LIQUIDITY DEFLATION:
Supply-Side Liquidity Trap, Deflation Bias and Flat Phillips Curve

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Abstract. The paper shows that Liquidity Deflation, a liquidity-drainage mechanism associated with Safe Asset Shortage, helps to rationalize supply-side liquidity trap and involuntary unemployment under wage-price flexibility, especially in economies that suffered massive liquidity destruction, like advanced economies in the wake of the Lehman crisis. Moreover, Liquidity Deflation is shown to imply deflation bias and flat Phillips curve in a New Keynesian model.

I. INTRODUCTION

The objective of this note is to show that what I am tempted to call the "crucial missing piece" in Keynes's General Theory, GT, namely, Liquidity Deflation — a liquidity-drainage mechanism, discussed in Calvo 2016 and 2017 — can help to give a rationale to at least two central puzzles in current macroeconomic debate, namely, (1) persistence of deflationary forces despite a large increase in the supply of central bank liquid liabilities, deflation bias, and (2) flattening of the Phillips curve.

To set the discussion on familiar grounds, let us consider the IS-LM apparatus in which fiat money $M$ is assumed to have no intrinsic value, and to be the only liquid asset (or, more precisely, the only asset providing liquidity services) — and, more importantly, that the liquidity of $M$ is independent of its supply. The latter is a critical assumption that Liquidity Deflation modifies by assuming that the liquidity of $M$ may be a decreasing function of the average market holdings of real monetary balances, $M/P$, where $P$ stands for the price level. Simple microfoundations are discussed in Calvo (2017), but an informal example could further help intuition.

Consider a representative-individual economy in which all individuals are ex ante alike. Thus, real monetary balances held by each individual would be $M/Pn$, where $n$ is the number of individuals. Therefore, if $n$ is sufficiently 'large', real monetary holdings by each individual could be 'small', even though, say, $M/P$ exceeds total economy-wide nonmonetary wealth. This may result in individuals feeling that $M$ is Safe. However, individuals' perception may change if $M/P$ becomes 'large'. This would be rational, for example, if liquidation of money holdings is subject to random systemic shocks. There would now be states of nature in which money is subject to "runs", making money less safe and deteriorating its liquidity services, implying that the quality of money as a liquid asset worsens as the economy-wide stock of money becomes 'large' in real terms. Clearly, this effect is formally equivalent to an
externality. Individual money holdings may have a negligible effect on the liquidity of money and, yet, the latter may significantly decline as the economy-wide real monetary balances, $M/P$, becomes 'large'.1,2

To capture this externality, I will write the LM equilibrium condition as follows:3

$$m + Z(m^e) = L(i, y), \quad Z' < 0, L_i < 0, L_y > 0,$$

where $i$ and $y$ denote, respectively, nominal interest rate and output (as in the standard IS-LM model); $m$ stands for the atomistic individual’s demand for real monetary balances $M/P$, while $m^e$ stands for the average market holding of real monetary balances — a negative externality from the point of view of the atomistic individual. Function $Z$ corresponds to Liquidity Deflation. Thus, expression (1) gives rise to a standard LM curve, except for the Liquidity Deflation term $Z(m^e)$. In equilibrium $m = m^e$ and, consequently, an increase in $m$ may fail to increase market liquidity if $Z'$ is large enough (in absolute value). This would give rise to what I will call Supply-Side Liquidity Trap.

Section II.1 will apply equation (1) to show the possibility of Supply-Side Liquidity Trap and involuntary unemployment, even if prices and wages are perfectly flexible. Section II.2, in turn, will couch the discussion in terms of a simple dynamic New Keynesian model and show that Liquidity Deflation may generate price deflation bias and a flattening of the Phillips curve. Section III summarizes and draws further links with current macroeconomic issues.

II. SIMPLE MODELS

1. Price/Wage Deflation

Consider equation (1), and suppose a closed economy with perfectly flexible prices. The latter is an important departure from the standard IS-LM apparatus that serves to illustrate that Supply-Side Liquidity Trap does not depend on the sticky-prices assumption.

