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# A combinatorial auction improves school meals in Chile: a case of OR in developing countries

Rafael Epstein[a], Lysette Henríquez[b,c], Jaime Catalán[a], Gabriel Y. Weintraub[a], Cristián Martínez[d,e] and Francisco Espejo[b]

[a]*Department of Industrial Engineering, University of Chile, República 701, Santiago, Chile,*
[b]*Junta Nacional de Auxilio Escolar y Becas ( JUNAEB), Chile,*
[c]*United Nations Development Programme (UNDP), Mexico,*
[d]*Supervision Department, JUNAEB, Chile,*
[e]*Antonio Varas 153, Providencia, Santiago, Chile*
*E-mail: repstein@dii.uchile.cl [Epstein]; lysette.henriquez@un.org.mx [Henríquez]; jcatalan@dii.uchile.cl [Catalán]; gweintra@dii.uchile.cl [Weintraub]; cmartinez@junaeb.cl [Martínez]; fespejo@junaeb.cl [Espejo]*

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**Abstract**

The Chilean State delivers essential meal services at schools for low-income students. Junta Nacional de Auxilio Escolar y Becas, the institution in charge of covering 1,300,000 children, leases the meal service to private enterprises. We developed an integer linear programming model to assign the meal contracts, in a process known as combinatorial auctions. The resulting model, which is NP-hard, led to significant improvements in efficiency and also contributed to making the process more transparent. The results are apparent in substantial improvements in quality and coverage of the service, and important savings to the country, which are equivalent to feeding 300,000 children in addition. We developed techniques to solve the combinatorial models and also to analyze and compare multiple scenarios to find robust solutions. For the objective function of this problem, we analyzed several options to consider different kinds of social benefits. In this paper, we describe the problem, the methodology and the results. We also present empirical results based on 6 years of experience. Finally, we discuss the relevance and impact of using operations research in these central issues in developing countries.

*Keywords:* Combinatorial auctions; public policy

## 1 Introduction

'There is no such thing as a free lunch', goes a saying beloved by economists. Nothing is free in this life. But Chile's National Board responsible for school grants and assistance (Junta Nacional


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de Auxilio Escolar y Becas (JUNAEB)), is an exception that proves the rule, providing daily breakfasts and lunches for 1,300,000 school children, on a budget of US$180 million.

JUNAEB Directors and Academics with the University of Chile's Department of Industrial Engineering have developed and applied an operational research tool that has made it possible to save US$40 million a year, improving ration quality and service coverage. JUNAEB applied this tool for the first time in 1997 and in every auction since.

Chile is a developing country with 15 million inhabitants. The average annual economic growth during the 1990s was 6.3%, which boosted per capita income from US$2,768 in 1990 to US$4,603 in 2000. During this same period, poverty fell from 39% to 20%, and from 50% to 30% in the case of minors under 18 years of age. Primary school coverage (children from 6 to 14 years of age) has reached 97%, while secondary school enrollment (young people from 15 to 18 years) is 84%. Illiteracy in Chile stands at 4.4%. In short, Chile has progressed significantly in the past decade and this has made it possible to improve the social conditions of the poorest of its people; nonetheless, 30% of children still live in poverty.

In this context, the 70% rise in state spending on education during the decade is understandable. Chile's development strategy is partly based on improving the education of the poorest children. The purpose of the JUNAEB, a public agency associated with the education sector, is to provide children from the lowest income households with meals at school, during the school day (breakfast, lunch and dinner). This national service, provided at no cost to students, has been crucial in reducing school dropout rates and in improving the academic achievements of the children receiving these benefits.

The JUNAEB's annual budget for all social programs is US$150 million (year 2000), with US$138 million spent on the school food program (Programa de Alimentación Escolar (PAE)), which benefits 1,200,000 children. The JUNAEB has other programs, for example providing eyeglasses to students, at a cost of US$3 million per year. The JUNAEB is also responsible for auctioning food programs associated with two other agencies, JUNJI and INTEGRA, which are responsible for child care (infants and children from 4 to 5 years). Together, these two agencies serve 126,000 children with an annual budget of US$46 million. These auctions are very important to the country, not only because of the amounts involved but also because of the social importance of the service involved: providing food to Chile's most vulnerable children studying in schools.

The JUNAEB has granted concessions to these food services to private companies through public auctions since 1980. The number of companies bidding has risen from three in 1980 to 35 in 2002. The country is divided into 90 territorial units (TUs) and each JUNAEB auctions off service to 30 of these, with contracts lasting 3 years.

These auctions are very complex. In the first place, different kinds of foods must be provided according to the children's age, the school and their economic situation. The different regional realities must also be considered, involving school access and food habits. Moreover, these auctions involve 3-year contracts but the unforeseen does occur. Finally, we must deal with very tight budgets on one hand and the need to maximize service quality on the other.

