The Informational Role of Sponsored Advertising on Online Retail Marketplaces

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Abstract

E-commerce platforms, such as Amazon, Alibaba and Flipkart, have transformed the retail sector by matching sellers and consumers at an unprecedented scale. These platforms operate their internal search engines to help buyers find relevant products from a huge number of sellers, and also allow sellers to advertise to consumers, via ad auctions, for positions in the search listing. Determining an optimal ranking of products in response to a search query is a challenging problem for the platform because sellers have certain private information about products they sell that the platform does not have. However, sellers’ bids in ad auctions can reveal this information, which can be used to refine the ranking of products in the organic listing. In this paper, we explore how a platform’s internal search engine should be designed, by modeling this ecosystem as a Bayesian game with multiple sellers on the platform, each of whom has private information about his product’s match with consumers’ needs. The platform can use the sellers’ bids for sponsored advertising to estimate this information and improve the ordering of its organic listings (“strategic listing” case). We compare this strategy against the benchmark case of ranking organic links independently from sellers’ bids (“independent listing” case). We find that while strategic listing tends to dominate, it also has a major impact on the platform’s choice of commission as well as on sellers’ pricing and participation. In particular, we find that strategic listing of organic links reduces price competition between sellers but increases advertising competition between them; the latter effect reduces sellers’ incentives to participate, and therefore the platform must balance this by charging a lower commission rate. Therefore, the platform may not always find it beneficial to use strategic listing. These effects are stronger when consumers’ search cost becomes higher, or when the product fits the consumers’ needs with an extreme probability. We also prescribe the optimal commission rate that a platform should charge, by balancing between the revenue from commissions and the revenue from advertising.

Keywords: E-commerce platform, search advertising, retailing, asymmetric information
1 Introduction

The explosive growth of e-commerce platforms in recent years has fundamentally changed retail structures all over the world. Most major markets consist of a single dominant e-commerce platform, competing with several brick and mortar establishments for a larger share of the overall retail market. The appeal of these platforms derives from the variety of products and sellers that they make available on a single website. Often these third-party sellers sell directly to the consumers, with the platform acting like an intermediate marketplace. In the US, Amazon makes up 49% of the e-commerce market share. A growing number of sellers on Amazon marketplace are third-party sellers, where Amazon acts as a pure intermediary. Based on recent data, more than two million third-party sellers are selling on Amazon, accounting for more than half of the paid units sold on Amazon. In China, Alibaba constitutes 51% of the e-commerce market share. It relies 100% on its third-party sellers (there are around 10M of them in total) without selling directly to consumers. In India, Flipkart constitutes 60% of the e-commerce market share, with the majority of sales coming from third-party sellers.

With such a large and increasing number of third party sellers, optimally ranking products in response to a consumer search query becomes a daunting task. This task could often be aided by access to some private information that third party sellers possess. This information may be temporally unstable, e.g., based on market trends which third-party sellers, being active in their respective markets, can detect before the platform can detect it in data. Sellers' external marketing activities can also affect their products' relevance but this information might not be easily accessible to the platform. For example, if a consumer is searching for a professional bag for women, it might be more relevant to show a brand that just got mentioned in a viral blog post “the 10 best professional bags for women in 2018”, or a seller that started a marketing campaign on Instagram which makes her brand more popular. These affects are not permanent, and change quickly, before the platform can learn them eventually. More broadly, sellers possess information about their product, consumer wants and needs, etc., that the platform can miss. Recent empirical papers (e.g. Sharma et al. [2018]) have documented the existence of information asymmetry between the marketplace and their third-party sellers. In conversations with us, managers and researchers at several platforms (e.g., Alibaba, Zomato and Flipkart) also expressed, in no uncertain terms, that this is a problem that they face.

In order to help consumers better find a product match, e-commerce platforms operate their own internal product search engines. Figure 1 illustrates a typical search page that a consumer will see after
entering the search query ‘bags women job” at Amazon. Similar to search engines, sponsored product listings on Amazon are assigned via a competitive auction and they appear at the top of users’ organic product search results. Indeed, when it comes to searches related to products, Amazon has traded places with Google as the preferred search engine for products in the US. Based on recent data in December 2017, 52% of product searches took place on Amazon, compared to 26% on search engines, including Google. Alibaba, the dominant online retailer in China, is making a similar play against Baidu, the “Google of China”. Alibaba’s mobile search engine grew its market share from just 4% in October 2017 to 34% in February 2018, and is threatening the dominance of Baidu in China’s online search market.

E-commerce platforms’ rise as the most popular destination for product search has caused marketers to rethink their paid search campaigns. The relevance of e-commerce platforms as advertising platforms is growing and, as a result, is posing the first real challenge to search engines, which have long dominated digital advertising budgets virtually unopposed. Amazon is building its own digital advertising business that could reach $4.5bn in 2018. Amazon’s ad revenue almost doubled that in 2017, and it is growing more than twice as fast as Google’s ad revenue. Alibaba and Flipkart, which are the dominant online retailers in China and India respectively, are considered pure-play marketplaces and yet, have earned

the bulk of their revenues from sellers ads.

While sponsored advertising is becoming an emerging revenue driver for e-commerce platforms, its role in the marketplace can grow beyond an independent revenue stream. In particular, sellers’ bids in ad auctions may reveal their private information about products that the platform does not have. There is potential for the platform to learn sellers’ relevance to the query from these bids, and use this information to improve the ranking of organic listings. Such strategies have been employed in industry; for instance, marketing agencies report that a higher ranking in ads listings on Amazon correlates with more visibility on the organic listings.

In this paper, we study the linkage between sponsored advertising and organic ranking. We ask the question: Can advertising act as an information source for the platform? Can incorporating information obtained from sellers’ bids for advertising positions to improve the organic listing positively impact the platform’s overall revenue? We also study how this marketplace design strategy effects the platform’s commission rates, sellers’ pricing decisions, and its implications on consumer welfare. Ultimately, we try to answer when, if at all, is such a strategy beneficial to the platform.

We model the e-commerce platform as a Bayesian game with private information, where the probability of a product’s match with consumers is \textit{a priori} only known to the seller. This assumption is reasonable when sellers have some information about the relevance of their products to consumers that the platform does not have but would like to have. Consumers are heterogeneous in their search strategies, as they differ in their responsiveness to advertising (i.e., are affected or unaffected by ads), and in their search costs (i.e., they are either non-shoppers who have high search costs, or shoppers with low search costs). The e-commerce marketplace allocates a sponsored ad slot via a second-price auction, followed by sellers’ organic listings. Sellers bidding decisions in the search ad auction may reveal information about the relevance of their products, which, in turn, can be exploited by the platform to improve the ordering of its organic listings (the strategic case). To isolate the effect of strategic listing, we establish a benchmark case where the platform ranks organic links independent of search ads (the independent case).

We find that exploiting information from bid auctions may help the platform identify the product’s relevance — we call this an “information effect”. Compared to independent listing, strategic listing leads to efficient ordering of organic results and thus results in more sales for the platform. It also intensifies

\footnote{https://content26.com/ecommerce-content-resources/ebooks/definitive-guide-amazon-marketing-services}
advertising competition and, in equilibrium, the more-relevant seller, who is the winner of the ad auction, gets a better placement in both ad and organic listings. This leads to higher advertising revenues for the platform. Both insights suggest that if seller participation is guaranteed, the platform is always better off with the strategic listing of organic search results over independent listing. A major drawback of strategic listing, however, is that it dis-incentivizes sellers from participating on the platform in the first place, what we call the “competition effect”. Intuitively, if the platform can identify sellers’ relevance from their bids, it is able to extract more surplus from sellers by rearranging firm positions in the organic listing. This leads sellers to have less incentive to participate in selling and bidding in the marketplace.

Overall, the platform may or may not have enough incentive to use bidding information to refine the organic rankings, depending on various factors in that particular market. The platform is more likely to benefit from strategic listing of organic listings when the percentage of consumers with high search costs is small, or when sellers’ average match probabilities are in the intermediate range. In both of these scenarios, refining organic ranking using bidding information can benefit the platform, without pushing sellers to abandon the platform. This can be explained as follows. As the number of shoppers in the marketplace increases, the low-fit seller can still realize a sale, and thus, is more likely to participate ex-ante. When sellers’ average match probabilities are intermediate, a seller is relevant enough to make a sale, yet not relevant enough to consider the cost of advertising competition too high.

Our analysis also provides insights into how a platform can leverage its advertising business, together with the commission rate, to maximize overall revenue. In particular, the platform can rely on advertising to extract more surplus from sellers who are a better fit to a particular search query. Meanwhile, the commission rate can be readjusted to maximize seller participation. We also provide guidance to e-commerce platforms regarding how to set up their commission rate optimally, taking into account sellers’ participation. We find that a platform should lower its commission rate when searching for a product becomes more difficult (i.e., the size of consumers with a high search cost is high), or for mass product categories (i.e., very high relevance on average), or for very niche product categories (i.e., very low relevance on average). In line with our results, Tianmao—Alibaba’s business-to-consumer marketplace—charges only 2% to 5%, and Taobao—Alibaba’s consumer-to-consumer marketplace—charges zero commission rate per transaction, since consumers’ search becomes more difficult in typically fragmented markets with many small sellers. This is much lower than the 15% that Amazon makes on their popular product categories.
In the above analyses, we keep sellers’ prices exogenous because we assume, realistically, that prices are set before a query is input by the consumer. As an extension, we discuss the case where sellers choose prices endogenously. In this scenario, platforms have another potential benefit from adopting strategic listing—it can reduce price competition. The intuition is as follows. Expecting that the more-relevant seller gets both the ad slot and the first organic slot, sellers will not reduce prices significantly to compete for shoppers. For consumers, however, the benefit from a better ordering of organic listings under the strategic case is dominated by the higher selling price, i.e., strategic ordering of search results lowers consumer surplus when sellers choose prices endogenously.

Our research question is unique in the context of an e-commerce marketplace and has not been studied in previous search advertising literature. First, the nature of organic and sponsored listings in the context of a pure search engine can be very different: an organic listing is more information-based, whereas a sponsored listing is more product oriented. At e-commerce marketplaces, however, both sponsored and organic listings are meant for product sales, therefore, the information revealed from a sponsored listing can directly contribute to organic rankings. Second, recently, search engines have been attempting to regain lost ground from retail platforms by monetizing consumers’ e-commerce activities. Google, for example, introduced the “Shopping Actions” program that enables retailers to list products across Google Search and charges a percentage of each sale from the Google Shopping listings, earning essentially a commission revenue from sales generated from these placements. Furthermore, to make sure that the new program will not affect its core reputation as a search engine, Google explicitly promises that no organic rankings will be impacted or changed. In other words, the listings appear in Google Shopping results and do not affect the rankings or listings of the regular organic Google search results. E-commerce platforms do not face such constraints, however. The primary role of E-commerce platforms is to match consumers with product sellers using the vast amount of transaction data generated by buyers and sellers.

Our paper contributes to practice by proposing an ecosystem that allows the e-commerce platform to design its internal search function to match consumers with the most relevant products, while maximizing its own revenues. Using the information effect in organic listings enables the e-commerce platform to distinguish between related products based on the bidding of their sellers, by utilizing the advertising strategies of the sellers to better understand where their products are more relevant. It also aids the

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platform to suggest new products to users when relevant, instead of falling prey to the cold start problem where they can’t recommend these products to anyone due to a lack of data. In both scenarios, utilizing the information effect gives the e-commerce platform an edge over search engines. By providing organic listings that are more relevant to the users they gain the consumer’s trust and become the de-facto mode of product search. We find that if consumers do not fully internalize the information revealed by ads listings, the platform can appropriate this benefit by strategic listing, yet not always. We establish the conditions under which the platform would benefit from exploiting information from ad auctions to better match consumers with sellers in organic listings, and how it would affect the platform’s commission rate, its advertising revenue, as well as consumer surplus. Our insights help explain the very different commission rates across platforms operating in different geographic regions. To the best of our knowledge, our paper is the first attempt to provide a comprehensive framework in this important, rapidly emerging area.

The theoretical contribution of our paper is two-fold. First, facing asymmetric information, the first-best solution for the platform is to propose a menu of two-part tariffs (with different combinations of upfront payments and commission rates) to induce sellers’ self-selection. However, this is not practical due to its complication. In practice, the platform is constrained with a single commission rate in order to keep the contract simple. Our paper presents, under such constrain, the optimal design parameters for the platform to maximize its overall revenue. Second, we identify a new force for mixed pricing equilibrium, as sellers balance between competing for shoppers by lowering price versus competing for favorable slots by raising price.

The rest of the paper is organized as follows. We discuss related literature in Section 2. Next, we formally describe the model in Section 3. We then analyze the implications of listing strategies on both sellers’ profit and consumer surplus in Section 4. In Section 5, we then turn to the platform and present the platform’s optimal search design. This includes the optimal choice of commission rates and listing strategies. In Section 6, we analyze two alternatives for modelling price competition between sellers, and discuss how price competition can moderate our main insights. Finally, in Section 7, we discuss the practical implications of our research and point out future research directions, both theoretical and empirical.
2 Relevant Literature

Broadly, our paper is related to two streams of literature: the literature on economics of platforms, and the literature on sponsored search advertising. We combine these two streams of literature by studying the interaction between sponsored advertising and organic listings in the presence of asymmetric information at e-commerce marketplaces.

First, a growing literature has focused on the economics of e-commerce platforms, especially on the matching between sellers and consumers. In particular, the informative role of advertising in e-commerce platforms facing asymmetric information has been supported by some recent empirical papers. For example, Sharma et al. [2018] employ a field experiment at the leading Indian online marketplace Flipkart to show that online marketplaces indeed suffer from the information asymmetry problem, i.e., independent sellers have relevant private information that the platform does not have. Furthermore, they show that search advertising on the platform can work as a screening mechanism to identify high-relevance products. Sahni and Nair [2018] conduct a field experiment on the online food delivery and restaurant search marketplace Zomato to show that advertising can serve as a signal that leads to enhanced evaluations of advertised restaurants. Our paper adds to these works by further examining how product relevance information revealed from sellers’ bids can help a platform to improve its organic listings. We note that sellers in our model do not use their bids to “signal” their types to the platform; instead, we use a mechanism design approach in which the seller types are revealed in equilibrium.