I will assume that at full-employment output, $y^F$, and $i = 0$ we have:

$$m + Z(m) < L(0, y^F)$$

1 This argument could be couched in terms of Diamond and Dybvig (1983). See examples in Goldstein and Pauzner (2005), Holmström (2015), and Gorton (2016).
2 See Calvo (2017) for a more explicit but quite different microfoundations of Liquidity Deflation.
3 In what follows, and to simplify the notation, I will assume an atomistic scenario in which the 'measure' of total population $n = 1$, constant over time.
This is a situation in which the economy has reached the Zero Lower Bound on interest rates, ZLB, but there still is excess demand for liquidity that prevents achieving full employment equilibrium and generates a situation labeled involuntary unemployment in Keynes (1936) General Theory, GT. Increasing money supply will fail to restore full employment because inequality (3) holds. Pumping more money into the economy may succeed in increasing real monetary balances, $m$, but fails to increase the supply of liquid assets. This gives rise to a phenomenon that could easily be mistaken for Liquidity Trap in GT, although the sources lie on the supply side, not on the existence of an infinitely elastic demand for money with respect to the nominal interest rate, a demand-side phenomenon emphasized in the GT. I will call the situation depicted by inequalities (2) and (3) Supply-Side Liquidity Trap. As a first approximation, recent events fit well into this kind of scenario. During the Great Recession, central banks resorted to increasing central bank liquidity, QE, but results, although positive, were largely disappointing and gave rise to attempts to lowering the demand for money by implementing negative interest rates on liquid government liabilities ($m$ in the present model). The latter is the right medication for a GT Liquidity Trap but, as I will argue next, there is no guarantee that it will work when the Liquidity Trap comes from supply-side problems.

What is interesting and, no doubt concerning, is that, unlike in standard models (including those that display GT Liquidity Trap) price deflation is no solution for escaping the maelstrom. Price deflation may become chronic unless one finds a way to create liquidity in a different manner. As pointed out in Calvo (2017), lowering the demand for liquidity by implementing negative interest rates on government’s liquid liabilities may actually be counterproductive. For example, it may lead to replacing $M$ with gold or bitcoin, objects which are not necessarily free from giving rise to SS Liquidity Trap. Moreover, in the context of the model, expansive fiscal policy may be ineffective unless, for example, it is accompanied by an increase in other forms of liquid assets (i.e., other than $m$ in the present model). Bond financed infrastructure projects, for example, could be effective in increasing aggregate demand if the public perceives these bonds as safe because they are collateralized by those projects — thus resulting in an increase in the supply of liquid assets. But just the sheer increase of aggregate demand through the IS curve, as in the standard IS-LM model, would fail to release the grip on output and employment provoked by the SS Liquidity Trap depicted in equations (2) and (3).

The model can be extended to take explicit account of the labor market. Let the production function be $f(s)$, where $s$ stands for employment, and function $f$ satisfies all standard properties. Let $\bar{y} < y^f \equiv f(s^f)$, where $s^f$ stands for full-employment labor, and assume

$$\max_m [m + Z(m)] = L(0,\bar{y}).$$  \hfill (4)
Moreover, let $\bar{s}$ be such that $f(\bar{s}) = \bar{y}$. Thus, in words, the above equations imply that the largest amount of labor that the economy can employ at ZLB, $\bar{s}$, is less than full employment $s^\ell$. If prices and wages are perfectly flexible, this will lead to chronic deflation. Downward wage flexibility will be of no help because their decline will quickly be followed by lower prices.

The model helps to rationalize GT Chapter 19 view, frequently quoted, but seldom modeled, according to which a fall in wages will not succeed in reestablishing full employment. For, in the present model, as long as equation (4) holds, aggregate demand will not be able to match full-employment output. A fall in nominal wages may temporarily raise employment and output, but the liquidity constraint implied in (4) prevents sales from rising.

A more realistic version of the model might include price/wage adjustment frictions, but the ghost of deflation is unlikely to entirely disappear: full employment may never be reached, unless markets are helped by structural change and/or a healthy infusion of liquid assets.

2. Deflation Bias and Flat Phillips Curve

The above model assumes that Liquidity Deflation is instantaneous. Here I will extend the model by assuming that the negative liquidity externality takes time and, thus, money printing is capable of increasing liquidity in the short run.

