Note Title 11/16/2004

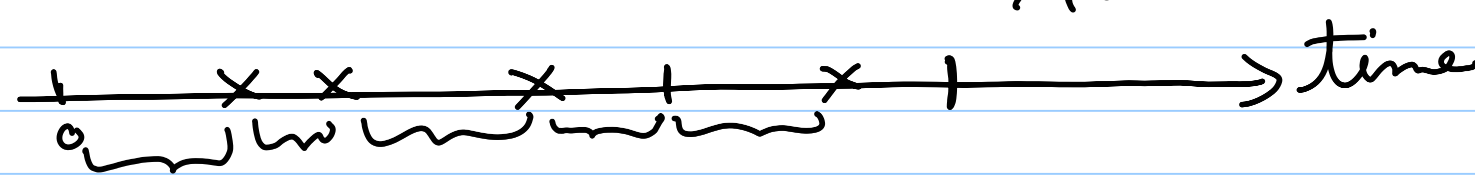
Stochastic Models with fixed plus linear ordering costs.

- $c(x) = \begin{cases} K + cx & \text{if } x > 0, \\ 0 & \text{otherwise.} \end{cases}$

- Similar to EOQ model
  Suggests batch ordering.

- Continuous Review Model
  - Monitor inventory position continuously
- Either Poisson process or compound Poisson.

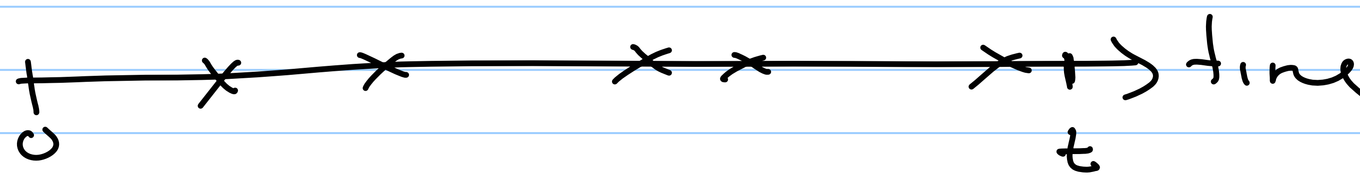
Poisson process
- Interarrival times IID Exponential R.V.



Each demand is for one unit

- Compound Poisson

$X_1 + X_2 + \quad X_3 + X_4 + \quad X_5$



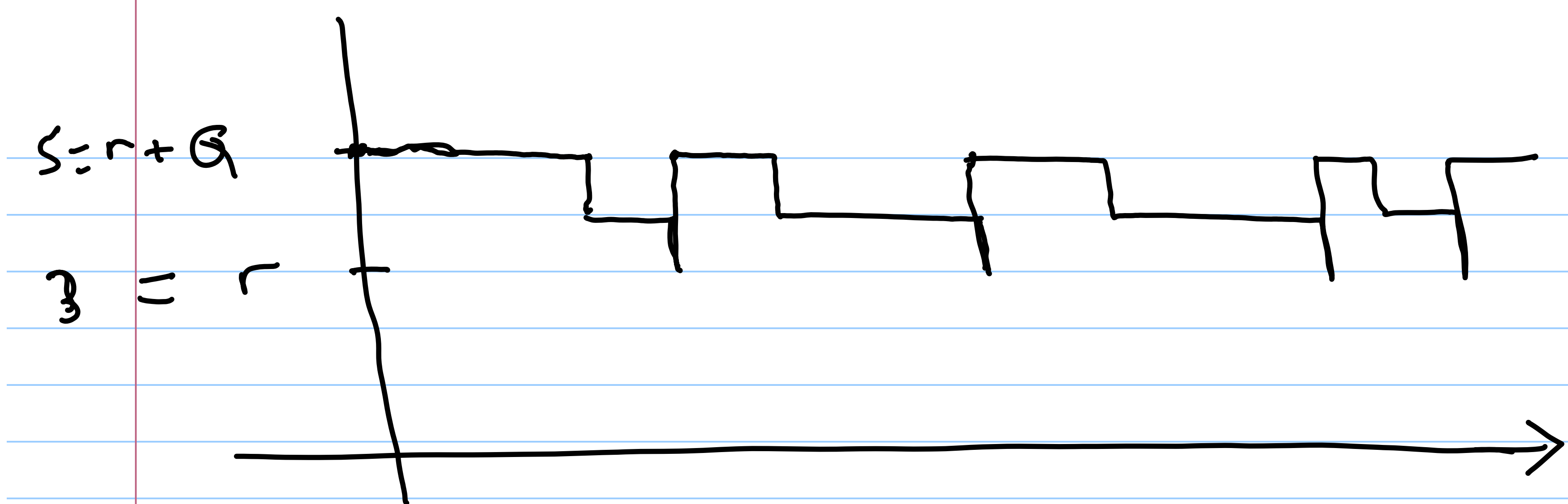
- (Q, r) policies
  (s, S) policies. } ordering policies

(Q, r)

- Monitor the IP and place an order of size Q whenever the inventory position falls to or below the reorder point r.