Several other related papers consider the optimal platform design problem by accounting for consumer search behaviors and sellers’ pricing responses. Choi and Mela [2016] study the optimal fee structures for an online marketplace, arguing that the pay-per-click structure (similar to the click-based advertising fee) outperforms pay-per-action structure (similar to the sales-based commission fee) for the online marketplace by realigning sellers’ economic incentives. Armstrong and Zhou [2011] consider intermediaries’ incentive to bias their advice in favor of firms from which they receive larger payments. Hagiu and Jullien [2011] analyze the incentive for an intermediary who has superior information about the match between consumers and stores to direct consumers to their least favorite store to increase consumer traffic and influence store pricing. Eliaz and Spiegler [2011] discuss an intermediary’s incentive to include more firms with low relevance to generate more clicks and increase selling price. Cornière and Taylor [2014] study the bias of a search engine to direct searchers towards publishers that display many ads to increase the price of sponsored links. Zhong [2016] considers the platform design problem in choosing...
the optimal search precision, when sellers set price strategically in response to the precision of search targeting. None of these papers, however, consider the platform’s revenue streams from both advertising and commissions, and associated with it, the information spillover from advertising to the organic side. Finally, we derive implications of how the platform should balance revenue from commissions on sales versus revenue from advertising fees, a question that is related to the literature on monetizing media platforms where the platform balances between generating revenue from charging for content or subsidizing the content by showing (and charging for) advertising [Godes et al. 2009, Lambrecht and Misra 2017].

For sponsored search advertising, mechanisms for position auctions and their equilibrium properties have been investigated extensively (Varian 2007, Edelman et al. 2007, Feng et al. 2007, Feng 2008, Chen and He 2011, Athey and Ellison 2011, Jerath et al. 2011, Xu et al. 2011, Zhu and Wilbur 2011, Gomes and Sweeney 2014, Sayedi et al. 2014, Sayedi et al. 2017). Among them, Jerath et al. 2011 studies the bidding strategies of vertically differentiated firms. Chen and He 2011 examine the informative role of paid placement to reveal seller’s private information about the relevance of their products, improving efficiency of consumer searches (we make a similar point in the ad auction part). However, in other aspects, our model is different. In particular, our work departs from this stream of literature by focusing less on the optimal auction design and more on how the ad auction interacts with organic listing.

The possible ways of how search ads can interact with organic listings has been studied by Katona and Sarvary 2010, Berman and Katona 2013, Yang and Ghose 2010, White 2008, Agarwal et al. 2015 and Liu and Viswanathan 2014. For example, Katona and Sarvary 2010 show that sites with lower attractiveness may obtain the top position in sponsored links when consumers are adverse to sponsored links and the marginal return of clicks is decreasing. White 2008 discusses the trade-off for a search engine to provide high quality organic results to attract users which may cannibalize the revenue from paid ads. Berman and Katona 2013 examine search engine optimization, proposing a situation where high-quality websites invest in search engine optimization while low-quality websites focus on winning the ad auction. Our work contribute to this stream of literature by studying such an interaction from the perspective of information spillover in the presence of asymmetric information. However, our focus is explicitly on e-commerce.

Finally, the trade off between the price competition induced by competing for shoppers and the profit
obtained by exploiting non-shoppers is discussed by Xu et al. [2011] in the context of search advertising, and is related to Varian [1980], Lal and Sarvary [1999], Iyer et al. [2005], Armstrong et al. [2009] more broadly. In Varian [1980]’s model, the first firm sampled by a consumer is random and firms compete by offering randomized prices. In contrast, the auction setting in our paper leads to a pure pricing equilibrium. Armstrong et al. [2009] discuss the pricing decision of a prominent firm in a consumer search setting, showing that a high-quality firm will set a high price. We reach similar results in the section for firms’ pricing. Our study is also related to the empirical work on search advertising regarding consumer search behaviors and search engine revenue (Agarwal et al. [2011], Chen and Yao [2016], Yao and Mela [2011]).

3 Model

We assume that multiple independent sellers sell products on a marketplace. We model this using a Bayesian game framework where the seller of a product holds some private information about the relevance of the product for consumers (“product fit”). This information asymmetry between the platform and sellers reflects the reality that it is impossible for the platform to correctly evaluate every listing on how well it matches a consumer’s search query, considering the huge number of sellers that introduce their products to the marketplace at any given time.

Sellers

For simplicity, in our model, we assume that there are two sellers $i = 1$ or 2. We assume that seller $i$’s product matches the consumer’s preference with probability $q_i$, $i \in \{1, 2\}$, where $q_1$ and $q_2$ are independent and $q_i$ is only known to seller $i$. We also assume that $q_i$ follows a uniform distribution, $q_i \sim U[q_L, 1]$ with $0 \leq q_L \leq 1$, where $q_L$ is common knowledge. A larger $q_L$ reflects a higher product fit on average. Each seller also has an exogenous outside option of value $u_0$ for selling its product. For example, the seller could sell through his online store directly rather than selling via the platform. If the seller decides to use the platform, it pays a commission $\phi$ to the platform on the total transaction revenue. Sellers can also advertise their products on the platform’s search engine. The marginal cost of the products is assumed to be zero for both sellers.
### Table 1: Possible organic lists when seller $i$ wins the advertising auction.

<table>
<thead>
<tr>
<th>Scenario (a)</th>
<th>Scenario (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ad: Seller $i$</td>
<td>Ad: Seller $i$</td>
</tr>
<tr>
<td>Organic 1: Seller $i$</td>
<td>Organic 1: Seller $j$</td>
</tr>
<tr>
<td>Organic 2: Seller $j$</td>
<td>Organic 2: Seller $i$</td>
</tr>
</tbody>
</table>

Platform

Consumers search for products on the platform using the platform’s search engine, which shows both an organic list and search ads to the consumers. For simplicity, we consider only one advertising slot that is placed above the organic search results list and is auctioned away between the sellers. Figure 1 represents the possible configurations for the organic list when seller $i$ wins the ad auction. Sellers’ bidding decisions in the search ad auction may reveal information about the relevance of their products to consumers (i.e., reveal information about $q_i$), which, in turn can be exploited by the platform. In our model, the platform can make the organic listings dependent on the search ad allocation, which we call the “strategic listing case”. We compare this with the benchmark “independent listing case” where the platform ranks organic links independent of the search ad.

The platform has revenue from two sources: (i) commissions on sales and (ii) revenues from advertising. As mentioned earlier, for each product sold, the platform charges a commission that corresponds to a percentage $\phi$ of the transaction revenue. In addition, the platform sells advertising space on its search engine where a unique ad slot is assigned to a seller through a second-price auction. We consider the pay-per-impression mechanism, where a seller pays the advertisement fee whenever a consumer searches the keyword and the seller’s link is displayed. Our results remain qualitatively the same for the pay-per-click auction mechanism, where a seller pays the advertisement fee only when its link is clicked by the consumer. 

We note that in this formulation we assume that only the sellers know the match probabilities for their products and, a priori, the platform cannot distinguish between the sellers, i.e., the platform can only identify the more-relevant seller, i.e., the seller carrying the product with higher $q_i$, with probability.

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5. Under the pay-per-click auction, the total advertising payment from the seller to the platform remains the same as under the pay-per-impression auction.
However, considering the fact that the platform has access to and can process large amounts of data on seller and consumer activity histories, it can be argued that the platform has good ability to identify the more-relevant seller. In keeping with this, in an extension of the model, we assume that the platform can identify the more-relevant seller with probability $\mu \geq 1/2$ (details of the analysis are available by request). In this case, we find that as long as $\mu < 1$, i.e., the platform cannot perfectly identify the more-relevant seller, all of our results hold qualitatively. In other words, as long as we assume that sellers have some private information about the match probabilities of their products, which we believe is a reasonable assumption, all of our insights continue to hold.

**Consumers**

Consumers only buy the product if they find a match. However, they cannot tell whether a product is a match upon viewing the search results only. They need to inspect the product’s page by clicking on a product’s link on the search result list. We assume consumers to be heterogeneous in certain characteristics that determine their search strategies. Specifically, they differ in their responsiveness to advertising (are “affected” or “unaffected” by ads) and in their search costs (they are “shoppers” or “non-shoppers”). We assume that consumers who are unaffected by search ads (proportion $1 - \beta$) only click on organic listings, while consumers who are affected by search ads (proportion $\beta$) view sponsored ads and organic listings indifferently. We also assume that non-shoppers (proportion $\lambda$) only click on their first considered listing and purchase if and only if a match is found. On the other hand, shoppers (proportion $1 - \lambda$) click on all available listings and purchase the matching product. If both sellers’ products match, shoppers choose randomly (or they choose the product with the lowest price when price competition is considered). Importantly, we assume that whether a consumer is a shopper or not is independent of whether or not she is affected by ads.

In our model, shoppers examine both products so their click behavior is optimal. We assume that non-shoppers only click on their first considered listing, which can be either the ad slot (with a probability of $\beta$) or the first organic slot (with a probability of $1 - \beta$). We note that this behavior is also optimal under strategic listing by the platform as it always leads to non-shoppers clicking on the more-relevant seller. This is because, in equilibrium, the more-relevant seller will be placed in both the advertising slot and the first organic slot.
Timing of the Game

The game’s timing is illustrated in Figure 2. First, the platform announces its commission rate $\phi$. Next, each seller makes its decision to join the platform. To join, a seller compares his expected profitability from selling on the platform accounting for all the possible realizations of relevance to possible search queries to the outside option $u_0$. If sellers join, consumers enter a search query, and each seller privately observes the realized value of his matching probability $q_i$ based on the search query. The sellers then submit bids $b_i(q_i)$ for the sponsored ad slot simultaneously. Each seller is unaware of the other seller’s matching probability when submitting the bid. Therefore, the equilibrium bid of a seller is a function of his matching probability $q_i$. The platform places the winner of the ad auction in the sponsored ad slot, followed by an ordered list of the organic slots, which can be influenced by the information revealed during the ad auction. Finally, consumers see the listings and the demand gets realized. Note that, in the benchmark case of independent listing, we assume that organic listings are independent of the ad auction.

We solve the game by backward induction. We first derive, in Section 4.1, sales outcomes given each arrangement of sellers’ listings. Then, in Section 4.2, we analyze sellers’ bidding decisions given the platform’s choice of commission rate. In Section 4.3, we discuss the implication of strategic listing on sellers’ profit and consumer surplus. Finally, in Section 5, we derive implications for optimal marketplace design wherein we consider the choice of commission rate and the choice of listing strategy while accounting for the sellers’ participation on the marketplace. In Section 6, we make the sellers’ pricing decisions endogenous and consider two possible alternatives: sellers choose their prices independent of, or conditional on, the search queries.
Table 2: Consumer choice corresponding to Scenario (a) in Figure 1 - see text for details.

4 Equilibrium Analysis

To assess the first order effect of advertising, in this section, we assume away price competition and simply normalize the selling price of each seller to 1, i.e., $p_i = p_j = 1$. This analysis also applies to the context when numerous other factors determine a product’s price and firms cannot tailor their prices to a single e-commerce platform.

4.1 Consumer Choice

We first derive the demand realization corresponding to each arrangement of the sellers’ listings in Figure 1. Table 2 illustrates the demand distribution in Scenario (a), where the same seller appears in the ad slot and in the first organic listing.

In Table 2 “NS-A” stands for the non-shoppers who are affected by search ads. This segment is of size $\beta \lambda$. They only click on the ad slot and purchase only if there is a match with seller $i$, which happens with probability $q_i$. The “S-A” segment of size $\beta(1 - \lambda)$ corresponds to shoppers who are affected by search ads and click on all positions, including the sponsored ad. They buy from seller $i$ if they find a match with seller $i$ but fail to find a match with seller $j$, or if they find a match with both sellers and then choose seller $i$ with probability $1/2$. Thus, the total probability of purchasing seller $i$’s product is $q_i(1 - q_j) + q_i q_j \frac{1}{2} = q_i(1 - \frac{1}{2} q_j)$. Similarly, these consumers buy from seller $j$ with probability $q_j(1 - \frac{1}{2} q_i)$.

Note that this group of consumers, if deciding to purchase from seller $i$, can either buy through clicking on his ad listing or on his organic listing. However, this does not affect our analysis, so without loss of generality, we assume that if they decide to buy from seller $i$, they buy through his ad listing. Consumers in the “NS-NA” segment of size $(1 - \beta)\lambda$ are non-shoppers who are unaffected by search ads. They only click on the first organic listing with seller $i$ and purchase in case of finding a match there. Finally, consumers in segment “S-NA” of size $(1 - \beta)(1 - \lambda)$ are shoppers who are not affected by search ads. They examine the two organic listings and choose product $i$ with probability $q_i(1 - \frac{1}{2} q_j)$ and choose
Table 3: Sellers’ revenues given the placement of their organic listings when seller $i$ wins the auction.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Sponsored Ad</th>
<th>Organic 1</th>
<th>Organic 2</th>
<th>Seller i’s Revenue</th>
<th>Seller j’s Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Seller $i$</td>
<td>Seller $i$</td>
<td>Seller $j$</td>
<td>$q_i (1 - \frac{1 - \lambda}{2} q_j) (1 - \phi)$</td>
<td>$q_j (1 - \lambda - \frac{1 - \lambda}{2} q_i) (1 - \phi)$</td>
</tr>
<tr>
<td>b</td>
<td>Seller $i$</td>
<td>Seller $j$</td>
<td>Seller $i$</td>
<td>$q_i (1 - \frac{1 - \lambda}{2} q_j - (1 - \beta) \lambda) (1 - \phi)$</td>
<td>$q_j (1 - \lambda - \frac{1 - \lambda}{2} q_i + (1 - \beta) \lambda) (1 - \phi)$</td>
</tr>
</tbody>
</table>

4.2 Sellers’ Advertising Bids

Given sellers’ revenues in each configuration, we can now assess their optimal bidding strategies for the advertising slot. At this stage, we assume that the commission rate $\phi$ is fixed. Under the strategic listing, a seller’s bid will depend on how the platform orders the organic links conditional on sellers’ bids, which in turn depends on how sellers bid. We solve the game assuming rational expectations: given the platform’s expectation about who wins the ad auction — if the platform ranks organic listings accordingly, then in equilibrium, the seller who is believed to win will indeed win the auction.

