IEOR 4000: Production Management

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Solution to HW 6, Fall 2004, Prepared by Lin Li

1 Describe how a (Q, r) policy operates.
Under a (Q, r) policy, we place an order of size Q whenever the inventory position falls to or below the reorder point r.

2 When will a (Q, r) policy operate as an (s, S) policy?
A (Q, r) policy operate as an (s, S) policy in either of following situation. $a)$ The demand is always for a single unit; $b)$ The order quantity $Q = 1$, and $r = s$, $S = r + 1$.

3 Write an expression for the average cost $c(Q, r)$ when demand is Poisson with parameter λ.
The average cost will still be

$$c(Q, r) = \frac{K\lambda}{Q} + \frac{1}{Q} \sum_{y=r+1}^{r+Q} G(y)$$

where $G(y) = hE(y - D)^+ + pE(D - y)^+$ and the demand D is Poisson.

4 Describe how to find an optimal r for a given Q in terms of the values of y corresponding to the Q smallest values of the G function.
For a given Q, let the Q smallest value of the G function be $(y_1, y_2, ..., y_Q)$ and the optimal r is

$$r(Q) = \min\{y_1, y_2, ..., y_Q\} - 1$$

5 Let $c(Q) = c(Q, r(Q))$ be the optimal cost for a given value of Q. Explain how to obtain $c(Q + 1)$ in terms of $c(Q)$ and G_{Q+1}.
Note that

$$c(Q+1) = \frac{K\lambda}{Q+1} + \frac{1}{Q+1} \sum_{k=1}^{Q+1} G_k = \frac{Q}{Q+1}\left(\frac{K\lambda}{Q} + \frac{1}{Q} \sum_{k=1}^{Q} G_k\right) + \frac{1}{Q+1} G_{Q+1}$$

Thus, $c(Q+1) = \frac{Q}{Q+1} c(Q) + \frac{1}{Q+1} G_{Q+1}$

6 Use the algorithm presented in class to find the optimal (Q, r) policy when $K = 10$, $h = 1$, $b = 9$ $\lambda = 10$, and $L = 0.2$
Note that the demand rate here is $\lambda L = 10 \times 0.2 = 2$. Use the excel for the (Q, r) policy. You will find the optimal $Q = 9$ and the cost is 9.53

7 Compare the cost to the upper bound presented in class.
The upper bound is $\sqrt{(c_d)^2 + \bar{G}_1^{\,2}} = \sqrt{2HK\lambda + \lambda Lhb} = 14.07$