1 Aggregate Production Planning

Aggregate production planning is concerned with the determination of production, inventory, and work force levels to meet fluctuating demand requirements over a planning horizon that ranges from six months to one year. Typically the planning horizon incorporate the next seasonal peak in demand. The planning horizon is often divided into periods. For example, a one year planning horizon may be composed of six one-month periods plus two three-month periods. Normally, the physical resources of the firm are assumed to be fixed during the planning horizon of interest and the planning effort is oriented toward the best utilization of those resources, given the external demand requirements.

Since it is usually impossible to consider every fine detail associated with the production process while maintaining such a long planning horizon, it is mandatory to aggregate the information being processed. The aggregate production approach is predicated on the existence of an *aggregate unit of production*, such as the "average" item, or in terms of weight, volume, production time, or dollar value. Plans are then based on aggregate demand for one or more aggregate items. Once the aggregate production plan is generated, constraints are imposed on the detailed production scheduling process which decides the specific quantities to be produced of each individual item.

The plan must take into account the various ways a firm can cope with demand fluctuations as well as the cost associated with them. Typically a firm can cope with demand fluctuations by:

(a) Changing the size of the work force by hiring and firing, thus allowing changes in the production rate. Excessive use of hiring and firing may limited by union regulations and may create severe labor problems.

(b) Varying the production rate by introducing overtime and/or idle time or outside subcontracting.

(c) Accumulating seasonal inventories. The tradeoff between the cost incurred in changing production rates and holding seasonal inventories is the basic question to be resolved in most practical situations.

(d) Planning backorders.

These ways of absorbing demand fluctuations can be combined to create a large number of alternative production planning options.

Costs relevant to aggregate production planning:

(a) Basic production costs: material costs, direct labor costs, and overhead costs. It is customary to divide these costs into variable and fixed costs.

(b) Costs associated with changes in the production rate: costs involved in hiring, training, and laying off personnel, as well as overtime compensations.

(c) Inventory related costs.

Aggregate production planning models may be valuable as decision support systems and to evaluate proposals in union negotiations. Here we limit our study to linear cost models and to a brief discussion of models with quadratic costs. . Before presenting a model based on linear programming we will discuss two extreme aggregate production plans: The *just-in-time* production plan and the *production-smoothing* plan.

- **The Just-in-time** production plan, also known as the chase plan, consists in changing the production rate to exactly satisfy demand. The idea is consistent with the JIT production philosophy and results in low holding costs but may result in high cost of adjusting the production rate, i.e., high firing and hiring costs or high idle times. As a consequence, the just-in-time plan is best suited to situations where the cost of changing the production rate is relatively inexpensive.

- **The production-smoothing** plan, also known as the stable plan, consists in keeping the production rate constant over time. This strategy minimizes the *production* cost when production costs are convex. A stable, make to stock, production strategy needs to build inventories to cope with peaks in demand and may result in high holding costs. As a result, the production-smoothing plan is best suited to situations where inventory carrying costs are low.

We will now present a simple example that illustrates the two extreme plans and a plan that results from solving a linear model.

Data: Initial conditions: 300 workers, 500 units of inventory at the end of December. Demand Forecast January–June: 1,280, 640, 900, 1,200, 2,000, and 1,400. Final conditions: 600 units of ending inventory in June. Cost of hiring one worker $500, cost of firing one worker $1,000, cost of holding one unit of inventory for one month $80. Backorders not allowed. Number of aggregate units produced by one worker in one day 0.14653.

The just-in-time plan, see Table 1 results in hiring 755 workers, firing 145 workers and carrying a total of 604 units of inventory for a total cost equal to $570,784. The production-smoothing plan, see Table 2, results in hiring 111 workers, firing 0 workers and carrying a total of 6561 units of inventory for a total cost equal to $580,363. Finally, the linear programming plan, see Table 3 uses a combination of hiring and firing and by building inventories in anticipation of demand peaks. The plan results in hiring 465 workers, firing 27 workers, and carrying a total of 1498 units of inventory for a total cost of $379,292. See the spreadsheet app-strategies.xls for details.

