LIQUIDITY DEFLATION:
Supply-Side Liquidity Trap, Deflation Bias and Flat Phillips Curve

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Abstract. The paper shows that Liquidity Deflation, a liquidity-drainage mechanism associated with Safe Asset Shortage, helps to rationalize supply-side liquidity trap and involuntary unemployment under wage/price flexibility, especially in economies that suffered massive liquidity destruction, like advanced economies in the wake of the Lehman crisis. Moreover, Liquidity Deflation generates deflation bias and flat Phillips curve in a New Keynesian model.

I. INTRODUCTION

The objective of this note is to show that what I am tempted to call the "crucial missing piece" in Keynes's General Theory, GT, namely, Liquidity Deflation — a liquidity-drainage mechanism, discussed in Calvo 2016 and 2018 — can help to resolve two central puzzles in current macroeconomic debate, namely, (1) persistence of deflationary forces despite a large increase in the supply of central bank liquid liabilities, deflation bias, and (2) flattening of the Phillips curve.

To set the discussion on familiar grounds, let us consider the IS-LM apparatus in which fiat money $M$ is assumed to have no intrinsic value, and to be the only liquid asset (or, more precisely, the only asset providing liquidity services). In contrast with the standard model I will include Liquidity Deflation by assuming that the liquidity of $M$ may be a decreasing function of the average market holdings of real monetary balances, $M/P$, where $P$ stands for the price level. Some microfoundations are discussed in Calvo (2018). However, given the relative unfamiliarity of Liquidity Deflation, it is worth to start rolling the ball with an informal example.

Consider a representative-individual economy in which all individuals are ex ante alike. Thus, real monetary balances held by each individual would be $M/Pn$, where $n$ is the number of individuals. Therefore, if $n$ is sufficiently 'large', real monetary holdings by each individual would be 'small'. This may result in individuals feeling that $M$ is Safe. However, individuals' perception may change if $M/P$ becomes 'large'. This would be rational, for example, if liquidation of money holdings is subject to random systemic shocks. Thus, there would now be states of nature in which money is subject to "runs", making money less safe and deteriorating its liquidity services, implying that the quality of money as a liquid asset worsens as the economy-wide stock of money becomes 'large' in real terms. Clearly, this effect is
formally equivalent to an externality. Individual money holdings may have a negligible effect on the liquidity of money and, yet, the latter may significantly decline as the economy-wide real monetary balances, $M/P$, becomes 'large'.

To capture this externality in a simple manner, I will write the money market LM equilibrium condition as follows:

$$m + Z(m^e) = L(i, y), \quad Z' < 0, L_i < 0, L_y > 0,$$

where $i$ and $y$ denote, respectively, nominal interest rate and output (as in the standard IS-LM model); $m$ stands for the atomistic individual’s demand for real monetary balances $M/P$, while $m^e$ stands for the average market holding of real monetary balances — a negative externality from the point of view of the atomistic individual. Function $Z$ corresponds to Liquidity Deflation. Thus, expression (1) gives rise to a standard LM curve, except for the Liquidity Deflation term $Z(m^e)$. In equilibrium $m = m^e$ and, consequently, an increase in $m$ may fail to increase market liquidity if $Z'$ is large enough (in absolute value). This would give rise to what I will call Supply-Side Liquidity Trap.

II. SIMPLE MODELS

1. Price/Wage Deflation

Consider equation (1), and suppose a closed economy with perfectly flexible prices. I will assume that at full-employment output, $y^F$, and $i = 0$ we have:

$$m + Z(m) < L(0, y^F)$$

and

$$1 + Z'(m) \leq 0.$$

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1 This argument could be couched in terms of Diamond and Dybvig (1983). See related examples in Goldstein and Pauzner (2005), Holmström (2015), and Gorton (2016).