Specifically, if the seller with better fit is expected to win, then the platform will place the winner’s organic listing on top of the loser’s. Anticipating this, the more-relevant seller will indeed overbid the less-relevant seller, and will win in equilibrium. Conversely, if the less-relevant seller is expected to win the ad auction, then the platform will place the loser’s organic listing on top of the winner’s, since this seller has better fit with consumers. Anticipating this, the less-relevant seller will indeed overbid the more-relevant seller, and will win in equilibrium. Finally, an equilibrium can exist in which, expecting that the two sellers bid the same, the platform randomly places one seller’s organic listing on top of the other’s (since the two sellers bid the same and the platform gets no additional information). Anticipating this, the two sellers will indeed bid the same. Below, we consider these cases one by one and identify which case happens in equilibrium.

**Case 1:** Suppose the bid function $b(q_i)$ increases with $q_i$. Then the more-relevant seller, i.e., the seller with higher $q_i$, will win the ad auction. Under the belief that the seller who bids higher has a higher matching probability, the platform will also place the winner’s listing in the first position in the organic listings to maximize his revenue. This corresponds to Scenario (a) in Table 3. We can then write...
down the expected profit of a seller with matching probability \( q_i \) who bids \( b(q_i') \) as,

\[
U_{q_i}(q_i') = \int_0^{q_i'} \left[ q_i \left( 1 - \frac{1 - \lambda}{2} q_j \right) (1 - \phi) - b(q_j) \right] dq_j + \int_{q_i'}^1 \left[ q_i \left( 1 - \lambda - \frac{1 - \lambda}{2} q_j \right) (1 - \phi) \right] dq_j. 
\]

The first term denotes a seller’s utility when he wins the auction, when the other seller \( j \)’s matching probability, \( q_j \), is smaller than \( q_i' \). In this case, seller \( i \) obtains both, the ad slot, and the first organic listing. The second term denotes seller \( i \)’s utility when he loses the auction, where \( q_j \) is larger than \( q_i' \). In this case, seller \( i \) obtains the second organic listing only. The above expression is maximized at \( q_i' = q_i \), i.e., if we take the derivative with respect to \( q_i' \) and let \( q_i' = q_i \), then the resulting expression should be equal to zero. Let \( b_i \) denote the per-impression ad price paid by seller \( i \). This implies that, in equilibrium, a seller bids

\[
b^M(q_i) = \lambda(1 - \phi)q_i, 
\]

where the \( M \) (for “main”) superscript denotes the strategic case. Since \( b^M(q_i) \) increases with a seller’s matching probability \( q_i \), the more-relevant seller will win the ad auction. This aligns with our initial assumption of this scenario. Therefore, the more-relevant seller winning the auction and also getting the first position in the organic listings can happen in equilibrium.

**Case 2:** Now suppose that the bid function \( b(q_i) \) decreases with \( q_i \), i.e., the less-popular seller is expected to win the ad auction. Conversely to the above case, the platform will place the loser of the ad auction (who is expected to be the more-relevant seller) first in the organic listings to maximize revenue. This corresponds to Scenario (b) in Table 3. Again, take seller \( i \) for an example. If a seller with matching probability of \( q_i \) deviates to bid \( b(q_i') \), then he obtains both, the ad slot, and the second organic listing, when the other seller \( j \)’s matching probability, \( q_j \), is larger than \( q_i' \). In the other case with \( q_j \) smaller than \( q_i' \), seller \( i \) obtains the first organic listing only.

Then, if deviating to bid \( b(q_i') \), the expected profit of a seller with matching probability \( q_i \) is,

\[
U_{q_i}(q_i') = \int_0^{q_i'} \left[ (1 - \beta \lambda - \frac{1 - \lambda}{2} q_j) q_i (1 - \phi) \right] dq_j + \int_{q_i'}^1 \left[ (1 - (1 - \beta) \lambda - \frac{1 - \lambda}{2} q_j) q_i (1 - \phi) - b_i \right] dq_j. 
\]

Based on the first order condition that \( U_{q_i}(q_i') \) is maximized at \( q_i = q_i' \), seller \( i \)’s equilibrium bid is given by

\[
b^M(q_i) = (2\beta - 1)\lambda(1 - \phi)q_i. 
\]
From the above expression, when $\beta > \frac{1}{2}$, the more-relevant seller wins the ad auction, contradicting our initial assumption for this case. When $\beta \leq \frac{1}{2}$, neither seller has an incentive to bid a positive amount and will not participate in the search ad auction in the first place. Therefore, we can rule out the case where the less-relevant seller wins the ad auction in equilibrium.

We can also rule out the pooling equilibrium where the bid function $b(q_i)$ is unaffected by $q_i$ similarly. If two sellers are expected to bid the same, then the platform will randomly choose one seller’s listing to be placed on top of the other’s. Anticipating this, the more-relevant seller will indeed bid higher than the less-relevant seller to win the ad slot, contradicting the assumption that the two sellers bid the same.

Combining all cases, we conclude that in the strategic case, sellers bid

$$b^M(q_i) = \lambda(1 - \phi)q_i$$

in equilibrium. The seller with the higher fit wins the ad auction and he also appears in the first organic slot. The other scenario where the less-relevant seller wins the ad auction cannot be supported in equilibrium. The intuition behind this is as follows. If the less-relevant bids higher, identifying that the winner of the ad auction is the less-relevant seller, the platform will place its listing in the second organic slot to maximize the platform’s revenue. Expecting that winning the ad slot will lead to a lower organic ranking, the less-relevant seller will not bid that high.

**Effect of Strategic Listing on Advertising Competition**

To study how strategic listing affects the advertising competition between sellers, we compare it against independent listing where organic results are ordered independently from sellers’ bids. In this benchmark case, seller $i$ or seller $j$ will appear in the first organic slot randomly, i.e., with probability $1/2$. We note that when the platform has some information about sellers, it can detect the more-relevant seller with a probability $\geq \frac{1}{2}$. Our model determines that as long as this probability is not 1, i.e., the sellers have some information to supplement to determine the optimal ranking, all results of the model qualitatively go through.

Since the bids do not affect the organic list, given that we have a single-slot second-price auction, each bidder will truthfully bid its value of winning the ad auction. This value is equal to the proportion of customers who are both affected by ads and who are non-shoppers, $\beta \lambda$, multiplied by the seller’s
margin per sale, $1 - \phi$, multiplied by his match probability, $q_i$ (the detailed analysis can be found in Appendix A2). As a result, a seller bids $b^I_i$ for the sponsored ad position (the superscript $I$ denotes “the independent case”) where,

$$b^I_i = \beta \lambda (1 - \phi) q_i.$$  

Since a seller’s bid increases with his matching probability, under independent listing, the more-relevant still wins the ad auction. However, the organic results will be ordered randomly.

Comparing $b^M_i$ with $b^I_i$, the bid in the strategic case is equivalent to that in the independent case by setting $\beta = 1$ (i.e., assuming that all consumers are affected by advertising). This is because, under the strategic case, as the winner of the ad slot also gets placed in the first organic slot, all consumers become affected by search ads indirectly. This shows that if no consumers are averse to ads, then strategic and independent listings are identical. We state this important insight in Lemma 1.

**Lemma 1.** Mathematically, strategic listing of organic results is equivalent to setting $\beta = 1$ in independent listing.

Following Lemma 1, the equilibrium bids in the independent case and the strategic case only differ by the multiplier $\beta$, and a seller’s bid in the strategic case, $b^M(q_i)$ is always higher than his bid in the independent case: $b^I(q_i) = \beta b^M(q_i)$. Intuitively, the sellers bid more aggressively in the strategic case than in the independent case anticipating that the winner of the ad slot will also get the top position in the organic listings. Proposition 1 summarizes this result.

**Proposition 1.** (*Effect of strategic listing on advertising competition*)

Strategic listing of organic results leads to higher bids from sellers. Given an exogenous commission rate, $\phi$, and assuming away price competition (i.e., $p_i = 1$), a seller with matching probability $q_i$ bids $b^M(q_i) = \lambda (1 - \phi) q_i$ when the platform orders the organic listings conditional on sellers’ bids, and he bids $b^I(q_i) = \beta \lambda (1 - \phi) q_i$ when the platform orders the organic listings independently of sellers’ bids.

### 4.3 Sellers’ Profit and Consumer Surplus

Having solved the bidding game, we can assess the impact of strategic listing on sellers’ profits and consumer surplus. We can derive a seller’s expected profit (denoted $U_s$) and expected consumer surplus (denoted $U_c$) under a given commission rate $\phi$, by integrating over the possible realizations of $q_i \sim$
We then compare sellers’ profit and consumer surplus under strategic ordering of organic search results (corresponding to $\beta = 1$) versus independent ordering (corresponding to $\beta < 1$). We first state the result in Corollary 1 (see Appendix A3 for details).

**Corollary 1. (Effect of strategic listing on sellers’ profit and consumer surplus)**

Given an exogenous commission rate, strategic listing leads to lower seller profit and higher consumer surplus.

To provide intuition for these outcomes consider the sellers first. A seller’s profit can be written as,

$$
U_s = E[q] \left( 1 - \frac{(1 + \beta)\lambda}{2} - \frac{1 - \lambda}{2} E[q] \right) (1 - \phi) + \beta \lambda E_q [E_{q_j} (q - q_j, q_j < q)] (1 - \phi).
$$

The first term stands for a seller’s organic revenue. The second term stands for a seller’s potential gain from winning the ad slot net of the advertising price paid. Using the uniform distribution as per the assumption $q \sim U[q_L, 1]$, it is easy to see that $\frac{\partial U_s}{\partial \beta} = -\lambda \frac{5q_L + 1}{12} (1 - \phi) < 0$, so the seller is always worse off when organic listings are ordered strategically (when $\beta$ is highest).

Intuitively, under strategic listing, the platform can learn about sellers’ relevance from their bids, and then exploit such information to extract more surplus from sellers. The less-relevant seller is worse off under the strategic listing case because he only gets placed in the second organic listing, whereas he appears in the first organic listing with $\frac{1}{2}$ probability under the independent listing case. The more-relevant seller may also be worse off under the strategic case, because of the less-relevant seller’s more aggressive bidding compared to the independent case. Overall, strategic ordering leads to a lower profits for both sellers.

For consumers, it is straightforward that strategic ordering makes consumers better off due to an efficient ordering of the organic listing. Consumer surplus simplifies to $U_c = \frac{1}{8} (3 - q_L)(1 + q_L) - \frac{1}{24} (1 - q_L)(3 + 3q_L - 2\beta)$, which increases with $\beta$.

## 5 Marketplace Design

After a discussion of the impact of the platform’s listing strategy on sellers and consumers, now we are ready to turn to the question of optimal marketplace design. In the following, we first assume that both sellers are on the platform and obtain insights regarding the platform’s choice of commission rate and
choice of listing strategy. Following this, we make the sellers’ decision to join the platform endogenous and derive the optimal commission rate and listing strategy.

5.1 Commission Rate and Listing Strategy Assuming Seller Participation

5.1.1 Commission Rate

In this section, we discuss how the platform’s revenue changes with its commission rate, $\phi$, by fixing the platform’s listing strategy and assuming that sellers’ participation is ensured (i.e., sellers’ outside option, $u_0$, is negligible). We first write the platform’s expected profit as a function of its commission rate $\phi$ by integrating over the possible realizations of $q_i$ and $q_j$. For independent listing, the platform’s profit is given by

$$\pi_p^I = E[q] \left( 1 - \frac{1 + \lambda}{2} - \frac{1 - \lambda}{2} E[q] \right) \phi + \beta \lambda E_q[qF(q)] \phi + \beta \lambda E_q [E_{q_j} [q_j, q_j < q]] \left( 1 - \phi \right).$$

Commissions from organic slots

Commissions from ad slot

Advertising revenue

For strategic listing, the expression for the platform’s profit is similar and is given by

$$\pi_p^M = E[q] \left( 1 - \lambda - \frac{1 - \lambda}{2} E[q] \right) \phi + \lambda E_q[qF(q)] \phi + \lambda E_q [E_{q_j} [q_j, q_j < q]] \left( 1 - \phi \right).$$

Commissions from organic slots

Commissions from ad slot

Advertising revenue

In both expressions, the first two terms account for the platform’s commission revenues (where the platform’s commission rate, $\phi$, appears as a multiplier)—the first term represents the platform’s commission revenue from organic slots, and the second term represents its commission revenue from advertising slots. The third term accounts for the platform’s ad revenue (the seller’s margin, $1 - \phi$, appears as a multiplier).

