LIQUIDITY DEFLATION:
Supply-Side Liquidity Trap, Deflation Bias and Flat Phillips Curve

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I. Introduction

The objective of this note is to show that what I am tempted to call the "crucial missing piece" in Keynes's General Theory, GT, namely, Liquidity Deflation — a liquidity-drainage mechanism, discussed in Calvo 2016 and 2017 — can help to give a rationale to at least two central puzzles in current macroeconomic debate, namely, (1) persistence of deflationary forces despite a large increase in the supply of central bank liquid liabilities, deflation bias, and (2) flattening of the Phillips curve.

To set the discussion on familiar grounds, let us consider the IS-LM apparatus in which money $M$ is assumed to be the only liquid asset and, more importantly, that the liquidity of $M$ is independent of money supply. Liquidity Deflation modifies the model by assuming that the liquidity of $M$ may be a decreasing function of the average market holdings of real monetary balances, $M/P$, where $P$ stands for the price level. Thus, the LM equilibrium condition can be written as

\[ m + Z(m^e) = L(i, y), \quad Z' < 0, L_i < 0, L_y > 0, \]  

(1)

where $i$ and $y$ denote, respectively, nominal interest rate and output (as in the standard IS-LM model); $m$ stands for the atomistic individual's demand for real monetary balances $M/P$, while $m^e$ stands for the average market holding of real monetary balances — a negative externality from the point of view of the atomistic individual. Function $Z$ corresponds to Liquidity Deflation. Thus, expression (1) gives rise to a standard LM curve, except for the Liquidity Deflation term $Z(m^e)$. In equilibrium $m = m^e$ and, consequently, an increase in $m$ may fail to increase market liquidity if $Z'$ is large enough (in absolute value). Calvo (2017) discusses this in greater detail and provides some microfoundations. Section II.1 will apply equation (1) to show the possibility of chronic deflation. Furthermore, Section II.2 will couch the discussion in terms of a simple dynamic New Keynesian model and show that Liquidity Deflation may generate price deflation bias and a flattening of the Phillips curve. Section III closes the note with some policy implications and reflections on macroeconomic theory and the different "textbook fate" of financial shocks and frictions.
II. Price Deflation Bias and Flat Phillips Curve: A Simple Model

1. Price/Wage Deflation

Consider equation (1), and suppose a closed economy with perfectly flexible prices and, recalling equation (1), let us assume that at full-employment output, \( y^F \), and \( i = 0 \) we have:

\[
m + Z(m) < L(0, y^F)
\]

and

\[
1 + Z'(m) \leq 0.
\]

This is a situation in which the economy has reached the Zero Lower Bound, ZLB, on interest rates, but there is still is excess demand for liquidity that prevents achieving full employment. Increasing money supply will fail to restore full employment because inequality (3) holds. Pumping more money into the economy may succeed in increasing real monetary balances, \( m \), but fails to increase the supply of liquid assets. This gives rise to a phenomenon that could be easily mistaken for GT Liquidity Trap, although the sources lie on the supply side, not on the existence of an infinitely elastic demand for money with respect to the nominal interest rate, a demand side phenomenon emphasized in the GT. I will call the situation depicted by inequalities (2) and (3) Supply-Side (SS) Liquidity Trap. As a first approximation, recent events fit well into this kind of scenario. During the great recession central banks resorted to increasing central bank liquidity, QE, but results, although positive, were largely disappointing and gave rise to attempts to lowering the demand for money by implementing negative interest rates on liquid government liabilities (\( m \) in the present model). The latter is the right medication for a GT Liquidity Trap but, as I will argue next, there is no guarantee that it will work when the Liquidity Trap comes from supply-side problems.

What is interesting and, no doubt concerning, is that, unlike in standard models (including those that display GT Liquidity Trap) price deflation is no solution for escaping the maelstrom. Price deflation may become chronic unless one finds a way to create liquidity in a different manner. As pointed out in Calvo (2017), lowering the demand for liquidity by implementing negative interest rates on government’s liquid liabilities may actually be counterproductive. For example, it may lead to replacing \( M \) with gold or bitcoin, objects which are not necessarily free from the SS Liquidity Trap. Moreover, in the context of the model, expansive fiscal policy may be ineffective unless, for example, it is accompanied by an increase in other forms of liquid assets (i.e., other than \( m \) in the present model). Bond financed infrastructure projects, for example, could be effective in increasing aggregate demand if the public perceives these bonds as safe because they are collateralized by those projects — thus resulting in an increase in the supply of liquid assets. But just the sheer increase of aggregate demand through the IS curve, as in the standard IS/LM model,
would fail to release the grip on output and employment provoked by the SS Liquidity Trap depicted in equations (2) and (3).