2 In what follows, and to simplify the notation, I will assume an atomistic economy in which the 'measure' of total population $n = 1$, and remains constant over time.
This is a situation in which the economy has reached the Zero Lower Bound on interest rates, ZLB, but there still is excess demand for liquidity that prevents achieving full employment equilibrium and generates a situation labeled involuntary unemployment in Keynes (1936) General Theory, GT. Increasing money supply will fail to restore full employment because inequality (3) holds. Pumping more money into the economy may succeed in increasing real monetary balances, \( m \), but fails to increase the supply of liquid assets. This gives rise to a phenomenon that could easily be mistaken for Liquidity Trap in GT, although the sources lie on the supply side, not on the existence of an infinitely elastic demand for money with respect to the nominal interest rate (a demand-side phenomenon emphasized in the GT). I will call the situation depicted by inequalities (2) and (3) Supply-Side Liquidity Trap, SSLT.

As a first approximation, recent events fit well into this kind of scenario. During the Great Recession, central banks resorted to increasing central bank liquidity, QE, but results, although positive, were largely disappointing. The example above shows that this situation could hold even though prices are perfectly flexible. Moreover, Calvo (2018) shows that attempts to increase the nominal interest rate by raising the rate of growth of money supply may not succeed (as suggested in Krugman 1998), because there may exist a rational-expectations equilibrium in which individuals accumulate the entire additional amount of money flow. In contrast, if money pays interest, \( i^m \) not subject to ZLB, then lowering \( i^m \) may succeed in pushing the economy back to full employment. However, low \( i^m \) may give rise to Currency Substitution (since lowering \( i^m \) is equivalent to the imposition of an inflation tax). In extreme cases, this may simply make central bank’s monetary policy arsenal useless for alleviating Liquidity Trap.

The model can be extended to take explicit account of the labor market. Let the production function be \( f(s) \), where \( s \) stands for employment, and function \( f \) satisfies all standard properties. Let \( \bar{y} < y^F \equiv f(s^F) \), where \( s^F \) stands for perfectly inelastic labor supply (= full-employment labor), and assume

\[
\max_m [m + Z(m)] = L(0, \bar{y}). \tag{4}
\]

Moreover, let \( \bar{s} \) be such that \( f(\bar{s}) = \bar{y} \). Thus, in words, the above equations imply that the largest amount of labor that the economy can employ at ZLB, \( \bar{s} \), is less than full employment \( s^F \). If prices and wages are perfectly flexible, this will lead to chronic deflation. Downward wage flexibility will be of no help because their decline will quickly be followed by lower prices.

The model helps to rationalize GT Chapter 19 view, frequently quoted, but seldom modeled, according to which a fall in wages will not succeed in reestablishing full employment. For, in the present model, as long as equation (4) holds, aggregate demand will not be able to match full-employment output. A fall in nominal wages
may temporarily raise employment and output, but the liquidity constraint implied in (4) prevents sales from rising.

A more realistic version of the model might include price/wage adjustment frictions, but the ghost of deflation is unlikely to entirely disappear: full employment may never be reached, unless markets are undergo structural change and/or there is a healthy infusion of liquid assets.

2. Deflation Bias and Flat Phillips Curve

The above model assumes that Liquidity Deflation is instantaneous. Here I will extend the model by assuming that the negative liquidity externality takes time and, thus, money printing is capable of increasing liquidity in the short run.

I will couch these assumptions in terms of a simple, perfect-foresight standard New Keynesian model with liquidity-in-advance constraint. Let $X^j_t$ denote nominal liquidity held by individual $j$ at time $t$. Following the previous discussion, I assume that

$$X^j_t = M^j_t + Z_t,$$  \hspace{1cm} (5)

where, $M^j_t$ is the stock of money held by individual $j$, and $Z_t$ stands for the Liquidity Deflation component, which, for simplicity is homogeneously and exogenously distributed across agents. Thus, $Z$ could be thought as a lump-sum liquidity endowment that could possibly be negative. I assume that $Z_t$ is predetermined at time $t$ (for all $t$) and, therefore, unlike the previous subsection, $Z_t$ is invariant to changes in the economy-wide demand for money at time $t$.

