Note Title 11/23/2004

(Q, r) & (s, S) Policies

- Algorithm for (Q, r) policies
- Excel implementation
- Sensitivity Analysis
- Next topic:

Compound Poisson

X = random demand size

Special case $P(X=1) = 1$
reduces to Poisson case

$$c(Q,r) = \frac{K \lambda\, E\min(Q,X) + \sum_{j=r+1}^{r+Q} G(j)}{Q}$$

$= 1$ in Poisson case

$r+j$; $\quad j \sim U\{1, \dots, Q\}$

$r+j-X \le r$ iff $X \ge j$

$$\underbrace{\frac{1}{Q}\sum_{j=1}^{Q} P(X \ge j)}_{=\frac{1}{Q} E \min(Q, X)} = \text{probability of placing an order}$$

- General result

  wy X descrete, non-negative r.v

  taking integer values

$$EX = 1 \cdot p_1 + 2p_2 + 3p_3 + \cdots$$

$$\begin{aligned} = p_1 &+ p_2 + p_3 + \cdots \\ &+ p_2 + p_3 + \cdots \\ &\qquad\quad p_3 + \cdots \\ &\qquad\qquad\quad p_4 + \cdots \end{aligned}$$

$$EX = P(X \geq 1) + P(X \geq 2) + \cdots$$

$$= \sum_{j=1}^{\infty} P(X \geq j)$$

$$\tilde{X} = \min(Q, X)$$

$$E\tilde{X} = P(\tilde{X} \geq 1) + P(\tilde{X} \geq 2) + \cdots$$

$$P(\tilde{X} \geq j) = \begin{cases} P(X \geq j) & j \leq Q \\ 0 & j > Q \end{cases}$$

$$\Rightarrow E\tilde{X} = E\min(Q, X)$$

$$= \sum_{j=1}^{Q} P(X \geq j)$$

Ordering cost (average)

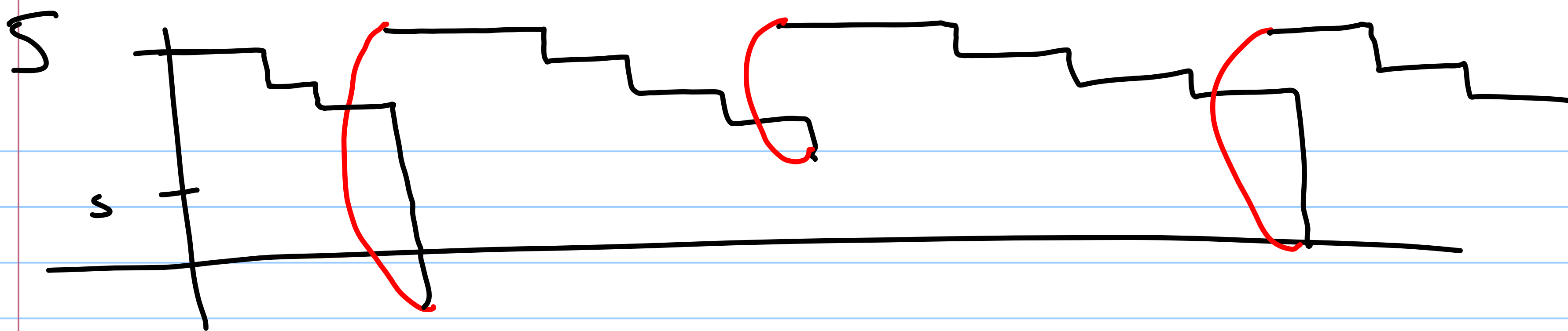
$$K\lambda \frac{1}{Q} \sum_{j=1}^{Q} P(X \geq j)$$

$$= K\lambda \frac{E \min(Q, X)}{Q}$$

$\blacksquare$

(s, S) policies

- Place an order to bring IP to S whenever it falls to or below s.



$\mathbb{P}$ has a distribution concentrated over $\{s+1, s+2, \ldots, S\}$ but it is not necessarily uniform.

$$c(s, S) = K \cdot \text{prob of placing an order}$$

$$+ \sum_{j=1}^{S-s} G(s+j)\, q_j$$

$$q_j = \Pr(IP = s+j) \quad j = 1, \ldots, S-s$$

depends on s, S.

Q: How do we find the distribution of the IP?