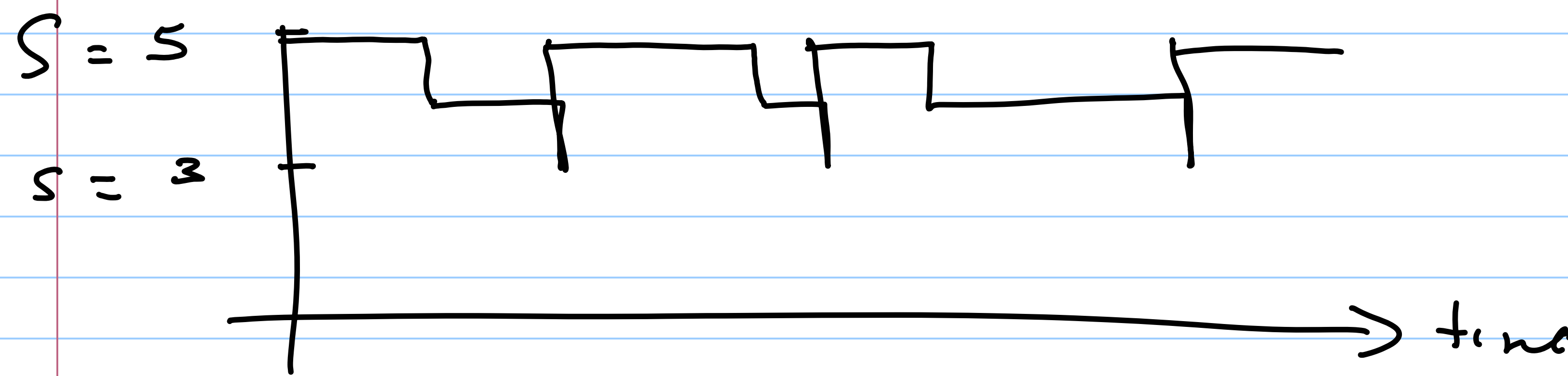
Example: $r = 3$, $Q = 2$

Demand is Poisson



(s, S) policy: Monitor the IP and place an order to restore the IP to S whenever it falls to or below s.

$s = 3$, $S = 5$, Poisson Demand



same as (Q, r)

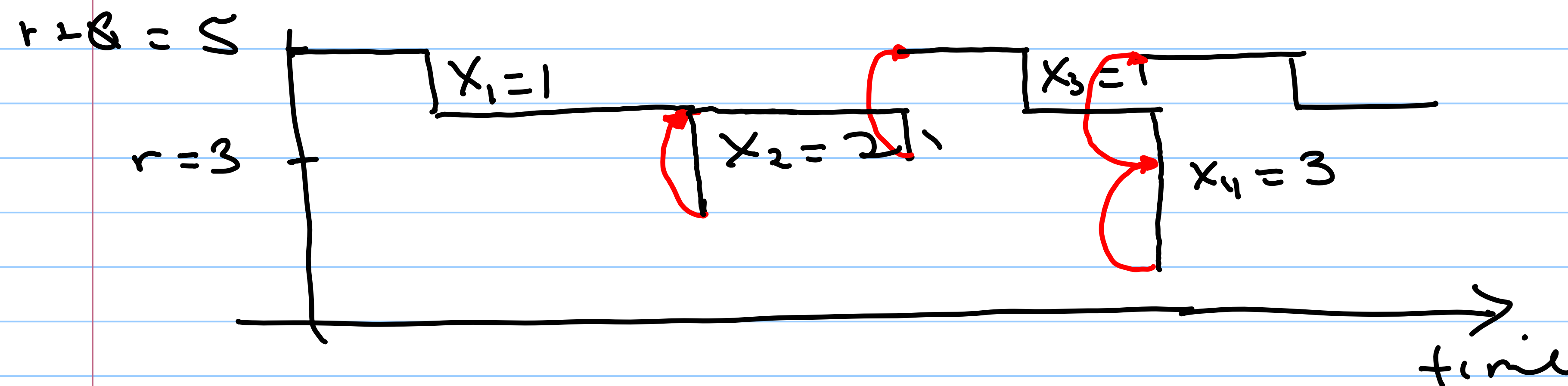
for Poisson demands

$s = r$, $S = r + Q$.