The model can be extended à la Modigliani (1944) to take explicit account of the labor market. Let the production be \( f(n) \), where \( n \) stands for employment, and function \( f \) satisfies all standard properties. Let \( \bar{y} < y^F \equiv f(n^F) \), where \( n^F \) stands for full-employment labor, and

\[
\max_m [m + Z(m)] = L(0, \bar{y}).
\]  

(4)

Moreover, let \( n^* \) be such that \( f(n^*) = \bar{y} \). Thus, in words, the above equations imply that the largest amount of labor that the economy can employ, \( n^* \), is less than full employment \( n^F \). If prices and wages are perfectly flexible, this will lead to chronic deflation. Downward wage flexibility will be of no help because their decline will quickly be followed by lower prices. The model helps to rationalize GT Chapter 19 statement, frequently quoted, but seldom modeled, according to which a fall in wages will not succeed in reestablishing full employment. A more realistic version of the model might include price/wage adjustment frictions, but the ghost of deflation is unlikely to entirely disappear: full employment may never be reached.

2. Flat Phillips Curve

The above model assumes that Liquidity Deflation externality is instantaneous. Here I will extend the model by assuming that the negative liquidity externality takes time and, thus, money printing increases liquidity is capable of increasing liquidity in the short run.

I will couch these assumptions in terms of a simple standard New Keynesian model with \textit{liquidity}-in-advance constraint. Let \( X_t \) denote nominal liquidity at time \( t \). I assume that

\[
\dot{X}_t = \dot{M}_t + \gamma (\ln \bar{x} - \ln x_t) X_t, \quad \gamma > 0, \bar{x} > 0,
\]  

(5)

where \( x = X/P = \text{real liquidity} \). Thus, nominal liquidity is fed by money supply but there exists an independent force, associated with Liquidity Deflation, which pushes real liquidity towards \( \bar{x} \). I will focus the discussion on cases in which full employment real liquidity would call for \( x > \bar{x} \), a phenomenon akin to Safe Asset Shortage.

I assume that money supply satisfies:

\[
\dot{M}_t = \mu X_t,
\]  

(6)

where \( \mu \) is a positive parameter. Notice that in the IS-LM model in which \( M = X, \mu \) would correspond to the rate of growth of money supply.
Hence, by equations (5) and (6), we have

\[
\dot{x}_t = (\mu - \pi_t)x_t + \gamma(\ln \bar{x} - \ln x_t)x_t,
\]

where \( \pi = \dot{P}/P = \text{instantaneous rate of inflation} \). Thus, denoting \( z = \ln x \), equation (7) can be expressed as

\[
\dot{x}_t = \mu - \pi_t + \gamma(\ln \bar{x} - z_t)
\]

(8)

This is a closed economy with homogeneous output. "Full employment" output is, again, denoted by \( y^F \). Equilibrium output is demand-determined as in New Keynesian macroeconomic models. Moreover, I will assume that consumption, \( c \), equals aggregate demand (investment and government expenditure are zero) and consumption is subject to the following liquidity-in-advance constraint:

\[
c_t = x_t,
\]

(9 a)

or, equivalently,

\[
\ln c_t = z_t.
\]

(9 b)

The price level is sticky and satisfies Calvo (1983) equation. Thus, recalling equation (9), I assume\(^1\)

\[
\pi_t = \beta(\ln y^F - z_t), \quad \beta > 0.
\]

(10)

Any equilibrium path must satisfy equations (8) and (10).\(^2\) Moreover, I will follow convention and assume that equilibrium paths must converge to steady state. These assumptions imply that, for all time \( t \), (log of) real liquidity \( z_t \) is predetermined. On the other hand, Calvo (1983) shows that the rate of inflation corresponding to an equilibrium path should be expected to be continuous for all \( t \). However, the "present" rate of inflation is free to jump. As is well known, the possibility that \( \pi_t \) can "jump" when the economy is at time \( t \), carries no non-uniqueness consequences if the equilibrium solution is unique. In turn, uniqueness is ensured if system (8)

\[^1\] If the reader wonders why I shift to logs instead of expressing variables in their natural units, the answer is that I want the system of differential equations to be linear. In this case the equilibrium Phillips curve has a straightforward solution. The inquisitive reader could easily verify that substantive implications are not affected by these math contortions.

