POLS W4292 Spring 2008

`lmer` examples

In all of the following examples, `group` is a group identifier and `data` is the data object.

- To estimate

$$y_{it} = \alpha_i + \delta x_{1it} + \beta x_{2it} + \varepsilon_{it} \tag{1}$$

where

$$\alpha_i \sim N(0, \sigma_\alpha) \tag{2}$$

do

```
lmer(y~x1+x2+(1|group), data)
```

- To estimate

$$y_{it} = \alpha_i + \delta x_{1it} + \beta_i x_{2it} + \varepsilon_{it} \tag{3}$$

where

$$\alpha_i \sim N(0, \sigma_\alpha) \tag{4}$$

$$\beta_i \sim N(0, \sigma_\beta) \tag{5}$$

do

```
lmer(y~x1+x2+(1|group)+(0+x2|group), data)
```

We do `(1|group)` and `(0+x2|group)` to make sure α_i and β_i are treated as independent normals. If we did something like

```
lmer(y~x1+x2+(1+x2|group), data)
```

then α_i and β_i are considered to be drawn from a bivariate normal, allowing for possible covariance between the parameters (thus, you are estimating an additional parameter).

- To estimate

$$y_{it} = \alpha_i + \delta x_{1it} + \beta_i x_{2it} + \varepsilon_{it} \tag{6}$$

where

$$\alpha_i \sim N(0, \sigma_\alpha) \tag{7}$$

$$\beta_i = \gamma z_i + \eta_i \Rightarrow \beta_i \sim N(\gamma z_i, \sigma_\beta) \Rightarrow \eta_i \sim N(0, \sigma_\beta) \tag{8}$$

you would first do something like

```
z.full <- z[index]
```

where `index` is a variable that indexes observations. This "blows up" the `z` variable in such a way so that there are now NT observations on it (instead of only N). This is similar to using the Kronecker product like we did to create vectors of 1s and 0s for LSDV. Then you could do

```
lmer(y~x1+x2+(1|group)+(0+x2|group)+z.full+x2:z.full, data)
```

- To estimate more than one variable with a random coefficient, such as

$$y_{it} = \alpha_i + \delta x_{1it} + \beta_{2i} x_{2it} + \beta_{3i} x_{3it} + \varepsilon_{it} \tag{9}$$

where

$$\alpha_i \sim N(0, \sigma_\alpha) \tag{10}$$
$$\beta_{2i} = \gamma_2 z_i + \eta_{2i} \tag{11}$$
$$\beta_{3i} = \gamma_3 z_i + \eta_{3i} \tag{12}$$

do something like

```
lmer(y~x1+x2+(1|group)+(0+x2|group)+(0+x3|group)+x2:z.full+x3:z.full, data)
```

- Note that the above models are a bit odd, because they do not include main effects for z_i. Consider the following model from the notes

$$y_{ij} = \beta_{0j} + \beta_{1j} x_{ij} + \varepsilon_{ij} \tag{13}$$
$$\beta_{0j} = \gamma_{00} + \gamma_{01} z_j + u_{0j} \tag{14}$$
$$\beta_{1j} = \gamma_{10} + \gamma_{11} z_j + u_{1j}, \tag{15}$$

which would give the reduced form

$$y_{ij} = \gamma_{00} + \gamma_{01} z_j + \gamma_{10} x_{ij} + \gamma_{11} z_j \cdot x_{ij} + u_{0j} + u_{1j} \cdot x_{ij} + \varepsilon_{ij}. \tag{16}$$

To estimate this model, we would do something like

```
lmer(y ~ x + z.full + x:z.full + (1 + x | group), data)
```

- Suppose we have a discrete variable z that is ordinal and we want to estimate different random effects for each value of z that we observe. The model would look something like

$$y_{itj} = \alpha_i + \delta x_{1it} + \beta_j x_{2j(it)} + \varepsilon_{it} \tag{17}$$

where

$$\alpha_i \sim N(0, \sigma_\alpha) \tag{18}$$
$$\beta_j = \gamma z_{j(it)} + \eta_{j(it)}. \tag{19}$$

The subscript $j(it)$ maps country observations to the different groups identified by the discrete values of z.

To estimate, we could do

```
lmer(y~x1+x2+(1|group)+(0+x2|z)+x2:z, data)
```

Note that `lmer` coerces the variable `z` to be a grouping factor.