Note Title 11/30/2004

Serial Systems

Demand

$$\xrightarrow{L_4} S_4 \xrightarrow{L_3} S_3 \xrightarrow{L_2} S_2 \xrightarrow{L_1} S_1$$

H. C $\quad h_4 < h_3 < h_2 < h_1$

Shortage cost $\quad b$

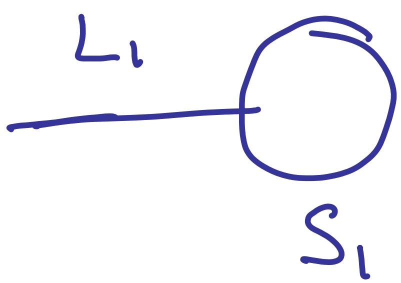
Questions:

- How to find optimal/good

policies?

- How to compute cost?
  Total / Each stage?
- Sensitivity analyses?
  How things change if "customer lead time" increases from zero?
  What if we are able to reduce a lead time?
  Who benefits?

Formulation



$$C_1(S_1) = h_1 E(S_1 - D_1)^+ + b E(D_1 - S_1)^+$$

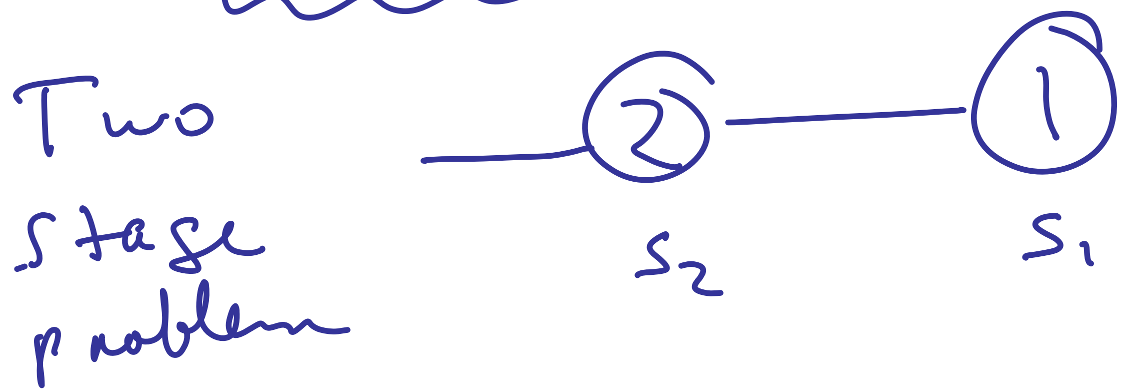
D_1 demand on L_1

D_1 LTD of stage 1.

News vendor problem.

$$S_1 = \min\left\{ s \in \mathbb{N} : P_r(D_1 \le s) \ge \frac{b}{b+h} \right\}$$

Single stage is just a NV problem — $\square$



What is the cost.

Need I_2, I_1 and B

$I_2 \to$ inv at ②

$I_1 \to$ inv at 1

$B \to$ backlogs

$$I_2 = (S_2 - D_2)^+ = \text{inv @ 2}$$

$$B_2 = (D_2 - S_2)^+ = \text{backlogs @ 2}$$

$$I_1 = (S_1 - D_1 - B_2)^+$$

$$B_1 = (B_2 + D_1 - S_1)^+$$

$$C(S_2, S_1) = E\{h_2 I_2 + h_1 I_1 + b B\}$$

How do we solve?

$$h_2 (S_2 - D_2)^+$$

$$\left.\begin{aligned} &+ h_1 \left(S_1 - (D_2 - S_2)^+ - D_1\right)^+ \\ &+ b \left(D_1 + (D_2 - S_2)^+ - S_1\right)^+ \end{aligned}\right\}$$

$$NV\left(h_1, b, D_1 + (D_2 - S_2)^+\right)$$

$$C_1(s_1-(D_2-s_2)^+)$$

$$C(s_2,s_1) = h_2 E(s_2-D_2)^+ + \underbrace{E\, C_1(s_1-(D_2-s_2)^+)}_{\text{Why?}}$$

$$h_1(s_1-(D_2-s_2)^+-D_1)^+ + b(D_1+(D_2-s_2)^+-s_1)^+$$

$$h_1 (s_1 - \tilde{D}_1)^+ + b(\tilde{D}_1 - s_1)^+$$

$$\tilde{D}_1 = D_1 + (D_2 - s_2)^+$$

$$h_1 (\tilde{s}_1 - D_1)^+ + b(D_1 - \tilde{s}_1)^+$$

$$\tilde{s}_1 = s_1 - (D_2 - s_2)^+$$

$$C_1(s) = h_1 E(s - D_1)^+ + b E(D_1 - s)^+$$

cost of $NV(h_1, b, D_1)$

$$C_1(s_1 - (D_2 - s_2)^+)$$

$$C(s_2, s_1) = h_2 E(s_2 - D_2)^+$$
$$+ E\, C_1(s_1 - (D_2 - s_2)^+)$$

Change of variable

$$s = s_1 + s_2$$
$$s_2 = x$$

$$c(x, s) = h_2 E(x - D_2)^+$$
$$+ E\, C_1(s - x - (D_2 - x)^+)$$

$$c_2(x, S) = h_2 E(x - D_2)^+ + E\, C_1(\min(S-x, S-D_2))$$

$$S - x - (D_2 - x)^+ = \begin{cases} S - x & \text{if } D_2 < x \\ S - D_2 & \text{o.w} \end{cases}$$

$$C_2(S) = \min_{x \in \{0, \ldots, S\}} c_2(x, S)$$

$\hookrightarrow$ optimal cost of allocating

s units between stages 2 and 1

$$x(s) = (s - s_1^*)^+$$

where $s_1^* = \min\{s \in \mathbb{N} : \Delta c_1(s) > h_2\}$

Unless $s \geq s_1^*$, the allocation to stage 2 would be zero.

