

Homework Assignment #1

Show all work. For those questions that require you to write R code, hand in the programs that you wrote. This assignment is due in class on Feb. 5th.

1. Fun with variance and covariance:

- (a) Show why $\text{var}(X - Y) = \text{var}(X) + \text{var}(Y)$, if X and Y are independent. (20 points)
- (b) Show why $\text{cov}(X, Y) = E[XY] - \mu_X \mu_Y$. (10 points)
- (c) Show why $\text{cov}(a + bX, c + dY) = bd \cdot \text{cov}(X, Y)$, where a, b, c , and d are constants. (10 points)
- (d) Using the equation for the population correlation coefficient on p. 13 of the notes, show that $\rho = -1$ if $Y = -X$. (20 points)

2. Fun with linear equations and R:

- (a) Write R code to solve the following systems of linear equations (10 points each):

i.

$$\begin{aligned} 4x - 2y &= 5 \\ -6x + 3y &= 1 \end{aligned}$$

ii.

$$\begin{aligned} 2x - 2y - z &= 5 \\ x + y - 2z &= 1 \\ x - z &= 4 \end{aligned}$$

iii.

$$\begin{aligned} x + 2y + 2z &= 2 \\ 3x - 2y - z &= 5 \\ 2x - 5y + 3z &= -4 \\ x + 4y + 6z &= 0 \end{aligned}$$

- (b) A person tells you that the following system of equations cannot be solved because if you square the elements of the first row of \mathbf{A} in the matrix representation, you obtain the elements in the second row. Explain why this person is not correct and then show this is the case using R. (10 points)

$$\begin{aligned} 5u - 4v &= 3 \\ 25u - 16v &= 27 \end{aligned}$$