	Dec	Jan	Feb	Mar	Apr	May	Jun	Totals
Working Days		20	24	18	26	22	15	
Demand		1280	640	900	1200	2000	1400	
Hiring		0	0	160	0	305	290	755
Firing		34	84	0	27	0	0	145
Workforce	300	266	182	342	315	620	910	
Production		780	640	902	1200	1999	2000	
Inventory	500	0	0	2	2	0	601	604

Table 1: The Just-in-Time Production Plan

	Dec	Jan	Feb	Mar	Apr	May	Jun	Totals
Working Days		20	24	18	26	22	15	
Demand		1280	640	900	1200	2000	1400	
Hiring		111	0	0	0	0	0	111
Firing		0	0	0	0	0	0	0
Workforce	300	411	411	411	411	411	411	
Production		1204	1445	1084	1566	1325	903	
Inventory	500	424	1230	1414	1780	1105	608	6561

Table 2: The Production-Smoothing Plan

	Dec	Jan	Feb	Mar	Apr	May	Jun	Totals
Working Days		20	24	18	26	22	15	
Demand		1280	640	900	1200	2000	1400	
Hiring		0	0	0	0	465	0	465
Firing		27	0	0	0	0	0	27
Workforce	300	273	273	273	273	738	738	
Production		800	960	720	1040	2378	1622	
Inventory	500	20	340	160	0	378	600	1498

Table 3: Linear Programming Production Plan

	Dec	Jan	Feb	Mar	Apr	May	Jun	Totals
Working Days		20	24	18	26	22	15	
Demand		1280	640	900	1200	2000	1400	
Hiring		0	0	0	105	195	0	300
Firing		0	0	0	0	0	0	0
Workforce	300	300	300	300	405	600	600	
Production		879	1055	791	1542	1934	1319	
Inventory	500	99	514	405	747	681	600	3047

Table 4: Modified Linear Programming Production Plan

Although the production plan based on linear programming resulted in lower cost over the planning horizon, it ended up with a relatively large workforce. If management judges the workforce to be too large, it can impose an additional constraint on the ending workforce and resolve the problem. If management wants the ending work force to be no larger than 600 workers, then solving the linear program with this additional constraint results in the plan given by Table IV and a cost of $393,768.

The model developed above allows hiring and firing, but does not allow overtime. We now present two two additional models. Model I has a fixed work force and allows overtime, while Model II has a variable work force, allows overtime and backorders.

1.1 Model 1: Fixed Work Force Model

Assumption : Hiring and firing are disallowed. Production rates can fluctuate only by using overtime.

Data:

c_{it} = unit production cost for product i in period t (exclusive of labor costs)
h_{it} = inventory carrying cost per unit of product i held in stock from period t to $t+1$.
r_t = cost per man-hour of regular labor in period t
o_t = cost per man-hour of overtime labor in period t
d_{it} = forecast demand for product i in period t
m_i = man-hours required to produce one unit of product i
$\bar{R}_t$ = total man-hours of regular labor available in period t
$\bar{O}_t$ = total man-hours of overtime labor available in period t
I_{i0} = initial inventory level for product i
T = time horizon in periods
N = total number of products

Decision Variables:

X_{it} = units of product i to be produced in period t
I_{it} = units of product i to be left over as an inventory in period t
R_t = man-hours of regular labor used during period t
O_t = man-hours of overtime labor used during period t

The linear program for this model is:

$$\text{Minimize} \quad \sum_{i=1}^{N}\sum_{t=1}^{T}[c_{it}X_{it} + h_{it}I_{it}] + \sum_{t=1}^{T}[r_tR_t + o_tO_t]$$

subject to:

$$\begin{array}{ll} X_{it} + I_{i,t-1} - I_{it} = d_{it} & \forall i,t \\ \sum_{i=1}^{N} m_iX_{it} - R_t - O_t = 0 & \forall t \\ 0 \leq R_t \leq \bar{R}_t & \forall t \\ 0 \leq O_t \leq \bar{O}_t & \forall t \\ X_{it} \geq 0, \quad I_{it} \geq 0 & \forall i,t. \end{array}$$

Example: We will assume a unit production rate. The data is given in Table 4.

	Jan	Feb	Mar	Apr	May	Jun
Demand	100	100	150	200	150	100
Unit Production Cost (Excluding Labor)	7	8	8	8	7	8
Unit Holding Cost	3	4	4	4	3	2
Unit Regular Labor Cost	15	15	18	18	15	15
Unit Overtime Labor Cost	22.5	22.5	27	27	22.5	22.5
Available Man-hours R Labor	120	130	120	150	100	100
Available Man-hours O Labor	30	40	40	30	30	30

Table 5: Data for Model I

The optimal solution is obtained in the file Aggre-Prod-Planning.xls. The optimal solution is displayed in Table 5 and has an optimal cost of $20,193.

	Dec	Jan	Feb	Mar	Apr	May	Jun
Man-hours R Labor		120	130	120	150	100	100
Man-hours O Labor		0	17	0	30	30	0
Production		120	147	120	180	130	100
Inventory	3	23	70	40	20	0	0

Table 6: Solution to Model I Example

Remark: Model 1 assumes that the number of regular man-hours used for production is a variable under our control. This would be the case, for example, if temporary workers are used. In many cases, the cost of regular labor is fixed, but we may decided not to use it. To model situations where this choice is not available we can impose a lower bound $\underline{R}_t$ on the use of the man-hours of regular labor used in period t. For example, the choice $\underline{R}_t = \overline{R}_t$ precludes the use of R_t as a decision variable.