I will couch these assumptions in terms of a simple, perfect-foresight standard New Keynesian model with liquidity-in-advance constraint. Let $X^j_t$ denote nominal liquidity held by individual $j$ at time $t$. Following the previous discussion, I assume that

$$X^j_t = M^j_t + Z_t,$$  \hspace{1cm} (5)

where, $M^j_t$ is the stock of money held by individual $j$, and $Z_t$ stands for the Liquidity Deflation component, which, for simplicity is homogeneously and exogenously distributed across agents. Thus, $Z$ could be thought as a lump-sum liquidity endowment that could possibly be negative. I assume that $Z_t$ is predetermined at time $t$ (for all $t$) and, therefore, unlike the previous subsection, $Z_t$ is invariant to changes in the economy-wide demand for money at time $t$.

The representative individual faces a perfect capital market and is thus free to choose the stock of money holdings, but the stock of liquidity is given by equation (5). I assume that $Z$ satisfies the following differential equation

$$Z_t = \gamma (\ln \bar{x} - \ln x_t) X_t, \hspace{0.5cm} \gamma > 0, \bar{x} > 0,$$  \hspace{1cm} (6)
where $X_t$ is economy-wide liquidity, $x = X/P$, and $\bar{x}$ is an exogenous constant. As will become clear in a moment, equation (6) introduces a force that pushes economy-wide real liquidity towards $\bar{x}$. Thus, time differentiating equation (5) and aggregating across (identical) individuals, we get, in equilibrium,

$$\dot{x}_t = \dot{M}_t + \gamma (\ln \bar{x} - \ln x_t) x_t.$$  

(7)

Thus, nominal liquidity is fed by money supply but there exists an independent force, associated with Liquidity Deflation, which gradually pushes real economy-wide liquidity towards $\bar{x}$. I will first examining cases in which full employment real liquidity would call for $x > \bar{x}$, a phenomenon akin to Safe Asset Shortage.

I will focus on monetary policy aimed at controlling monetary aggregates (as QE). I assume that money supply satisfies:

$$\dot{M}_t = \mu X_t,$$

(8)

where $\mu$ is a positive parameter. Notice that in the IS-LM model in which $M = X$, $\mu$ would correspond to the rate of growth of money supply.

Hence, by equations (7) and (8), we have

$$\dot{x}_t = (\mu - \pi_t) x_t + \gamma (\ln \bar{x} - \ln x_t) x_t,$$

(9)

where $\pi = \dot{P}/P =$ instantaneous rate of inflation. Thus, denoting $h = \ln x$, equation (9) can be expressed as

$$\dot{h}_t = \mu - \pi_t + \gamma (\ln \bar{x} - h_t)$$

(10)

This is a closed economy with homogeneous output. "Full employment" output is, again, denoted by $y^F$. Equilibrium output is demand-determined as in New Keynesian macroeconomic models. Moreover, I will assume that consumption, $c$, equals aggregate demand (investment and government expenditure are zero) and consumption is subject to the following liquidity-in-advance constraint:

$$c_t = x_t,$$

(11 a)

or, equivalently,

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4 The lag may be the result of signal-extraction problems (Lucas 1972), some form of Inattention (Mankiw and Reis 2002, Sims 2003), or familiar money illusion (Shafir, Diamond and Tversky 1997). The latter is the more appropriate interpretation for the present model because I assume that all individuals lag behind current realizations in lockstep.
\[ \ln c_t = h_t. \quad (11 \text{b}) \]

The price level is sticky and satisfies Calvo (1983) equation. Thus, recalling equation (11 b), I assume\(^5\)

\[ \pi_t = \beta (\ln y^f - h_t), \quad \beta > 0. \quad (12) \]

Any equilibrium path must satisfy equations (10) and (12).\(^6\) Moreover, I will follow convention and assume that equilibrium paths must converge to steady state. These assumptions imply that, for all time \( t \), (log of) real liquidity \( z_t \) is predetermined. On the other hand, Calvo (1983) shows that the rate of inflation corresponding to an equilibrium path should be expected to be continuous for all \( t \). However, the "present" rate of inflation is free to jump. As is well known, the possibility that \( \pi_t \) can "jump" when the economy is at time \( t \), carries no non-uniqueness consequences, and uniqueness is ensured if system (10) and (12) displays saddle-path stability, which in this instance is satisfied because the determinant of the corresponding steady-state Jacobean matrix is negative.

