Note Title 10/29/2004

Lecture notes 10/28/04

Compound Demands

$$D = \sum_{k=1}^{N} X_k$$

- N = Random # of customers
  X_k = demand from customer k

- N = Lead time (days)
  X_k = daily demand
- Find the mean & variance of D and use:
  - Normal approximation
  - Lognormal "
  - Worst case "

How to find mean & variance of $D = \sum_{k=1}^{N} X_k$?

$$E[D] = E_N E[D|N]$$

$$= E_N[N\mu_x]$$

$$\mu_d = \mu_x \mu_N$$

$$\mathrm{Var}[D] = \mathrm{Var}[E[D|N]] + E[\mathrm{Var}[D|N]]$$

$$\boxed{\sigma_d^2 = \mu_x^2 \sigma_N^2 + \mu_N \sigma_x^2}$$

- $\mu_x = E[X_k] \quad \forall k$

  $\sigma_x^2 = \mathrm{Var}[X_k] \quad \forall k$

  $\mu_N = E[N]$

  $\sigma_N^2 = \mathrm{Var}[N]$

$$\boxed{\mu_d = \mu_x \mu_N}$$

$$CV_d = \frac{\sigma_d}{\mu_d} = \sqrt{CV_N^2 + \frac{1}{\mu_N} CV_x^2}$$

- Special case ($N \sim \text{Poisson}(\lambda)$)

$$\boxed{\sigma_d^2 = \mu_x^2 \sigma_N^2 + \mu_N \sigma_x^2}$$

$$\sigma_N^2 = \mu_N = \lambda$$

$$\sigma_d^2 = \mu_x^2 \lambda + \lambda \sigma_x^2 = \lambda(\mu_x^2 + \sigma_x^2)$$

$$\sigma_d^2 = \lambda E[X^2]$$

$$\boxed{\mu_d = \mu_x \mu_N}$$

$$\mu_d = \lambda EX$$

$$CV_d = \frac{\sigma_d}{\mu_d} = \sqrt{CV_N^2 + \frac{1}{\mu_N} CV_x^2}$$

$$CV_N^2 = \left(\frac{\sqrt{\lambda}}{\lambda}\right)^2 = \frac{1}{\lambda}$$

$$\mu_N = \lambda$$

$$CV_d = \sqrt{\frac{1}{\lambda} + \frac{1}{\lambda} CV_x^2}$$

$$CV_d = \frac{1}{\sqrt{\lambda}} \sqrt{1 + CV_x^2}$$

$\blacksquare$

Worst Case Approximation

$$G(S) = h\,E(S-D)^+ + b\,E(D-S)^+$$

Game:

- $\min \quad G(S)$
- worst dist D with mean μ and variance σ^2.

$$(D-Q)^+ = \frac{1}{2}\left[(D-Q) + |D-Q|\right]$$

$$E(D-Q)^+ = \frac{1}{2}\left[(\mu-Q) + E|D-Q|\right]$$

$$\leq \frac{1}{2}\left[(\mu-Q) + \sqrt{E(D-Q)^2}\right]$$

$$E(D-Q)^2 = E(D-\mu)^2 + (Q-\mu)^2$$

$$E(D-Q)^+ \leq \frac{1}{2}\left[(\mu-Q) + \sqrt{\sigma^2 + (Q-\mu)^2}\right]$$

$$Q^s = \mu + \frac{\sigma}{2}\left(\sqrt{\frac{b}{h}} - \sqrt{\frac{h}{b}}\right)$$

$$G(Q^s) \leq \sqrt{bh}\,\sigma$$

$$= \sqrt{(p-c)(c-s)}\,\sigma$$

$$G(\mu) \leq \frac{1}{2}(b+h)\sigma$$

Remarks:

- $Q^s = 0$ if

$$1 - \sqrt{\frac{h}{b}}\,\frac{\sigma}{\mu} < 0$$

$$\boxed{\sqrt{\frac{b}{h}} < \frac{\sigma}{\mu}}$$

Fill rate:

$$\alpha = \frac{E \min(D, Q^s)}{E D}$$

$$\geq 1 - \frac{1}{2}\sqrt{\frac{h}{b}}\,\frac{\sigma}{\mu} \qquad \blacksquare$$

Solution in

- Normal,
- Poisson,
- Lognormal,
- Compound Demand,
- Worst Case.