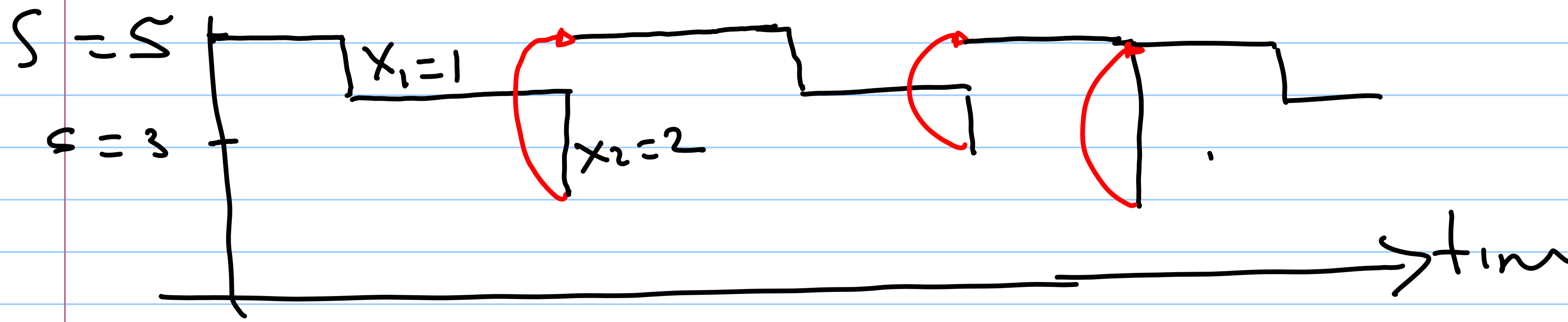
Q: What if demand is Compound Poisson?

$$X = \begin{cases} 1 & 1/3 \\ 2 & 1/3 \\ 3 & 1/3 \end{cases} \quad \text{Demand Size}$$