Renewal Theory

$y \in \{s+1, \ldots, S\}$

$K(s, y)$ = total expected cost until the next order is placed starting from $IP = y$

$M(j)$ = expected time until an order is placed starting from $IP = s + j$, $j = 1, \ldots, S - s$.

$$c(s,S) = \frac{k(s,S)}{M(S-s)}$$

$$k(s,y) = G(y) + K\,P_r(D \geq y-s) + \sum_{j=0}^{y-s-1} k(s,y-j)\,P_r(D=j)$$

$$k(s,y) = G(y) + K \sum_{j \geq y-s} p_j + \sum_{j=0}^{y-s-1} k(s,y-j)\,p_j .$$

$M(j) = 0$ if $j = 0$

$j \geq 1$

$$M(j) = 1 + 0 \Pr(D \geq j) + \sum_{i=1}^{j-1} M(j-i) \Pr(D = i).$$

Solution is

$$\boxed{M(j) = M(j-1) + m(j-1)}$$

$m(0) = \frac{1}{1-p_0}$,

$$m(j) = \sum_{k=0}^{j} p_k \, m(j-k) \qquad j = 1, 2, \ldots$$

$$K(s, y) = K + \sum_{j=0}^{y-s-1} m(j) \, G(y-j)$$

$$c(s, S) = \frac{K + \sum_{j=0}^{S-s-1} m(j) \, G(S-j)}{M(S-s)}$$

$$= \frac{K + \sum_{j=1}^{S-s} G(s+j) \, m(S-s-j)}{M(S-s)}$$

$$\frac{1}{M(S-s)} = \text{prob of placing an order}$$

$$\boxed{\frac{m(S-s-j)}{M(S-s)} = \Pr(IP = s+j) =: q_j}$$

- Algorithm

  Key idea:

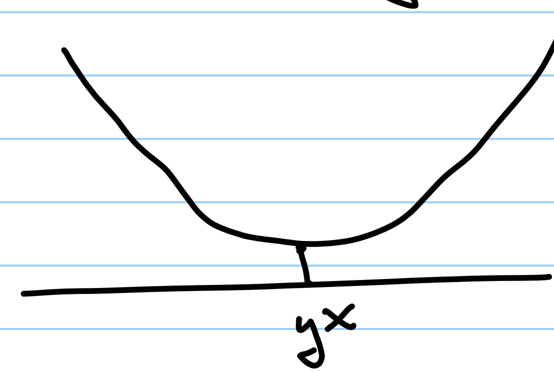
$c(s-1, S)$ in terms of $c(s, S)$ and $G(s)$

$$c(s-1, S) = \alpha_n \, c(s, S) + (1-\alpha_n) \, G(s)$$

$$\alpha_n = \frac{M(n)}{M(n+1)} \; ; \; n = S - s.$$

Algorithm

Let y^* be a minimizer of $G(y)$.



Step 0

$S^0 = y^*$, $s = y^* - 1$

DO WHILE $c(s, S^0) > G(s)$

$s = s - 1;$

```
ENDO;
c° = c(s, S°),  S = S° + 1
Step 1
  DO UNTIL  G(S) > c°;
  IF c(s, S) < c°
   S° = S
   DO WHILE c(s, S°) ≤ G(s+1)
    s = s + 1;
   ENDO;
   c° = c(s, S°)
```

```
ENDIF
S = S+1
ENDO;
END
```

Questions?

Single Location Models

- Newsvendor
  - Finite horizon
  - Infinite horizon
- Added Fixed costs
  - (Q, r) models
  - (s, S) "

Multiechelon Models

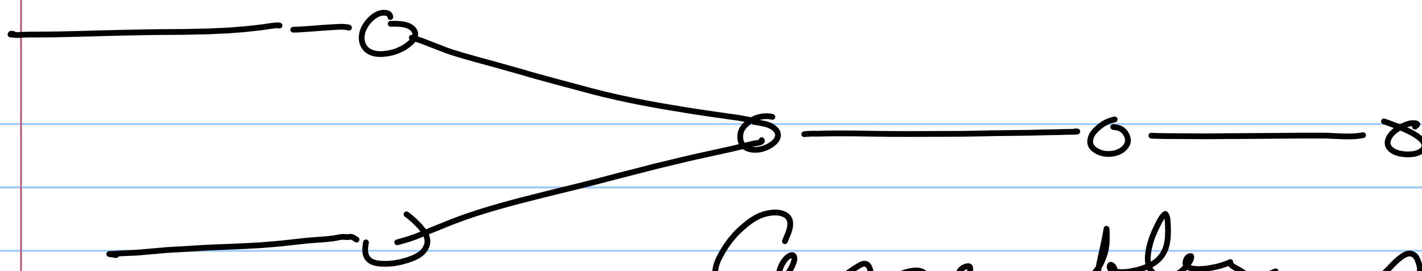
$$\begin{tikzcd}
 & & & & b \\
\text{Outside supplier} \arrow[r, "L_4"] & 4 \arrow[r, "L_3"] & 3 \arrow[r, "L_2"] & 2 \arrow[r, "L_1"] & 1 \\
 & h_4 & h_3 & h_2 & h_1
\end{tikzcd}$$

