_i^m} \tag{A16}$$

Divide (A16) by the same equation for the previous year, take logs, and re-arrange:

$$\frac{\ln \left( \frac{\widetilde{W}_{it}^*}{\widetilde{W}_{it-1}^*} \right) - \ln \left( \frac{\frac{W_{iht}}{\alpha_{iht}}}{\frac{W_{iht-1}}{\alpha_{iht-1}}} \right)}{\ln \left( \frac{S_{iht}^{m*}}{S_{iht-1}^{m*}} \right)} = \frac{1}{\sigma_i^m - 1}$$

Multiply both sides by  $S_{iht}^{m*} - S_{iht-1}^{m*}$  and sum over the common goods:

$$\sum_{h \in \Omega_{it}^{m*}} (S_{iht}^{m*} - S_{iht-1}^{m*}) \frac{\ln \left( \frac{\widetilde{W}_{it}^*}{\widetilde{W}_{it-1}^*} \right) - \ln \left( \frac{\frac{W_{iht}}{\alpha_{iht}}}{\frac{W_{iht-1}}{\alpha_{iht-1}}} \right)}{\ln \left( \frac{S_{iht}^{m*}}{S_{iht-1}^{m*}} \right)} = \left( \frac{1}{\sigma_i^m - 1} \right) \sum_{h \in \Omega_{it}^{m*}} (S_{iht}^{m*} - S_{iht-1}^{m*}) = 0$$

where the second equality follows because  $\sum_{h \in \Omega_{it}^{m*}} S_{iht}^{m*} = \sum_{h \in \Omega_{it}^{m*}} S_{iht-1}^{m*} = 1$ . This implies:

$$\sum_{h \in \Omega_{it}^{m*}} \left( \frac{S_{iht}^{m*} - S_{iht-1}^{m*}}{\ln S_{iht}^{m*} - \ln S_{iht-1}^{m*}} \right) \ln \left( \frac{\widetilde{W}_{it}^*}{\widetilde{W}_{it-1}^*} \right) = \sum_{h \in \Omega_{it}^{m*}} \left( \frac{S_{iht}^{m*} - S_{iht-1}^{m*}}{\ln S_{iht}^{m*} - \ln S_{iht-1}^{m*}} \right) \ln \left( \frac{\frac{W_{iht}}{\alpha_{iht}}}{\frac{W_{iht-1}}{\alpha_{iht-1}}} \right)$$

Since  $\ln \left( \frac{\widetilde{W}_{it}^*}{\widetilde{W}_{it-1}^*} \right)$  does not vary with  $h$ , this can be re-written as:

$$\ln \left( \frac{\widetilde{W}_{it}^*}{\widetilde{W}_{it-1}^*} \right) = \sum_{h \in \Omega_{it}^{m*}} \delta_{iht} \ln \left( \frac{W_{iht}}{W_{iht-1}} \right) - \sum_{h \in \Omega_{it}^{m*}} \delta_{iht} \ln \left( \frac{\alpha_{iht}}{\alpha_{iht-1}} \right) \tag{A17}$$

where  $\delta_{iht}$  is as defined in (11) above. Combining (A15) and (A17), we have:

$$\ln \left( \frac{\widetilde{W}_{it}}{\widetilde{W}_{it-1}} \right) = \sum_{h \in \Omega_{it}^{m*}} \delta_{iht} \ln \left( \frac{W_{iht}}{W_{iht-1}} \right) - \sum_{h \in \Omega_{it}^{m*}} \delta_{iht} \ln \left( \frac{\alpha_{iht}}{\alpha_{iht-1}} \right) - \frac{1}{\sigma_i^m - 1} \ln \left( \frac{\chi_{it-1,t}^m}{\chi_{it,t-1}^m} \right) \quad (\text{A18})$$

which is (10) in the main text.

### A.2.3 Quantity Index Log Change

We start by noting that (A12) implies

$$W_{iht} = \widetilde{W}_{it} \left( \frac{\widetilde{M}_{it}}{\widetilde{M}_{iht}} \right)^{\frac{1}{\sigma_i^m}} \frac{\sigma_i^{m-1}}{\alpha_{iht}}$$

Hence:

$$\ln \left( \frac{W_{iht}}{W_{iht-1}} \right) = \ln \left( \frac{\widetilde{W}_{it}}{\widetilde{W}_{it-1}} \right) + \frac{1}{\sigma_i^y} \ln \left( \frac{\widetilde{M}_{it}}{\widetilde{M}_{it-1}} \right) - \frac{1}{\sigma_i^y} \ln \left( \frac{W_{iht}}{W_{iht-1}} \right) + \frac{\sigma_i^y}{\sigma_i^y - 1} \ln \left( \frac{\alpha_{iht}}{\alpha_{iht-1}} \right)$$

Plugging this into (A18), re-arranging, and using the fact that  $\sum_{h \in \Omega_{it}^{m*}} \delta_{iht} = 1$  gives the log change in  $\widetilde{M}_{it}$ :

$$\ln \left( \frac{\widetilde{M}_{it}}{\widetilde{M}_{it-1}} \right) = \sum_{h \in \Omega_{it}^{m*}} \delta_{iht} \ln \left( \frac{M_{iht}}{M_{iht-1}} \right) + \sum_{h \in \Omega_{it}^{m*}} \delta_{iht} \ln \frac{\alpha_{iht}}{\alpha_{iht-1}} + \frac{\sigma_i^m}{\sigma_i^m - 1} \ln \left( \frac{\chi_{it-1,t}^m}{\chi_{it,t-1}^m} \right)$$

which is (12) in the main text. The fact that  $\widetilde{W}_{it}^* \widetilde{M}_{it}^* = E_{it}^*$  can be shown as in (A13), using just common goods.

### A.3 Variance Correction for $\beta_k$ in Levels-Equation Estimation

Our sequential production function estimation belongs to a general class of two-step M-Estimators discussed for instance in Wooldridge (2002, Section 12.4) (and previously in Newey (1984)). The results there can be applied directly. Under our assumptions, our first-step estimates  $\widehat{\beta}_m$  and  $\widehat{\beta}_l$  and their standard errors are consistently estimated. The levels-equation estimate of  $\beta_k$ , call it  $\widehat{\widehat{\beta}}_k$ , can be calculated by solving:

$$\sum_{t=1}^T \sum_{i=1}^N \Delta k_{it-1} \left( \left( \widetilde{y}_{it}^{SV} - \widehat{\beta}_m \widetilde{m}_{it}^{SV} - \widehat{\beta}_l l_{it} \right) - \widehat{\widehat{\beta}}_k k_{it} \right) = 0. \quad (\text{A19})$$

As noted in the main text (see footnote 38), the consistency of  $\widehat{\beta}_m$  and  $\widehat{\beta}_l$  is sufficient to guarantee the consistency of  $\widehat{\widehat{\beta}}_k$ . In the special case when  $E(\Delta k_{it-1} \widetilde{m}_{it}^{SV}) = 0$  and  $E(\Delta k_{it-1} l_{it}) = 0$ , the first step estimation can be ignored when computing the asymptotic variance of  $\widehat{\widehat{\beta}}_k$ .<sup>3</sup> If those

<sup>3</sup>The score function corresponding to the levels-equation IV estimation is  $s(a_{it}, \beta_k; \beta_m, \beta_l) = \Delta k_{it-1} (\widetilde{y}_{it}^{SV} - \beta_m \widetilde{m}_{it}^{SV} - \beta_l l_{it} - \beta_k k_{it})$ , where  $a_{it} = (\widetilde{y}_{it}^{SV}, \widetilde{m}_{it}^{SV}, l_{it}, k_{it}, \Delta k_{it-1})$ . If  $E(\Delta k_{it-1} \widetilde{m}_{it}^{SV}) = 0$  and  $E(\Delta k_{it-1} l_{it}) = 0$  then the gradient of the score function with respect to  $\beta_m$  and  $\beta_l$  is zero and equation 12.37 of Wooldridge (2002) holds,



conditions do not hold, then we need to use a corrected expression for the asymptotic variance of  $\widehat{\beta}_k$ , which takes into account that  $\widehat{\beta}_m$  and  $\widehat{\beta}_l$  were estimated in a previous step. A consistent estimate of the corrected asymptotic variance for  $\widehat{\beta}_k$ , call it  $\widehat{V}_{\beta_k}$ , is given by Newey and McFadden (1994):<sup>4</sup>

