Solution to Assignment 8

April 13, 2000

1 Problem 1:

Since $N = 5, n = 2$, we then know a simple random sample of size 2 consisting of selecting one of the ten subsets of size 2:

$(1, 2), (1, 4), (1, 8), (2, 2), (2, 4), (2, 8), (2, 4), (2, 8), (4, 8)$, where each of the subsets has equal probability of being selected, i.e., $1/10$.

Hence, we have

\[
\begin{align*}
P(X_2 = 3/2) &= \frac{2}{10}; \\
P(X_2 = 4/2) &= \frac{4}{10}; \\
P(X_2 = 5/2) &= \frac{5}{10}; \\
P(X_2 = 6/2) &= \frac{6}{10}; \\
P(X_2 = 9/2) &= \frac{9}{10}; \\
P(X_2 = 10/2) &= \frac{10}{10}.
\end{align*}
\]

It is easy to know that the mean and variance of $X_2$ are 3.4 and 2.34 respectively.

2 Problem 2:

2.1 Part a:

No.
2.2 Part b:
No.

2.3 Part c:
Yes.

2.4 Part d:
No.

2.5 Part e:
Yes.

2.6 Part f:
No.

2.7 Part g:
Yes.

3 Problem 3:

By the definition of a 95% confidence interval for $\mu$, we mean that there exist two random variables, $f_1(X_1, X_2, ..., X_n)$ and $f_2(X_1, X_2, ..., X_n)$, such that $P(f_1(X_1, X_2, ..., X_n) < \mu < f_2(X_1, X_2, ..., X_n)) = 0.95$, where $X_1, X_2, ..., X_n$ are samples. Before we get the exact value of $f_1$ and $f_2$, the only thing we can say is that the interval, $(f_1, f_2)$, covers $\mu$ with probability 0.95. However, if we know the observed value of these samples, then we can compute $f_1$ and $f_2$. Once we get these two values, we then know for sure that the interval either covers $\mu$ or not.
4 Problem 4:

4.1 Part a:
The unbiased estimate of the population mean is \( \bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \), where \( n = 30 \).

4.2 Part b:
From the lecture notes, we know the unbiased estimate of the population variance is \( \frac{N-1}{N} S^2 = \frac{N-1}{n} \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 \), and the unbiased estimate of \( \text{Var}(\bar{X}) \) is \( \frac{S^2}{\bar{X}} = \frac{N-n}{N} \frac{S^2}{n} \), where \( n = 30 \) and \( N = 2000 \).

4.3 Part c:
From the lecture notes, the approximate 95% confidence intervals for the population mean and total are \((\bar{X} - Z_{\frac{\alpha}{2}} \sqrt{\frac{N-n}{N} \frac{S^2}{n}}), \bar{X} + Z_{\frac{\alpha}{2}} \sqrt{\frac{N-n}{N} \frac{S^2}{n}}\) and \((N \bar{X} - NZ_{\frac{\alpha}{2}} \sqrt{\frac{N-n}{N} \frac{S^2}{n}}), N \bar{X} + NZ_{\frac{\alpha}{2}} \sqrt{\frac{N-n}{N} \frac{S^2}{n}}\) respectively, where \( n = 30 \), \( N = 2000 \), \( \alpha = 0.05 \).