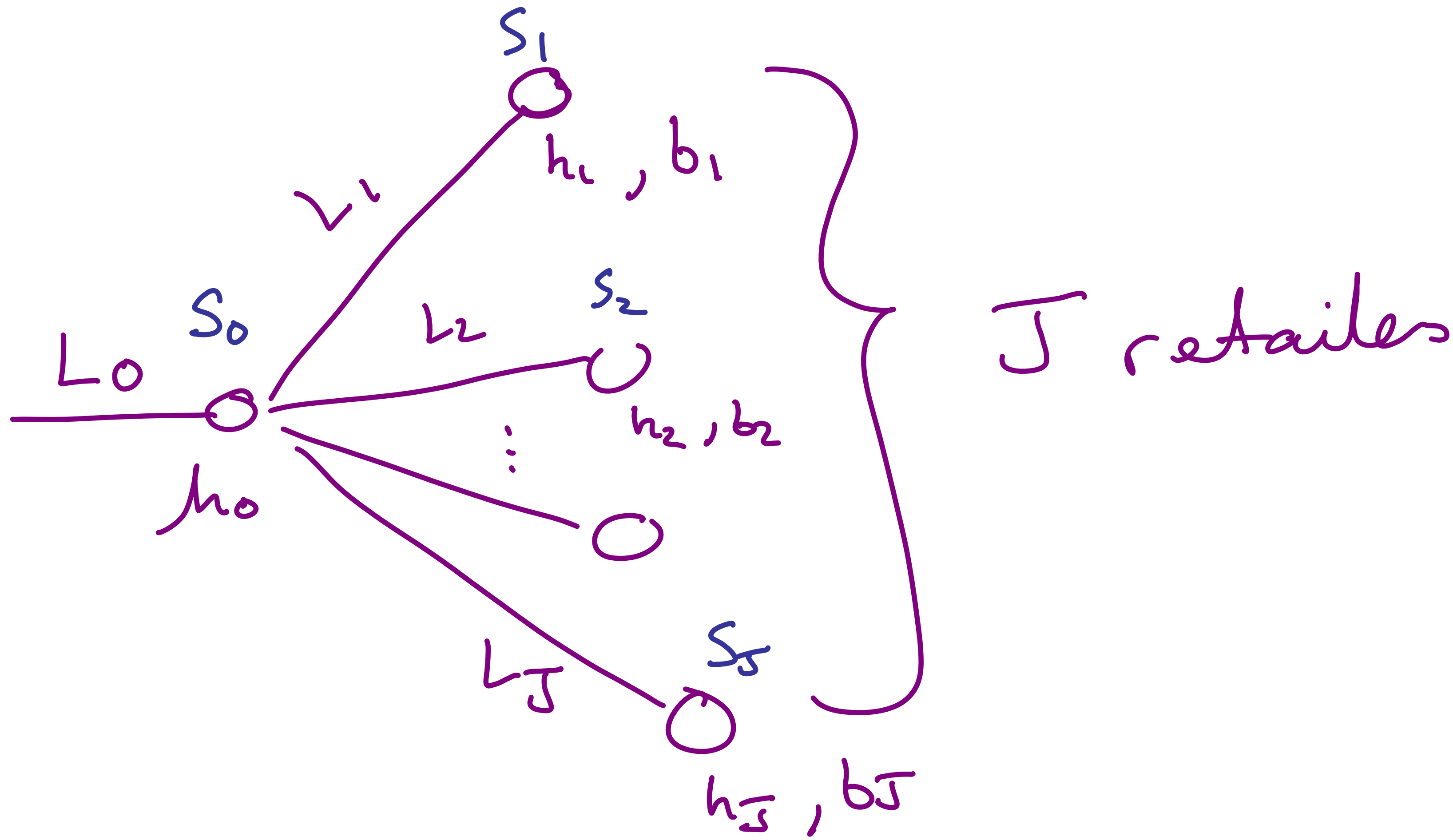
Note Title 12/9/2004

Distribution Systems



Local control

- Each stage follows base stock policy
- Warehouse use FIFO

Central control

- Warehouse may use smarter allocation rules.

Poisson Demands

λ_j = demand rate at retailer $j \in \{1, \ldots, J\}$

$\lambda_0 = \sum_{j=1}^{J} \lambda_j$ overall demand rate

S_j = base stock level at retailer $j \in \{1, \ldots, J\}$

S_0 = base stock at warehouse

$D_j \sim \text{Poisson}(\lambda_j L_j) \quad j \in \{1, \ldots, J\}$

$D_0 \sim \text{Poisson}(\lambda_0 L_0)$ warehouse

Backlogs & Inventory at warehouse

$$B_0 = (D_0 - S_0)^+$$

$$I_0 = (S_0 - D_0)^+$$

B_{0j} = backorders at warehouse due to retailer j

$$B_0 = \sum_{j=1}^{J} B_{0j}.$$

B_j = backorders at retailer j

$$B_j = (B_{0j} + D_j - S_j)^+$$

$$I_j = (S_j - D_j - B_{0j})^+$$

$$C(s_0, s_1, \dots, s_J) = h_0 E[I_0] + \sum_{j=1}^{J} c_j(s_0, s_j)$$

$$c_j(s_0, s_j) = h_j E[I_j] + b_j E[B_j].$$

Given s_0 the problem of selecting $S_j(s_0)$ is a Newsvendor problem $N(h_j, b_j, \underbrace{B_{0j} + D_j})$