$$\widehat{V}_{\beta_k} = \frac{(T \times N)^{-1} \left( \sum_{t=1}^T \sum_{i=1}^N \left( \widehat{s}_{it} + \widehat{F} \widehat{\psi}_{it} \right)^2 \right)}{\widehat{G}^2} \quad (\text{A20})$$

where

$$\begin{aligned} \widehat{G} &= -\frac{1}{NT} \sum_{t=1}^T \sum_{i=1}^N \Delta k_{it-1} k_{it} \\ \widehat{F} &= -\frac{1}{NT} \sum_{t=1}^T \sum_{i=1}^N \Delta k_{it-1} [\widetilde{m}_{it}^{SV}, l_{it}, 0] \\ \widehat{s}_{it} &= \Delta k_{it-1} \left( \widetilde{y}_{it}^{SV} - \widehat{\beta}_m \widetilde{m}_{it}^{SV} - \widehat{\beta}_l l_{it} - \widehat{\beta}_k k_{it} \right) \\ \widehat{\psi}_{it} &= - \left( \widehat{H}' \widehat{W} \widehat{H} \right)^{-1} \widehat{H}' \widehat{W} \widehat{m}_{it} \end{aligned}$$

and the terms in  $\widehat{\psi}_{it}$  are defined as:

$$\begin{aligned} \widehat{H} &= \frac{1}{NT} \sum_{t=1}^T \sum_{i=1}^N \left[ \Delta \widehat{w}_{it}^{imp}, \Delta z_{it}, k_{it-2}, \widetilde{m}_{it-2}^{SV}, l_{it-2} \right] \left[ \Delta \widetilde{m}_{it}^{SV}, \Delta l_{it}, \Delta k_{it} \right]' \\ \widehat{W} &= \frac{1}{NT} \sum_{t=1}^T \sum_{i=1}^N \left[ \Delta \widehat{w}_{it}^{imp}, \Delta z_{it}, k_{it-2}, \widetilde{m}_{it-2}^{SV}, l_{it-2} \right] \left[ \Delta \widehat{g}_{it}, \Delta z_{it}, k_{it-2}, \widetilde{m}_{it-2}^{SV}, l_{it-2} \right]' \\ \widehat{m}_{it} &= \left[ \Delta \widehat{g}_{it}, \Delta z_{it}, k_{it-2}, \widetilde{m}_{it-2}^{SV}, l_{it-2} \right]' \left( \Delta \widetilde{y}_{it}^{SV} - \widehat{\beta}_m \Delta \widetilde{m}_{it}^{SV} - \widehat{\beta}_l \Delta l_{it} - \widehat{\beta}_k \Delta k_{it} \right) \end{aligned}$$

We report the corresponding corrected standard errors when we report  $\widehat{\beta}_k$ .

#### A.4 Construction of Alternative Quantity Indexes

On the input side, following standard formulations (see e.g. Dodge (2008)), we define the Laspeyres input quantity index for  $t-1$  and  $t$  as:

$$\widetilde{M}_{it,t-1}^{Lasp} = \frac{\sum_{h \in \Omega_{it}^{m*}} W_{iht-1} M_{iht}}{\sum_{h \in \Omega_{it}^{m*}} W_{iht-1} M_{iht-1}} \quad (\text{A21})$$

and the Paasche input quantity index as:

$$\widetilde{M}_{it,t-1}^{Paas} = \frac{\sum_{h \in \Omega_{it}^{m*}} W_{iht} M_{iht}}{\sum_{h \in \Omega_{it}^{m*}} W_{iht} M_{iht-1}}, \quad (\text{A22})$$

implying that we can ignore the first step in calculating the asymptotic variance of  $\widehat{\beta}_k$ .

<sup>4</sup>See also Proposition 2 of Kripfganz and Schwarz (2019).

The Tornqvist quantity index is defined as:

$$\widetilde{M}_{it,t-1}^{Torn} = \prod_{h \in \Omega_{it}^{m*}} \left( \frac{M_{iht}}{M_{iht-1}} \right)^{\frac{1}{2}(S_{iht}^{m*} + S_{iht-1}^{m*})} \quad (\text{A23})$$

where  $S_{iht}^{m*}$  and  $S_{iht-1}^{m*}$  are as defined in footnote 22 of the main text.

Note that the Laspeyres quantity index is related to the Paasche price index, and vice-versa. If we define the Laspeyres price index as:

$$\widetilde{W}_{it,t-1}^{Lasp} = \frac{\sum_{h \in \Omega_{it}^{m*}} W_{iht} M_{iht-1}}{\sum_{h \in \Omega_{it}^{m*}} W_{iht-1} M_{iht-1}}. \quad (\text{A24})$$

and the Paasche price index as:

$$\widetilde{W}_{it,t-1}^{Paas} = \frac{\sum_{h \in \Omega_{it}^{m*}} W_{iht} M_{iht}}{\sum_{h \in \Omega_{it}^{m*}} W_{iht-1} M_{iht}} \quad (\text{A25})$$

then the common-input expenditure ratio between  $t$  and  $t - 1$  is the product of the Laspeyres price index and the Paasche quantity index and also the product of the Laspeyres quantity index and the Paasche price index:

$$\frac{E_{it}^*}{E_{it-1}^*} = \frac{\sum_{h \in \Omega_{it}^{m*}} W_{iht} M_{iht}}{\sum_{h \in \Omega_{it}^{m*}} W_{iht-1} M_{iht-1}} = \widetilde{M}_{it,t-1}^{Lasp} \times \widetilde{W}_{it,t-1}^{Paas} = \widetilde{M}_{it,t-1}^{Paas} \times \widetilde{W}_{it,t-1}^{Lasp}$$

The definition of the alternative output quantity indexes is analogous to the definition of the input quantity indexes (A21), (A22) and (A23).

## B Data Appendix

### B.1 Manufacturing Survey

The *Encuesta Anual Manufacturera* (EAM, Annual Manufacturing Survey), carried out by the Colombian national statistical agency, *Departamento Nacional de Estadística* (DANE), can be considered a census of manufacturing plants with 10 or more employees. The sample includes plants with fewer employees but value of production above a certain level (which has changed over time). Also, once plants are in the survey, they typically are kept in the sample, even if employment or value of production fall below the cutoffs.

The survey distinguishes between the value of output produced and output sold (which may differ because of holding inventories) and the value of materials consumed and materials purchased. We use value of output produced and value of materials consumed and refer to these, with some looseness of language, as sales (or revenues) and material expenditures.

For each plant, we construct capital stock using the perpetual-inventory method with a depreciation rate of 0.05, using information only on machinery and equipment, including transportation equipment. That is, we calculate  $K_{it} = K_{i,t-1} \times (0.95) + I_{i,t-1}$  where  $K_{it}$  is the capital stock of plant  $i$  in year  $t$  and  $I_{i,t-1}$  is investment in machinery and equipment by plant  $i$  in year  $t-1$ . We set the initial value for each plant,  $K_{i0}$ , using the book value of machinery and equipment reported by the plant in its first year in the sample. We deflate both initial book value and investment by a price index for gross fixed capital formation calculated by Colombia’s central bank. We sum capital stock across plants to get a firm-level measure.

DANE assigns plants to 4-digit industrial categories (International Standard Industrial Classification (ISIC) revision 2) in each year based on the sectors in which they have the most output. To each firm, we assign the 4-digit industry in which the firm has the most output over our study period, given DANE’s plant-year-level assignments.

The EAM contains employment and wage-bill information for broad occupational categories and contractual status (permanent vs. temporary). Employment is average employment over the year, and the wage bill is the total wage bill for the year. The employment measure we use as a covariate is the total number of workers, including temporary workers. When calculating the average monthly earnings at the firm level (for use in comparing to the monthly minimum wage in the “bite” measure — see Subsection 2.4.1 in the main text), we use only permanent workers, since dividing annual earnings by twelve arguably gives a sensible measure of monthly earnings only for permanent workers, who have a higher likelihood of working 12 months per year.