The phase diagram for equations (10) and (12) is depicted in Figure 1, where the arrowed line pointing to the steady state is the equilibrium path. Thus, in equilibrium there exists a positive association between \( h \) and \( \pi \), i.e., recalling equation (11b), a positive association between output and inflation. Hence, the model’s reduced-form Phillips curve displays the conventional slope, despite the addition of the new Liquidity Deflation ingredients.

\(^5\) If the reader wonders why I shift to logs instead of expressing variables in their natural units, the answer is that I want the system of differential equations to be linear. In this case the equilibrium Phillips curve is linear and can easily be computed. The inquisitive reader could easily verify that substantive implications are not affected by these math contortions.

\(^6\) The liquidity-in-advance assumption simplifies the analysis enormously compared to Calvo (1983), for example: There is no need to refer to utility functions! As far as I know, Reinhart (1992) is the first paper that explores this assumption. However, for completeness, Appendix A1 shows the missing equations.
At steady state where \( \dot{h} = \dot{\pi} = 0 \), we have, by equation (12), that \( h = \ln y^F \), and, by equation (10):\(^7\)

\[
\pi = \mu + \gamma (\ln \bar{x} - \ln y^F). \tag{13}
\]

If \( \bar{x} = y^F \), the rate of inflation equals the rate of growth of liquidity driven exclusively by the expansion of money supply (recall equation (8)). However, if \( \bar{x} < y^F \), inflation at steady state \( \pi < \mu \), reflecting a "deflation bias." This is an interesting case because it brings us closer to the current deflation debate. In addition, the larger is the Liquidity Deflation parameter \( \gamma \), the smaller will be the rate of inflation at steady state, and chronic deflation cannot be discounted, even if the central bank takes an aggressive QE stance (i.e., a high \( \mu \) in the model)! So much for the formal analysis; now it is a good time to reflect on the two new parameters \( \bar{x} \) and \( y \). By equation (10), the lower is \( \bar{x} \), the less effective is money supply in increasing market liquidity. On the other hand, if \( \bar{x} < y^F \), the higher is \( \gamma \), the stronger is the Liquidity Deflation effect on steady-state inflation. These results can help to cast some light on recent events. For instance, several analyses of the Lehman crisis point to the fact that it involved a massive destruction of Safe Assets (e.g., Gorton 2010, Caballero et al 2016 and 2017), which in the present setup could be interpreted as resulting in a lower steady state capacity to supply liquidity services. This could be interpreted as a fall in \( \bar{x} \). On the other hand, the massive

\(^7\) This implies that in the long run the economy converges to full employment. However, this could be modified introducing elements highlighted in Section II.1.
destruction of liquidity may have made the market more aware of the relevance of the liquidity of collateral behind presumed Safe Assets and, therefore, become more sensitive to departures of \( x \) from \( \bar{x} \), giving rise to a higher \( \gamma \). Thus, the Lehman crisis may have enhanced the relevance of Liquidity Deflation and its consequent deflation bias. I will show below that an increase in \( \gamma \) offers a new insight for the existence of a flat Phillips curve.\(^8\)

A related implication of the model is that deflationary bias goes hand in hand with a decline in the velocity of circulation of money, \( m \), another salient fact in developed economies after the Lehman crisis. I will show that the implication holds across steady states. By equation (8), (11 a,b), and (12), we have that if \( x \) and \( \pi \) are at steady state\(^9\)

\[
\frac{m_t}{m_t} = \mu \frac{y^F}{\mu} - \pi. \tag{14}
\]

Therefore, by equation (14), if \( \mu > 0 \), \( m \) also converges to its steady state. Consider the case in which \( m \), \( x \) and \( \pi \) are simultaneously at steady state, i.e., \( \dot{m} = \dot{x} = \dot{\pi} = 0 \). By equations (11 a, b), (12) and (13), it follows that

\[

\text{velocity of circulation of money at steady state} =
\]

\[
= \frac{y^F}{m} = \frac{\pi}{\mu} = \frac{\mu + \gamma (\ln \bar{x} - \ln y^F)}{\mu}, \tag{15}
\]

