

Homework Assignment #4

Show all work. For those questions that require you to write R code, hand in the code that you wrote. This assignment is due on 4/3.

1. Suppose $x_1, \dots, x_i, \dots, x_n$ is a sample from a population which follows a Bernoulli distribution:

$$f(x_i) = p^{x_i} q^{1-x_i}, \quad 0 \leq p \leq 1, \quad q = 1 - p$$

The population mean and variance for the Bernoulli distribution are p and pq , respectively.

- (a) Find the maximum likelihood estimator of the population mean. (25 points)
 - (b) What is the variance of the maximum likelihood estimator of the population mean? (10 points)
 - (c) State how you would find the maximum likelihood estimator of the population variance. You do not actually have to work through the math on this. (10 points)
2. Working with the R code contained in `ml_ex.r`, do the following (use plots where they will be helpful):
- (a) Show how to compute the likelihood (not the log-likelihood) value for the sample drawn using `rnorm`. (10 points)
 - (b) Change the mean of the distribution that you are drawing the sample from. What happens? Increase the variance of the distribution. What happens? (10 points)
 - (c) Set the mean value for `rnorm` back to zero, the variance back to 1, and redraw the sample. Conduct a likelihood ratio test for $\mu = .05$. Conduct a likelihood ratio test for $\mu = 4$. Are the results of these tests affected if you substantially increase the variance of the distribution that you are drawing from? Explain why they are or are not affected. (25 points)
 - (d) Increase the sample size by a factor of 10. How does this affect the maximum likelihood estimates? Is this what you expected? Why or why not? (10 points)