IEOR 4000: Production Management

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Solutions to HW 1, Fall 2004, Prepared by Lin Li

1 **(Problem** 5.1**)** A is changed from 3.20 to 12.80, so we have,

a) 1)

$$EOQ\text{in units} = \sqrt{\frac{2 \times 12.8 \times 2400}{0.4 \times 0.24}} = 800 \quad \text{units}$$

2)

$$EOQ\text{in dollars} = 800 \times 0.4 = \$320$$

3)

$$T_{EOQ} = \frac{12 \times 800}{2400} = 4 \quad \text{month}$$

And the EOQ as a month supply is $\frac{800}{4} = 200$ units

Yes, the effect of the change in A makes sense. As the setup cost increases we expect the EOQ to go up.

b) r is increased from $0.24/yr$ to $0.30/yr$, similarly, we have,

$$EOQ\text{in units} = \sqrt{\frac{2 \times 3.2 \times 2400}{0.4 \times 0.3}} \approx 358 \quad \text{units}$$

$$EOQ\text{in dollars} = 358 \times 0.4 = \$143.11$$

$$T_{EOQ} = \frac{12 \times 358}{2400} = 1.79 \quad \text{month}$$

And the EOQ as a month supply is $\frac{358}{1.79} = 200$ units

Yes, the effect of the change in r makes sense. As the carrying charge becomes more expensive, it is reasonable to order less every month.

2 **(Problem** 5.3**)** If the firm loads and packages its own, it would be in the situation of having a finite replenishment rate. Let production cost be $K_1 = 20$, inventory carrying charge is$I = 0.24$, value of the product is $c_1 = 1.23$ per roll, demand rate is $\lambda = 10000/yr$, production rate is $\mu = 500 \times 365 = 182500/yr$ and $\rho = \frac{\lambda}{\mu}$. The total annual cost would be the variable cost of $12,300$ plus the average holding and setup cost which is given by

$$C_1 = \sqrt{2 \times K_1 \times I \times c_1 \lambda}\sqrt{1-\rho} = \sqrt{2 \times 20 \times 0.24 \times 1.23 \times 10000}\sqrt{1 - \frac{10000}{182500}} = 334.08$$

So, the total cost is $C_1^{sum} = C_1 + 12,300 = \$12634.08$

The alternative would be to buy prepacked rolls. The fixed ordering charge is $K_2 = 3$ per order and value of the product is $c_1 = 1.26$ per roll. This will have the variable cost of $12,600$ and the holding and ordering cost of

$$C_2 = \sqrt{2 \times K_2 \times I \times c_2 \lambda} = \sqrt{2 \times 3 \times 0.24 \times 1.26 \times 10000} = 134.7$$

And the total cost is $C_2^{sum} = C_2 + 12,600 = \$12734.7$

Since $C_2^{sum} > C_1^{sum}$, the company would choose to load and package their own brand of film.

3 **(Problem** 5.9**)**

a) Use the algorithm provided in Section 5.5, we have,

Step 1 $EOQ(discount) = \sqrt{\frac{2\times25\times40\times52}{0.26\times9.7}} = 203$ units

Step 2 Since $EOQ(disount) = 203.07 < 300 = Q_b$, we go to step 3

Step 3 Compute the value of $TRC(EOQ)$ and $TRC(Q_b)$,

$$TRC(EOQ) = \sqrt{2 \times 25 \times 40 \times 52 \times 10 \times 0.26} + 40 \times 52 \times 10 = \$21320$$

$$TRC(Q_b) = \frac{25 \times 40 \times 52}{300} + 40 \times 52 \times 9.7 + \frac{300 \times 9.7 \times 0.26}{2} = \$20727.63$$

Note that $TRC(EOQ) > TRC(Q_b)$, we should use $Q = Q_b = 300$ units

b) Let v_1 be the unknown maximum unit cost.

$$TRC(500) = \frac{500 \times v_1 \times 0.26}{2} + \frac{25 \times 40 \times 52}{500} + 40 \times 52 \times v_1 = 2145v_1 + 104$$

The mining company will be indifferent when $TRC(500) = TRC(300) = 20727.63$, That is $2145v_1 + 104 = 20727.63$, so we get $v_1 = \$9.61$

4 **(Problem** 5.19**)** The most appropriate strategy is to both reduce the set up cost A and use the EOQ. The reason is :

Case 1: Under the current A value,

$$EOQ^1 = \sqrt{\frac{2 \times 100 \times 5000}{0.28 \times 2}} = 1336 \quad units$$

$$TRC(EOQ^1) = \sqrt{2 \times 100 \times 5000 \times 0.28 \times 2} = \$748.33/yr$$

$$TRC(500) = \frac{500 \times 2 \times 0.28}{2} + \frac{100 \times 5000}{500} = \$1140/yr$$

Case 2 When A is changed to \$25,

$$EOQ^2 = \sqrt{\frac{2 \times 25 \times 5000}{0.28 \times 2}} = 668 \quad units$$

$$TRC(EOQ^2) = \sqrt{2 \times 25 \times 5000 \times 0.28 \times 2} = \$374.17/yr$$

$$TRC(500) = \frac{500 \times 2 \times 0.28}{2} + \frac{25 \times 5000}{500} = \$390/yr$$

5 a) Let (85 46 30 33 32 28 24 24) be the vector of number of orders per year. This choice results in approximately 1461.5 setup hours and an average cycle stock \$69,958.12. Notice that this represents about a 21.9% reduction in N, the number of setup hours.

b) Let (111 59 39 43 42 37 32 29) be the vector of number of orders per year. This choice results in approximately 1900 setup hours and an average cycle stock \$53,813.46. Notice that this represents about a 22.2% reduction in the ACS, the average cycle stock.

c) EOQ formula for each item is:

$$EOQ_i = \sqrt{\frac{2A_i\lambda_i}{h_i}}$$

where

- $A_i = 15s_i$, $h_i = 0.2c_i$,
- number of orders =(Demand for item i)/EOQ_i
- total setup hours of item i per year =(number of orders) × (setup hours per order)
- average capital tied up in inventory = EOQ_i×(unit cost of item i)/2

d) The exchange curve is

$$ACS = \frac{(\sum_{i=1}^{8} \sqrt{c_i \lambda_i s_i})^2}{2N} = \frac{102,215,683}{N}$$

Take point ($N = 1850, ACS = 55251.72$) on the curve. Then

$$\frac{ACS}{N} = \frac{a}{I} = 29.87$$

Use the formula for economic order interval, we have

$$T_i^* = \sqrt{\frac{2s_i}{c_i \lambda_i} \frac{a}{I}}$$

So the vector of order intervals is (3.39 6.31 9.61 8.64 8.92 10.18 11.83 12.88) days.