Note Title 11/10/2004

Multi-period models

11/9/04

- Finite horizon
- Periodic review
- $D_1, D_2, \ldots, D_T$

  demands in periods $1, \ldots, T$
- Unit cost c_t, $t = 1, \ldots, T$
- Zero lead times

  Anything ordered at the BOP t arrives during period t
- x_t inventory level at BOP t, $t = 0, 1, \ldots, T$

  $x_t > 0$ means inventory

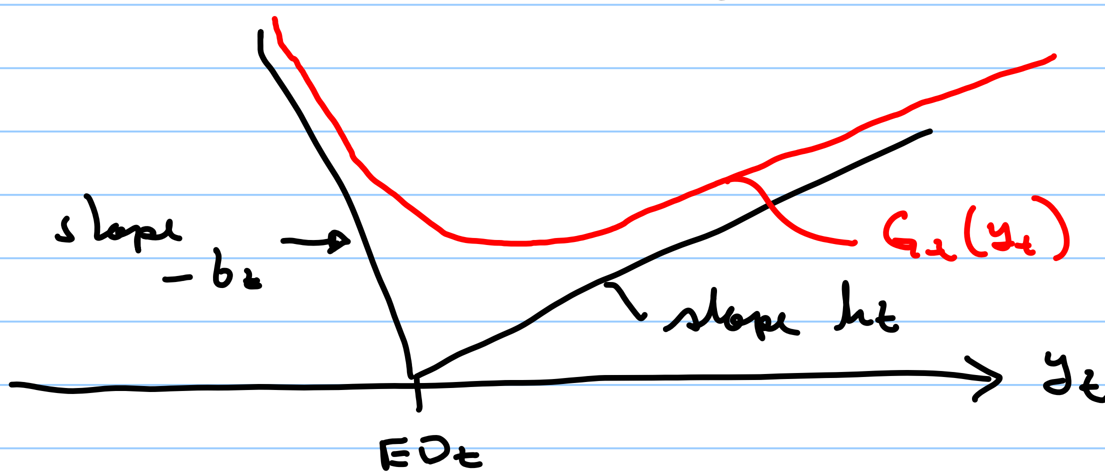
  $x_t < 0$ " backorders
- $y_t - x_t \geq 0$ order quantity placed at the BOP t
- $x_{t+1} = y_t - D_t$

How about underage and overage costs?

if $y_t - D_t > 0$ overage

if $y_t - D_t < 0$ underage

$$G_t(y_t) = h_t E(y_t - D_t)^+ + b_t E(D_t - y_t)^+$$



- G_t is convex
- G_t is coercive

$G_t(y_t) \to \infty$ as $|y_t| \to \infty$

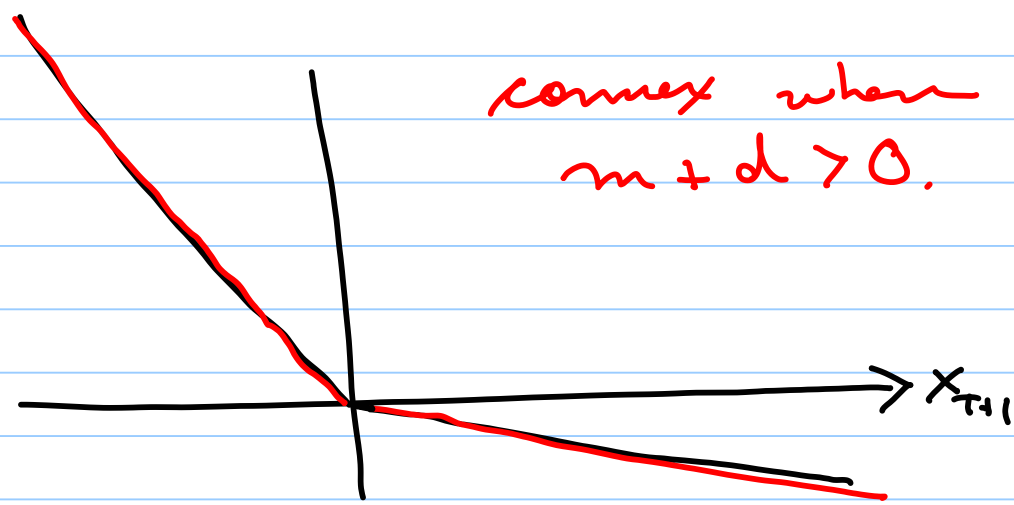
Write a DP (Dynamic Program)

Terminal cost

- $C_{T+1}(x_{T+1})$ = cost of ending period T with x_{T+1}.