SERIAL SYSTEM

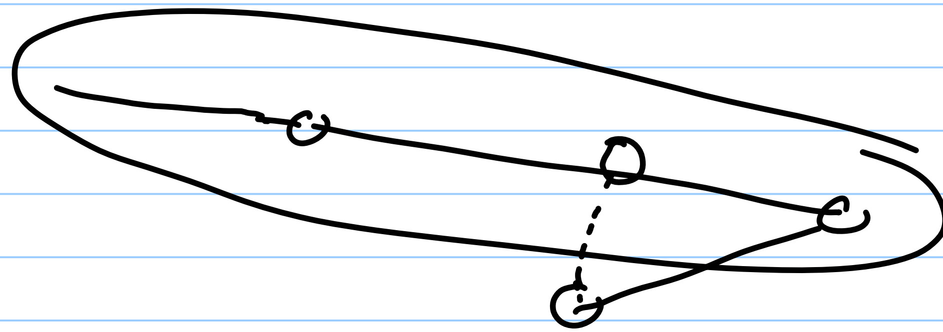
Base stock policies are optimal when set up costs are zero.

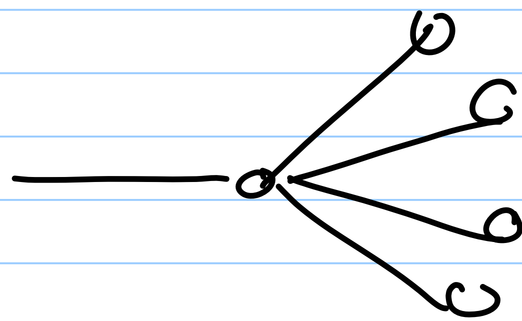
$S_4 \qquad S_3 \qquad S_2 \qquad S_1$

Series system



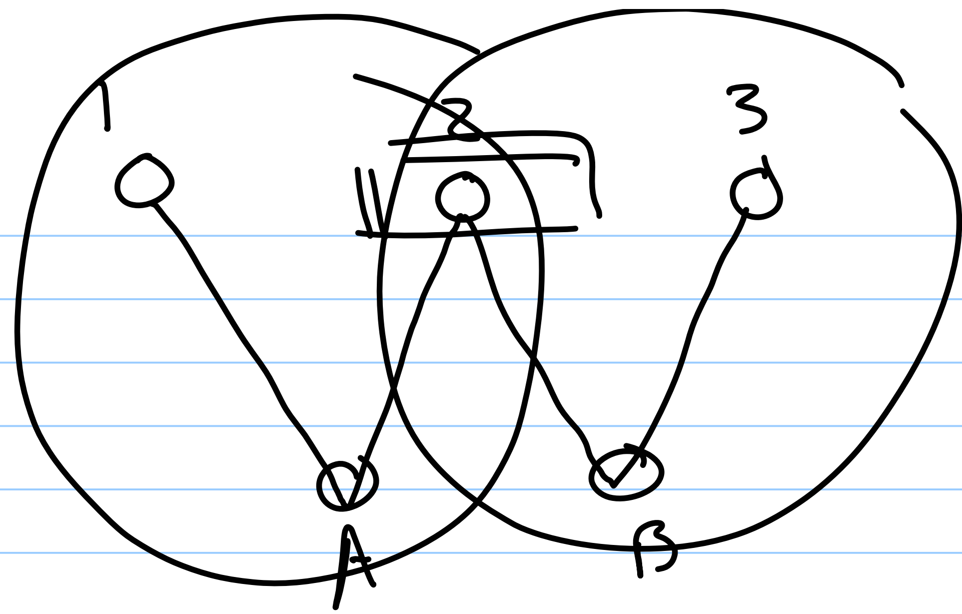
Assembly system





Distribution Systems

- Approx / Heuristic


1
2
3
A
B