IEOR 4000: Production Management

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Solutions to HW 2, Fall 2004, Prepared by Lin Li

1 **(Problem 1)**

a)

$$e(\frac{a}{T_1^*}) = e(\frac{a}{T_n^*}) \quad \Rightarrow \quad \frac{a}{T_1^*} + \frac{T_1^*}{a} = \frac{a}{T_n^*} + \frac{T_n^*}{a}$$

$$\Rightarrow \quad (T_n^* - T_1^*)a^2 = T_n^* T_1^* (T_n^* - T_1^*) \quad \Rightarrow \quad a^2 = T_n^* T_1^* \quad (\text{since} T_n^* > T_1^*)$$

Thus, $a = \sqrt{T_1^* T_n^*}$.

$$T_1^* + T_n^* - 2\sqrt{T_1^* T_n^*} = (\sqrt{T_1^*} - \sqrt{T_n^*})^2 \geq 0 \quad \Rightarrow \quad \sqrt{T_1^* T_n^*} \leq \frac{T_1^* + T_n^*}{2}$$

So, a is below the average of T_1^* and T_n^*.

$$e(\frac{a}{T_1^*}) = \frac{1}{2}(\frac{\sqrt{T_n^*}}{\sqrt{T_1^*}} + \frac{\sqrt{T_1^*}}{\sqrt{T_n^*}})$$

b) **Claim:** Given the information of T_1^* and T_n^*, the best way to split the items is by $\sqrt{T_1^* T_n^*}$.
Proof: Suppose we will split the items at point x, then the optimal reorder interval for groups $\{T_1^*, T_2^*, ...x\}$ and $\{x, ..., T_n^*\}$ are respectively $a = \sqrt{T_1^* x}$ and $b = \sqrt{T_n^* x}$. To minimize the maximum cost, let $e(a/T_1^*) = e(b/T_n^*)$. Thus,

$$\frac{a}{T_1^*} + \frac{T_1^*}{a} = \frac{b}{T_n^*} + \frac{T_n^*}{b} \quad \Rightarrow \quad \sqrt{\frac{x}{T_1^*}} + \sqrt{\frac{T_1^*}{x}} = \sqrt{\frac{x}{T_n^*}} + \sqrt{\frac{T_n^*}{x}}$$

And, we get $x = \sqrt{T_1^* T_n^*}$. *Q.E.D*
Follow the result in part a), the optimal reorder intervals are
$a = \sqrt{T_1^* \sqrt{T_1^* T_n^*}} = (T_1^*)^{3/4}(T_n^*)^{1/4}$, and $b = \sqrt{T_n^* \sqrt{T_1^* T_n^*}} = (T_n^*)^{3/4}(T_1^*)^{1/4}$

2 **(Problem 2)** Use the ELSP calculator,

a) The lower bound on the long run average ordering and holding cost is \$4,895.33
b) The optimal rotation order interval is: 0.425yr
c) The cost penalty of RS to the lower bound is 10.48%