Therefore, velocity falls as deflationary conditions get exacerbated, i.e. if \( \gamma (\ln \bar{x} - \ln y^F) \) goes down.\(^{10}\)

On the other hand, in a standard New Keynesian model, the nominal interest rate on non-liquid assets, \( i \), at steady state satisfies (see Appendix A1 for details):

\[
i = \rho + \pi = \rho + \mu + \gamma (\ln \bar{x} - \ln y^F), \tag{16}
\]

where \( \rho \) is the subjective rate of discount, and the right-hand equality follows from equation (13). Thus, stronger deflationary conditions result in lower nominal interest rates, another phenomenon observed in the wake of the Lehman crisis.

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\(^8\) The Flat Phillips curve issue is attracting wide attention in macroeconomic circles. See, for instance, Borio 2017, Blanchard and Summers 2017.

\(^9\) Notice that the equilibrium paths of \( x \) and \( \pi \) converge to their respective steady state equilibrium values (recall Figure 1), irrespective of \( m \).

\(^{10}\) Notice that in the realistic case in which \( m \) is constrained to be positive, we must assume that \( \mu \) must be large enough so that the numerator of equation (16) is positive.
I will now turn to examine the slope of the Phillips curve. System (10) and (12) is linear. Therefore, the equilibrium saddle path can easily be computed. Let \( B \) denote the slope of the reduced-form Phillips curve (i.e., \( B = \frac{d\pi}{d\bar{r}} \)) along the arrowed equilibrium path in Figure 1. It can be shown (see Appendix A2) that

\[
B = \frac{-\gamma+\sqrt{\gamma^2+4\beta}}{2}.
\]  

(17)

Hence, by equation (17), proof that the Phillips curve becomes flatter as the Liquidity Deflation parameter increases can be inferred from the following expressions:

\[
\frac{dB}{d\gamma} < 0 \text{ and } \lim_{\gamma \to \infty} B = 0.
\]  

(18)

Moreover, by Appendix A2,

\[
\hat{h}_t = -(B + \gamma)(\hat{h}_t - \ln y^F) = -\frac{-\gamma+\sqrt{\gamma^2+4\beta}}{2} (\hat{h}_t - \ln y^F).
\]  

(19)

Hence, the speed at which output converges to full employment is invariant to the liquidity drag \( \bar{x} \), and accelerates as the drag becomes stronger, i.e., \( \gamma \) goes up. Therefore, Liquidity Deflation fails to slow down convergence to full employment, and may actually accelerate it! This may sound implausible but these results are tied to the assumption that full employment output, \( y^F \), is invariant to liquidity frictions. Relaxing this assumption, however, need not eliminate Deflation Bias and Flat Phillips curves, as is illustrated by the case in which \( y^F \) suffers a once-and-for-all negative shock.

As it stands, the model implies that a stock change in money supply could place the economy smack into full employment. This contradicts the view that liquidity of money supply declines with its stock in real terms. However, this can easily be remedied by modifying equation (7) and, for instance, replace \( \dot{M} \) by a concave function of \( \dot{M} \) subject to an upper bound.

III. FINAL WORDS

The paper explores financial frictions that give rise to chronic deflation and a flat Phillips curve, and give rise to situations that are observationally equivalent to a Keynesian Liquidity Trap. Moreover, this holds even if the demand for money is interest elastic. The approach is relevant for situations in which, say, the economy suffered a massive destruction of liquidity that interferes with trade flows, and restoration of liquidity to "normal" levels cannot be rapidly achieved by pumping State liquidity, e.g., high-powered money. The reason for this the paper attributes to the fact that the liquidity quality of liquid assets may deteriorate as the real value of those assets becomes large. This effect is a result of a negative externality rooted in
the fact that by its very nature liquidity is opened to self-fulfilling runs à la Diamond-Dybvig (1983), especially if the supply of liquid assets increase without being accompanied by an equivalent rise of real collateral. An instance of this sort is the large increase in high-powered money in the wake of the Lehman crisis. Thus, the economy may, at least in the short run, end up in a situation that several authors have labeled Safe Asset Shortage (e.g., Caballero et al 2016, 2017, Gorton 2016, Barro et al 2017) in which, beyond a certain point, liquidity cannot be increased by easy-money policy, like QE. It is a Supply-Side Liquidity Trap that can even hold under perfectly flexible prices and wages, and may not be alleviated by expansionary fiscal or monetary policy unless it results in an increase in market liquidity.