### B.2 Trade Data

The Colombian customs agency, *Dirección Nacional de Impuestos y Aduanas Nacionales* (DIAN), registers firm-level international trade transactions. Every registry corresponds to a purchase (import) or to a sale (export) by a Colombian firm and includes information on the date of the transaction, country of origin or destination, quantities purchased or sold, net weight of the shipment (in Kilograms) and total value of the transaction at the product 10 digits Harmonized System (HS) level. We exclude from our analysis the following: (1) Transactions with zero or negative total monetary value. (2) Transactions with zero or negative quantities. (3) Transactions with missing origin or destination. (4) Transactions made through a Free Trade Zone (*Zona Franca*). (5) Transactions of goods temporarily going out of the country for modifications and then coming back in. (6) Domestic transactions that are subject to taxes. (7) Transactions involving products

corresponding to the HS 2-digit classifications: 27 (Mineral fuels, mineral oils and products of their distillation; bituminous substances; mineral waxes), 84 (Nuclear reactors, boilers, machinery and mechanical appliances; parts thereof) and 85 (Electrical machinery and equipment and parts thereof; sound recorders and reproducers, television image and sound recorders and reproducers, and parts and accessories of such articles). After making these exclusions we rank countries according to the total value of imports by Colombian firms for the period 1992-2009. We keep only transactions (imports or exports) between Colombian firms and foreign firms located in the top 100 countries of this ranking.

### B.3 Household Survey Data

To construct the histogram of real wages in Appendix Figure A3, we use household surveys collected by DANE, the statistical agency. (In unreported results, we have constructed similar histograms by year for the entire 1992-2009 period.) We combine three different waves of surveys to compute monthly average wages at the individual level: *Encuesta Nacional de Hogares* (ENH) from 1992-Q2 to 2002-Q2, *Encuesta Continua de Hogares* (ECH) from 2002-Q3 to 2006-Q2 and the *Gran Encuesta Integrada de Hogares* (GEIH) from 2006-Q3 to 2009-Q4. When the survey reports daily or weekly wages we obtain monthly wages by multiplying the reported daily wage by 20.4 (approximate number of working days per month) or the reported weekly wage by 4.2 (approximate number of weeks per month). We restrict our analysis to wages reported by individuals employed by manufacturing firms with 11 or more workers and use the survey's individual sampling weights to compute the average monthly wage across locations and individuals.<sup>5</sup>

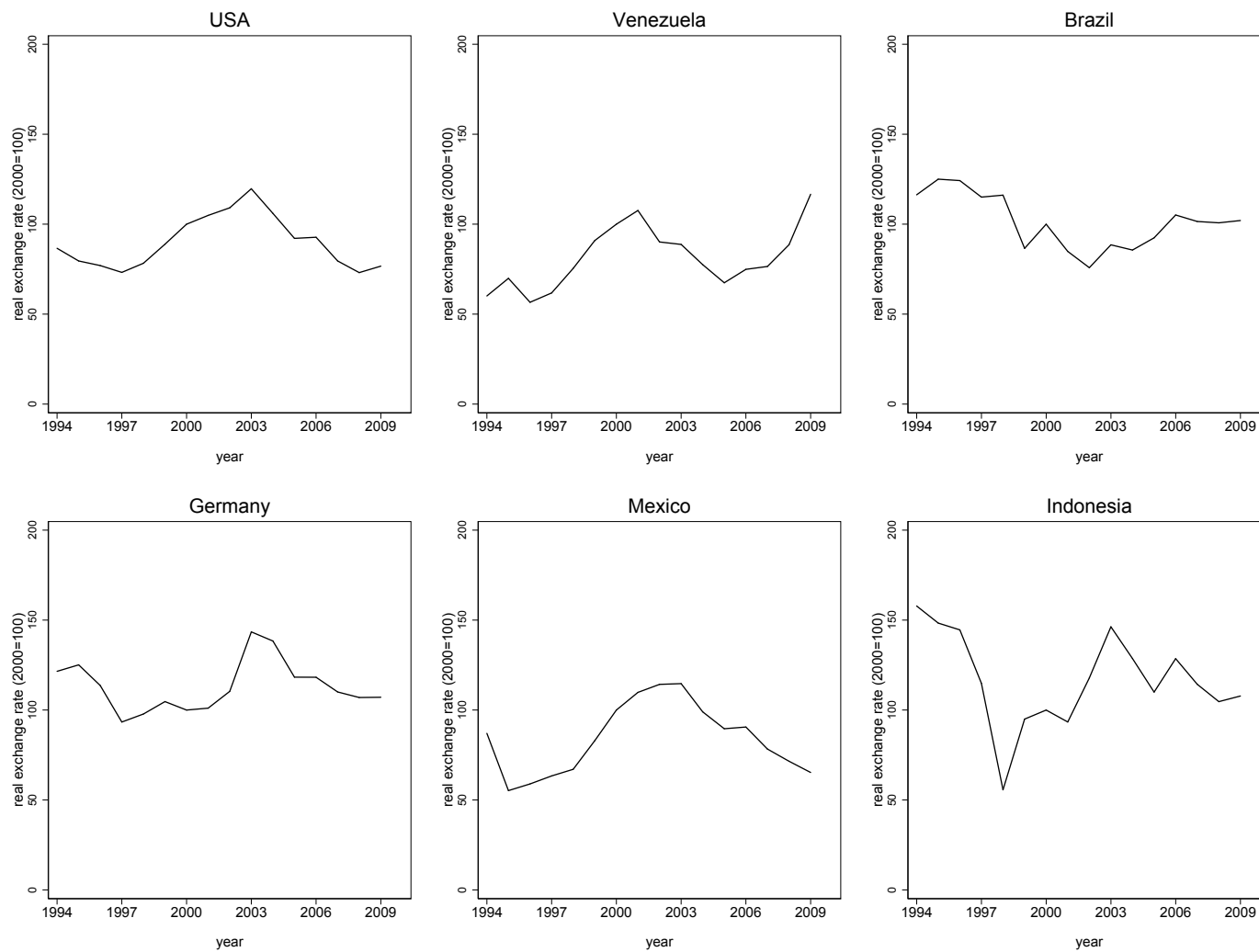
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<sup>5</sup>In Colombia, in addition to the monthly minimum salary, employers are also required to pay a transport subsidy of approximately 9% of the minimum salary to workers who earn less than 2 times the minimum wage. The instructions in the household survey ask respondent not to include travel expenses (*viáticos*) in their wage reports. It appears that some respondents include the transport subsidy when reporting their wage and some do not; that appears to be why we see bunching in Appendix Figure A3 both at the minimum wage (203,826 nominal pesos, approximately 247,000 pesos in real terms (2000 pesos)) and at the minimum plus the transport subsidy (224,526 nominal pesos, approximately 272,000 pesos in real terms (2000 pesos)).

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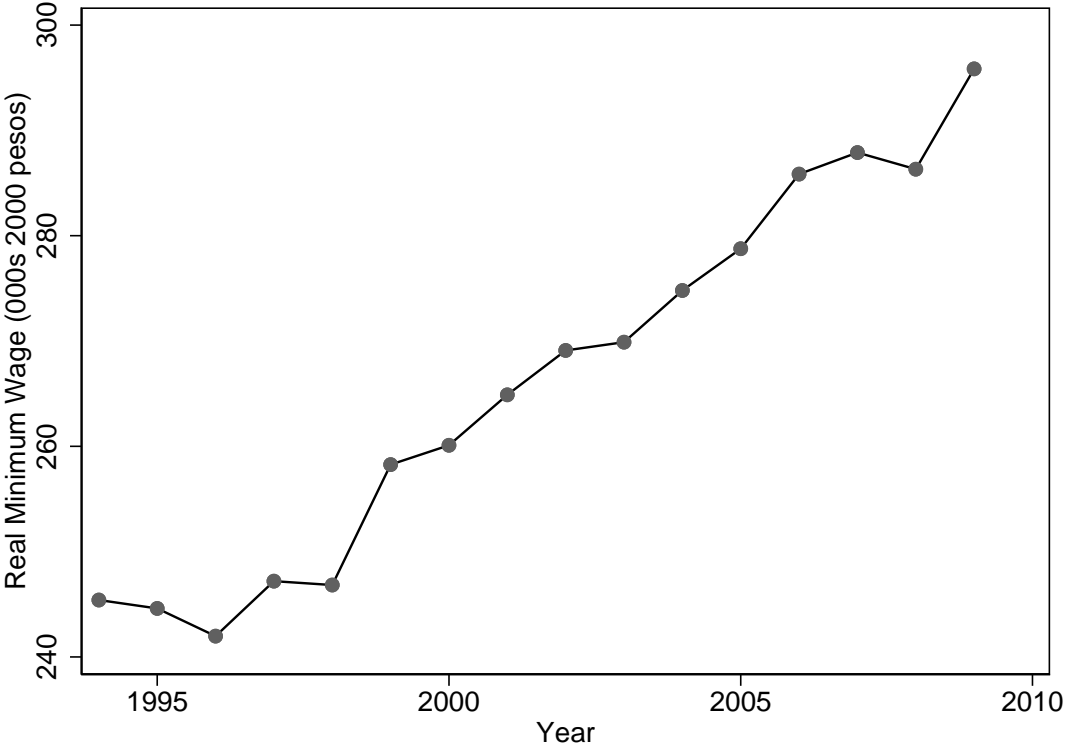
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Figure A1. Real Exchange Rate Variation, 1994-2009