The representative individual faces a perfect capital market and is thus free to choose the stock of money holdings, but the stock of liquidity is given by equation (5). I assume that $Z$ satisfies the following differential equation

$$Z_t = \gamma (\ln x - \ln x_t)X_t, \hspace{0.5cm} \gamma > 0, \bar{x} > 0,$$  \hspace{1cm} (6)

where $X_t$ is economy-wide liquidity, $x = X/P$, and $\bar{x}$ is an exogenous constant. As will become clear in a moment, equation (6) introduces a force that pushes economy-wide real liquidity towards $\bar{x}$. Thus, time differentiating equation (5) and aggregating across (identical) individuals, we get, in equilibrium,

$$X_t = \dot{M}_t + \gamma (\ln \bar{x} - \ln x_t)X_t.$$  \hspace{1cm} (7)

Thus, nominal liquidity is fed by money supply but there exists an independent force, associated with Liquidity Deflation, which gradually pushes real economy-
wide liquidity towards \( \bar{x} \). I will first examine cases in which full employment real liquidity would call for \( x > \bar{x} \), a phenomenon akin to Safe Asset Shortage.

I will focus on monetary policy aimed at controlling monetary aggregates (as QE). I assume that money supply satisfies:

\[
\dot{M}_t = \mu X_t, \tag{8}
\]

where \( \mu \) is a positive parameter. Notice that in the IS-LM model in which \( M = X, \mu \) would correspond to the rate of growth of money supply.

Hence, dividing both sides of equation (7) by \( X \) and taking equation (8) into account, we have

\[
\dot{x}_t = (\mu - \pi_t)x_t + \gamma (\ln \bar{x} - \ln x_t)x_t, \tag{9}
\]

where \( \pi = \dot{P}/P = \) instantaneous rate of inflation. Thus, denoting \( \dot{h} = \ln x \), equation (9) can be expressed as

\[
\dot{h}_t = \mu - \pi_t + \gamma (\ln \bar{x} - \dot{h}_t). \tag{10}
\]

This is a closed economy with homogeneous output. "Full employment" output is, again, denoted by \( y^F \). Equilibrium output is demand-determined as in New Keynesian macroeconomic models. Moreover, I will assume that consumption, \( c \), equals aggregate demand (investment and government expenditure are zero) and consumption is subject to the following liquidity-in-advance constraint:

\[
c_t = x_t, \tag{11 a}
\]

or, equivalently,

\[
\ln c_t = h_t. \tag{11 b}
\]

The price level is sticky and satisfies Calvo (1983) equation. Thus, recalling equation (11 b), I assume

\[\text{The lag may be the result of signal-extraction problems (Lucas 1972), some form of Inattention (Mankiw and Reis 2002, Sims 2003), or familiar money illusion (Shafr, Diamond and Tversky 1997). The latter is the more appropriate interpretation for the present model because I assume that all individuals lag behind current realizations in lockstep.}

\[\text{If the reader wonders why I shift to logs instead of expressing variables in their natural units, the answer is that I want the system of differential equations to be linear. In this case the equilibrium Phillips curve is linear and can easily be}\]
\[ \dot{\pi}_t = \beta (\ln y^F - h_t), \quad \beta > 0. \] (12)

Any equilibrium path must satisfy equations (10) and (12).\(^5\) Moreover, I will follow convention and assume that equilibrium paths must converge to steady state. These assumptions imply that, for all time \( t \), (log of) real liquidity \( h_t \) is predetermined. On the other hand, Calvo (1983) shows that the rate of inflation corresponding to an equilibrium path should be expected to be continuous for all \( t \). However, the "present" rate of inflation is free to jump. As is well known, the possibility that \( \pi_t \) can jump when the economy is at time \( t \), carries no non-uniqueness consequences, and uniqueness is ensured if system (10) and (12) displays saddle-path stability, which in this instance is satisfied because the determinant of the corresponding steady-state Jacobean matrix is negative.

