IEOR 4000: Production Management

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Solutions to HW 3, Fall 2004, Prepared by Lin Li

1 **(Problem 1)** This is the production planning with time-varying demand and convex costs. Use the time-vary-dem excel, we get the taut-string solutions as follows;

a) For the case that $I_0 = 0$, we get production plan: $X = (0, 2, 6, 8, 12)$ with total cost of 16.38.

b) When $I_0 = 1$, we have a production plan: $X = (0, 1, 5, 7, 11)$ with total cost of 15.56.

c) If we require that $I_4 = 2$, then the production plan will be $X = (0, 2, 6.8, 9.2, 14)$ with total cost of 17.86.

2 **(Problem 2)** This is a capacitated production planning problem with concave costs. $\sum_{i=1}^{7} r_i = 10$, and $c = 2$. So we have $k = 10/2 = 5$ and $q = 0$. The candidate for the optimal production plan is $x = (2, 2, 2, 2, 2, 0, 0)$. It is easy to check that this plan satisfies all the constraints. Thus, it is optimal with total cost of $c(x) = 60$.

3) Wagner-Whitin Problem.

a) First, construct the cost matrix C.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1		25	33	77	149	197	227	251	293	421	637	897	985
2			25	47	95	131	155	175	211	323	515	749	829
3				25	49	73	91	107	137	233	401	609	681
4					25	37	49	61	85	165	309	491	555
5						25	31	39	57	121	241	397	453
6							25	29	41	89	185	315	363
7								25	31	63	135	239	297
8									25	41	89	167	199
9										25	49	101	125
10											25	51	67
11												25	33
12													25

Second, use Dynamic Programming, let $f_{13} = 0$ and work backwards. We get

f_13	f_12	f_11	f_10	f_9	f_8	f_7	f_6	f_5	f_4	f_3	f_2	f_1
0	25	33	58	82	99	113	123	138	160	185	207	218

An optimal production plan can be

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
60	0	110	180	0	80	0	0	200	0	170	0

with total cost of \$218

b) For $K = 50$, apply the similar method as in part a). We get
The optimal production plan is

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
170	0	0	260	0	0	0	0	200	0	170	0

with total cost of \$344

c) For $K = 10$, we get
The optimal production plan is

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
60	0	110	120	90	0	50	0	80	120	170	0

with total cost of \$108