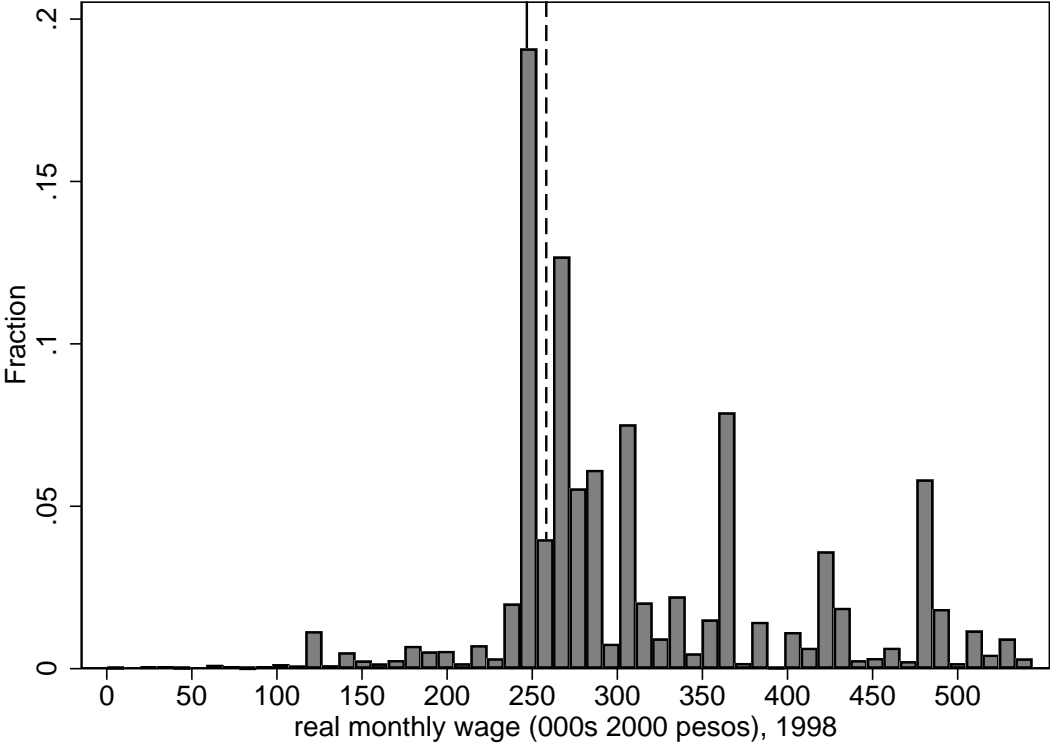
Notes: Figure plots real exchange rate (RER), normalized to 100 in 1994, calculated as in equation (17) in text, for top 6 import origins for rubber and plastics sectors. An RER increase reflects a real appreciation in the trading partner.

Figure A2. Real Minimum Monthly Wage, 1994-2009



Notes: Figure plots Colombian national real monthly minimum wage, in thousands of 2000 pesos, for 1994-2009. Average 2000 exchange rate is approximately 2,000 pesos/USD.

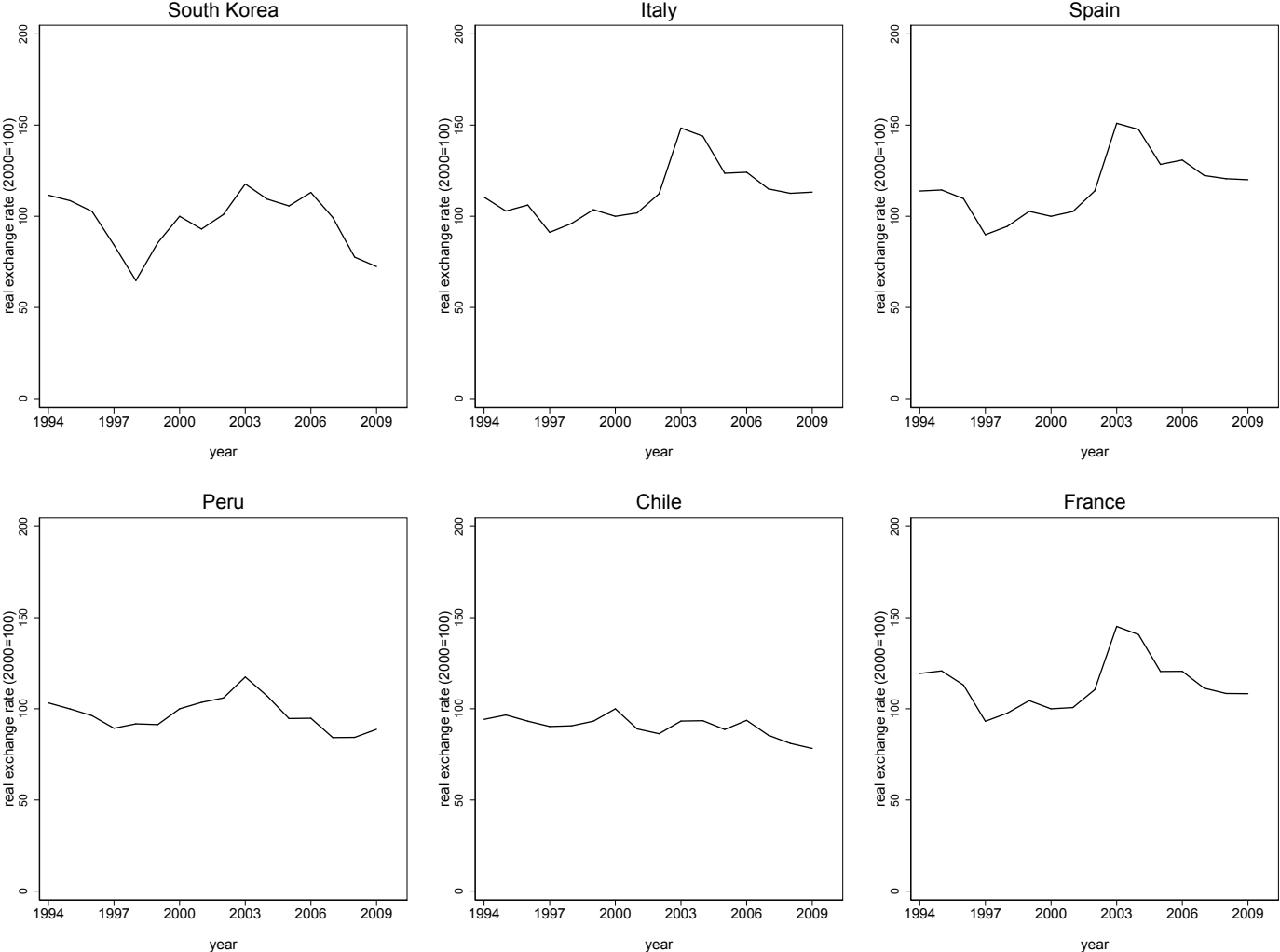
Figure A3. Histogram of Real Wages from Household Survey, 1998



Notes: Histogram of real monthly wages in 1998, in thousands of 2000 pesos, from Encuesta Nacional de Hogares (ENH, National Household Survey). See Appendix B.3 for details. Bins are 10,000 pesos wide. Solid vertical line is national minimum wage in 1998, dashed vertical line is national minimum wage in 1999. Average 2000 exchange rate is approximately 2,000 pesos/USD.