1.2 Model 2: Variable Work Force Model

Assumption : Hiring and firing are allowed in addition to using overtime from the regular work force. Backorders are allowed.

Data:

c_{it} = unit production cost for product i in period t (exclusive of labor costs)
h_{it} = inventory carrying cost per unit of product i held in stock from period t to $t+1$
π_{it} = backorder cost per unit of product i carried from period t to $t+1$
r_t = cost per man-hour of regular labor in period t
o_t = cost per man-hour of overtime labor in period t
h_t = cost of hiring one man-hour in period t
f_t = cost of firing one man-hour in period t
d_{it} = forecast demand for product i in period t
m_i = man-hours required to produce one unit of product i
p = fraction of regular hours allowed as overtime
I_{i0} = initial inventory level for product i
T = time horizon in periods
N = total number of products

Decision Variables:

X_{it} = units of product i to be produced in period t
I^+_{it} = units of product i to be left over as an inventory in period t
I^-_{it} = units of product i backordered at the end of period t
H_t = man-hours of regular work force hired in period t
F_t = man-hours of regular work force fired in period t
R_t = man-hours of regular labor used during period t
O_t = man-hours of overtime labor used during period t

The linear program for this model is

$$\text{Minimize} \sum_{i=1}^{N}\sum_{t=1}^{T}[c_{it}X_{it} + h_{it}I^+_{it} + \pi_{it}I^-_{it}] + \sum_{t=1}^{T}[r_tR_t + o_tO_t + h_tH_t + f_tF_t]$$

$$
\begin{aligned}
&\text{subject to:} \\
X_{it} + I^+_{i,t-1} - I^+_{it} - I^-_{i,t-1} + I^-_{it} &= d_{it} && \forall i, t \\
\sum_{i=1}^{N} m_i X_{it} - R_t - O_t &\le 0 && \forall t \\
R_t - R_{t-1} - H_t + F_t &= 0 && \forall t \\
O_t - pR_t &\le 0 && \forall t \\
X_{it} \ge 0, \quad I^+_{it} \ge 0, \quad I^-_{it} &\ge 0 && \forall i, t \\
R_t, O_t, H_t, F_t &\ge 0 && \forall t
\end{aligned}
$$

Example: We will assume a unit production rate, that overtime is at most 25% of regular labor, and the data from Table 6.

	Jan	Feb	Mar	Apr	May	Jun
Demand	100	100	150	200	150	100
Unit Production Cost (Excluding Labor)	7	8	8	8	7	8
Unit Holding Cost	3	4	4	4	3	2
Unit Backorder Cost	20	25	25	25	20	15
Unit Regular Labor Cost	15	15	18	18	15	15
Unit Overtime Labor Cost	22.5	22.5	27	27	22.5	22.5
Hiring Cost	20	20	20	20	20	20
Firing Cost	20	20	20	20	20	15

Table 7: Data for Model II

The optimal solution is obtained in the file Aggre-Prod-Planning.xls. The optimal solution, given in Table 7, and has an optimal cost of $19,784.

	Dec	Jan	Feb	Mar	Apr	May	Jun
Man-hours Hired		129	0	0	0	0	0
Man-hours Fired		0	0	0	0	0	7
Man-hours of R Labor		129	129	129	129	129	121
Man-hours of O Labor		0	0	0	32	0	0
Production		129	129	129	161	129	121
Inventory	3	32	60	39	0	0	0
Backlogs		0	0	0	0	21	0

Table 8: Solution to Model II

1.3 Advantages and Disadvantages of Linear Cost Models

The main advantage of linear cost models is that they generate linear programs which can be solved by readily available and efficient codes. The shadow price information can be of assistance in identifying opportunities for capacity expansions, marketing penetration strategies, new product introductions, and so on. The most serious disadvantage of linear programming models is their failure to deal with demand uncertainties explicitly.

1.4 Linear Decision Rules

Some authors have proposed using quadratic costs to penalize deviation of key variables from target levels. The advantage of using quadratic functions is that they result in linear production rules. For example, the optimal production in period t may be of the form

$$P_t = \sum_{s=t}^{T} a_{s-t} D_s + bR_{t-1} + cI_{t-1} + d.$$

While quadratic costs lead to linear decision rules there is no guarantee that the solution will be non-negative. In addition, a quadratic cost imposes a symmetric penalty above and below target levels. In practice, it may be significantly more expensive to fire than to hire and it may be more expensive to incur backorders than to incur holding costs.

1.5 Disaggregating Aggregate Plans

An aggregate plan determines work force levels over a specified planning horizon, and the size of the workforce imposes a constraint on the the detailed planning of individual items. It is important that the resulting master production schedule be consistent with the aggregate production plan.