The phase diagram for equations (10) and (12) is depicted in Figure 1, where the arrowed line pointing to the steady state is the equilibrium path. Thus, in equilibrium there exists a positive association between \( h \) and \( \pi \), i.e., recalling equation (11b), a positive association between output and inflation. Hence, the model’s reduced-form Phillips curve displays the conventional slope, despite the addition of the new Liquidity Deflation ingredients.

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5 The liquidity-in-advance assumption simplifies the analysis enormously compared to Calvo (1983), for example: There is no need to refer to utility functions! As far as I know, Reinhart (1992) is the first paper that explores this assumption. However, for the sake of completeness, Appendix A1 shows the missing equations.
At steady state where $\dot{h} = \dot{\pi} = 0$, we have, by equation (12), that $h = \ln y^F$, and, by equation (10):

$$\pi = \mu + \gamma (\ln \bar{x} - \ln y^F).$$

If $\bar{x} = y^F$, the rate of inflation equals the rate of growth of liquidity driven exclusively by the expansion of money supply (recall equation (8)). However, if $\bar{x} < y^F$, inflation at steady state $\pi < \mu$, reflecting a “deflation bias.” This is an interesting case because it brings us closer to the current deflation debate. In addition, the larger is the Liquidity Deflation parameter $\gamma$, the smaller will be the rate of inflation at steady state, and chronic deflation cannot be discounted, even if the central bank takes an aggressive QE stance (i.e., a high $\mu$ in the model)!

So much for the formal analysis; now it is a good time to reflect on the two new parameters $\bar{x}$ and $\gamma$. By equation (10), the lower is $\bar{x}$, the less effective will be money supply in increasing market liquidity. On the other hand, if $\bar{x} < y^F$, the higher is $\gamma$, the stronger is the Liquidity Deflation effect on steady-state inflation. These results can help to cast some light on recent events. For instance, several analyses of the Lehman crisis point to the fact that it involved a massive destruction of Safe Assets (e.g., Gorton 2010, Calvo 2012, Caballero et al 2016 and 2017), which in the present setup could be interpreted as resulting in a lower steady state capacity to supply liquidity services. This could be interpreted as a fall in $\bar{x}$. On the other hand, the massive destruction of liquidity may have made the market more aware of the relevance of the liquidity of collateral behind presumed Safe Assets and, therefore, become more sensitive to departures of $x$ from $\bar{x}$, giving rise to a higher $\gamma$. Thus, the Lehman crisis may have enhanced the relevance of Liquidity Deflation and its consequent deflation bias. I will show below that an increase in $\gamma$ offers a new insight for the existence of a flat Phillips curve.

A related implication of the model is that deflationary bias goes hand in hand with a decline in the velocity of circulation of money, $m$, across stable steady states—another salient fact in developed economies after the Lehman crisis. By equation (8), (11 a and b), and (12), we have that if $x$ and $\pi$ are at steady state

$$\frac{\dot{m}_t}{m_t} = \mu \frac{y^F}{m_t} - \pi.$$

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6 This implies that in the long run the economy converges to full employment. However, this could be modified introducing elements highlighted in Section II.1.

7 The Flat Phillips curve issue is attracting wide attention in macroeconomic circles. See, for instance, Borio 2017, Blanchard and Summers 2017.

8 Notice that the equilibrium paths of $x$ and $\pi$ converge to their respective steady state equilibrium values (recall Figure 1), irrespective of $m$. 

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Therefore, by equation (14), a necessary condition existence and stability of a steady state for real monetary balances, m, is $\mu > 0$, and $\pi > 0$. Consider the case in which m, x and $\pi$ are simultaneously at steady state, i.e., $\dot{m} = \dot{x} = \dot{\pi} = 0$. By equations (11 a, b), (12), (13) and (14), it follows that

$$velocity\ of\ circulation\ of\ money\ at\ steady\ state =$$

$$\frac{\pi^F}{m} = \frac{\pi}{\mu} = \frac{\mu + \gamma (\ln x - \ln y^F)}{\mu}.$$  \hspace{1cm} (15)

Therefore, across stable steady states, velocity falls as deflationary conditions get exacerbated, i.e. if $\gamma (\ln x - \ln y^F)$ goes down.

