Monte Carlo Analysis

- Monte Carlo (MC) analysis offers a way to understand/assess the performance of estimators.
- We are often interested in comparing the performance of different estimators in various research situations. Theory does not always give us a clear sense of what estimator will work the best.
- Large sample v. small sample properties.
- Draws on frequentist notion of repeated samples.
- Experiments: we control the conditions under which the data are generated and evaluated.
- Basic steps for MC analysis:
  1. Define the DGP (specification, functional form, distribution, parameter values).
     - Tip: keep it simple (always have external validity issues, no matter how complex you make the DGP).
  2. Simulate the data: use random number generator to draw values for RHS quantities $(\mathbf{X}, \varepsilon)$ and compute LHS values $(\mathbf{y})$.
  3. Compute the estimator(s) of interest and save the results (coefficients, standard errors).
  4. Repeat these steps numerous times ($\sim$ 500–1000).
  5. Examine the distribution of the estimates produced to assess performance/compare estimators (e.g., are results consistent w/ theory, asymptotic or otherwise?).

- Criteria for evaluating performance:
  - Bias: average performance of point estimates across simulations (e.g., $\bar{\hat{\beta}} = S^{-1} \sum_{s=1}^{S} \hat{\beta}_s$; $S^{-1} \sum_{s=1}^{S} |\beta - \hat{\beta}_s|$; $\bar{\hat{\beta}} - \beta$).
  - Variance of sampling distribution (e.g., $\sum_{s=1}^{S} (\hat{\beta}_s - \bar{\hat{\beta}})^2/(S-1)$).
  - Mean squared error (e.g., $\sum_{s=1}^{S} (\hat{\beta}_s - \beta)^2/S$).
  - Coverage/confidence interval length (e.g., proportion of times $\hat{\beta}_s$ is located within 95% CI).
  - "Optimism": assessment of performance of standard errors; E.g.,

$$100 \times \frac{\sum_{s=1}^{S} \left(\hat{\beta}_s - \bar{\hat{\beta}}\right)^2}{\sum_{s=1}^{S} \left[\mathrm{SE}\left(\hat{\beta}_s\right)\right]^2},$$

    where "SE" refers to the estimated standard errors.
    * Values above 100 indicate that true sampling variability is greater than the reported estimate of that variability, while values less than 100 indicate that the estimate overstates true variability.
    * Understating variability means that we might reject the null of a zero coefficient when the null is true (i.e., commit a Type I error).
    * Overstating variability implies that we might not reject the null of a zero coefficient when that null is false, leading us to conclude that a variable does not have effects when in fact it does (i.e., increases the risk of Type II errors).