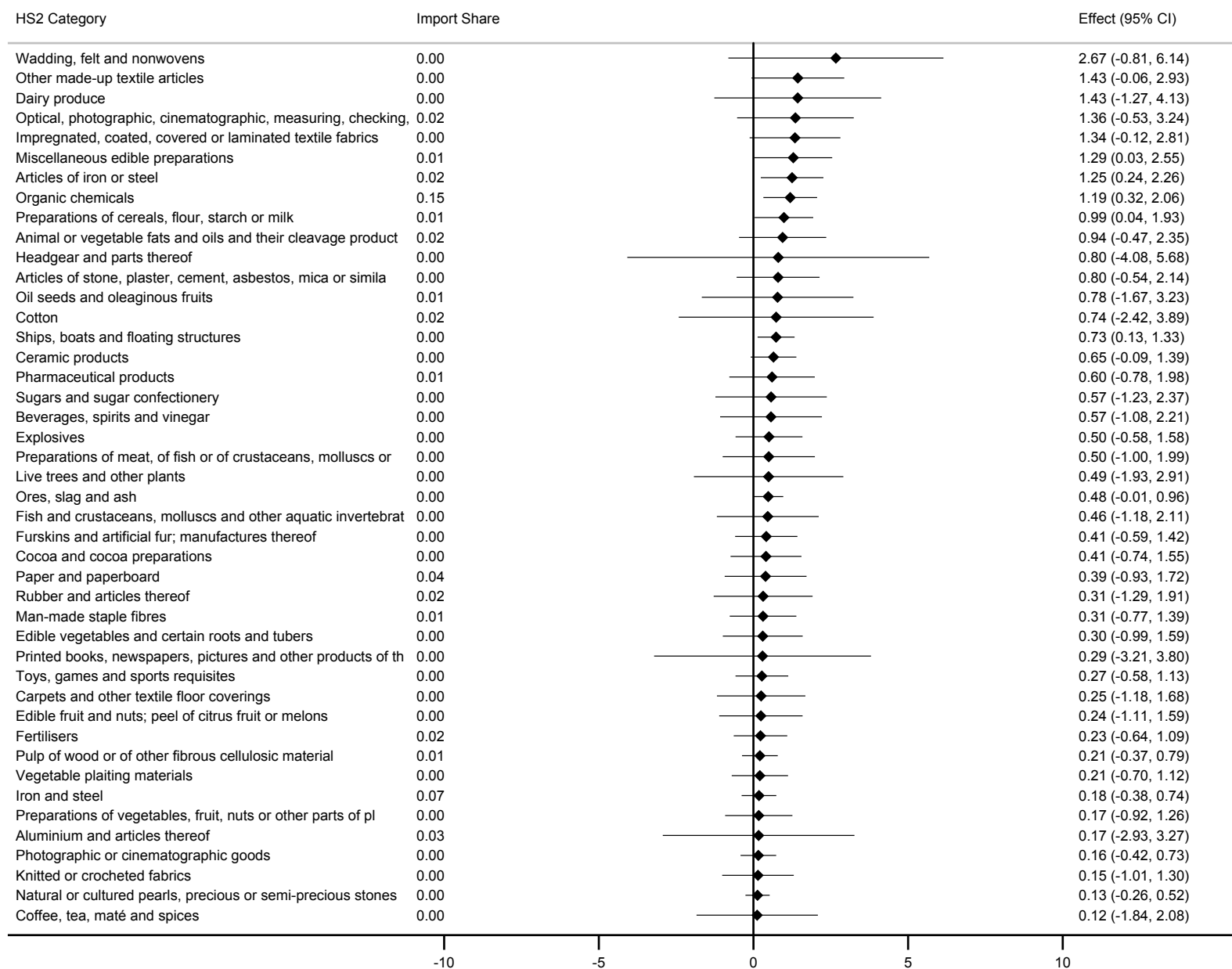


Figure A3. Real Exchange Rate Variation, 1994-2009 (cont.)



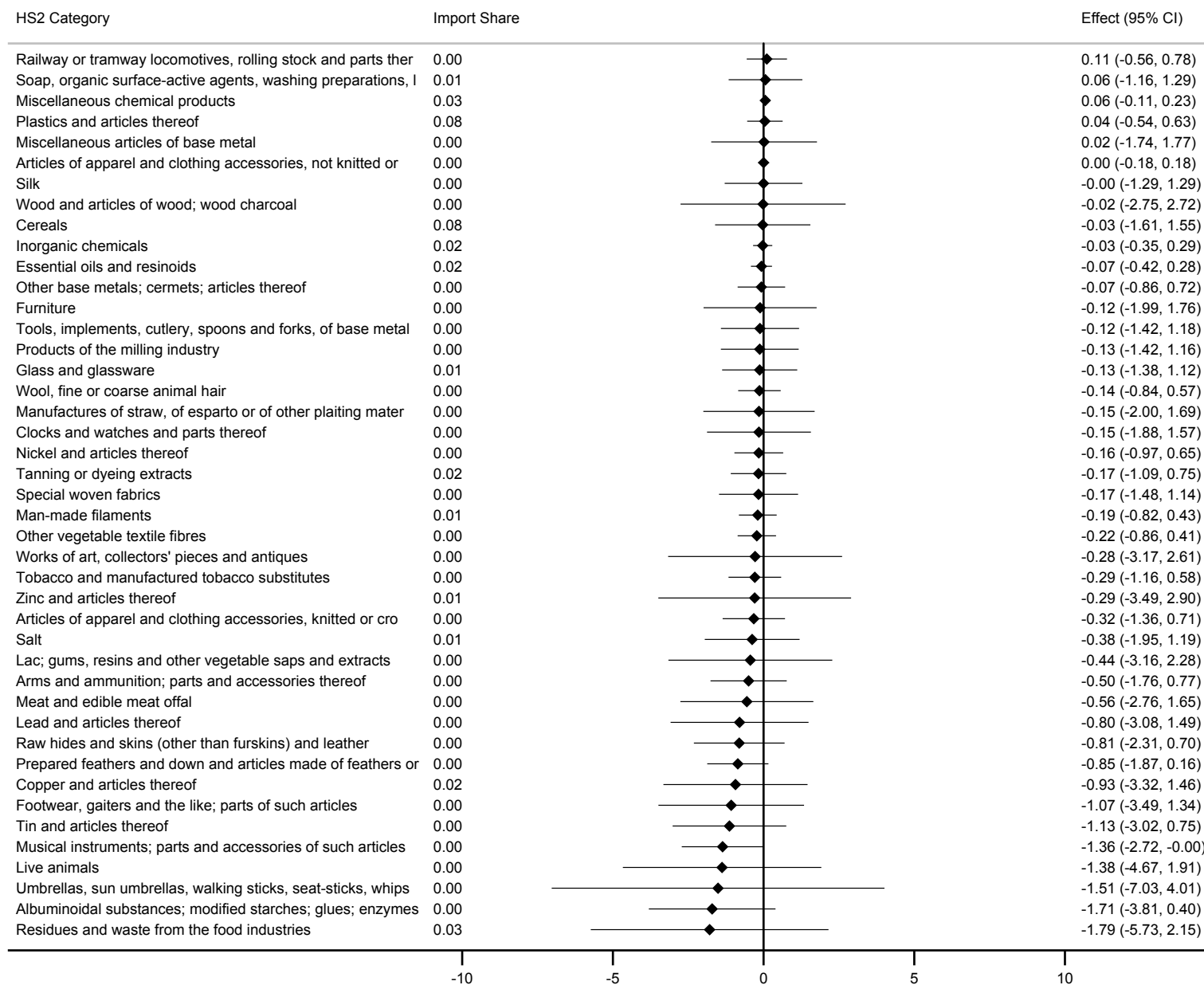
Notes: Figure plots real exchange rate (RER), normalized to 100 in 1994, calculated as in equation (17) in text, for import origins ranked 7-12 for rubber and plastics sectors. (See Fig. A1 for ranks 1-6.) An RER increase reflects a real appreciation in the trading partner.

Figure A4. Coefficients from Import-Price Regressions



Notes: Average sector-specific coefficients from estimating equation (19) in main text. We generate 362 sets of leave-one-out coefficient estimates, then average both estimates and standard errors across firms and years. All Harmonized System 2-digit categories except petroleum products, machinery and equipment (HS2 categories 27, 84 and 85) included; see Section 2.4.1 for details. Import share calculated as imports in HS2 category over total imports for 1994-2009 period.

Figure A4. Coefficients from Import-Price Regressions (cont.)



Notes on first page of table.