In closing, it is worth pointing out that the effects highlighted in this paper hold when liquidity is scarce relative to "normal" situations. This is worth keeping in mind, because, for instance, the slope of the Phillips curve could steepen sharply as there is greater confidence in the stability of the financial sector. Unfortunately, return to normality is surrounded by a large degree of uncertainty, and predictions are likely to be a poor guide for policy. But central banks could still play a valuable role if they succeed in better understanding and managing new liquid assets that, as shown by recent events, could cause inefficient frictions or massive liquidity crunch.
11

Here I will sketch out the microfoundations of the model considered in Section II.2. The model is close to the one in Calvo (1983), except that money is removed from the utility function and replaced for a cash-in-advance condition, and the Liquidity Deflation effect defined in the main text.

The representative consumer’s utility function from the perspective of time 0 satisfies:

\[
\int_0^\infty u(c_t^j) e^{-\rho t} dt, \tag{A1.1}
\]

where, again, \( j \) refers to individual \( j \), function \( u \) is defined on the positive real line; it is increasing, strictly concave, and twice-continuously differentiable everywhere. Parameter \( \rho > 0 \) stands for the subjective rate of discount.

The budget constraint satisfies (assuming 0 initial wealth),

\[
\int_0^\infty [y_t^j + g_t - c_t^j - i_t m_t^j] e^{-\int_0^t r_s ds} dt \geq 0, \tag{A1.2}
\]

where \( g \) stands for lump-sum subsidy employed to rebate central bank seigniorage to the public, a standard assumption in monetary theory; \( r \) is the instantaneous real rate of interest, and the nominal interest rate \( i = r + \pi \).

Equations (5) and (11) imply that

\[
c_t^j = x_t^j = m_t^j + z_t, \tag{A1.3}
\]

implying that

\[
m_t^j = c_t^j - z_t. \tag{A1.4}
\]

Inserting equation (A1.4) in budget constraint (A1.2) yields

\[
\int_0^\infty [y_t^j + g_t - c_t^j - i_t (c_t^j - z_t)] e^{-\int_0^t r_s ds} dt \geq 0. \tag{A1.5}
\]

Variable \( z \) is exogenous to the representative individual. Therefore, the first-order condition for consumption, taking into account (A1.1) and (A1.5), takes the following familiar form:

\[11\] Variables defined in the main text, will not be defined here unless necessary for clarity.
\[
u'(c_t^l)e^{-\rho t} = \lambda (1 + i_t)e^{-\int_0^t r_s ds}, \tag{A1.6}
\]

where \( \lambda \) is the time-invariant Lagrange multiplier. The model can now be complemented by the \( z \) dynamics discussed in the text.

**APPENDIX A2**

Consider equations (10) and (12). The system is linear and, hence, there are parameters denoted \( D \) and \( B, B > 0 \), such that along the saddle path depicted in Figure 1, we have

\[
\pi = D + Bh, \text{ for all } h. \tag{A2.1}
\]

Dropping time subscripts, equations (10), (12) and (B1) imply:

\[
\dot{\pi} = B\dot{h} = B[\mu - D - Bh + \gamma (\ln \bar{x} - h)] = \beta (\ln y^F - h). \tag{A2.2}
\]

The rightmost equation must hold for all \( h \), which implies:

\[
B^2 + B\gamma - \beta = 0. \tag{A2.3}
\]

Solving for \( B \) and recalling that, by Figure 1, \( B > 0 \), yields equation (17) in the text.

To solve for parameter \( D \) in (A2.2), set \( h = \ln y^F \). It follows that

\[
D = \mu - B \ln y^F + \gamma (\ln \bar{x} - \ln y^F). \tag{A2.4}
\]

By equations (A2.2) and (A2.4), one can derive equation (19) in the text.
REFERENCES