Example:

$$C_{T+1}(x_{T+1}) = -c_T(1-d)x_{T+1}^{+} + c_T(1+m)x_{T+1}^{-}$$



$C_t(x_t)$ = optimal discounted expected cost over periods $t, \dots, T$ starting with inventory level x_t.

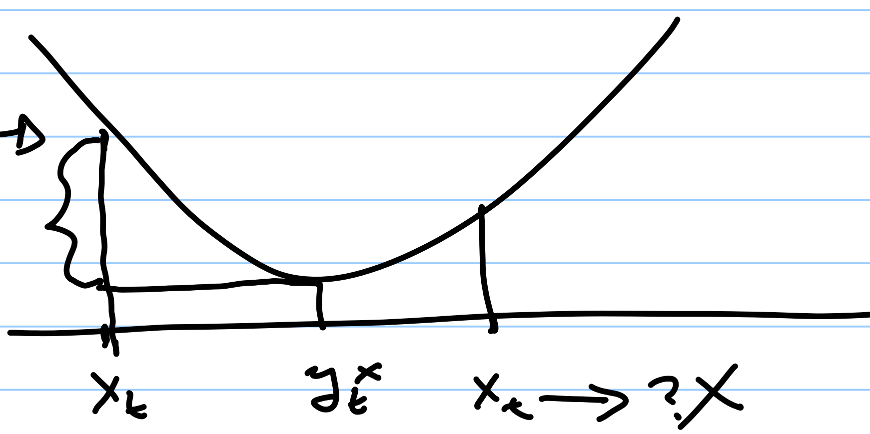
$$C_t(x_t) = \min_{y_t \geq x_t} \left\{ c_t(y_t - x_t) + G_t(y_t) + \alpha E\, C_{t+1}(y_t - D_t) \right\}$$

Fact: if C_{t+1} is convex

then

$c_t y_t + G_t(y_t) + \alpha E\, C_{t+1}(y_t - D_t)$

is a convex function of y_t



- if $x_t < y_t^*$, order $y_t^* - x_t$.

if $x_t \geq y_t^*$, order zero

There is a y_t^* such that an optimal policy is to order

$$(y_t^* - x_t)^+$$

in period t.

- Order up-to policy (aka base-stock policies)

- Policy $(y_1^*, \ldots, y_T^*)$ not difficult to compute for some distributions.

- Is there a simpler way?

Myopic Policies

- Sometimes optimal
- Easy to compute

$$C_1(x_1) = \min_{y_t \geq x_t} E\left[\sum_{t=1}^{T} \alpha^{t-1}\left\{c_t(y_t - x_t) + G_t(y_t)\right\} + \alpha^T C_{T+1}(x_{T+1})\right]$$

$$x_t = y_{t-1} - D_{t-1}$$

$$x_{t+1} = y_t - D_t$$

$$C_1(x_1) = \min_{y_t \geq x_t} E \sum_{t=1}^{T} \alpha^{t-1}\left\{(c_t - \alpha c_{t+1}) y_t + G_t(y_t)\right\} - c_1 x_1 + E \sum_{t=1}^{T} \alpha^{t-1} c_{t+1} D_t$$

Myopic policy minimizes

$$(c_t - \alpha c_{t+1}) y_t + G_t(y_t) \quad \text{wrt } y_t$$

$$(c_t - \alpha c_{t+1}) y_t + \overbrace{h_t E(y_t - D_t)^+ + b_t E(D_t - y_t)^+}^{''}$$

$$(c_t - \alpha c_{t+1}) + h_t \Pr(D_t \leq y_t) - b_t \Pr(D_t > y_t) = 0$$

$$\Pr(D_t \leq y_t) = \frac{b_t - m_t}{h_t + b_t}$$

where $m_t = c_t - \alpha c_{t+1}$.

Let y_t^m be the myopic policy.

Order $(y_t^m - X_t)^+$ in period t.

Is it ever optimal?

Yes, if $X_{t+1} = y_t^m - D_t \le y_{t+1}^m$

$$y_t^m - D_t \le y_{t+1}^m$$

A sufficient condition is $y_t^m \le y_{t+1}^m$.