**Table A1. Primary Outputs and Inputs, Glass Products Producers**

CPC code	Share of total revenues/expenditures	CPC description
<b>A. Outputs</b>		
3719102	0.21	Glass bottles for soft drinks
3719103	0.18	Glass bottles of a capacity not exceeding 1 liter
3711502	0.18	Safety glass
3711201	0.12	Unworked flat glass
3719104	0.11	Glass bottles of a capacity exceeding 1 liter
3711503	0.06	Safety glass for motor car, windshield glass and similar articles
3719101	0.03	Small glass jars for perfumery, pharmacy and laboratory
3712204	0.01	Glass wool sheet
3719309	0.01	Glass vases
2799704	0.01	Asphalt fabrics
4299942	0.01	Wire rods and rings, for brassieres
3719302	0.01	Glasswares of a kind used for table and kitchen
3712203	0.01	Fiberglass ducts
3719503	0.01	Glass ampoules
3712101	0.01	Fiberglass
3711601	0.01	Unframed mirror
3712907	0.01	Fiberglass bathtubs
3712908	0.01	Fiberglass tanks
3711501	0.00	Tempered glass
3719903	0.00	Glass screens
<b>B. Inputs</b>		
3711201	0.30	Unworked flat glass
3424501	0.22	Sodium carbonate
3711103	0.10	Waste and scrap of glass
3633019	0.07	Plastic fabric
3633007	0.05	Polyvinyl film
1531201	0.05	Siliceous sands and gravels
1639902	0.03	Feldspar
3219702	0.03	Printed labels
3474002	0.02	Polyester resins
1512004	0.02	Crushed or ground limestone
3215308	0.02	Partitions and dividers of carboard for boxes
3215302	0.01	Corrugated cardboard boxes
4151203	0.01	Angles, shapes and sections of copper
3511104	0.01	Anticorrosive bases and paints
3712101	0.01	Fiberglass
3170101	0.01	Wooden packaging box
4299942	0.01	Wire rods and rings, for brassieres
3170105	0.01	Pallets
3424202	0.01	Sodium sulfate
3641002	0.01	Unprinted plastic film in tubular form

Notes: Sample is producers of glass products (ISIC rev. 2 category 362). Shares calculated as revenues from output over total revenues (outputs) or expenditures on input over total expenditures (inputs) for 2000-2009 period, pooling firms and years.

**Table A2. Summary Statistics, Glass Products**

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<i>A. Period: 1996-2009</i>	
Number of Observations	410
Number of Firms	34
Number of Workers	122.97
Production value (billions 2000 pesos)	16.41
Earnings per year, permanent workers (millions 2000 pesos)	7.06

<i>B. Period: 2000-2009</i>	
<i>Input variables</i>	
No. inputs per firm in average firm-year	9.43
Share of firms that import	0.65
No. inputs per firm in avg. firm-year, cond. on importing	9.50
Fraction of expenditure on imported inputs	0.46
No. imported HS8 categories in avg. firm-year, cond. on importing	24.74
<i>Output variables</i>	
No. outputs per firm in average firm-year	2.92
Share of firms that export	0.53
No. outputs per firm in avg. firm-year, cond. on exporting	3.48
Fraction of revenues from exported outputs	0.25
No. exported HS8 categories in avg. firm-year, cond. on exporting	5.31

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Notes: Sample is producers of glass products (ISIC rev. 2 category 362). Exports and imports available in EAM data only in 2000-2009. Average 2000 exchange rate is approximately 2,000 pesos/USD.

**Table A3. Difference Equation, Quantity Indexes, GMM-Style Instruments**

	Dep. var.: $\Delta \tilde{y}_{it}^{SV}$		
	(1)	(2)	(3)
$\Delta \tilde{m}_{it}^{SV}$	0.580*** (0.129)	0.523*** (0.094)	0.434*** (0.077)
$\Delta \log \text{ labor } (\Delta \ell_{it})$	0.442*** (0.166)	0.466*** (0.134)	0.441*** (0.099)
$\Delta \log \text{ capital } (\Delta k_{it})$	-0.015 (0.111)	0.005 (0.070)	0.044 (0.050)
N	4,247	4,247	4,247
Lag Limit	2	3	all
Number of excluded instruments	42	81	315
Hansen test	41.59	71.73	306
Hansen p-value	0.358	0.678	0.584
F - SW $\Delta \tilde{m}_{it}^{SV}$	1.538	1.355	1.770
F - SW $\Delta \log \text{ labor } (\Delta \ell_{it})$	2.186	1.797	1.814
F - SW $\Delta \log \text{ capital } (\Delta k_{it})$	3.324	3.100	2.328
KP LM statistic (underidentification)	50.78	104.4	387.9
KP Wald F-stat (weak instruments)	1.266	1.411	1.775

Notes: Table reports GMM estimation of our difference equation, (16), where further lags have been added “GMM-style” (Holtz-Eakin et al., 1988; Roodman, 2009), using only available lags and allowing separate coefficients in each period. Lags are included just to  $t - 2$  in Column 1, to  $t - 3$  in Column 2, and to the firm’s initial year in Column 3. The first-stage coefficients are not reported, but number of instruments and the Sanderson-Windmeijer F-statistics corresponding to the first-stage regressions are reported in each column. Robust standard errors in parentheses. \*10% level, \*\*5% level, \*\*\*1% level.

**Table A4. Differences (Step 1): First Stage, Alternative Aggregators**

	$\Delta \tilde{m}_{it}^{Torn}$	$\Delta \ell_{it}$	$\Delta k_{it}$	$\Delta \tilde{m}_{it}^{Lasp}$	$\Delta \ell_{it}$	$\Delta k_{it}$	$\Delta \tilde{m}_{it}^{Paas}$	$\Delta \ell_{it}$	$\Delta k_{it}$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\tilde{m}_{it-2}^{Torn}$	-0.015** (0.006)	0.011*** (0.004)	0.026*** (0.005)						
$\tilde{m}_{it-2}^{Lasp}$				-0.021*** (0.007)	0.012*** (0.004)	0.025*** (0.005)			
$\tilde{m}_{it-2}^{Paas}$							-0.015** (0.006)	0.013*** (0.004)	0.025*** (0.005)
$\ell_{it-2}$	0.012 (0.010)	-0.030*** (0.007)	0.042*** (0.010)	0.016 (0.010)	-0.031*** (0.007)	0.042*** (0.010)	0.010 (0.009)	-0.031*** (0.007)	0.043*** (0.010)
$k_{it-2}$	0.005 (0.006)	0.008** (0.004)	-0.049*** (0.007)	0.009 (0.006)	0.008** (0.004)	-0.049*** (0.007)	0.005 (0.006)	0.008** (0.004)	-0.049*** (0.007)
$\Delta$ pred. import price index ( $\Delta \hat{w}_{it}^{imp}$ )	-0.210** (0.099)	-0.047 (0.063)	0.118 (0.102)	-0.249** (0.098)	-0.048 (0.063)	0.116 (0.102)	-0.260*** (0.099)	-0.046 (0.063)	0.120 (0.102)
$\Delta$ log min. wage x “bite” ( $\Delta z_{it}$ )	-1.770** (0.885)	-1.738*** (0.495)	-2.025*** (0.600)	-1.914** (0.860)	-1.729*** (0.496)	-2.019*** (0.599)	-1.680* (0.871)	-1.743*** (0.495)	-2.049*** (0.600)
Year effects	Y	Y	Y	Y	Y	Y	Y	Y	Y
N	4,247	4,247	4,247	4,247	4,247	4,247	4,247	4,247	4,247
R squared	0.024	0.038	0.042	0.027	0.038	0.041	0.026	0.039	0.041
F - statistic	3.209	6.860	14.496	4.337	7.164	13.538	3.095	7.143	14.035
F - SW	5.326	11.605	18.842	6.803	11.849	17.27	5.050	12.131	17.928
KP LM test (underidentification)		14.440			17.890			14.000	
KP Wald F-test (weak insts.)		3.010			3.830			2.927	

Notes: Specifications similar to Columns 7-9 of Table 5 but using alternative quantity indexes (Tornqvist, Lapeyres, Paasche) defined in Appendix A.4. Dependent variables at tops of columns. SW refers to Sanderson and Windmeijer (2016), KP to Kleibergen and Paap (2006). The F-statistic is the standard F for a test that the coefficients on the excluded instruments (indicated at left) are zero. The KP statistics, LM test for under-identification and Wald F test for weak instruments, are for each IV model as a whole, and are not specific to Columns 2, 5, 8. Robust standard errors in parentheses. \*10% level, \*\*5% level, \*\*\*1% level.