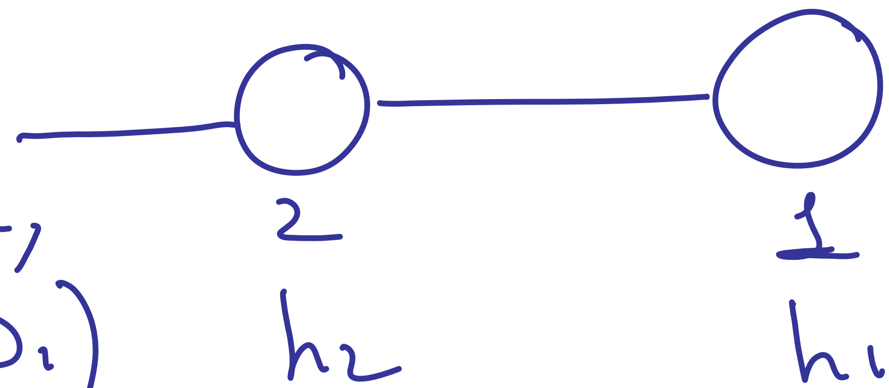
$$\Delta c_1(s) = c_1(s+1) - c_1(s)$$

$$S_1^* = \min\left\{ S \in \mathbb{N} : P_r(D_1 \le S) \geq \frac{b+h_2}{b+h_1} \right\}$$

" optimal base stock for stage 1 in a 2 stage system.

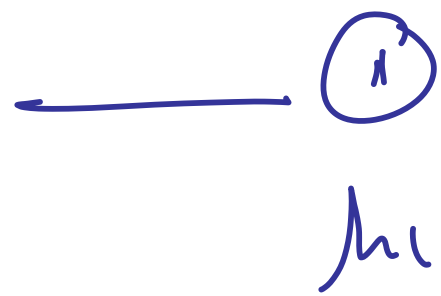
- Also optimal for n-stage system $n \geq 2$
- Independent of $D_2, D_3, \ldots$

$NV(h_1 - h_2, b + h_2, D_1)$



$$\frac{b+h_2}{b+h_1}$$

$NV(h_1, b, D_1)$



$$\frac{b}{b+h_1}$$

Easy to solve for S_i^*

How about s_2^* ?

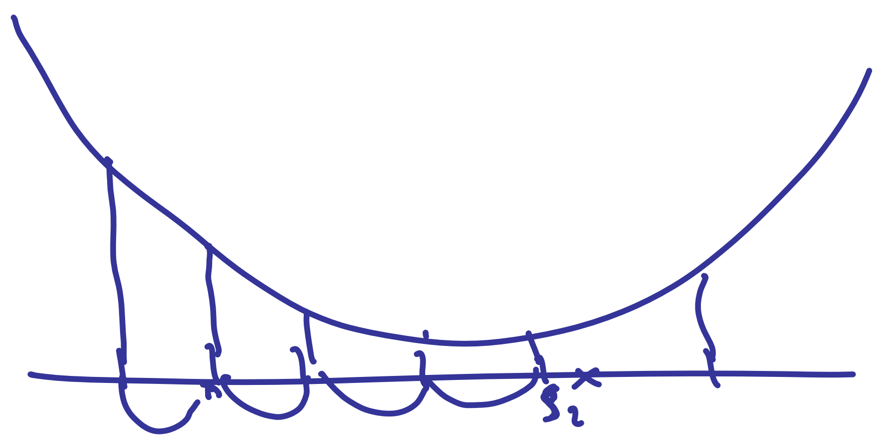
$$C_2(s) = \min_{x \in \{0,\dots,s\}} \Big\{ h_2 E(x-D_2)^+ + E\, C_1(\min(s-x,\ s-D_2)) \Big\}$$

$$x = (s-s_1^*)^+$$

$$C_2(s) = h_2 E\left((s-s_1^*)^+ - D_2\right)^+ + E\, C_1\left(\min\left(s_1^*,\ s-D_2\right)\right)$$

For a two stage system

$$S_2^{\times} = \min\{s \in \mathbb{N} : \Delta C_2(s) > 0\}$$



More than two Stages?

$$C_j(x;s) = E\left\{h_j(x-D_j)^+ + C_{j-1}(\min(s-x,\ s-D_j))\right\}$$

→ how to allocate between stage j and stages $\{j-1, \ldots, 1\}$

$$s_j^* = \min\{s \in \mathbb{N} : \Delta C_j(s) > h_{j+1}\}$$

$$h_{J+1} = 0.$$

Next Time

- Quick & Dirty Heuristics
  close to optimal
- Simulations.