IEOR 4000: Production Management

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Solution to HW 5, Fall 2004, Prepared by Lin Li

1 Note that X is lognormal with mean μ and standard deviation σ. We have

$$\mu = E(X) = \exp(\nu + \frac{\tau^2}{2}) \qquad (1)$$

$$\sigma^2 = E(X^2) - (E(X))^2 = \exp(2\nu + 2\tau^2) - \exp(2\nu + \tau^2) = \mu^2(\exp(\tau^2) - 1)$$

From these, we get,

$$cv^2 = (\frac{\sigma}{\mu})^2 = \exp(\tau^2 - 1)$$

which implies that $\tau^2 = \ln(cv^2 + 1)$, thus $\tau = \sqrt{\ln(1 + cv^2)}$
Plug the result of τ into equation (1), we can find ν which is $\nu = \ln\mu - \ln\sqrt{1 + cv^2}$

2 Since D is lognormal with mean μ and variance σ^2, $\ln(D)$ is normal with mean ν and variance τ^2.

$$Pr(D \leq x) = Pr(\ln D \leq \ln x) = Pr(\frac{\ln D - \nu}{\tau} \leq \frac{\ln x - \nu}{\tau}) = \Phi(\frac{\ln x - \nu}{\tau})$$

Use the formula of ν and τ in problem 1,

$$Pr(D \leq x) = \Phi(\frac{\ln(\frac{x}{\mu}) + \ln(\sqrt{1 + cv^2})}{\sqrt{\ln(1 + cv^2)}})$$

With μ fixed, as $\sigma^2 \to \infty$, the $\sqrt{\ln(1 + cv^2)} \to \infty$ and

$$\frac{\ln(\frac{x}{\mu}) + \ln(\sqrt{1 + cv^2})}{\sqrt{\ln(1 + cv^2)}} \to \infty$$

Thus, $Pr(D \leq x) \to 1$.

3 In the lognormal model, denote the base stock level as$S$, then

$$\frac{\ln(\frac{x}{\mu}) + \ln(\sqrt{1 + cv^2})}{\sqrt{\ln(1 + cv^2)}} = z_\beta$$

Solve for S, we have

$$S = \mu \exp(\sqrt{ln(1 + cv^2)} z_\beta - \ln(\sqrt{1 + cv^2}))$$

In this case, as $\sigma^2 \to \infty$, the $\sqrt{ln(1 + cv^2)} \to \infty$
$\sqrt{ln(1 + cv^2)} z_\beta - \ln(\sqrt{1 + cv^2}) = \sqrt{ln(1 + cv^2)}(z_\beta - \frac{1}{2}\sqrt{ln(1 + cv^2)}) \to -\infty$, thus, $S \to 0$.

4 For normal model, $S = \mu + z_\beta \sigma$, so

$$\frac{S}{\mu} = 1 + z_\beta cv$$

For lognormal model, $S = \mu \exp(\sqrt{ln(1 + cv^2)} z_\beta - \ln(\sqrt{1 + cv^2}))$, so

$$\frac{S}{\mu} = \exp(\sqrt{ln(1 + cv^2)} z_\beta - \ln(\sqrt{1 + cv^2}))$$

See the excel file for solution.