**Table A5. Differences (Step 1): Second Stage, Alternative Aggregators**

	$\Delta \log$ Tornqvist output index (1)	$\Delta \log$ Laspeyres output index (2)	$\Delta \log$ Paasche output index (3)
$\Delta \log$ Tornqvist materials index ( $\Delta \tilde{m}_{it}^{Torn}$ )	0.470** (0.206)		
$\Delta \log$ Laspeyres materials index ( $\Delta \tilde{m}_{it}^{Lasp}$ )		0.400** (0.166)	
$\Delta \log$ Paasche materials index ( $\Delta \tilde{m}_{it}^{Paas}$ )			0.375* (0.195)
$\Delta \log$ labor ( $\Delta \ell_{it}$ )	0.414** (0.200)	0.340* (0.189)	0.434** (0.189)
$\Delta \log$ capital ( $\Delta k_{it}$ )	-0.203 (0.131)	-0.146 (0.129)	-0.216* (0.122)
Year effects	Y	Y	Y
N	4,247	4,247	4,247
R-squared	0.205	0.247	0.218
Materials Robust (LC) Conf. Interval 90%	[0.202 - 0.737]	[0.183 - 0.616]	[0.121 - 0.629]
Labor Robust (LC) Conf. Interval 90%	[0.154 - 0.674]	[0.095 - 0.586]	[0.189 - 0.679]
Materials Robust (LC) Conf. Interval 95%	[0.151 - 0.788]	[0.141 - 0.658]	[-0.129 - 0.678]
Labor Robust (LC) Conf. Interval 95%	[0.104 - 0.724]	[0.048 - 0.633]	[0.142 - 0.726]
Arellano-Bond AR(2) statistic	0.359	0.364	0.340
Arellano-Bond p-value	0.720	0.716	0.734

Notes: Specifications similar to Column 3 of Table 5 but using alternative quantity indexes as defined in Appendix A.4. Robust standard errors in parentheses. \*10% level, \*\*5% level, \*\*\*1% level.



**Table A6. Levels (Step 2): Second Stage, Alternative Aggregators**

	Dep. var.: $\tilde{y}_{it} - \hat{\beta}_m \tilde{m}_{it} - \hat{\beta}_\ell \ell_{it}$		
	Tornqvist (1)	Laspeyres (2)	Paasche (3)
log capital $k_{it}$	0.104 (0.089) [0.209]	0.222 (0.095) [0.188]	0.169 (0.083) [0.197]
Year effects	Y	Y	Y
N	4,247	4,247	4,247
R-squared	0.081	0.192	0.146

Notes: The output and input aggregates used to construct the dependent variable are indicated at the top of each column. Columns correspond to Appendix Table A5. The first stage of this levels (Step 2) IV model is identical to that reported in Table 7. Uncorrected robust standard errors in parentheses. Corrected robust standard errors in brackets. See Section 2.4.2 for details. Tornqvist, Paasche and Laspeyres quantity indexes are defined in Appendix A.4. \*10% level, \*\*5% level, \*\*\*1% level.

**Table A7. Differences (Step 1): First Stage, Including Export Price Index**

	$\Delta \tilde{m}_{it}^{SV}$ (1)	$\Delta \ell_{it}$ (2)	$\Delta k_{it}$ (3)
$\tilde{m}_{it-2}^{SV}$	-0.018*** (0.006)	0.013*** (0.004)	0.026*** (0.005)
$\ell_{it-2}$	0.012 (0.009)	-0.031*** (0.007)	0.042*** (0.010)
$k_{it-2}$	0.007 (0.006)	0.008** (0.004)	-0.049*** (0.007)
$\Delta$ pred. import price index ( $\Delta \hat{w}_{it}^{imp}$ )	-0.254*** (0.099)	-0.056 (0.063)	0.134 (0.102)
$\Delta$ log min. wage x “bite” ( $\Delta z_{it}$ )	-1.793** (0.862)	-1.740*** (0.495)	-2.018*** (0.601)
$\Delta$ pred. export price index ( $\Delta \hat{w}_{it}^{exp}$ )	0.007 (0.094)	0.103* (0.062)	-0.163 (0.100)
Year effects	Y	Y	Y
N	4,247	4,247	4,247
R-squared	0.026	0.039	0.042
F - statistic	3.611	7.437	13.640
F - SW	5.724	12.564	16.414
KP LM test (underidentification)		15.410	
KP Wald F-test (weak insts.)		3.248	

Notes: Dependent variables at tops of columns. SW refers to Sanderson and Windmeijer (2016), KP to Kleibergen and Paap (2006). The F-statistic is the standard F for a test that the coefficients on the excluded instruments (indicated at left) are zero. The KP statistics, LM test for under-identification and Wald F test for weak instruments, are for each IV model as a whole, and are not specific to Columns 2, 5, 8. Robust standard errors in parentheses. \*10% level, \*\*5% level, \*\*\*1% level.

**Table A8. Differences (Step 1): Second Stage, Including Export Price Index**

	Dep. var.: $\Delta \log$ output index ( $\Delta \tilde{y}_{it}^{SV}$ ) (1)
$\Delta \tilde{m}_{it}^{SV}$	0.375** (0.180)
$\Delta \log$ labor ( $\Delta \ell_{it}$ )	0.395** (0.183)
$\Delta \log$ capital ( $\Delta k_{it}$ )	-0.182 (0.125)
$\Delta$ pred. export price index ( $\Delta \hat{w}_{it}^{exp}$ )	-0.040 (0.095)
N	4,247
R-squared	0.238
Materials Robust (LC) Conf. Interval 90%	[ 0.141 - 0.609]
Labor Robust (LC) Conf. Interval 90%	[ 0.157 - 0.633]
Materials Robust (LC) Conf. Interval 95%	[-0.090 - 0.653]
Labor Robust (LC) Conf. Interval 95%	[ 0.112 - 0.679]
Arellano-Bond AR(2) statistic	0.338
Arellano-Bond p-value	0.735

Notes: Corresponding first-stage estimates are in Appendix Table A7. Robust standard errors in parentheses. Weak-instrument-robust confidence intervals are based on LC test of Andrews (2018), implemented by Stata `twostepweakiv` command. Arellano-Bond statistic and p-value test for serial correlation in residual, based on Arellano and Bond (1991). \*10% level, \*\*5% level, \*\*\*1% level.

**Table A9. Differences (Step 1): First Stage, Alternative Samples**

	plastics-only			including glass		
	$\Delta \tilde{m}_{it}^{SV}$ (1)	$\Delta \ell_{it}$ (2)	$\Delta k_{it}$ (3)	$\Delta \tilde{m}_{it}^{SV}$ (4)	$\Delta \ell_{it}$ (5)	$\Delta k_{it}$ (6)
$\tilde{m}_{it-2}^{SV}$	-0.022*** (0.007)	0.011*** (0.004)	0.026*** (0.006)	-0.020*** (0.006)	0.014*** (0.004)	0.027*** (0.005)
$\ell_{it-2}$	0.010 (0.009)	-0.031*** (0.008)	0.045*** (0.011)	0.011 (0.009)	-0.032*** (0.007)	0.041*** (0.010)
$k_{it-2}$	0.011* (0.006)	0.009** (0.004)	-0.050*** (0.007)	0.010* (0.005)	0.008** (0.003)	-0.046*** (0.006)
$\Delta$ pred. import price index ( $\Delta \hat{w}_{it}^{imp}$ )	-0.297*** (0.106)	-0.107 (0.067)	0.101 (0.114)	-0.235*** (0.079)	-0.039 (0.049)	0.099 (0.078)
$\Delta$ log min. wage x “bite” ( $\Delta z_{it}$ )	-1.426 (0.960)	-1.641*** (0.518)	-2.161*** (0.650)	-1.692** (0.809)	-1.644*** (0.467)	-1.816*** (0.556)
Year effects	Y	Y	Y	Y	Y	Y
Observations	3,693	3,693	3,693	4,657	4,657	4,657
R-squared	0.030	0.037	0.041	0.028	0.040	0.041
F - statistic	3.847	6.147	11.922	4.626	8.235	15.575
F - SW	5.316	9.719	11.419	6.695	13.576	16.517
KP LM test (underidentification)		14.040			18.450	
KP Wald F-test (weak insts.)		2.958			3.852	

Notes: Table similar to Table 5, Columns 7-9, for alternative samples of (a) plastics producers only (Columns 1-3) and (b) rubber, plastic, and glass product producers (Columns 4-6). Robust standard errors in parentheses. \*10% level, \*\*5% level, \*\*\*1% level.