From the expressions for the platform’s revenues $\pi_p$, we can see that a higher commission rate leads to higher revenue through commissions but lower revenue through advertising. The latter is because sellers bid lower when the platform takes a larger proportion of their revenue. Taking derivatives of the platform’s revenue $\pi_p$ with respect to the commission rate $\phi$, we have $\frac{\partial \pi_p}{\partial \phi} > 0$. This implies that if the platform charges a higher commission rate, the loss in the revenue through advertising is a second order effect compared with the gain in revenue through commissions. As a result, the platform’s profit increases with its commission rate, $\phi$. Lemma 2 summarizes the above discussion.
Lemma 2. Given a fixed listing strategy and assuming away the sellers' participation constraint, the platform’s total revenue increases with the commission rate, $\phi$.

The intuition for Lemma 2 is that the platform earns commission revenue from both the ad slot and the organic slots, whereas it earns advertising revenue only from the ad slot. Thus the increase in commission revenue dominates the losses in ad revenue for the platform when $\phi$ increases.

5.1.2 Listing Strategy

In this section, we discuss the choice of listing strategy of the platform assuming a fixed commission rate and assuming that both sellers are on the platform. To do this, we compare the platform’s revenue from strategic listing and independent listing. Lemma 3 states the result.

Lemma 3. Given a fixed commission rate and assuming both sellers join, the platform always chooses strategic listing of organic results.

It is straightforward to see that the platform always benefits from ordering organic listings strategically. Indeed, the platform’s revenue can be simplified to

$$\pi_P = \left(\frac{1}{8} (3 - q_L)(1 + q_L) - \frac{\lambda}{24} (1 - q_L)(3 + 3q_L - 2\beta)\right) \phi + \beta \lambda \frac{1+2q_L}{6} (1 - \phi),$$

which increases with $\beta$. Intuitively, first, strategic listing increases commission revenues. This is because a more efficient ordering of the organic listings (i.e., the more-relevant seller being placed above the less-relevant seller in organic listings) increases the likelihood of sales. In the absence of price competition, the increase in commission revenues is given by $\frac{1-q_L}{12} (1 - \beta) \lambda \phi$, which is equal to the expected incremental sales from non-shoppers who are unaffected by search ads. The size of this segment is $(1 - \beta) \lambda$ and their probability of purchase increases by $\frac{1-q_L}{12}$ when they see the more-relevant seller rather than the less-relevant one being placed in the first organic slot.

Second, the platform also earns more advertising revenue due to more aggressive bidding from the sellers because the winner of the ad auction will not only be placed in the ad slot but also in the first organic slot. The increase in advertising revenue is given by $(1 - \beta) \lambda \frac{1+2q_L}{6} (1 - \phi)$, i.e., the incremental sales for the seller who wins the ad auction and also occupies the first organic slot, $\frac{1+2q_L}{6} (1 - \beta) \lambda$, multiplied by the seller’s margin, $1 - \phi$.

To summarize, exploiting information from the search ad auction leads to an efficient ordering of organic listings and induces aggressive bidding in the search ad auctions (information effect). This suggests that strategic listing is beneficial for the platform, keeping all other factors fixed.
5.2 Marketplace Design Considering Sellers’ Participation

In the previous section, we showed that the platform prefers a higher commission rate and the strategic listing strategy without considering sellers’ participation. However, both a higher commission rate and strategic listing lead to reduced profit for the sellers, which reduces their incentive to be on the platform. Therefore, in this section, we allow sellers to endogenously decide whether they want to be on the platform, given that they have an outside option $u_0$. Given this, we determine the platform’s marketplace design parameters, specifically, the optimal commission rate on sales and the listing strategy for organic results.

Recall that at the time the sellers make their joining decisions, they do not know the realization of their products’ match probabilities and are identical in every way. Therefore, both sellers face the same participation constraint and the platform must choose its commission rate to maximize its profit while ensuring the sellers’ participation. Since we have shown that the platform’s profit increases with the commission rate $\phi$ (Lemma 2), at the optimum, the platform sets its commission rate to make the sellers’ participation constraint binding, extracting all the surplus from sellers. In particular, the optimal commission rate is $\phi^* = 1 - \frac{u_0}{U_s(\phi=0)}$, where $U_s(\phi=0)$ is each seller’s profit given zero commission rate (as derived in Section 4.3). Figure 3 presents the optimal commission rate under the strategic case and the independent case respectively, as a function of $\beta$, $\lambda$ and $q_L$. We state the results in the following proposition and subsequently discuss in more detail.

**Proposition 2. (Optimal commission rate)**

The optimal commission rate weakly decreases in $\beta$ (the proportion of consumers affected by ads) and $\lambda$ (the proportion of non-shoppers). In addition, the optimal commission rate increases in $q_L$ for $\lambda < \frac{3}{5}$, and decreases in $q_L$ for $\lambda > \frac{3}{5}$.

Strategic listing leads to a lower commission rate than independent listing (as shown in Figure 3(a)), the equilibrium commission rate is decreasing in $\beta$. This is because given the same commission rate, the seller’s profit is lower under the strategic case (Corollary 1). Thus, the highest commission rate that the platform can charge while ensuring the seller’s participation will be lower under the strategic case. The comparative statics of the equilibrium commission rates against $\lambda$ and $q_L$ are given in Figure 3(b) and 3(c) respectively. Importantly, the comparative statics of equilibrium commission rates under the strategic case directly affect the optimal listing strategy for the platform. Therefore, we focus on the
Figure 3: Optimal commission rates. The default parameters are $u_0 = 0.25$, $q_L = 0.5$, $\lambda = 0.5$, $\beta = 0.5$.

First, under strategic listing, the optimal commission rate decreases with $\lambda$. Intuitively, under strategic listing, a seller’s profit decreases when there are more non-shoppers in the market ($\lambda$ increases): while the net gain of winning the ad slot increases for a seller, the baseline likelihood of sales decreases, and this latter effect dominates the former increase in net gain from advertising. As a result, as $\lambda$ increases, the platform needs to lower the commission rate to ensure sellers’ participation. When $\lambda$ becomes large enough, sellers may not participate even if the platform does not charge any commission rate.

Turning to $q_L$, for sellers, the effect of $q_L$ is two-fold. When $q_L$ increases from a small value, it leads to a higher matching probability in expectation, thus increasing a seller’s baseline sales. Consequently, the platform can raise the commission rate without driving sellers away from the platform. However, increasing $q_L$ will also lead to stronger advertising competition, as sellers bid more aggressively when they have a higher match probability. This effect can dominate the first positive effect when $q_L$ increases beyond a threshold, specifically, $\frac{3-5\lambda}{3-3\lambda}$. Then, the platform needs to lower the commission rate to ensure sellers’ participation. Overall, when $\lambda < \frac{3}{5}$, the optimal commission rate presents an inverted-U shape against $q_L$. When $\lambda > \frac{3}{5}$, the second effect always dominates and the optimal commission rate decreases in $q_L$.

The above analysis shows that there are two components for the platform to maximize its overall revenue. On one hand, the platform relies on sponsored advertising to extract surplus from sellers who fit a particular search query better. On the other hand, the platform readjusts its commission rate to ensure sellers’ participation.

Now we turn to the platform’s preference of listings strategies. In particular, comparing the two cases, an interesting finding is that the platform may or may not benefit from exploiting ad information to refine
organic ranking when the commission rate is endogenized. Furthermore, strategic listing outperforms independent listing for the platform when $\lambda$ is small or when $q_L$ and $\lambda$ are intermediate. This finding is stated in Proposition 3 and illustrated in Figure 4.

**Proposition 3.** (Search design considering sellers’ participation)

*If sellers’ participation constraint is non-negligible and the platform chooses the commission rate endogenously, then the platform prefers strategic listing when the size of non-shoppers, $\lambda$, is small or when the average match probability is intermediate ($q_L$ is intermediate).*

The intuition is as follows. On one hand, exploiting information from the search ad auction leads to an efficient ordering of organic listings and induces aggressive bidding in the search ad auctions (information effect). On the other hand, the strategic ordering will lead to the more-relevant seller occupying favorable placements in both ad slots and organic listings; this disincentives sellers from joining the platform in the first place and results in lower commission rates (competition effect).

Indeed, the information effect dominates the competition effect so the platform is better off under the strategic case as long as the seller’s participation is ensured under the strategic case (Region I in Figure 4). Compared with independent listing, the platform earns more ad revenue, while it may earn more or less commission revenue (because the optimally chosen commission rate is lower). Overall, the
gain in ad revenue dominates and the platform is better off in the strategic case. In Region II, exploiting advertising information to refine organic ranking will lead sellers to ultimately abandon the platform. Under this region, the sellers will not participate in the strategic case even with a zero commission rate.

Based on Figure 4, it is counter-intuitive that the platform is less likely to adopt strategic listing when the size of non-shoppers is large (i.e., \( \lambda \) is large). We may expect that as more consumers only click on the top position, the platform would have a larger incentive to exploit advertising information to give the more-relevant seller a better placement. However, this will make sellers abandon the platform. When there are more shoppers who examine multiple slots (i.e., \( \lambda \) is small), in contrast, even a low-fit seller has a good chance to make a sale. Under this scenario, the platform can benefit from strategic listing without driving sellers away. In terms of \( q_L \), the platform is more likely to benefit from strategic listing when \( q_L \) is intermediate. In this case, a seller’s product matches consumers need with a high probability. However, the match probability is not too high such that the intense ad competition drives sellers expected profit below the outside option.

Proposition 3 shows that it is critical to consider sellers’ outside options in the platform’s search design problem which results in a non-linear choice of commission rates and distinct listing strategies depending on consumer search behaviors and product characteristics.

In practice, we indeed observe very different commission rates across platforms operating in different geographies. For example, Alibaba charges a zero commission rate for each transaction on its consumer-to-consumer marketplace, Taobao and it charges a commission rate of 2% to 5% for each transaction from its business-to-consumer marketplace, Tianmao. Amazon takes around 15% out of the popular product categories. Moreover, search engines like Google explicitly promise that organic listings will be kept independent from search ads. Our results in this section shed light on some factors that can drive these choices of platforms.

6 Making Seller Pricing Endogenous

So far we abstracted away from price competition between sellers and normalized the products’ selling price to 1. Indeed, firms’ pricing is determined by a variety of factors, only one of which may be firms’ advertising strategy on a particular distribution platform. However, as a platform such as Amazon becomes increasingly dominant for many sellers, it may well represent the primary way to reach consumers,
therefore influencing pricing as well. In this section, we explore how taking into account sellers’ pricing modifies the platform’s optimal design.

We discuss two alternatives to model the price competition between sellers. In Section 6.1, a seller offers his product at the same price, regardless of the particular search query. We will show that the platform has an additional incentive to adopt strategic listing: strategic listing can reduce price competition between sellers. However, consumers can become worse off: although organic results are ordered more efficiently, consumers also face a higher selling price on average.

Alternatively, sellers may be able to vary their selling prices based on each search query. Even though this practice is not observed at major platforms, it is interesting as a theoretical thought experiment. In this case, we find that the platform can learn sellers’ relevance from their selling prices, and there is no informational role of advertising.

**Consumer Demand with Price Competition.** In the presence of price competition, the demand distribution is affected by both the placements of the sellers’ listings and their selling prices. Consider the case where seller \( i \) gets placed in the ad slot and the first organic listing, and the price of seller \( i \) is higher than the price of seller \( j \). In this scenario, the shoppers purchase the lower-priced product \( j \) as long as they find a match with it (with probability \( q_j \)), whereas they purchase product \( i \) only if they find a match with product \( i \) but fail to find a match with product \( j \) (with probability \( q_i(1-q_j) \)). Denote by \( p_i \) the price of seller \( i \). Table 4 presents the demand realization under this scenario. Demand realizations and the two sellers’ revenues under the other possible scenarios can be derived similarly and are presented in Appendix A1.

### 6.1 Search-independent Pricing

In this section, we assume that sellers decide prices before the query is searched. In other words, a seller offers his product at the same price regardless of the query that is being searched, which is a faithful representation of reality. This implies that a seller chooses his selling price before observing the match

<table>
<thead>
<tr>
<th>Position</th>
<th>Seller</th>
<th>NS-A</th>
<th>S-A</th>
<th>NS-NA</th>
<th>S-NA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sponsored Ad</td>
<td>( i )</td>
<td>( q_i )</td>
<td>( q_i(1-q_j) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Organic 1</td>
<td>( i )</td>
<td></td>
<td>( q_i )</td>
<td>( q_i(1-q_j) )</td>
<td></td>
</tr>
<tr>
<td>Organic 2</td>
<td>( j )</td>
<td></td>
<td>( q_j )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Consumer choice corresponding to Scenario (a) in Figure 1 with \( p_i > p_j \).
Platform chooses commission rate \( \phi \) Sells \( T = 0 \)

Sellers decide to join or not and choose prices \( p_i \) if join

Consumers enter a query and sellers observe own \( q_i \) \( T = 1 \)

Sellers bid \( b_i(q_i, p_i) \)

Platform arranges ad and organic slots \( T = 3 \)

Demand realizes \( T = 6 \)

Figure 5: Timing of the game (search-independent pricing).

probability for his product. In this case, a seller’s bid will not only depend on his observation of the match probability but also his selling price. In all other respects, the game remains the same as in Section 4. The timing is illustrated in Figure 5. We explore the sellers’ bidding and pricing decisions before turning to the platform.

**Sellers’ Bidding Strategy**

When prices are endogenous, sellers’ bids will depend on prices. However, we can show that the intuition that strategic listing leads to higher advertising competition between sellers remains valid. We focus on the efficient equilibrium where a seller’s bid is increasing in his matching probability and his selling price. Based on the intuition from the bid function in Section 4.2, we find the equilibrium by guessing that \( b(q_i, p_i) \) is a function of \( q_i \cdot p_i \), i.e., \( b(q_i, p_i) = b(q_i \cdot p_i) \). Under the assumption that \( b(q_i, p_i) = b(q_i \cdot p_i) \), the platform will place the seller with the higher bid in the first organic listing, since he is also the seller which generates a higher expected revenue for the platform. We show that there exists a unique solution determined under this initial guess (see Appendix A4.1 for details). Using similar logic as in Section 4.2, we show that a seller’s bid in the independent case and the strategic case only differ by the multiplier \( \beta \):

\[
\begin{align*}
b^I(q_i, p_i) &= \beta \lambda (1 - \phi) q_i p_i, \\
b^M(q_i, p_i) &= \lambda (1 - \phi) q_i p_i.
\end{align*}
\]

We state the above results in Proposition 4.