\[^2\] The liquidity-in-advance assumption simplifies the analysis enormously compared to Calvo (1983), for example. There is no need to refer to utility functions. As far as I know, Reinhart (1992) is the first paper that explores this assumption.
and (10) displays local saddle-path stability, which in this instance is satisfied because the determinant of the corresponding Jacobean matrix is negative.

The phase diagram for equations (8) and (10) is depicted in Figure 1, where the arrowed line pointing to the steady state is the equilibrium path. Thus, in equilibrium there exists a positive association between \( z \) and \( \pi \), i.e., a positive association between output and inflation. Hence, the model’s reduced-form Phillips curve displays the conventional slope, despite the addition of the new Liquidity Deflation ingredients.

**Figure 1. Equilibrium Determination**

At steady state where \( \dot{z} = \dot{\pi} = 0 \), we have, by equation (10), that \( z = \ln y^F \), and, by equation (8):\(^3\)

\[
\pi = \mu + \gamma (\ln \bar{x} - \ln y^F). \tag{11}
\]

If \( \bar{x} = y^F \), the rate of inflation equals the rate of growth of liquidity driven exclusively by the expansion of money supply (recall equation (5)). However, if \( \bar{x} < y^F \), inflation at steady state \( \pi < \mu \), reflecting a "deflation bias." This is an interesting case because it brings us closer to the current deflation debates. In addition, the larger is the Liquidity Deflation parameter \( \gamma \), the smaller will be the rate of inflation at steady state, and chronic deflation cannot be discounted, even if the central bank takes an aggressive QE stance (a high \( \mu \) in the model)!

\(^3\) This implies that in the long run the economy converges to full employment. However, this can be modified introducing elements highlighted in Section II.1.
So much for the formal analysis; now it is a good time to reflect on the two new parameters $\bar{x}$ and $\gamma$. By equation (5), the lower is $\bar{x}$, the less effective is money supply in increasing market liquidity. On the other hand, the higher is $\gamma$, the stronger is this Liquidity Deflation effect (if $\bar{x} < y^F$). Several analyses of the Lehman crisis point to the fact that it involved a massive destruction of Safe Assets (e.g., Gorton 2010, Caballero et al. 2016 and 2017), which in the present setup could be interpreted as resulting in a lower steady state capacity to supply liquidity services. This can then be formalized as a fall in $\bar{x}$. On the other hand, the massive destruction of liquidity may have made the market more aware of the relevance of the liquidity of collateral behind Safe Assets and, therefore, become more sensitive to departures of $x$ from $\bar{x}$, giving rise to a higher $\gamma$. Thus, the Lehman crisis may have enhanced the relevance of Liquidity Deflation and its consequent deflation bias. I will show next that an increase in $\gamma$ offers a new conjecture for the apparent flattening of the Phillips curve.4

A related implication of the model is that deflationary bias goes hand in hand with a decline in the velocity of money, $m$, another salient fact in developed economies after the Lehman crisis. I will show that the implication holds across steady states. By equation (5), we have

$$\frac{\dot{m}_t}{m_t} = \mu \frac{x_t}{m_t} - \pi_t. \quad (12)$$

Previous results imply that the equilibrium paths of $x$ and $\pi$ are independent of $m$, and converge to their respective steady state equilibrium values (recall Figure 1). Therefore, by equation (12) and $\mu > 0$, $m$ also converges to its steady state. Consider the case in which $m, x$ and $\pi$ are simultaneously at steady state, i.e., where $\dot{m} = \dot{x} = \dot{\pi} = 0$. By equations (9 a, b), (10) and (11), it follows that

$$velocity \ of \ money \ at \ steady \ state = \frac{y^F}{m} = \frac{\mu + \gamma(\ln \bar{x} - \ln y^F)}{\mu}. \quad (13)$$

Therefore, velocity falls as deflationary conditions get exacerbated, i.e., $\bar{x}$ falls below full employment, $y^F$, and $\gamma$ rises when the latter holds.

