1. The lognormal distribution is often used to model non-negative random variables such as lifetimes. It is also very frequently used by to model random returns in finance. A random variable X is said to have the lognormal distribution, with parameters ν and τ, if $\ln(X)$ has the normal distribution with mean ν and standard deviation $\tau \geq 0$. Equivalently, $X = \exp(Y)$ where Y is normally distributed with mean ν and standard deviation τ. It is well known that

$$E(X^n) = \exp(n\nu + n^2\tau^2/2).$$

Thus,

$$\mu = \exp(\nu + \tau^2/2)$$

and

$$\sigma^2 = \mu^2(\exp(\tau^2) - 1).$$

Suppose that X is lognormal with mean μ and variance σ^2. Show that X is lognormal with parameters

$$\nu = \ln \mu - \ln \sqrt{1 + cv^2}$$

and

$$\tau = \sqrt{\ln(1 + cv^2)},$$

where $cv = \sigma/\mu$ is the coefficient of variation of X.

2. Show that D is lognormal with mean μ and variance σ^2 then for all $x > 0$,

$$Pr(D \leq x) = \Phi\left(\frac{\ln(\frac{x}{\mu}) + \ln(\sqrt{1 + cv^2})}{\sqrt{\ln(1 + cv^2)}}\right).$$

Use this to show that

$$Pr(D \leq x) \rightarrow 1$$

as $\sigma^2 \rightarrow \infty$.

3. Notice that while the normal model prescribes a base stock level

$$S = \mu + z_\beta \sigma$$

to achieve a service level $100\beta\%$, where $z_\beta = \Phi^{-1}(\beta)$, the lognormal model would prescribe a base stock level

$$S = \mu \exp\left(\sqrt{\ln(1 + cv^2)} z_\beta - \ln(\sqrt{1 + cv^2})\right).$$

Argue that in the lognormal case $S \rightarrow 0$ as $\sigma^2 \rightarrow \infty$.

4. Compute and then contrast the ratios S/μ for the normal and for the lognormal for different values of σ, keeping μ fixed. Try this for $cv \in \{1, 2, 3, 4, 5, 10, 15, 20, 25, 50\}$ for $\beta = 0.90$ and for $\beta = 0.95$.