In this problem, the parameters are, unit cost $c = 150$, selling price $p = 210$, salvage value $s = 150$. Demand is uniform distributed between 75 and 125, $D \sim U(75,125)$. $Q$ is the order quantity.

(a) The total profit is

$$\pi(Q) = E[p \cdot \min(Q,D) + (1 - 0.15)s \cdot (Q - D)^+] - cQ$$

The optimal order quantity $Q^*$ satisfies

$$Q^* = \min\{Q : Pr(D < Q) \geq \beta = \frac{c-p}{0.85s-p} = \frac{150-210}{0.85 \times 150 - 210} \approx 0.73\}$$

which implies that $Q^* = 0.73$ (since $D$ follows uniform distribution). A straightforward calculation gives

$$Q^*_1 = 50 \times 0.73 + 75 = 111.5 \approx 112$$

(b) If the demand is Normally distributed, then $\Phi(Q) = Pr(D < Q) = \beta = 0.73$. Check the table of standard normal distribution, we get $z_\beta \approx 0.61$. So the optimal order quantity is

$$Q^*_2 = \mu + \sigma z_\beta = 110 + 15 \times 0.61 = 119$$

The total profit with order quantity $Q^*_2 = 119$ is

$$\pi(Q^*_2) = (p-c)\mu_1 - G(Q^*_2)$$

$$= (p-c)\mu - (p - 0.85s)\sigma\phi(z_\beta)$$

$$= (210 - 150) \times 110 - (210 - 0.85 \times 150) \times 15 \times \phi(0.6)$$

$$= $6187.63$$

We know that the distribution is normal. If we use the order quantity $Q^*_1 = 112$, then $z_\beta = (112 - 110)/15 = 0.133$ and $\beta = \Phi(0.133) = 0.553$. Let $h = c - 0.85s$, $b = p - c$. So the total profit is:

$$\pi(Q^*_1) = (p-c)\mu_1 - G(Q^*_1)$$

$$= (p-c)\mu - [h\sigma z_\beta + (h + b)\sigma(\phi(z_\beta - (1 - \beta)z_\beta))]$$

$$= (210 - 150) \times 110 - 460.65$$

$$= $6139.35$$

This inside information worth $\pi(Q^*_2) - \pi(Q^*_1) = 6187.63 - 6139.35 = $48.28$

(b) If the demand is Normally distributed, then $\Phi(Q) = Pr(D < Q) = \beta = 0.73$. Check the table of standard normal distribution, we get $z_\beta \approx 0.61$. So the optimal order quantity is

$$Q^*_2 = \mu + \sigma z_\beta = 110 + 15 \times 0.61 = 119$$

The total profit with order quantity $Q^*_2 = 119$ is

$$\pi(Q^*_2) = (p-c)\mu_1 - G(Q^*_2)$$

$$= (p-c)\mu - (p - 0.85s)\sigma\phi(z_\beta)$$

$$= (210 - 150) \times 110 - (210 - 0.85 \times 150) \times 15 \times \phi(0.6)$$

$$= $6187.63$$

We know that the distribution is normal. If we use the order quantity $Q^*_1 = 112$, then $z_\beta = (112 - 110)/15 = 0.133$ and $\beta = \Phi(0.133) = 0.553$. Let $h = c - 0.85s$, $b = p - c$. So the total profit is:

$$\pi(Q^*_1) = (p-c)\mu_1 - G(Q^*_1)$$

$$= (p-c)\mu - [h\sigma z_\beta + (h + b)\sigma(\phi(z_\beta - (1 - \beta)z_\beta))]$$

$$= (210 - 150) \times 110 - 460.65$$

$$= $6139.35$$

This inside information worth $\pi(Q^*_2) - \pi(Q^*_1) = 6187.63 - 6139.35 = $48.28$

(b) If the demand is Normally distributed, then $\Phi(Q) = Pr(D < Q) = \beta = 0.73$. Check the table of standard normal distribution, we get $z_\beta \approx 0.61$. So the optimal order quantity is

$$Q^*_2 = \mu + \sigma z_\beta = 110 + 15 \times 0.61 = 119$$

The total profit with order quantity $Q^*_2 = 119$ is

$$\pi(Q^*_2) = (p-c)\mu_1 - G(Q^*_2)$$

$$= (p-c)\mu - (p - 0.85s)\sigma\phi(z_\beta)$$

$$= (210 - 150) \times 110 - (210 - 0.85 \times 150) \times 15 \times \phi(0.6)$$

$$= $6187.63$$

We know that the distribution is normal. If we use the order quantity $Q^*_1 = 112$, then $z_\beta = (112 - 110)/15 = 0.133$ and $\beta = \Phi(0.133) = 0.553$. Let $h = c - 0.85s$, $b = p - c$. So the total profit is:

$$\pi(Q^*_1) = (p-c)\mu_1 - G(Q^*_1)$$

$$= (p-c)\mu - [h\sigma z_\beta + (h + b)\sigma(\phi(z_\beta - (1 - \beta)z_\beta))]$$

$$= (210 - 150) \times 110 - 460.65$$

$$= $6139.35$$

This inside information worth $\pi(Q^*_2) - \pi(Q^*_1) = 6187.63 - 6139.35 = $48.28$
(a) The expected demand is \( \mu = E(D) = 100 \sum_{j=3}^{8} (jp(j)) = 540 \)