**Proposition 4.** A seller with matching probability \( q_i \) and selling price \( p_i \) bids \( b^M(q_i, p_i) = \lambda (1 - \phi) q_i p_i \) when the platform orders the organic listings conditional on sellers’ bids, and he bids \( b^I(q_i, p_i) = \beta \lambda (1 - \phi) q_i p_i \) when the platform orders the organic listings independently of sellers’ bids.
Seller’s Pricing Strategy

We now solve for the equilibrium price. Sellers balance between competing for shoppers by lowering price versus competing for favorable slots by raising price, and we expect a mixed pricing equilibrium. Denote that the equilibrium price follows the cumulative density function $G(p)$ and the probability density function $g(p)$. Below, we sketch the solution for the mixed pricing equilibrium.

By substituting the equilibrium bid function into a seller’s profit function, we can write down the expected profit of a seller given its selling price $p_i$ and matching probability $q_i$. If a seller raises his price, for the seller, there is a direct effect through changing the selling price, and an indirect effect through changing the probability that he wins the ad auction via the bid function. Considering each of the six possible scenarios involving the arrangement of sellers’ listings and sellers’ prices (see Table 3), this leads to the following expression for the seller’s profit:

$$U(p_i) = E_{q_i,q_j} \left[ (1 - \lambda)(1 - \phi)q_i p_i \left( 1 - q_j \int_{p_L}^{p_i} g(p_j) dp_j \right) \right] + E_{q_i,q_j} \left[ \lambda(1 - \phi) \int_{p_L}^{q_i p_i \left( q_i p_i - q_j p_j \right) g(p_j) dp_j \right]$$

In the above expression, the first term represents seller $i$’s revenue from organic slots taking into account price competition (i.e., the marginal loss for seller $i$ by pricing lower than seller $j$, as the shoppers may choose seller $j$ for a lower price). The second term reflects the potential net gain from winning the ad slot for seller $i$.

We can solve the mixed pricing equilibrium by noting that a seller would be indifferent towards pricing at any level between the lowest bound $p_L$ and the upper bound 1. It follows that the cumulative distribution function of equilibrium prices is (see Appendix A4.2 for details),

$$G(p) = a + 1 - \frac{1}{p} (a + bE[q]) + bpE[\frac{1}{p}], \text{ for } p_L \leq p \leq 1,$$

where $a = \frac{1}{1 - \lambda E[q]} - 1$ and $b = \frac{1}{2} \frac{\beta \lambda E[q^2]}{1 - \lambda E[q]} - \frac{1}{q_L}$. The results on comparative statics for the expected equilibrium price are summarized in Proposition 5 (for an illustration see Figure 6).

**Proposition 5.** When sellers set price endogenously and prices are independent of search queries, a seller prices higher in expectation as the size of consumers affected by ads ($\beta$) increases and the size of non-shoppers ($\lambda$) increases. Furthermore, it first decreases then increases against the average match
Figure 6: Expected selling prices. The default parameters are \( q_L = 0.5, \lambda = 0.5, \beta = 0.5 \).

Proposition 5 says that price competition is reduced when \( \beta \) increases or when \( \lambda \) increases. This is because as \( \beta \) increases (i.e., as more consumers are affected by ads), a seller benefits more from winning the ad auction, and the increased benefit from winning the advertising competition reduces the price competition between sellers. Similarly, when \( \lambda \) increases (i.e., there are more non-shoppers in the market), fewer customers will examine multiple slots before making a purchase decision; this also reduces price competition. The result with respect to \( q_L \) is less intuitive: as \( q_L \) first increases, a seller’s product is more likely to be a “fit” in expectation, shoppers have a larger chance to choose the lower-priced product (i.e., there is more overlap in “fit” between sellers), and price competition is intensified. However, as \( q_L \) increases further, sellers have a higher probability of finding a match with non-shoppers, which helps reduce price competition. It can further dominate the first force when \( q_L \) increases beyond a certain threshold, resulting in a higher expected price.

Based on Proposition 5, sellers price higher in the strategic case (represented by solid lines in Figure 6) than in the independent case (represented by dashed lines in Figure 6). Intuitively, under strategic ordering, the more-relevant seller gets a favorable placement in both the sponsored listing and the organic listings on average. Expecting this, he does not need to reduce price that much. To further understand this, we discuss the boundary cases with \( \lambda = 0 \) and \( \lambda = 1 \) where a seller prices the same under independent listing and under strategic listing. In the extreme case with \( \lambda = 0 \), i.e., when all consumers are shoppers, the ordering of organic listings wouldn’t affect the demand distribution. Then, the equilibrium price is unaffected by \( \beta \), i.e., a seller prices the same under independent listing and under strategic listing. In the other extreme case with \( \lambda = 1 \), i.e., all consumers are non-shoppers, reducing prices does not bring more sales to a seller. When \( \lambda \) lies between 0 and 1, we have that strategic listing
leads to higher expected prices from sellers. We summarize this insight in Corollary 2.

**Corollary 2. (Effect of strategic listing on price competition)**

*When sellers set price endogenously and prices are independent of search queries, strategic listing of organic results reduces price competition and leads to a higher expected prices than independent listing of organic results.*

**Sellers’ Profit and Consumer Surplus**

Having solved the equilibrium bidding and pricing decisions, we then compare sellers’ profits and consumer surplus under strategic ordering of organic search results versus independent ordering. We first state the result in Proposition 6.

**Proposition 6.** *When sellers set price endogenously and prices are independent of search queries, strategic listing leads to lower sellers’ profit and lower consumer surplus.*

To provide intuition, assuming away price competition, we have shown that sellers are worse off when organic listings are ordered strategically. When price competition is taken into account, this does *not* change. Although sellers price higher under the strategic case, the effect of strategic ordering on reducing price competition is dominated by its effect on intensifying advertising competition for the sellers. Overall, strategic ordering still leads to a lower profit for the sellers.

In terms of consumer surplus, if price competition is not taken into account, consumers are better off due to an efficient ordering of the organic listing. However, the negative effect of higher selling prices on consumers more than offsets the benefit of an ordered organic listing, therefore, consumers are worse off under strategic ordering when sellers set price strategically. The intuition is that only non-shoppers benefit from the efficient ordering of organic listing. However, all consumers are hurt by the higher selling price.

**Marketplace Design with Endogenous Pricing**

Turing to the platform, when the sellers’ outside option, \( u_0 \), is negligible, since sellers price higher in the strategic case, as a higher price leads to higher bids for the ad slot from sellers and also results in higher commission revenue, there is an additional motivation for the e-commerce platform to use strategic listing. When the sellers’ outside option, \( u_0 \), is non-negligible, the platform’s revenue can be higher in
the independent case than in the strategic case when the commission rate is endogenized. However, the analysis for when the platform benefits from strategic listing becomes more involved. Generally speaking, the platform is more likely to benefit from strategic listing when the size of non-shoppers is intermediate, and when the average match probabilities is extreme, as Figure 7 illustrates.

With endogenous pricing, distinct from Section 5.2, the platform may not benefit from strategic listing when the size of non-shoppers is too small as the price competition is strong. Moreover, the platform may not have incentive to adopt strategic listing when the average match probability is intermediate as sellers compete in pricing aggressively.

6.2 Search-specific Pricing

In this section, we discuss the scenario in which sellers set their prices after the query is searched and each of them observe the respective match probabilities of their products. Another way to understand this is that $q_i$ represents vertical differentiation due to product quality or service level. Therefore, a seller may have a higher match probability on average compared with other sellers. In this modified game, a seller chooses his selling price privately after observing the match probability for his product and before submitting his bid in the ad auction. Note that the seller does not know the match probability of the other seller’s product or the other seller’s selling price when he submits the bid. Figure 8 presents the
Platform chooses commission rate $\phi$ and sellers decide to join or not. Consumers enter a query and sellers observe own $q_i$. Sellers choose prices $p_i(q_i)$ and bids $b_i(q_i)$ and the platform observes and arranges slots. Demand realizes.

Figure 8: Timing of the game (search-specific pricing).

time line under search-specific pricing.

Under this scenario, the seller with the higher match probability will also price higher. Appendix A4.3 shows that in equilibrium, a seller with a match probability of $q_i$ prices at,

$$p(q_i) = Ce \left( \frac{(1-\lambda)q_i f(q_i)}{1 - \frac{1+\beta}{2} \lambda + \beta \lambda F(q_i) - (1-\lambda) \int q_j f(q_j) dq_j} \right),$$

where $C$ is a constant pinned down by the boundary condition that $p_i = 1$ when $\lambda = 1$. Closed-form solution of the equilibrium price for the case when $q_i \sim U[q_L, 1]$ can be obtained and the corresponding expression is given in Appendix A4.3.

The complexity of these expressions does not allow an analytic approach to studying comparative statics. However, we can do so via numerical analysis because we can cover the entire parameter space of the model. Importantly, $p(q_i)$ increases with $q_i$, implying that the platform can infer sellers’ relevance from their prices alone. Consequently, there is no informational role of advertising when sellers are allowed to price based on the search query, or when sellers’ private information about their match probability stems from vertical differentiation.

7 Discussion

Advertising on e-commerce platforms is an increasingly pervasive and economically important phenomenon. In the US, Amazon is wielding an increasing amount of influence as a product search engine, while also becoming a more compelling advertising platform, challenging the duopoly of Google and Facebook in digital advertising; in China and India, Alibaba and Flipkart, which are the dominant online retailers, respectively, are pure-play marketplaces but still earn the bulk of their revenues from sellers’ ads. In this paper, we study this trend and provide timely guidance to the various industry players in this ecosystem.
We highlight and analyze the phenomenon that on online retailing marketplaces, sponsored advertising is not merely a separate revenue driver, in fact, it has a huge potential to help the platform better arrange organic results by using information revealed through sponsored advertising. The marketplace design strategy presented in our paper is relevant to e-commerce platforms, to help rank sellers in terms of relevance to search queries: Using the information effect in organic listings enables distinguishing between closely related products based on the bidding of their sellers, by utilizing the advertising strategies of the sellers to better understand where their products are more relevant. The search design strategy we propose also helps to address the cold start problem in recommendation systems, where the platform has little information about new sellers or products. Instead of falling prey to the cold start problem, where these products can’t be recommended to anyone due to lack of data, the platform can suggest new products to users when relevant, by exploiting information revealed from sellers’ bids for sponsored listings.

Our model helps understand why, and identify under what conditions, the e-commerce platform benefits from taking search advertising as a strategic tool to improve the relevance of its product listings in order to better serve consumers. The platform benefits from strategic ordering because of a higher probability of sales (due to more efficient ordering of organic listings) and also from more intense advertising competition (higher advertising bids) and a weakened price competition between sellers (higher prices). The downside of strategic listing however, is that it disincentivizes sellers from joining the platform in the first place. Therefore, if sellers’ participation is at stake, then it is in the e-commerce platform’s interest to commit to take search advertising solely as an independent revenue source.

Our analysis sheds light on different strategies followed by different retail platforms dominating distinct geographies, the following being an example. Our model predicts that as consumer search through products becomes more difficult, the platform should charge a lower commission rate and relay more on advertising revenue. Indeed, commission rates charged by platforms can be markedly different, and also vary on the same platform across product categories. For instance, Alibaba, relies primarily on advertising revenue while charging a minimal commission rate for each transaction (0% – 5%), while Amazon charges much higher commission rates (~ 15%). Considering that Alibaba hosts about five times more number of sellers (~ 10 million) compared to Amazon (~ 2 million), searching for products is indeed expected to be more difficult on Alibaba. Therefore, the fact that Alibaba has lower commission rates than Amazon is consistent with our theoretical results.
Utilizing the information effect also gives the e-commerce platform an edge over search engines, as by providing organic results that are more relevant to the users they gain the consumers trust and become the de-facto mode of product search. An important insight of the model is that advertising on an e-commerce platform may benefit consumers by resulting in efficient ordering of organic listings thus increasing total sales. However, consumers should be aware that being exposed to the most-relevant product first may not necessarily benefit them. If the platform acts as the major distribution channel and sellers also set prices strategically as a response to the platform’s search design, this positive effect can be offset by the higher selling price that consumers will face. In this sense, our work provides a framework for regulators to analyze these trade-offs and its impact on consumers. The implication for sellers is that sellers at retailing marketplaces should determine their valuation for ads based on the impact on the organic listing as well.

Finally, the informational role of advertising exists when a seller’s fit relative to other sellers varies with the specific search query. This applies well to the scenario where sellers’ private information about their product fit stems from recent market trend or external marketing activities by sellers. However, if such private information is in regard with vertical differentiation like product quality or service levels, then the platform can rely on pricing to refine organic ranking. In other words, if sellers set prices after learning the match probabilities of their products, then there is no informational role of advertising.

In formulating our model, we have made a number of assumptions. Some of these assumptions are not critical. For instance, we only consider ad auctions of one advertising slot. However, our main findings should carry through if we consider an ad auction with multiple slots as the same set of forces will be at play. However, there may be other assumptions that could be revisited. For instance, while we give solutions for a general distribution of sellers’ fit, we only study their properties under a particular (uniform) distribution. This is a simple setting that enables us to analyze key insights from the strategic role of advertising in e-commerce platforms. It would interesting to investigate other distributions.

Our theoretical model can generate a number of hypotheses that can be tested empirically. First, our model predicts that for product categories with an intermediate match probability (medium $q_L$), the platform chooses to order organic listings strategically. In this case, sellers with higher fit get a favorable placement in both the ad listings and the organic listings. Then, we expect a positive correlation between sellers’ rankings in the ad listings and in the organic listings in this scenario.