$$(Q, r) = (2, 3)$$

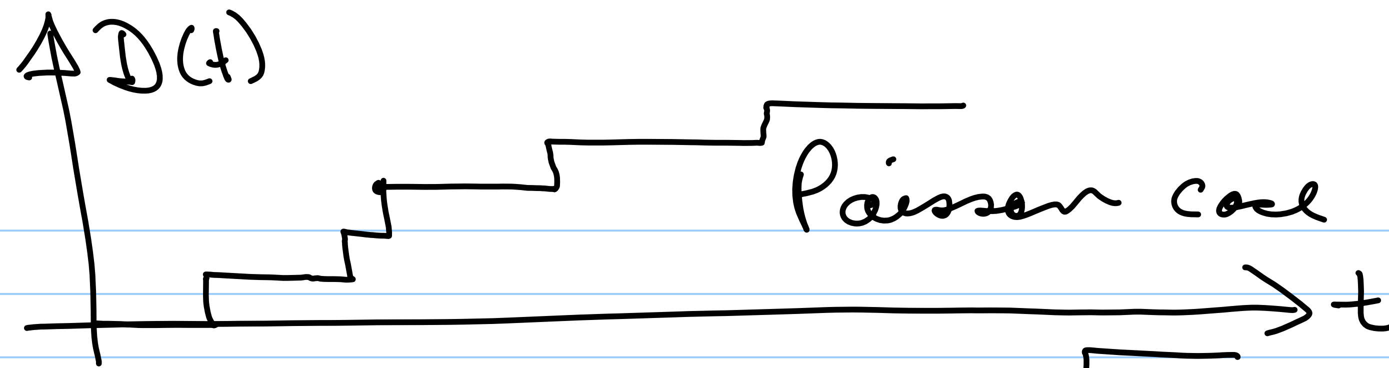


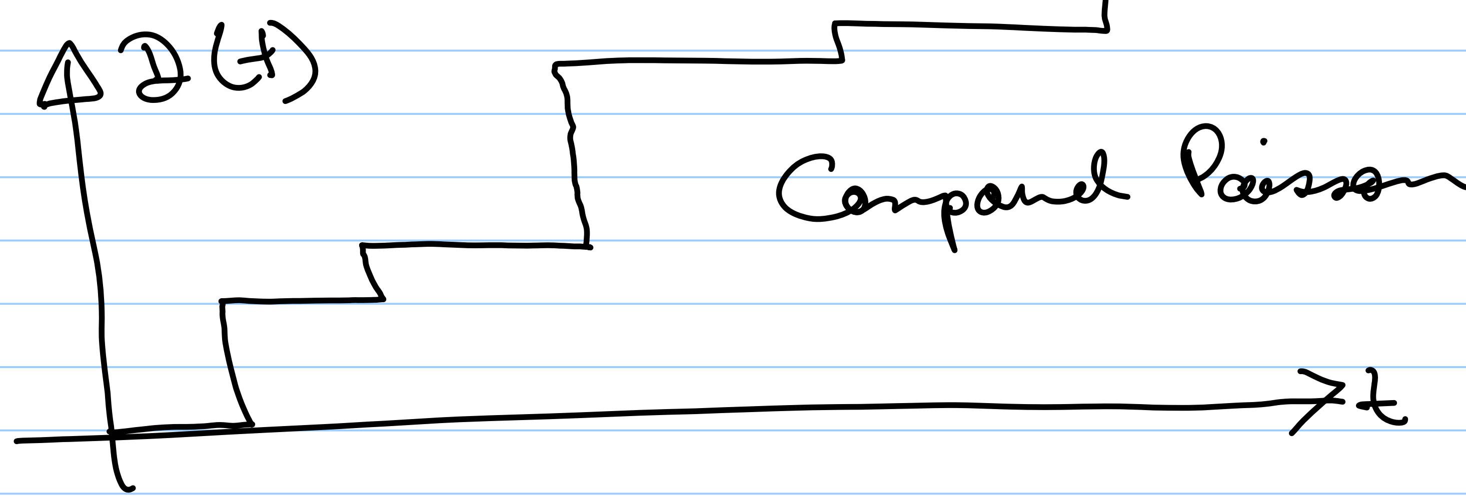
$(s, S) = (3, 5)$



Definitions:

$D(t)$ = cumulative demand up to time t.





- L lead time
  (known non-negative number)

- $D(t|L) = D(t) - D(t-L)$

  $= \text{demand over } (t-L, t]$

- As $t \to \infty$, $D(t|L) \longrightarrow D$

  → lead time demand in steady state

- Examples:
  - $D(t) \sim \text{Poisson}(\lambda t)$

    $D(t|L) \sim \text{Poisson}(\lambda L)$

    independent of t

$$D \sim \text{Poisson}(\lambda L)$$

- $D(t) \sim \mu t + \sigma\sqrt{t}\, B$

$$D = D(t|L) \sim \mu L + \sigma\sqrt{L}\, Z$$

$$Z \sim N(0,1)$$

- $D(t) \sim$ Compound Poisson

$$D = D(t|L) = \sum_{k=1}^{N(L)} X_k$$

$$N(L) \sim \text{Poisson}(\lambda L)$$

$I(t)$ = inventory on hand at time t

$B(t)$ = backorders at time t

$IN(t)$ = net inventory at time t

$IO(t)$ = inventory on order at time t

$IP(t)$ = inventory position at time t

$$IN(t) = I(t) - B(t)$$

$$I(t) = IN(t)^+$$

$$B(t) = IN(t)^-$$

$$IP(t) = I(t) + IO(t) - B(t)$$

$$= IN(t) + IO(t)$$

Agenda: Compute key

performance measures under a (Q, r) policy and then optimize over Q and r.

Key performance measures:

- I = inventory
- B = backorders
- $\Pr(B > 0)$

- $Pr(B=0)$
  - Frequency of orders.

Roadmap:

- First study $Q=1$
  Base stock policies
  $(Q,r) \Leftarrow (s,S)$ equiv.
- $Q>0$ (Q,r), (s,S) equiv
  for Poisson

- Sensitivity Analyses
  - How does the optimal cost relate to:
    - Newsvendor cost
    - EOQ cost
  - How sensitive is the cost to changes in Q?
    - Less sensitive than EOQ formula

$$\frac{C(Q)}{C(Q^*)} \leq \frac{1}{2}\left(\frac{Q}{Q^*} + \frac{Q^*}{Q}\right)$$

- Closed form bounds
- Heuristics.