The model is also consistent with situations in which m grows without bound (implying that velocity steadily declines towards 0), which is an extreme case of Liquidity Trap that I have discussed in terms of a simpler model (see Calvo 2018). To illustrate, consider the case in which $\mu > 0$, and steady state $\pi < 0$. By equation (13), steady state price deflation arises if Liquidity Deflation is strong enough (i.e., if $\gamma (\ln x - \ln y^F)$ is negative enough to offset $\mu$). Interestingly, the resulting path does not violate transversality conditions because real monetary balances are not a "fundamental". Its role have been appropriated by real liquidity, i.e., x!

On the other hand, as in a standard New Keynesian model, the nominal interest rate on non-liquid assets, $i$, satisfies at steady state (see Appendix A1 for details):

$$i = \rho + \pi = \rho + \mu + \gamma (\ln x - \ln y^F),$$  \hspace{1cm} (16)

where $\rho$ is the subjective rate of discount, and the right-hand equality follows from equation (13). Thus, stronger deflationary conditions result in lower nominal interest rates, another phenomenon observed in the wake of the Lehman crisis.

I will now turn to examine the slope of the Phillips curve. System (10) and (12) is linear. Therefore, the equilibrium saddle path can easily be computed. Let B denote the slope of the reduced-form Phillips curve (i.e., $B = \frac{d\pi}{dn}$ along the arrowed equilibrium path in Figure 1). It can be shown (see Appendix A2) that

$$B = -\gamma + \sqrt{\gamma^2 + 4B^2}.$$  \hspace{1cm} (17)

Hence, by equation (17), proof that the Phillips curve becomes flatter as the Liquidity Deflation parameter increases can be inferred from the following expressions:

$$\frac{dB}{d\gamma} < 0 \ and \ \lim_{\gamma \to \infty} B = 0.$$  \hspace{1cm} (18)
Moreover, by Appendix A2,
\[
\dot{h}_t = -(B + \gamma)(h_t - \ln y^F) = -\frac{\gamma + \sqrt{\gamma^2 + 4\beta}}{2}(h_t - \ln y^F).
\]  
(19)

Hence, the speed at which output converges to full employment is invariant to the liquidity drag \(\bar{x}\), and accelerates as the drag becomes stronger, i.e., \(\gamma\) goes up. Therefore, Liquidity Deflation fails to slow down convergence to full employment, and may actually accelerate it! This sounds implausible but these results are tied to the assumption that full employment output, \(y^F\), is invariant to liquidity frictions. Relaxing this assumption, however, need not eliminate Deflation Bias and Flat Phillips curves, as would be illustrated by the case in which \(y^F\) suffers a once-and-for-all negative shock.

As it stands, the model implies that a stock change in money supply could place the economy smack into full employment. This contradicts the view that liquidity of money supply declines with its stock in real terms. However, this can easily be remedied by modifying equation (7) and, for instance, replacing \(\dot{M}\) by a concave function of \(\dot{M}\) subject to an upper bound.

### III. FINAL WORDS

The paper explores financial frictions that give rise to chronic deflation, a flat Phillips curve, and situations that are observationally equivalent to a Keynesian Liquidity Trap. Moreover, this holds even if the interest elasticity of the demand for money is bounded. The approach is relevant for situations in which, say, the economy suffered a massive destruction of liquidity that interferes with trade flows, and restoration of liquidity to "normal" levels cannot be rapidly achieved by pumping central bank liquidity, e.g., high-powered money. The reason for this, the paper attributes to the fact that the liquidity quality of liquid assets may deteriorate as the real value of those assets becomes large. This effect is a result of a negative externality rooted in the fact that by its very nature liquidity is opened to self-fulfilling runs à la Diamond-Dybvig (1983), especially if the supply of liquid assets increases without being accompanied by an equivalent rise of real collateral. An instance of this sort is the large increase in high-powered money in the wake of the Lehman crisis. Thus, the economy may, at least in the short run, end up in a situation that several authors have labeled Safe Asset Shortage (e.g., Caballero et al 2016, 2017, Gorton 2016, Barro et al 2017) in which, beyond a certain point, liquidity cannot be increased by QE. It is a Supply-Side Liquidity Trap that can even hold under perfectly flexible prices and wages, and may not be alleviated by expansionary fiscal or monetary policy unless it results in an increase in market liquidity.