**Hypothesis 1.** For product categories with an intermediate match probability, there is a positive cor-
relation between a seller’s ranking in the sponsored list and in the organic list. Otherwise, there is no significant correlation.

Second, our model predicts that the platform’s choice of commission rate reduces as the proportion of consumers with high search cost increases. This indeed aligns with the industry evidence, for example, Alibaba charges a minimal commission rate since consumer search is difficult as consumers have to search through many sellers. More formally,

**Hypothesis 2.** The platform charges a lower commission rate as the proportion of consumers with high search cost increases.

Finally, our model provides a rich framework to analyze sellers’ pricing driven by several forces which can be tested structurally. While we leave the work to future research, one simple hypothesis to be tested would be the following.

**Hypothesis 3.** Sellers appearing in the top position in the ad listings price higher than an average seller in the same product category.

To conclude, in addition to providing valuable theoretical and managerial insights about an increasingly important problem, our model provides a rich framework for empirical work on retail platforms.
References


Navdeep S Sahni and Harikesh Nair. Does advertising serve as a signal? evidence from field experiments in mobile search. 2018.


Appendix

A1 Consumer Choice Given the Placement of Sellers’ Listings

Please see Tables A1, A2, A3, A4 and A5.

A2 Equilibrium Bids under Independent Listing

First, assume that \( b(q_i) \) increases with \( q_i \). A bidder who bids \( b(q'_i) \) obtains the ad slot when the other seller \( j \)'s matching probability, \( q_j \), is smaller than \( p'_i \). We first write down the expected profit of a seller with matching probability \( q_i \) who bids \( b(q'_i) \) as,

\[
U_{q_i}(b(q'_i)) = \int_0^{q'_i} \left( \frac{1}{2} q_i \left( 1 - (1 - \lambda) \frac{q_j}{2} \right) (1 - \phi) \right) dq_j + \int_{q'_i}^1 \left( \frac{1}{2} q_i \left( 1 - (1 - \lambda) \frac{q_j}{2} \right) (1 - \phi) \right) dq_j,
\]

\[= \int_0^{q'_i} \left( q_i \left( 1 - (1 - \lambda) \frac{q_j}{2} - \frac{1}{2} (1 - \beta) \lambda \right) (1 - \phi) - b(q_j) \right) dq_j + \int_{q'_i}^1 q_i \left( 1 - (1 - \lambda) \frac{q_j}{2} + \frac{1}{2} (1 - \beta) \lambda \right) (1 - \phi) dq_j.
\]

The first term denotes a seller’s utility when he wins the auction. The second terms denotes his utility when he loses the auction. In either case, because organic listings are randomly ordered, a seller’s utility is obtained by averaging his utility when he is ranked first and second in the organic listings. The utility function can be further simplified into,

\[
U_{q_i}(b(q'_i)) = \frac{3}{4} q_i (1 - \lambda)(1 - \phi) + \beta \lambda q_i q'_i (1 - \phi) - \int_0^{q'_i} b(q_j) dq_j.
\]

The above expression is maximized at \( q'_i = q_i \), i.e., if we take the derivative with respect to \( q'_i \) and let \( q'_i = q_i \), then the resulting expression should be equal to zero. That is,

\[
\beta \lambda q_i (1 - \phi) - b(q_i) = 0.
\]

Therefore, we have that when organic listings are ordered randomly, in equilibrium, a seller bids

\[
b^i(q_i) = \beta \lambda q_i (1 - \phi).
\]

Second, assume that \( b(q_i) \) decreases with \( q_i \). Under the assumption that \( b(q_i) \) decreases with \( q_i \), a bidder who bids \( b(q'_i) \) obtains the ad slot when the other seller \( j \)'s matching probability, \( q_j \), is larger than
Table A1: Consumer choice corresponding to scenario (b) in Figure 1 with $p_i = p_j$.

<table>
<thead>
<tr>
<th>Position</th>
<th>Seller</th>
<th>NS-A</th>
<th>S-A</th>
<th>NS-NA</th>
<th>S-NA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sponsored Ad</td>
<td>$i$</td>
<td>$q_i$</td>
<td>$q_i(1 - \frac{q_j}{2})$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Organic 1</td>
<td>$j$</td>
<td>$q_j(1 - \frac{q_i}{2})$</td>
<td>$q_j$</td>
<td>$q_j(1 - \frac{q_i}{2})$</td>
<td></td>
</tr>
<tr>
<td>Organic 2</td>
<td>$i$</td>
<td></td>
<td></td>
<td>$q_i(1 - \frac{q_j}{2})$</td>
<td></td>
</tr>
</tbody>
</table>

Table A2: Consumer choice corresponding to scenario (a) in Figure 1 with $p_i < p_j$.

<table>
<thead>
<tr>
<th>Position</th>
<th>Seller</th>
<th>NS-A</th>
<th>S-A</th>
<th>NS-NA</th>
<th>S-NA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sponsored Ad</td>
<td>$i$</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Organic 1</td>
<td>$i$</td>
<td></td>
<td>$q_i$</td>
<td>$q_i$</td>
<td></td>
</tr>
<tr>
<td>Organic 2</td>
<td>$j$</td>
<td>$q_j(1 - q_i)$</td>
<td>$q_j$</td>
<td>$q_j(1 - q_i)$</td>
<td></td>
</tr>
</tbody>
</table>

Table A3: Consumer choice corresponding to scenario (b) in Figure 1 with $p_i > p_j$.

<table>
<thead>
<tr>
<th>Position</th>
<th>Seller</th>
<th>NS-A</th>
<th>S-A</th>
<th>NS-NA</th>
<th>S-NA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sponsored Ad</td>
<td>$i$</td>
<td>$q_i$</td>
<td>$q_i(1 - q_j)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Organic 1</td>
<td>$j$</td>
<td>$q_j$</td>
<td>$q_j$</td>
<td>$q_j$</td>
<td></td>
</tr>
<tr>
<td>Organic 2</td>
<td>$i$</td>
<td></td>
<td></td>
<td>$q_i(1 - q_j)$</td>
<td></td>
</tr>
</tbody>
</table>

Table A4: Consumer choice corresponding to scenario (b) in Figure 1 with $p_i < p_j$.

<table>
<thead>
<tr>
<th>Position</th>
<th>Seller</th>
<th>NS-A</th>
<th>S-A</th>
<th>NS-NA</th>
<th>S-NA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sponsored Ad</td>
<td>$i$</td>
<td>$q_i$</td>
<td>$q_i$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Organic 1</td>
<td>$j$</td>
<td>$q_j(1 - q_i)$</td>
<td>$q_j$</td>
<td>$q_j(1 - q_i)$</td>
<td></td>
</tr>
<tr>
<td>Organic 2</td>
<td>$i$</td>
<td></td>
<td></td>
<td>$q_i$</td>
<td></td>
</tr>
</tbody>
</table>

Table A5: Sellers’ revenue given the placement of their organic listings when seller $i$ wins the auction.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Sponsored Ad</th>
<th>Organic 1</th>
<th>Organic 2</th>
<th>Price</th>
<th>Seller $i$’s Revenue</th>
<th>Seller $j$’s Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Seller $i$</td>
<td>Seller $i$</td>
<td>Seller $j$</td>
<td>$p_i &gt; p_j$</td>
<td>$[1 - (1 - \lambda)q_i]q_ip_i(1 - \phi)$</td>
<td>$(1 - \lambda)q_ip_i(1 - \phi)$</td>
</tr>
<tr>
<td>2</td>
<td>Seller $i$</td>
<td>Seller $i$</td>
<td>Seller $j$</td>
<td>$p_i &lt; p_j$</td>
<td>$q_ip_i(1 - \phi)$</td>
<td>$(1 - \lambda)(1 - q_i)q_ip_i(1 - \phi)$</td>
</tr>
<tr>
<td>3</td>
<td>Seller $i$</td>
<td>Seller $j$</td>
<td>Seller $i$</td>
<td>$p_i &gt; p_j$</td>
<td>$[1 - (1 - \beta)\lambda - (1 - \lambda)q_i]q_ip_i(1 - \phi)$</td>
<td>$(1 - \beta)q_iq_ip_i(1 - \phi)$</td>
</tr>
<tr>
<td>4</td>
<td>Seller $i$</td>
<td>Seller $j$</td>
<td>Seller $i$</td>
<td>$p_i &lt; p_j$</td>
<td>$[1 - (1 - \beta)\lambda]q_ip_i(1 - \phi)$</td>
<td>$(1 - \beta)(1 - \lambda)q_iq_ip_i(1 - \phi)$</td>
</tr>
</tbody>
</table>

Table A5: Sellers’ revenue given the placement of their organic listings when seller $i$ wins the auction.
The expected profit of a seller with matching probability \( q_i \) who bids \( b(q_i) \) is,

\[
U_s(b(q_i)) = \int_0^{q_i} \left( \frac{1}{2} \left[ q_i \left( 1 - \beta \lambda - \frac{q_j}{2} \right) (1 - \phi) \right] + \frac{1}{2} \left[ q_i (1 - \lambda) \left( 1 - \frac{q_j}{2} \right) (1 - \phi) \right] \right) dq_j \\
+ \int_{q_i}^1 \left( \frac{1}{2} \left[ q_i \left( 1 - \frac{1 - \lambda}{2} q_j \right) (1 - \phi) \right] + \frac{1}{2} \left[ q_i \left( 1 - \frac{1 - \lambda}{2} \right) q_j \left( 1 - \phi \right) (1 - \phi) \right] - b(q_j) \right) dq_j,
\]

\[
= \int_0^{1/2} \left( 1 - \frac{1 - \lambda}{2} q_j \right) q_i (1 - \phi) dq_j + \int_{q_i}^1 \left( 1 - (1 - \beta) \lambda - \frac{1 - \lambda}{2} q_j \right) q_i (1 - \phi) dq_j \\
- \int_0^{q_i} \beta \lambda q_i (1 - \phi) dq_j - \int_{q_i}^1 b(q_j) dq_j.
\]

The first term denotes a seller’s utility when he loses the auction. The second terms denotes his utility when he wins the auction. Taking the derivative with respect to \( q_i \) and let the resulting expression equal to zero when \( q_i = q_i \), we again have,

\[
b^i(q_i) = \beta \lambda q_i (1 - \phi).
\]

This contradicts with the assumption that \( b(q_i) \) decreases with \( q_i \). Therefore \( b(q_i) \) decreasing with \( q_i \) cannot be in equilibrium. Overall, in the independent case, a seller with matching probability of \( q_i \) bids \( \beta \lambda q_i (1 - \phi) \) in equilibrium.

### A3 Comparative Statics

#### A3.1 Sellers’ Profit

When prices are normalized to 1, taking expectation over the possible realizations of \( q_i \) and \( q_j \), a seller’s profit can be written down as

\[
U_s = \int_0^1 \left( \int_0^{q_i} u^W(q_j)dq_j + \int_{q_i}^1 u^L(q_j)dq_j \right) f(q_i)d(q_i)
\]

\[
= E[q] \left( \frac{1}{2} \left( 1 + \frac{1 + \beta}{2} \lambda - \frac{q_L + 1}{2} \right) \left( 1 - \phi \right) + \beta \lambda E[q F(q)] \right)
\]

where \( u^W \) and \( u^L \) are given in Section 4.2. Using the uniform distribution as per the assumption \( q \sim U[q_L, 1] \):

\[
U_s = \left( -\frac{1 - \lambda}{2} \left( \frac{q_L + 1}{2} \right)^2 + \left( 1 - \frac{1 + \beta}{2} \lambda \right) \frac{q_L + 1}{2} \right) \left( 1 - \phi \right) + \beta \lambda \frac{1 - q_L}{6} \left( 1 - \phi \right).
\]

The first term, \( -\frac{1 - \lambda}{2} \left( \frac{q_L + 1}{2} \right)^2 + \left( 1 - \frac{1 + \beta}{2} \lambda \right) \frac{q_L + 1}{2} \left( 1 - \phi \right) \), stands for a seller’s baseline revenue without considering search advertising. The second term, \( \beta \lambda \frac{1 - q_L}{6} \left( 1 - \phi \right) \), stands for a seller’s gain from winning the ad slot net of the advertising price paid. It is easy to see that \( \frac{\partial U_s}{\partial \lambda} = -\lambda \frac{5q_L + 1}{12} (1 - \phi) < 0 \), so the seller is always worse off when organic listings are ordered strategically (when \( \beta \) is highest).

The comparative statics of seller profit with exogenous pricing is presented below:

- Sellers’ profit decreases with \( \lambda \).
Sellers’ profit increases with $q_L$ when $\beta < \frac{3}{5}$; it first increases then decreases with $q_L$ when $\beta > \frac{3}{5}$ and $\lambda < \frac{3}{5} \beta$ and it decreases with $q_L$ when $\lambda > \frac{3}{5}$.

### A3.2 Platform Revenue

Turning to the platform, its revenue is the sum of the commission revenue and the advertising revenue, which are given by

$$
\pi_p = \left( E[q] \left( 1 - \frac{\lambda}{2} - \frac{\beta}{2} \frac{E[q]}{2} + \beta \lambda E[qF(q)] \right) + \beta \lambda E[q_F(q)] \right) \phi + \beta \lambda E[E[q_j | q_j < q]] (1 - \phi).
$$

Using the uniform distribution $q \sim U[q_L, 1]$:

$$
\pi_p = \left( \frac{1}{8}(3 - q_L)(1 + q_L) - \frac{\lambda}{24}(1 - q_L)(3 + 3q_L - 2\beta) \right) \phi + \beta \lambda \frac{1 + 2q_L}{6}(1 - \phi),
$$

where the first term, $\frac{1}{8}(3 - q_L)(1 + q_L) - \frac{\lambda}{24}(1 - q_L)(3 + 3q_L - 2\beta) \phi$ is the platform’s revenue from commission and the second term, $\beta \lambda \frac{1 + 2q_L}{6}(1 - \phi)$, is its revenue from advertising.

