Homework Assignment #1

Show all work. For those questions that require you to write R code, hand in the programs that you wrote. This assignment is due in class on Feb. 5th.

1. Fun with variance and covariance:
   (a) Show why $\text{var}(X - Y) = \text{var}(X) + \text{var}(Y)$, if $X$ and $Y$ are independent. (20 points)
   (b) Show why $\text{cov}(X, Y) = E[XY] - \mu_X \mu_Y$. (10 points)
   (c) Show why $\text{cov}(a + bX, c + dY) = bd \cdot \text{cov}(X, Y)$, where $a$, $b$, $c$, and $d$ are constants. (10 points)
   (d) Using the equation for the population correlation coefficient on p. 13 of the notes, show that $\rho = -1$ if $Y = -X$. (20 points)

2. Fun with linear equations and R:
   (a) Write R code to solve the following systems of linear equations (10 points each):
      i. \[
      \begin{align*}
      4x - 2y &= 5 \\
      -6x + 3y &= 1
      \end{align*}
      \]
      ii. \[
      \begin{align*}
      2x - 2y - z &= 5 \\
      x + y - 2z &= 1 \\
      x - z &= 4
      \end{align*}
      \]
      iii. \[
      \begin{align*}
      x + 2y + 2z &= 2 \\
      3x - 2y - z &= 5 \\
      2x - 5y + 3z &= -4 \\
      x + 4y + 6z &= 0
      \end{align*}
      \]
   (b) A person tells you that the following system of equations cannot be solved because if you square the elements of the first row of $A$ in the matrix representation, you obtain the elements in the second row. Explain why this person is not correct and then show this is the case using R. (10 points)
      \[
      \begin{align*}
      5u - 4v &= 3 \\
      25u - 16v &= 27
      \end{align*}
      \]