Note Title 11/12/2004

Infinite Horizon Model

- c_t, h_t, b_t time independent
- $D_1, D_2, \ldots$ are i.i.d
- Myopic Policy is Optimal

$$P(D_t \le y) = \frac{p - c(1-\alpha)}{p + h}$$

$$y^* = \mu + \sigma\,\Phi^{-1}\left(\frac{p - c(1-\alpha)}{p+h}\right)$$

= optimal base stock level.

$$C(x) = c(\underbrace{y^* - \mu}_{\text{safety stock}} - x) + \frac{c\mu + G(y^*)}{1-\alpha}$$

cost of safety stock minus cx

average purchasing cost

average inv. cost

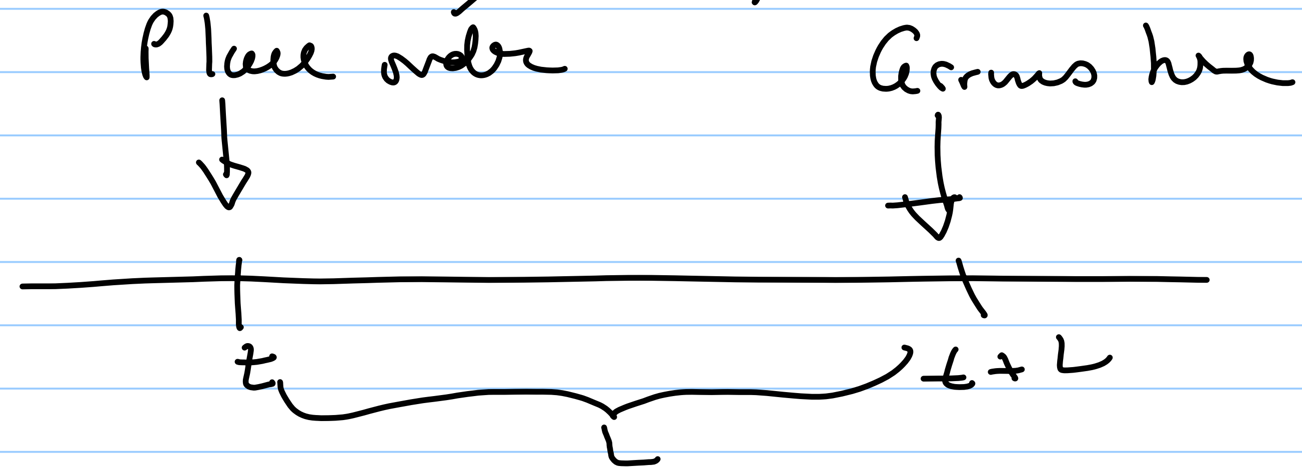
present value of purchasing

+ inv. cost

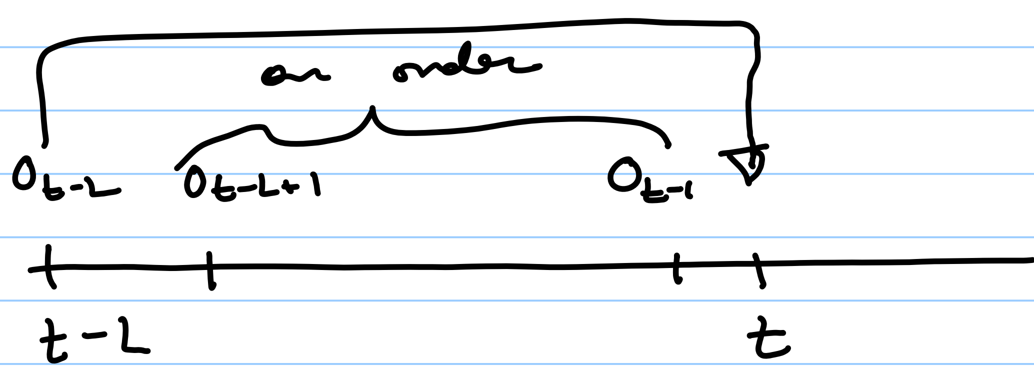
Finite Horizon with positive Lead Times.

Previously $L = 0$

Now, L positive integer



New definition of X_t



X_t = inventory on hand

+ (inventory on order)

− backorders

≐ inventory level + on order

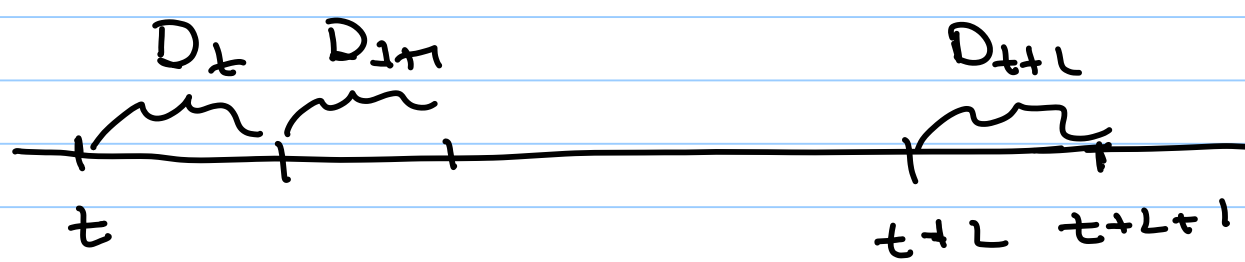
= inventory position.

Order $y_t - x_t \geq 0$ units at bop t.

Brings $\underline{IP}_t = y_t$ (after ordering)

$\underline{IP}_t^- = x_t$ (before ")

y_t



What is the inventory level at end of period $t+L$?

$$= y_t - D_t - \cdots - D_{t+L}$$

$$= y_t - D[t, t+L]$$

$D[t, t+L] = \sum_{s=t}^{t+L} D_s$ = lead time demand

Inventory Level at EOP $t+L$

= Inventory Position (after ordering)

in period t minus the lead time demand.

$$G_t(y) = h_t E\left(y - D[t, t+L]\right)^+ + b_t E\left(D[t, t+L] - y\right)^+$$

$$C_t(x) = \min_{y_t \geq x_t} \left\{ c_t(y_t - x_t) + G_t(y_t) + \alpha E\, C_{t+1}(y_t - D_t) \right\}$$

- Solve exact by BDP

- Myopic Policy

$$y_t^m : \quad \Pr\big(\overbrace{D[t, t+L]}^{LTD} \le y\big) = \frac{p_t - m_t}{p_t + h_t}$$

$$m_t = c_t - \alpha c_{t+1}$$

- Random Lead Times???

- No optimal policy

- Good heuristic is

to use myopic policy
with lead time demand

$D[t, t+L]$

random

Demand per period is random

μ_d = mean demand per period

σ_d = stdev

μ_ℓ = mean lead time

σ_ℓ = stdev

$$E[D[t, t+L]] = \mu_d \mu_\ell$$

$$\text{stdev}[D[t, t+L]] = \sqrt{\mu_\ell \sigma_d^2 + \sigma_\ell^2 \mu_d^2}$$

$\mu_d = 80$, $\sigma_d = 20$

Supplier 1: $\mu_\ell = 5$, $\sigma_\ell = 4$

stdev of LTD = 323

Supplier 2:

$$\mu_e = 25 \quad , \quad \sigma_e = 0$$

Stdv of LTD = 100

Next Topic

- Random Demands
- Positive Setup Cost.