(b) The variance of demand is

\[
Var(D) = 100^2 Var(d) = 100^2 [E(d^2) - (E(d))^2] = 100^2 [31 - 5.4^2]
\]

So the standard deviation is \( \sigma_D = \sqrt{Var(D)} = 100 \times \sqrt{31 - 5.4^2} \approx 135.6 \)

(c) \( \beta = \frac{p-c}{p-s} = \frac{100-60}{100-51} = 0.816 \) The optimal order quantity \( Q^* \) should be

\[
Q^* = \min\{Q : Pr(D < Q) \geq \beta\}
\]

Calculate the c.d.f of demand \( D \), it is easy to see that the optimal quantity is \( Q^* = 700 \)

(d) The expected profit is,

\[
\pi(Q^*) = (p-c)\mu - G(Q^*)
\]

\[
= (100-60)540 - [(60-51) \times 100 \times \sum_{j=3}^{6} (7-j)p(j) + (100-60) \times (800-700) \times 0.1]
\]

\[
= 21600 - 1930 = \$19670
\]

(e) If Smith decides to use a normal distribution, then the optimal order quantity is

\[
Q^* = \mu + \sigma z_\beta = 540 + 135.6 \times 0.9 \approx 662
\]

Rounded to the nearest hundred units, we have \( Q = 700 \).

(f) Since the order quantity are the same under these two models, there is no penalty for using normal distribution.

10.11 (a) Using the formula \( \hat{x} = E(S) = \sum_{x_0 > Q}(x_0 - Q)p_x(x_0) \), we have

<table>
<thead>
<tr>
<th>( x_0 )</th>
<th>( E(S)if(Q = x_0) )</th>
<th>( E[P(Q)] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4 \times 1 + 0.35 \times 2 = 1.1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

The mean of demand is \( \mu = 2.1 \). Use equation (10.3), we get

\[
E[P(Q)] = (p-g)\mu - (v-g)Q - (p-g+B)E(S) = 9 \times 2.1 - 4Q - 10E(S)
\]

Thus,

<table>
<thead>
<tr>
<th>( x_0 )</th>
<th>( E(S)if(Q = x_0) )</th>
<th>( E[P(Q)] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.1</td>
<td>3.9</td>
</tr>
<tr>
<td>2</td>
<td>0.35</td>
<td>7.4</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>6.9</td>
</tr>
</tbody>
</table>

The best order quantity is \( Q = 2 \).

Use equation (10.10), we need smallest \( Q \) such that

\[
Pr(D < Q) \geq \frac{p-v+B}{p-g+B} = 0.6
\]

Note that \( Pr(D \leq 1) = 0.25 \) and \( Pr(D \leq 2) = 0.65 > 0.6 \). So the optimal \( Q = 2 \) which is the same as above.
(b)

\[ E[P(Q)] = \sum_{x_0} P(Q, x_0) p_x(x_0) \]

\[ = \sum_{x_0 \leq Q} (p x_0 + g(Q - x_0) - Qv)p_x(x_0) + \sum_{x_0 \geq Q} (pQ - B(x_0 - Q) - Qv)p_x(x_0) \]

\[ = -Qv + gQ \sum_{x_0 \leq Q} (p_x(x_0)) + (p - g) \sum_{x_0 \leq Q} x_0 p_x(x_0) + pQ \sum_{x_0 > Q} p_x(x_0) \]

\[ -B \sum_{x_0 > Q} (x_0 - Q)p_x(x_0) \]

\[ = -Qv + gQ \sum_{x_0 \leq Q} (p_x(x_0)) + (p - g) \left[ \sum_{x_0} x_0 p_x(x_0) - \sum_{x_0 > Q} (x_0 - Q)p_x(x_0) \right] \]

\[ -Q \sum_{x_0 > Q} p_x(x_0) \right] + pQ \sum_{x_0 > Q} p_x(x_0) - B \sum_{x_0 > Q} (x_0 - Q)p_x(x_0) \]

\[ = -Qv + gQ \sum_{x_0 \leq Q} (p_x(x_0)) + (p - g) \left[ \hat{x} - E(S) - Q \sum_{x_0 > Q} p_x(x_0) \right] \]

\[ + pQ \sum_{x_0 > Q} p_x(x_0) - BE(S) \]

\[ = (p - g)\hat{x} - (v - g)Q - (p - g + B)E(S) \quad Q,E,D. \]

10.15 Selling price is \( p = 425 \), order cost is \( c = 300 \) and salvage value is \( s = 250 \). Let \( h = c - s \) and \( b = p - c \) The optimal order quantity should be

\[ Q^* = \min\{Q : Pr(D < Q) \geq \beta\} \]

where \( \beta = \frac{p-c}{p-s} = \frac{125}{175} = 0.714 \) Use the probability distribution, the optimal is \( Q^* = 3 \).