**Table A10. Differences (Step 1): Second Stage, Alternative Samples**

	Dep. var.: $\Delta \log$ output index ( $\Delta \tilde{y}_{it}^{SV}$ )	
	plastics-only (1)	including glass (2)
$\Delta \log$ materials index ( $\Delta \tilde{m}_{it}^{SV}$ )	0.351** (0.175)	0.409** (0.170)
$\Delta \log$ labor ( $\Delta \ell_{it}$ )	0.328* (0.199)	0.439** (0.175)
$\Delta \log$ capital ( $\Delta k_{it}$ )	-0.158 (0.123)	-0.188 (0.122)
Year effects	Y	Y
Observations	3,693	4,657
R-squared	0.268	0.220
Materials Robust (LC) Conf. Interval 90%	[0.123 - 0.579]	[0.188 - 0.631]
Labor Robust (LC) Conf. Interval 90%	[0.070 - 0.587]	[0.212 - 0.666]
Materials Robust (LC) Conf. Interval 95%	[-0.102 - 0.623]	[0.146 - 0.673]
Labor Robust (LC) Conf. Interval 95%	[0.020 - 0.636]	[0.169 - 0.709]
Arellano-Bond AR(2) statistic	0.414	0.803
p-value Arellano-Bond test	0.679	0.422

Notes: Corresponding first-stage estimates are in Table A9. Samples are (a) plastics producers only (Column 1) and (b) rubber, plastic, and glass product producers (Column 2). Robust standard errors in parentheses. Weak-instrument-robust confidence intervals are based on LC test of Andrews (2018), implemented by Stata `twostepweakiv` command. Arellano-Bond statistic and p-value test for serial correlation in residual, based on Arellano and Bond (1991). \*10% level, \*\*5% level, \*\*\*1% level.

**Table A11. Levels (Step 2): First & Second Stages, Alternative Samples**

		Dep. var.: log capital ( $k_{it}$ )	
		plastics-only (1)	including glass (2)
$\Delta k_{it-1}$		0.616*** (0.110)	0.767*** (0.110)
Year effects		Y	Y
N		3,693	4,657
R-squared		0.026	0.030
Kleibergen-Paap LM test		31.889	48.426
Kleibergen-Paap Wald F-test		31.409	43.404
<hr/>			
		Dep. var.: $\tilde{y}_{it}^{SV} - \hat{\beta}_m \tilde{m}_{it}^{SV} - \hat{\beta}_\ell \ell_{it}$	
		plastics-only (1)	including glass (2)
log capital $k_{it}$		0.214 (0.103) [0.204]	0.195 (0.073) [0.180]
Year effects		Y	Y
N		3,693	4,657
R-squared		0.242	0.161

Notes: Uncorrected robust standard errors in parentheses. Corrected robust standard errors in brackets. See Section 2.4.2 for details. \*10% level, \*\*5% level, \*\*\*1% level.

**Table A12. System GMM, Weak IV Diagnostics**

Covariates	Differences			Covariates	Dep. var.: log sales <sub>it</sub> (4)
	Dep. var.: $\Delta \log \text{sales}_{it}$ (1)	(2)	(3)		
$\Delta \log \text{sales}_{it-1}$	0.264** (0.106)	0.277*** (0.091)	0.183*** (0.060)	log sales <sub>it-1</sub>	0.680*** (0.108)
$\Delta \log \text{expenditure}_{it}$	0.270** (0.113)	0.387*** (0.092)	0.397*** (0.053)	log expenditure <sub>it</sub>	0.556*** (0.172)
$\Delta \log \text{expenditure}_{it-1}$	-0.142* (0.081)	-0.115* (0.065)	-0.073 (0.045)	log expenditure <sub>it-1</sub>	-0.274*** (0.103)
$\Delta \log \text{labor} (\Delta \ell_{it})$	0.290 (0.183)	0.339** (0.144)	0.341*** (0.070)	log labor <sub>it</sub> ( $\ell_{it}$ )	-0.432* (0.239)
$\Delta \log \text{labor} (\Delta \ell_{it-1})$	-0.077 (0.142)	0.069 (0.116)	0.046 (0.062)	log labor <sub>it-1</sub> ( $\ell_{it-1}$ )	0.481** (0.209)
$\Delta \log \text{capital} (\Delta k_{it})$	0.009 (0.141)	-0.003 (0.081)	-0.004 (0.053)	log capital <sub>it</sub> ( $k_{it}$ )	0.016 (0.127)
$\Delta \log \text{capital} (\Delta k_{it-1})$	-0.200* (0.117)	-0.146* (0.084)	-0.084* (0.046)	log capital <sub>it-1</sub> ( $k_{it-1}$ )	0.010 (0.125)
N	4,247	4,247	4,247		4,247
R-squared	0.166	0.203	0.264		0.961
Lag Limit	2	3	all		NA
Number of excluded instruments	56	108	420		56
SW F-stat log sales <sub>it</sub>	1.858	2.070	2.233		3.970
SW F-stat log expenditure <sub>it</sub>	2.164	2.034	2.473		1.845
SW F-stat log expenditure <sub>it-1</sub>	2.149	2.334	3.869		2.094
SW F-stat log labor ( $\ell_{it}$ )	1.796	1.643	1.985		1.238
SW F-stat log labor ( $\ell_{it-1}$ )	2.149	2.334	3.869		1.392
SW F-stat log capital ( $k_{it}$ )	1.549	2.120	1.970		1.339
SW F-stat log capital ( $k_{it-1}$ )	1.728	2.208	1.855		1.400
KP LM test (underidentification)	75.270	123.959	444.033		51.838
KP Wald test (weak instruments)	1.425	1.462	1.835		0.968

Notes: Table reports IV estimates corresponding to differences (Columns 1-3) and levels (Column 4) equations of System GMM, with weak-instrument diagnostic statistics. Robust standard errors in parentheses. \*10% level, \*\*5% level, \*\*\*1% level.

**Table A13. System GMM, Using Sato-Vartia Quantity Indexes**

	Dep. var.: log output index ( $\tilde{y}_{it}^{SV}$ )		
	(1)	(2)	(3)
$\tilde{y}_{it-1}^{SV}$	0.867*** (0.031)	0.862*** (0.030)	0.826*** (0.027)
$\tilde{m}_{it}^{SV}$	0.357*** (0.082)	0.398*** (0.078)	0.415*** (0.055)
$\tilde{m}_{it-1}^{SV}$	-0.306*** (0.068)	-0.325*** (0.069)	-0.297*** (0.053)
log labor ( $\ell_{it}$ )	0.253* (0.152)	0.200 (0.150)	0.217** (0.095)
log labor ( $\ell_{it-1}$ )	-0.204 (0.142)	-0.150 (0.135)	-0.183** (0.078)
log capital ( $k_{it}$ )	0.225** (0.114)	0.151* (0.087)	0.101** (0.047)
log capital ( $k_{it-1}$ )	-0.199** (0.099)	-0.139* (0.076)	-0.098** (0.043)
N	4,247	4,247	4,247
Lag limit	2	3	All
Hansen test	124.1	191.3	348.9
Hansen p-value	0.099	0.052	1.000

Notes: Table is similar to Table 8 but using CES output and input quantity indexes in places of log sales and log expenditures. The numbers of instruments are as indicated in Appendix Table A12. Robust standard errors in parentheses. \*10% level, \*\*5% level, \*\*\*1% level.



**Table A14. Correlation of TFP Measures**

	TSIV	OLS-SV	SysGMM	OP	LP	Wool- dridge	GNR	ACF
	(1)	(2)	(3)	(5)	(6)	(7)	(8)	(9)
TSIV	1.00							
OLS-SV	0.54	1.00						
SysGMM	0.94	0.77	1.00					
OP	0.34	0.92	0.61	1.00				
LP	0.45	0.95	0.70	0.99	1.00			
Wooldridge	0.76	0.92	0.92	0.87	0.92	1.00		
GNR	0.59	0.99	0.79	0.88	0.92	0.91	1.00	
ACF	0.38	0.79	0.57	0.80	0.81	0.75	0.77	1.00

Notes: Table reports pairwise correlation coefficients (using all available observations for each pair) of TFPR in levels defined as in equation (28) in footnote 59, using coefficient estimates as follows: baseline estimates from Table 6, Column 3 and Table 7, Panel B, Column 2 (TSIV); OLS using Sato-Vartia quantity indexes with year effects, as in Table 4, Panel B, Column 2 (OLS-SV); System GMM using all available lags, as in Table 8, Column 3 (SysGMM); Olley and Pakes (1996), Levinsohn and Petrin (2003), Wooldridge (2009), and Gandhi et al. (2020), as in Table 9 (OP, LP, W, and GNR, respectively); Akerberg et al. (2015) using a value-added production function (ACF), estimated using Stata command `prodest` (Rovigatti and Mollisi, 2018).