1. The lognormal distribution is often used to model non-negative random variables such as lifetimes. It is also very frequently used to model random returns in finance. A random variable $X$ is said to have the lognormal distribution, with parameters $\nu$ and $\tau$, if $\ln(X)$ has the normal distribution with mean $\nu$ and standard deviation $\tau \geq 0$. Equivalently, $X = \exp(Y)$ where $Y$ is normally distributed with mean $\nu$ and standard deviation $\tau$. It is well known that 

$$E(X^n) = \exp(n\nu + n^2\tau^2/2).$$ 

Thus, 

$$\mu = \exp(\nu + \tau^2/2)$$ 

and 

$$\sigma^2 = \mu^2(\exp(\tau^2) - 1).$$ 

Suppose that $X$ is lognormal with mean $\mu$ and variance $\sigma^2$. Show that $X$ is lognormal with parameters 

$$\nu = \ln \mu - \ln \sqrt{1 + cv^2}$$ 

and 

$$\tau = \sqrt{\ln(1 + cv^2)},$$ 

where $cv = \sigma/\mu$ is the coefficient of variation of $X$.

2. Show that $D$ is lognormal with mean $\mu$ and variance $\sigma^2$ then for all $x > 0$, 

$$Pr(D \leq x) = \Phi \left( \frac{\ln(\frac{x}{\mu}) + \ln(\sqrt{1 + cv^2})}{\sqrt{\ln(1 + cv^2)}} \right).$$ 

Use this to show that 

$$Pr(D \leq x) \to 1$$ 

as $\sigma^2 \to \infty$.

3. Notice that while the normal model prescribes a base stock level 

$$S = \mu + z\beta \sigma$$ 

to achieve a service level 100$\beta$%, where $z\beta = \Phi^{-1}(\beta)$, the lognormal model would prescribe a base stock level 

$$S = \mu \exp \left( \sqrt{\ln(1 + cv^2)} z\beta - \ln(\sqrt{1 + cv^2}) \right).$$ 

Argue that in the lognormal case $S \to 0$ as $\sigma^2 \to \infty$.

4. Compute and then contrast the ratios $S/\mu$ for the normal and for the lognormal for different values of $\sigma$, keeping $\mu$ fixed. Try this for $cv \in \{1, 2, 3, 4, 5, 10, 15, 20, 25, 50\}$ for $\beta = 0.90$ and for $\beta = 0.95$. 

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