Without considering price competition, as $\lambda$ increases, it earns less commission revenue but more ad revenue. Overall, $\frac{\partial \pi_p}{\partial \lambda} = -\frac{1}{8}(1 - q_L^2)\phi + \frac{\beta}{12}(2 - \phi - q_L(5\phi - 4))$ is positive, if and only if, $\phi < \frac{4\beta(1 + 2q_L)}{3(1 - q_L^2) + 2\beta(1 + 5q_L)}$. This indicates that when the commission rate is small, the increase in ad revenue dominates, and the platform revenue increases with $\lambda$; when the commission rate is large, the decrease in commission revenue dominates, and the platform revenue decreases with $\lambda$.

Turning to $q_L$, without considering price competition, as $q_L$ increases, the sellers’ baseline revenue becomes higher, thus the platform earns more commissions; in addition, as the sellers compete more aggressively for the advertising slot, the platform also earns higher ad revenue. Since the two forces of increasing $q_L$ work in the same direction for the platform, its total revenue increases with $q_L$. Consumer surplus also always increases with $q_L$ as the likelihood of sales increases. Furthermore, as $q_L$ increases, the presence of price competition tends to bring down the platform revenue.

The comparative statics of platform revenue without considering price competition are presented below:

- The platform’s revenue increases with $\lambda$ when $\phi < \frac{4\beta(1 + 2q_L)}{3(1 - q_L^2) + 2\beta(1 + 5q_L)}$ and it decreases with $\lambda$ when $\phi > \frac{4\beta(1 + 2q_L)}{3(1 - q_L^2) + 2\beta(1 + 5q_L)}$.
- The platform’s revenue increases with $q_L$

### A3.3 Consumer Surplus

For consumers, if we normalize the net utility of consuming each product for the consumer to 1, then the consumer surplus is equal to the expected number of sales, i.e.,

$$
U_c = E[q] \left( 1 - \frac{\lambda}{2} - \frac{\beta}{2} \frac{E[q]}{2} + \beta \lambda E[qF(q)] \right) + E[(\beta \lambda E[q_F(q)])].
$$

Using the uniform distribution $q \sim U[q_L, 1]$,

$$
U_c = \frac{1}{8}(3 - q_L)(1 + q_L) - \frac{\lambda}{24}(1 - q_L)(3 + 3q_L - 2\beta),
$$
which increases with $\beta$, implying that consumers are better off under the strategic case, where $\beta = 1$. Furthermore, compared with the independent case, consumer surplus increases by $\frac{1-q_L}{12}(1-\beta)\lambda$.

In terms of $\lambda$, with exogenous pricing, consumer surplus always decreases with $\lambda$ as the sales likelihood decreases: $\frac{\partial U_c}{\partial \lambda} = -\frac{1-q_L}{24}(3+3q_L-2\beta) < 0$. Since an increasing $\lambda$ leads to higher prices, consumer surplus will further decrease with $\lambda$ with endogenous pricing. With exogenous pricing, consumer surplus also always increases with $q_L$ as the likelihood of sales increases.

The comparative statics of consumer surplus are summarized below:

- Consumer surplus decreases with $\lambda$.
- Consumer surplus increases with $q_L$.

### A4 Solution for Price Competition

#### A4.1 Bayes-Nash Analysis for Solving Equilibrium Bids with Price Competition

To solve the equilibrium bid, we write down the expected profit of a seller with matching probability $q_i$ and price $p_i$. A Nash equilibrium is given if the seller’s profit when bidding $b(q_i, p_i')$ is maximized at $q_i' = q_i$ so that a seller with matching probability with $q_i$ will not deviate to bid at other matching probabilities. Denote $f(q_i)$ as the distribution of seller $i$’s matching probability, and denote $g_q(p_i)$ as the price distribution of seller $i$’s given that his matching probability is $q_i$. Note that Under the search-independent pricing, $g_q(p_i) \equiv g(p_i)$. Given a possible placement of the sellers’ listings, we write down the expected profit of seller $i$ based on the demand distribution given in Table 3. In particular, given
matching probability $q_i$ and selling price $p_i$, seller $i$'s profit by bidding at $b(q'_i, p_i)$ is

$$U_{q_i, p_i}(b(q'_i, p_i)) =$$

$$\int_0^{p_i} \int_0^q q'_i \left( \frac{1}{2} [q_i (1 - (1 - \lambda) q_j) p_i (1 - \phi)] + \frac{1}{2} [q_i (\beta \lambda + (1 - q_j) (1 - \lambda)) p_i (1 - \phi)] - b(q_j, p_j) \right) g_{q_i}(p_j) f(q_j) dq_j dp_j$$

$$+ \int_{p_i}^1 \int_0^q q'_i \left( \frac{1}{2} [q_i p_i (1 - \phi)] + \frac{1}{2} [q_i (\beta \lambda + 1 - \lambda) p_i (1 - \phi)] - b(q_j, p_j) \right) g_{q_i}(p_j) f(q_j) dq_j dp_j$$

$$+ \int_0^{p_i} \int_0^{q_i} q'_i \left( \frac{1}{2} [q_i (1 - (1 - \lambda) q_j) p_i (1 - \phi)] + \frac{1}{2} [q_i (\beta \lambda + (1 - q_j) (1 - \lambda)) p_i (1 - \phi)] - b(q_j, p_j) \right) g_{q_i}(p_j) f(q_j) dq_j dp_j$$

$$+ \int_{p_i}^1 \int_0^{q_i} q'_i \left( \frac{1}{2} [q_i (1 - (1 - \lambda) q_j) p_i (1 - \phi)] + \frac{1}{2} [q_i (\beta \lambda + (1 - q_j) (1 - \lambda)) p_i (1 - \phi)] - b(q_j, p_j) \right) g_{q_i}(p_j) f(q_j) dq_j dp_j,$$

$$= \int_0^{p_i} \int_0^q q'_i q_i p_i \left[ 1 - \frac{1 + \beta}{2} \lambda - (1 - \lambda) q_j \right] g_{q_i}(p_j) f(q_j) dq_j dp_j (1 - \phi)$$

$$- (1 - \lambda) q_i p_i \int_0^{p_i} \int_0^q q_j g_{q_i}(p_j) f(q_j) dq_j dp_j (1 - \phi) - \frac{1}{2} \int_0^{p_i} q'_i g_{q_i}(p_j) f(q_j) dq_j dp_j(1 - \phi)$$

$$- (1 - \lambda) q_i p_i \int_0^{p_i} \int_0^q q_j g_{q_i}(p_j) f(q_j) dq_j dp_j (1 - \phi) - \frac{1}{2} \int_0^{p_i} q'_i g_{q_i}(p_j) f(q_j) dq_j dp_j(1 - \phi)$$

In arriving at a seller's profit, we use the assumption that under $b(q_i, p_i) = b(q_i, p_j)$ the platform will place the seller with the higher bid in the first organic listing, since he is also the seller which generates a higher expected revenue for the platform. In the first step, the first two terms denote seller $i$'s net profits when he wins the auction, which requires $q_i p_i > q_j p_j$. The last two terms denote seller $i$'s profit when he loses the auction, which requires $q_i p_i < q_j p_j$. The following steps are obtained by the change of variables and combining multiple integrals.

The above expression is maximized at $q'_i = q_i$, i.e., if we take the derivative with respect to $q'_i$ and...
let \( q'_i = q_i \), then the resulting expression should be equal to zero. That is,

\[
\frac{\partial U_{q_i,p_i}(b(q'_i,p_i))}{\partial q'_i}|_{q'_i=q_i} = \beta \lambda q_i p_i (1 - \phi) \int_0^1 \frac{p_i}{p_j} g_{\frac{\phi}{p_j}}(p_j) f(p_j) q'_i dp_j - \int_0^1 \frac{p_i}{p_j} \frac{b(p_i,q'_i,p_j) g_{\frac{\phi}{q'_i}}(p_j)}{q'_i} f(p_j) q'_i dp_j
\]

\[= \int_0^1 \left[ \beta \lambda q_i p_i (1 - \phi) - b(q'_i, p_i) \right] \frac{p_i}{p_j} g_{\frac{\phi}{q'_i}}(p_j) f(p_j) q'_i dp_j \]

\[= \int_0^1 \left[ \beta \lambda q_i p_i (1 - \phi) - b(q_i, p_i) \right] \frac{q_i}{q_j} g_{\frac{\phi}{q_j}}(p_j) f(p_j) q'_i dp_j, \]

\[= 0.\]

This leads to the equilibrium bid solution:

\[b(q_i, p_i) = \beta \lambda q_i p_i (1 - \phi).\]

Again, the solution to the strategic case corresponds to setting \( \beta = 1.\)

### A4.2 Solution for Search-independent Pricing

Substituting the equilibrium bid into a seller’s profit function derived in the above section, we get a seller’s profit given price \( p_i \) and matching probability \( q_i \) as,

\[
U_{q_i}(p_i) = \int_0^1 \int_0^1 \left( 1 - \frac{1 + \beta}{2} \lambda \right) q_i p_i f(q_j) g(p_j) dq_j dp_j (1 - \phi) + \beta \lambda q_i p_i \int_0^1 \int_0^1 \int_0^1 \frac{p_i}{p_j} f(q_j) g(p_j) dq_j dp_j (1 - \phi)
\]

\[- (1 - \lambda) q_i p_i \int_0^1 \int_0^1 q_j g(p_j) f(q_j) dq_j dp_j (1 - \phi) - \int_0^1 \int_0^1 \int_0^1 \frac{p_i}{p_j} \beta \lambda q_j p_j f(q_j) g(p_j) dq_j dp_j (1 - \phi), \]

\[= \left( 1 - \frac{1 + \beta}{2} \lambda \right) q_i p_i (1 - \phi) - (1 - \lambda) q_i p_i \int_0^1 \int_0^1 q_j g(p_j) f(q_j) dq_j dp_j (1 - \phi)
\]

\[+ \beta \lambda q_i p_i \int_0^1 \int_0^1 \int_0^1 \frac{p_i}{p_j} f(q_j) g(p_j) dq_j dp_j (1 - \phi) - \beta \lambda \int_0^1 \int_0^1 \int_0^1 q_j p_j f(q_j) g(p_j) dq_j dp_j (1 - \phi), \]

\[= \left( 1 - \frac{1 + \beta}{2} \lambda \right) q_i p_i (1 - \phi) - (1 - \lambda) q_i p_i q_j G(p_i) (1 - \phi)
\]

\[+ \beta \lambda q_i p_i \int_0^1 \int_0^1 \int_0^1 \frac{p_i}{p_j} f(q_j) g(p_j) dq_j dp_j (1 - \phi) - \beta \lambda \int_0^1 \int_0^1 \int_0^1 q_j p_j f(q_j) g(p_j) dq_j dp_j (1 - \phi), \]

\[= \left( 1 - \frac{1 + \beta}{2} \lambda \right) q_i p_i (1 - \phi) - (1 - \lambda) q_i p_i q_j G(p_i) (1 - \phi)
\]

\[+ \beta \lambda q_i p_i \int_0^1 \int_0^1 \frac{p_i}{p_j} q_j f(p_j) dp_j (1 - \phi) - \beta \lambda \int_0^1 \int_0^1 \int_0^1 q_j p_j f(q_j) g(p_j) dq_j dp_j (1 - \phi). \]
With \( q_i \sim U[q_L, 1] \), the above expression is equivalent to,

\[
U_q(p_i) = \left( 1 - \frac{1 + \beta}{2} \lambda \right) q_i p_i (1 - \phi) - (1 - \lambda) q_i p_i q_j G(p_i)(1 - \phi)
+ \beta \lambda q_i p_i \int_0^1 F \left( \frac{p_i}{q_j} \right) g(p_j) dp_j (1 - \phi) - \beta \lambda \int_0^1 \left[ \frac{1}{2} p_j^2 q_i^2 - \frac{q_i^2}{2} p_j \right] \frac{1}{1 - q_L} g(p_j) dp_j (1 - \phi),
\]

\[
= \left( 1 - \frac{1 + \beta}{2} \lambda \right) q_i p_i (1 - \phi) - (1 - \lambda) q_i p_i q_j G(p_i)(1 - \phi)
+ \beta \lambda q_i p_i \int_0^1 F \left( \frac{p_i}{q_j} \right) g(p_j) dp_j (1 - \phi) - \beta \lambda \frac{1}{2} p_i^2 q_i^2 E \left[ \frac{1}{p_j} \right] \frac{1}{1 - q_L} + \beta \frac{\lambda q_i^2}{2} E[p_j] \frac{1}{1 - q_L} (1 - \phi),
\]

\[
= \left( 1 - \frac{1 + \beta}{2} \lambda \right) q_i p_i (1 - \phi) - (1 - \lambda) q_i p_i q_j G(p_i)(1 - \phi)
+ \beta \lambda q_i p_i \int_0^1 F \left( \frac{p_i}{q_j} \right) g(p_j) dp_j (1 - \phi) - \beta \lambda \frac{1}{2} p_i^2 q_i^2 E \left[ \frac{1}{p_j} \right] \frac{1}{1 - q_L} + \beta \frac{\lambda q_i^2}{2} E[p_j] \frac{1}{1 - q_L} (1 - \phi),
\]

\[
= \left( 1 - \frac{1 + \beta}{2} \lambda \right) q_i p_i (1 - \phi) - (1 - \lambda) q_i p_i q_j G(p_i)(1 - \phi)
+ \beta \frac{1}{2} p_i^2 q_i^2 E \left[ \frac{1}{p_j} \right] - \beta \lambda q_i q_j \frac{q_L}{1 - q_L} \frac{1}{1 - q_L} + \beta \frac{\lambda q_i^2}{2} E[p_j] \frac{1}{1 - q_L} (1 - \phi).
\]