The deflationary bias highlighted above depends on the system being unable to reach full employment, i.e., $\bar{x} < y^F$. Thus, the bias may no longer be relevant for an economy like the US, which appears to have achieved full employment. However, as I show next, existence of a flat Phillips curve can hold even if the system is consistent with full employment in the long run (i.e., $\bar{x} = y^F$).

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4 The issue is attracting wide attention in macroeconomic circles. See, for instance, Borio 2017, Blanchard and Summers 2017.
System (8) and (10) is linear. Therefore, the equilibrium saddle path can easily be computed. Let $B$ denote the slope of the reduced-form Phillips curve (i.e., $B = \frac{dx}{dz}$ over the equilibrium path, arrowed curve in Figure 1). It can be shown that

$$B = \frac{-y + \sqrt{y^2 + 4B}}{2}. \quad (14)$$

Hence, by equation (14), proof that the Phillips curve becomes flatter as the Liquidity Deflation parameter increases can be inferred from the following expressions:

$$\frac{dB}{dy} < 0 \text{ and } \lim_{y \to \infty} B = 0. \quad (15)$$

III. Final Words

These notes show that Liquidity Deflation, a plausible financial friction that interferes with market liquidity, could give rise to a phenomenon that on the surface resembles very much a Liquidity Trap. The difference between the SS Liquidity Trap discussed here and the traditional GT Liquidity Trap is that the determining factors of the former lie on the supply side. This implies that super-low interest rates or aggressive QEs may be unable to free the economy from SS Liquidity Trap. Supply Side problems are structural (see Phelps 1994) and in this instance one should explore the financial sector for a solution. This may imply fixing the financial sector or bypassing it and creating new forms of reliable liquid assets (e.g., Safe Assets). The latter, as mentioned above, could be achieved by involving the government in large profitable enterprises, like infrastructure projects, that the current financial system is not capable supporting. Unfortunately, these types of projects are difficult to implement, especially in emerging markets where the payoff is likely the largest but emerging markets’ ability to generate Safe liabilities is scant, unless supported by international financial institutions, like the World Bank. Without fixing those supply-side problems, recession triggered by SS Liquidity Trap can be long lasting and perhaps vulnerable to crises triggered by whimsical market perceptions. If these problems remain untreated, SS Liquidity Trap could get exacerbated because the set of Safe Assets may shrink. This topic is truly global, given the expanse and incidence of the modern financial sector, and should occupy a pride of place in the next G20 meetings.

Let me close with some remarks on theory.

- In my opinion, as hinted in the Introduction, Liquidity Deflation is the missing piece in the GT tour de force, a piece that I guess would have met with Keynes’s approval. He makes it clear in Chapter 19 that he was not entirely happy with the price stickiness assumption, and tried to get rid of it in that chapter. Unfortunately, the discussion is somewhat muddled (see De Vroey 2016) and, for instance, resorts to issues like income distribution that
played no role in previous GT chapters. Thus, Chapter 19 reads a bit like a stream of conscience based on strong intuitions that do not amount to a cohesive theory. His Liquidity Trap got close of the mark but it constituted an easy target for Keynes's 'Classicals' (recall the Pigou Effect, discussed in Calvo 2017). In contrast, SS Liquidity Trap is free from price stickiness assumptions and, as argued in Section II.1 above, can help to rationalize involuntary unemployment under perfectly flexible prices and wages.

- Current literature has been using the terms "financial shocks" and "financial frictions" interchangeably. However, as a general rule, "shocks" are random, while "frictions" need not (at a macro level, that is). In the GT, for instance, "animal spirits" give rise to shocks, e.g., exogenous contraction of aggregate demand, while "price stickiness" can be modeled in absence of random shocks and qualifies as a "friction."

- Shocks are much harder to model than frictions. Few economists would question price stickiness nowadays, but shocks have elicited a wide variety of different opinions (see Borio 2017, and Blanchard and Summers 2017). This bodes well for the durability of financial frictions in the economics literature. As a proof, and referring to the GT, price stickiness kept its presence in textbooks since WWII, while Animal Spirits was left mostly for experts of the Great Depression. True, it has resurfaced with the Great Recession but I do not expect a long-lived revival.

- Liquidity Deflation is a financial friction according to my dictionary. Thus, its chances of textbook survival are less than ephemeral. However, I suspect that once the world economy steps out of Liquidity Trap territory it will tend to be forgotten.
References