→ requires convolution

$$S_j(s_0) = \min\left\{ S_j \in \mathbb{Z} : \Pr\left(D_j + B_{0j} \le S_j\right) \geq \frac{b_j}{b_j + h_j} \right\}$$

$B_{0j} \sim \mathrm{bin}\left(\theta_j, B_0\right) \qquad \theta_j \equiv \lambda_j / \lambda_0$

$$\underbrace{h_0 E[I_0] + \sum_{j=1}^{J} c_j(s_0, S_j(s_0))}_{\text{function of } s_0 \text{ alone}}$$

- Not convex in s_0
- Search over $LB \le s_0 \le UB$ and find optimal solution.

Heuristics ?

CD heuristic

$s_0 = 0$

$$B_0 = (D_0 - s_0)^+$$
$$= D_0 = \sum_{j=1}^{J} D_{0j}$$
$$D_{0j} \sim \text{Poisson}(\lambda_j L_0)$$

$$B_{0j} = D_{0j}$$

$$B_{0j} + D_j \sim \text{Poisson}(\lambda_j L_0 + \lambda_j L_j)$$

$$\sim \text{Poisson}(\lambda_j (L_0 + L_j))$$

Simply solve J univariate problems

$$N(h_j, b_j \text{ Poisson}(\lambda_j (L_0 + L_j)))$$

Stock-pooling heuristic

$$B_j = (B_{0j} + D_j - S_j)^+$$
$$\leq B_{0j} + (D_j - S_j)^+$$

Cost reduces to $J+1$ newsvendor problems:

Retailer j $N(h_j, b_j, D_j)$
"as if the warehouse had ample supply"

Warehouse solves

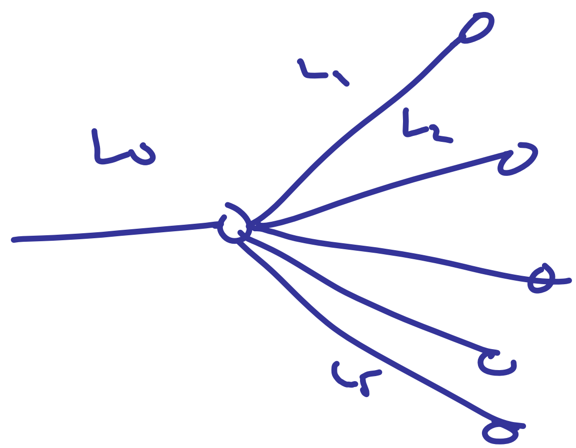
$$N\Big(h_0, \underbrace{\sum_{j=1}^{J} b_j \theta_j}_{= b_0}, D_0\Big)$$

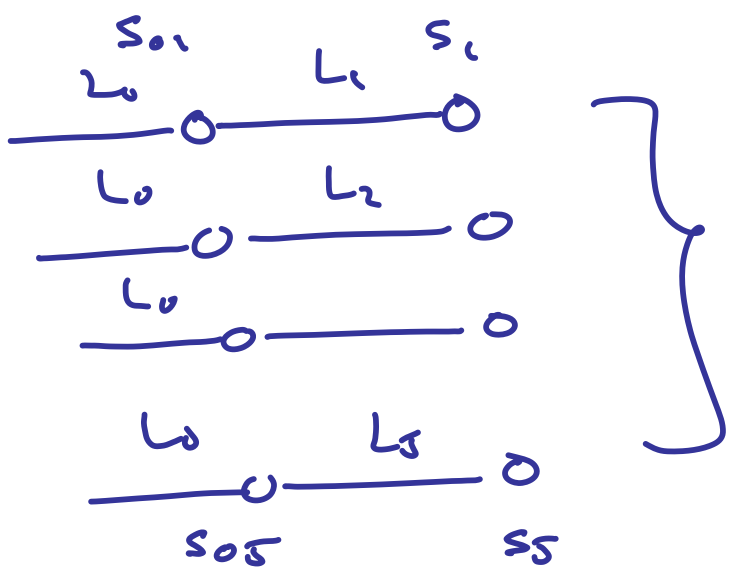
$$S_j = \min\left\{s \in \mathbb{Z} : \Pr(D_j \le s) \ge \frac{b_j}{b_j + h_j}\right\}$$

$$S_0 = \min\left\{s \in \mathbb{Z} : \Pr(D_0 \le s) \ge \frac{b_0}{b_0 + h_0}\right\}$$

solving $J+1$ newsvendor problems.

Projection Heuristic





Solve 5
two stage
serial systems

Find s_0 such that

$$\mathbb{E}(D_0 - S_0)^+ = \sum_{j=1}^{J} (D_{0j} - S_{0j})^+$$

For each $j \in \{1, \dots, J\}$ find

$$S_j(S_0) = \min\left\{ s \in \mathbb{Z} : \Pr(B_{0j} + D_j \le s) \geq \frac{b_j}{b_j + h_j} \right\}.$$

Require one convolution.

$$\underbrace{h_0 \mathbb{E}[I_0] + \sum_{j=1}^{J} C_j(s_0, S_j(s_0))}_{}$$