Integrated over \( q_i \sim U[q_L, 1] \), a seller’s expected profit by pricing at \( p_i \) is given by,

\[
U(p_i) = E_q[U_q(p_i)]
= \left( 1 - \frac{1 + \beta}{2} \lambda \right) E[q_i] p_i (1 - \phi) - (1 - \lambda) E^2[q_i] p_i G(p_i)(1 - \phi)
+ \beta \lambda \frac{1}{2} E[q_i^2] p_i^2 E \left[ \frac{1}{p_j} \right] - \beta \lambda p_i E[q_i] \frac{q_L}{1 - q_L} + \beta \frac{\lambda q_i^2}{2} E[p_j] \frac{1}{1 - q_L} (1 - \phi),
\]

\[
= \left( 1 - \frac{1 + \beta}{2} \lambda \right) q_i p_i (1 - \phi) - (1 - \lambda) E^2[q_i] p_i G(p_i)(1 - \phi)
+ \beta \frac{\lambda E[q_i^2]}{2} p_i^2 E \left[ \frac{1}{p_j} \right] + \beta \frac{\lambda q_i^2}{2} E[p_j] \frac{1}{1 - q_L} (1 - \phi),
\]

\[
= \left[ 1 - \left( 1 + \frac{1 + q_L}{1 - q_L} \right) \frac{\lambda}{2} \right] E[q_i] p_i (1 - \phi) - (1 - \lambda) E^2[q_i] p_i G(p_i)(1 - \phi)
+ \beta \frac{\lambda E[q_i^2]}{2} p_i^2 E \left[ \frac{1}{p_j} \right] + \beta \frac{\lambda q_i^2}{2} E[p_j] \frac{1}{1 - q_L} (1 - \phi).
\]

Since a seller would be indifferent towards pricing between \( p_L \leq p_i \leq 1 \) under the mixed pricing equilibrium, we have that the seller expects a profit of \( U(1) \) for any \( p_L \leq p_i \leq 1 \), where

\[
U(1) = \left[ 1 - \left( 1 + \frac{1 + q_L}{1 - q_L} \right) \frac{\lambda}{2} \right] E[q_i] (1 - \phi) - (1 - \lambda) E^2[q_i] (1 - \phi)
+ \beta \frac{\lambda E[q_i^2]}{2} E \left[ \frac{1}{p_j} \right] + \beta \frac{\lambda q_i^2}{2} E[p_j] (1 - \phi).
\]
From \( U(p_i) \equiv U(1) \), we find that the cumulative distribution function of equilibrium prices is,

\[
G(p) = a + 1 - \frac{1}{p}(a + bE[p]) + bpE[p],
\]

where \( a = \frac{1 - \left(1 + \frac{1 + q_L}{1 - q_L}\right)\lambda}{E[q]} - 1 \) and \( b = \frac{\beta \lambda}{2} E[q^2] \). Next, we find the closed-form cumulative distribution function by solving the following two simultaneous equations. The first equation is derived from evaluating \( E[p] \), i.e.,

\[
E[p] = \int_{p_L}^{1} \frac{1}{p} g(p) dp = \int_{p_L}^{1} \frac{1}{p} \left[ \frac{1}{p^2} (a + bE[p]) + bE[p] \right] dp = \left[ \frac{1}{2} - \frac{a + bE[p]}{p} + bE[p]\ln(p) \right]_{p_L}^{1},
\]

\[
= (a + bE[p]) \left( \frac{1}{2p_L} - \frac{1}{2} \right) - bE[p]\ln(p_L).
\]

The second equation can be derived from \( G(p_L) = 0 \), i.e.,

\[
(a + 1)p_L - (a + bE[p]) + p_L^2 bE[p] = 0
\]

Combining the above two equations, the equilibrium price distribution can be solved analytically. The expected selling price is,

\[
E[p] = \int_{p_L}^{1} p g(p) dp = \int_{p_L}^{1} p \left[ \frac{1}{p^2} (a + bE[p]) + bE[p] \right] dp = \left[ \ln(p)(a + bE[p]) + bE[p]^2/2 \right]_{p_L}^{1},
\]

\[
= -(a + bE[p])\ln(p_L) + bE[p]\frac{1 - p_L^2}{2}.
\]

Substituting the solution into \( U(1) \), we can find the seller’s equilibrium profit as,

\[
U(1) = \left[ 1 - \left(1 + \beta \frac{1 + q_L}{1 - q_L}\right)\frac{\lambda}{2} \right] E[q_i](1 - \phi) - (1 - \lambda)E^2[q_i](1 - \phi) + \beta \lambda E[q^2] \frac{E[p]}{p_j} + \frac{\beta \lambda}{2} \frac{q_L^2}{1 - q_L} \left[ -(a + bE[p])\ln(p_L) + bE[p]\frac{1 - p_L^2}{2} \right] (1 - \phi).
\]

Consumer surplus given a seller with price \( p_i \) and math probability \( q_i \) comes as,

\[
U_c(p_i, q_i) = \left(1 - \frac{1 + \beta}{2}\frac{\lambda}{2} \right) q_i(1 - p_i)(1 - \phi) - (1 - \lambda)E^2[q_i](1 - p_i)G(p_i)(1 - \phi) + \beta \lambda q_i(1 - p_i) \left[ \frac{p_iq_i}{1 - q_L} E[p] \frac{1}{p_j} - \frac{q_L}{1 - q_L} \right].
\]

Therefore, the expected consumer surplus is,

\[
U_c = E[U_c(p_i, q_i)] = \left(1 - \frac{1 + \beta}{2}\frac{\lambda}{2} \right) E[q_i](1 - E[p])(1 - \phi) - (1 - \lambda)E^2[q]E_p[(1 - p)G(p)](1 - \phi) + \beta \lambda \left[ E[q^2] \frac{E[p]}{1 - q_L} E[p] + E[q]\frac{q_L}{1 - q_L} (1 - E[p]) \right].
\]

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A4.3 Solution for Search-dependent Pricing

Substituting the equilibrium bid into a seller’s profit function derived in the above section, we get a seller’s profit given matching probability $q_i$ and price $p(q_i)$ as,

\[
U_{q_i}(p(q_i)) = \int_0^1 \int_0^1 \left(1 - \frac{1 + \beta}{2}\right) q_i p_i g_i(p_j) f(q_j) dq_j dp_j (1 - \phi) + \beta \lambda q_i p_i \int_0^1 \int_0^{q_i} g_i(p_j) f(q_j) dq_j dp_j (1 - \phi) \\
- (1 - \lambda) q_i p_i \int_0^1 \int_0^{p_i} q_j g_i(p_j) f(q_j) dq_j dp_j (1 - \phi) - \int_0^1 \int_0^{q_i} \beta \lambda q_i p_j (1 - \phi) g_i(p_j) f(q_j) dq_j dp_j,
\]

\[
= \int_0^1 \int_0^1 \left(1 - \frac{1 + \beta}{2}\right) q_i p_i g_i(p_j) f(q_j) dq_j dp_j (1 - \phi) + \beta \lambda q_i p_i \int_0^1 \int_0^{q_i} q_j g_i(p_j) f(q_j) dq_j dp_j (1 - \phi) \\
- (1 - \lambda) q_i p_i \int_0^1 \int_0^{q_i} q_j g_i(p_j) f(q_j) dq_j dp_j (1 - \phi) - \beta \lambda \int_0^1 \int_0^{q_i} q_j g_i(p_j) f(q_j) dq_j dp_j (1 - \phi).
\]

We can indeed prove that there does not exist mixed price equilibrium, thus $p(q_i)$ degenerates into a fixed-point function of $q_i$, i.e., $g_i(p_j) \equiv 1$ for $p_i = p(q_i)$ and $g_i(p_j) \equiv 0$ otherwise. Then the probability that seller $i$ wins the ad auction, i.e., the integral of $p_j$ from 0 to $\frac{p_i}{q_i}$ in the second and the fourth terms, is equivalent to the integral of $p_j$ from 0 to $q_i$. The above seller profit can be simplified into,

\[
U_{q_i}(p(q_i)) = \int_0^1 \left(1 - \frac{1 + \beta}{2}\right) q_i p_i f(q_j) dq_j (1 - \phi) - (1 - \lambda) q_i p_i \int_0^{q_i} q_j f(q_j) dq_j (1 - \phi) \\
+ \beta \lambda q_i p_i \int_0^{q_i} q_j f(q_j) dq_j (1 - \phi) \\
= \int_0^1 \left(1 - \frac{1 + \beta}{2}\right) q_i p_i f(q_j) dq_j (1 - \phi) - (1 - \lambda) q_i p_i \int_0^{q_i} q_j f(q_j) dq_j (1 - \phi) \\
+ \beta \lambda \int_0^{q_i} (q_i p_i - q_j p_j) f(q_j) dq_j (1 - \phi).
\]

If a seller with matching probability $q_i$ deviates to bid at $p(q_i')$, the seller’s profit is,

\[
U_{q_i}(p(q_i')) = \left(1 - \frac{1 + \beta}{2}\right) q_i p_i (q_i') (1 - \phi) - (1 - \lambda) q_i p_i \int_0^{q_i'} q_j f(q_j) dq_j (1 - \phi) \\
+ \beta \lambda \int_0^{q_i'} \left[ q_i p_i (q_i') - q_j p(q_j) \right] f(q_j) dq_j (1 - \phi).
\]

Taking derivatives of $U_{q_i}(p(q_i'))$ against $q_i$ based on Leibniz integral rule, we have

\[
\frac{\partial U_{q_i}(p(q_i'))}{\partial q_i} = \left(1 - \frac{1 + \beta}{2}\right) q_i p'(q_i') (1 - \phi) - (1 - \lambda) q_i \left[ q_i f(q_j) dq_j (1 - \phi) + p(q_i') q_i f(q_i') (1 - \phi) \right] \\
+ \beta \lambda \int_0^{q_i'} q_i p'(q_i') f(q_j) dq_j (1 - \phi) + \beta \lambda \left[ q_i p(q_i') - q_i p'(q_i') \right] f(q_j) (1 - \phi).
\]
At $q_i' = q_i$, the last terms cancels out and the above expression is evaluated as

$$\frac{\partial U_{q_i}(p(q_i))}{\partial q_i} \bigg|_{q_i' = q_i}$$

$$= \left(1 - \frac{1 + \beta}{2}\lambda\right) q_i p'(q_i)(1 - \phi) - (1 - \lambda) q_i \left[p'(q_i) \int_0^{q_i} q_j f(q_j) dq_j (1 - \phi) + p(q_i) q_i f(q_i) (1 - \phi)\right]$$

$$+ \beta \lambda \int_0^{q_i} q_i p'(q_i) f(q_j) dq_j (1 - \phi) + \beta \lambda [q_i p(q_i) - q_i p(q_i)] f(q_i) (1 - \phi),$$

$$= \left(1 - \frac{1 + \beta}{2}\lambda\right) q_i p'(q_i)(1 - \phi) - (1 - \lambda) q_i \left[p'(q_i) \int_0^{q_i} q_j f(q_j) dq_j (1 - \phi) + p(q_i) q_i f(q_i) (1 - \phi)\right]$$

$$+ \beta \lambda \int_0^{q_i} q_i p'(q_i) f(q_j) dq_j (1 - \phi),$$

$$= q_i (1 - \phi) \left[\left(1 - \frac{1 + \beta}{2}\lambda\right) p'(q_i) - (1 - \lambda) q_i \left[p'(q_i) \int_0^{q_i} f(q_j) dq_j + p(q_i) f(q_i)\right] + \beta \lambda \int_0^{q_i} p'(q_i) f(q_j) dq_j\right],$$

$$= q_i (1 - \phi) \left[p'(q_i) \left(1 - \frac{1 + \beta}{2}\lambda - (1 - \lambda) \int_0^{q_i} q_j f(q_j) dq_j + \beta \lambda F(q_i)\right) - (1 - \lambda) q_i p(q_i) f(q_i)\right].$$

In equilibrium, the seller maximizes his profit at $q_i' = q_i$, that is, $\frac{\partial U_{q_i}(p(q_i))}{\partial q_i} \bigg|_{q_i' = q_i} = 0$. This leads to the following differentiation equation,

$$p'(q_i) = \frac{(1 - \lambda) q_i f(q_i)}{1 - \frac{1 + \beta}{2}\lambda + \beta \lambda F(q_i) - (1 - \lambda) \int_0^{q_i} q_j f(q_j) dq_j} p(q_i).$$

Solving the above differential equation, we get,

$$p(q_i) = C e^{\frac{(1 - \lambda) q_i f(q_i)}{1 - \frac{1 + \beta}{2}\lambda + \beta \lambda F(q_i) - (1 - \lambda) \int_0^{q_i} q_j f(q_j) dq_j}},$$

where $C$ is a constant pinned down by the boundary condition that $p_i = 1$ when $q_i = 1$.

When $q$ is distributed as $q \sim U[q_L, 1]$, the equilibrium price under the independent listing is given by

$$p(q) = \left(1 - p_L\right)(1 - p_L + p_L \lambda + \beta \lambda) e^{\frac{2 \beta \lambda \arctan\left[\frac{\beta \lambda - q(1 - \lambda)}{A}\right]}{2(1 - p_L) + p_L^2 - q^2 - \lambda + p_L \lambda - q^2 \lambda - \lambda - p_L \beta \lambda + 2 q \beta \lambda}},$$

where $A = \sqrt{-2 - p_L^2 (1 - \lambda)^2 + (3 - \beta) \lambda - (1 + \beta + \beta^2) \lambda^2 + p_L (1 - \lambda)(2 - \lambda + \beta \lambda)}$. 