In closing, it is worth pointing out that the effects highlighted in this paper hold when liquidity is scarce relative to "normal" situations. This is important because, for instance, the slope of the Phillips curve could steepen sharply as there is greater
confidence in the stability of the financial sector. Unfortunately, return to normality is surrounded by a large degree of uncertainty, and forecasts are likely to be a poor guide for policy. But central banks could still play a valuable role if they succeed in better understanding and managing new liquid assets that, as shown by recent events, could cause inefficient frictions or massive liquidity crunch.
APPENDIX A1

Here I will sketch out the microfoundations of the model considered in Section II.2. The model is close to the one in Calvo (1983), except that money is removed from the utility function and replaced for a cash-in-advance condition, and the existence of Liquidity Deflation.

The representative consumer’s utility function from the perspective of time 0 satisfies:

\[ \int_0^\infty u(c_t^j) e^{-\rho t} dt, \]  

where, again, \( j \) refers to individual \( j \), function \( u \) is defined on the positive real line; it is increasing, strictly concave, and twice-continuously differentiable everywhere. Parameter \( \rho > 0 \) stands for the subjective rate of discount.

The budget constraint satisfies (assuming 0 initial wealth),

\[ \int_0^\infty [y_t^j + s_t - c_t^j - i_t m_t^j] e^{-\int_0^t r_v dv} dt \geq 0, \]  

where \( s \) stands for lump-sum subsidy employed to rebate central bank seigniorage to the public, a standard assumption in monetary theory; \( r \) is the instantaneous real rate of interest, and the nominal interest rate \( i = r + \pi \).

Equations (5) and (11) imply that

\[ c_t^j = x_t^j = m_t^j + z_t, \]  

implying that

\[ m_t^j = c_t^j - z_t. \]  

Inserting equation (A1.4) in budget constraint (A1.2) yields

\[ \int_0^\infty [y_t^j + s_t - c_t^j - i_t (c_t^j - z_t)] e^{-\int_0^t r_v dv} dt \geq 0. \]  

Variable \( z \) is exogenous to the representative individual. Therefore, the first-order condition for consumption, taking into account (A1.1) and (A1.5), takes the following familiar form:

\[ \text{Variables defined in the main text will not be defined here, unless necessary for the sake of clarity.} \]
\[ u'(c^l_t)e^{-\rho t} = \lambda(1 + i_t)e^{-\int_0^t r_v dv}, \quad \text{(A1.6)} \]

where \( \lambda \) is the time-invariant Lagrange multiplier. The model can now be complemented by the \( z \) dynamics discussed in the text.

**APPENDIX A2**

Consider equations (10) and (12). The system is linear and, hence, there are parameters denoted \( D \) and \( B, B > 0 \), such that along the saddle path depicted in Figure 1, we have

\[ \pi = D + B h, \text{ for all } h. \quad \text{(A2.1)} \]

Dropping time subscripts, equations (10), (12) and (A2.1) imply:

\[ \dot{\pi} = B \dot{h} = B[\mu - D - B h + \gamma (\ln \bar{x} - h) = \beta (\ln y^F - h). \quad \text{(A2.2)} \]

The rightmost equation must hold for all \( h \), which implies:

\[ B^2 + B \gamma - \beta = 0. \quad \text{(A2.3)} \]

Solving for \( B \) and recalling that, by Figure 1, \( B > 0 \), yields equation (17) in the text.

To solve for parameter \( D \) in (A2.2), set \( h = \ln y^F \). It follows that

\[ D = \mu - B \ln y^F + \gamma (\ln \bar{x} - \ln y^F). \quad \text{(A2.4)} \]

By equations (A2.2) and (A2.4), one can derive equation (19) in the text.
REFERENCES