Before 1997, contracts were assigned by a committee applying rather rudimentary and subjective criteria. Basically, a series of filters based on financial and technical criteria were applied, which eliminated some offers. Thus, an iterative method was used to eliminate possible options until a solution, supposedly of good quality, was obtained. Given the complexity of auctions, this solution was obviously not very efficient and, even worse, permitted all kinds of pressure on the committee. In auctions of this scope, it is essential to have objective, transparent procedures in place to reduce possible pressures, be they legitimate or otherwise, and force companies to compete in quality and prices (Klemperer, 2002).

Every company participating in an auction provides a technical offer and several financial offers. Each financial offer includes a set of TUs and a set of prices for the services bid on. Each financial offer is accepted or rejected as a whole and companies can provide all the financial offers they wish. Freedom to offer on different packages of TUs, gives the auction a combinatorial nature.

In technical terms, we developed a mixed integer programming model in which the offers to be selected are the decision variables. This problem is hard to solve. In mathematical terms, it belongs to the class NP-hard and includes classical combinatorial structures: the set-covering problem, the knapsack problem, and the uncapacitated facility-location problem. We use sophisticated optimization techniques to solve real instances and the system runs on a personal computer, using the CPLEX optimization program.

The methodology consists of analyzing many scenarios, differentiated by cost, quality, price bands, menus and other operating aspects. Using the mathematical model, we found the optimal solution for each scenario. The inter-ministerial committee that decides the auction evaluates the different scenarios and, based on the budget available and long-term policies, chooses an optimal solution and scenario for the auction result. The committee pays special attention to ensure that the solution, which is the set of offers selected, is coherent and robust for the country as a whole, that is, promises efficiency in every scenario that could appear during the 3-year contract.

The new auction system encourages efficiency, transparency and fairness. For example, the possibility that companies have to develop suitable territorial offers has allowed them to reflect on the economies of scale in the prices they offer, and thus make the auction more efficient. These savings occur for many reasons: already depreciated infrastructure, transportation, discounts for higher volumes, knowledge of the region, and so on. The system, moreover, is efficient because given the assumptions, the mathematical model makes it possible to identify the best combination of offers.

For this process to be transparent, every step of the process must be objective, specifically, the financial evaluation of companies, performance evaluations, standardization of technical offers, adjudication and other criteria involved. Each step can be audited and the whole process duplicated, which guarantees seriousness, a factor that companies appreciate.

Standardizing technical offers involves defining the composition of food rations. Additional services, such as more frequent fruit, are valued separately in the financial offer. Thus, the adjudication committee can compare products with similar characteristics, and if the budget permits, choose enriched rations. In the past, the quality of offers and prices varied, making it difficult to compare. Today, all the companies offer the same products.

Finally, the issue of fairness arises, as a consequence of efficiency and transparency. Companies must perceive that the process is fair, that is, that everyone had the same chance and that the decision was made in terms of the common good. To guarantee fairness it is vital to select the best solution according to auction rules. For example, if the rules call for choosing the least expensive option, it would be unfair to select anything but the least expensive offer. Under the new auction system, the possibilities of exercising undue pressure on staff are virtually non-existent and companies must concentrate on improving their efficiency to offer the best price and service. This transparency favors the entry of new companies into the system, because they can evaluate this business possibility objectively.

Of the savings reported by the JUNAEB, 60% arise due to greater transparency and healthy competition among companies. The other 40% reflects greater efficiency introduced into the process, through territorial flexibility and optimization models.

This application has stimulated interest in other developing countries that face very similar problems. The United Nations' Food and Agriculture Organization considers the JUNAEB an international example of good management and efficiency. The Chilean government consider JUNAEB excellent. Moreover, the ILOG technology firm, owner of the CPLEX optimization software, is publicizing this experience as a successful case and example for other combinatorial auctions.

In Section 2, we review the state of the art of combinatorial auctions, evaluating the main applications. Section 3 provides a detailed description of the auction process. In Section 4 we formulate and analyze the mathematical programming model. In Section 5, we provide details of implementation, and Section 6 offers results and conclusions. Finally, Section 7 shows how this work is related to OR in development.

## 2 Bibliographic review

In the so-called 'combinatorial' auctions, bidders are encouraged to present offers for different product combinations, to take advantage of their complementary nature (e.g., rights to a single transmission frequency for neighboring geographic regions) or substitution (e.g., rights to different transmission frequencies in a single geographic region) for different products, or economies of scale (e.g., volume discounts, geographic proximity, transport efficiency). This type of auction generally arises when each resource is just one part of a possible solution and only product packages have any real value. We now review the applications reported in the literature.

Combinatorial auctions have been of increasing interest in the literature in the last 10 years since the spectrum auction done by the FCC of the United States (McMillan, 1994). Publications done by Vickrey (1961) in the 1960s and Jackson (1976) in the 1970s, for adjudicating radio frequency rights, are pioneering works in the area.

Despite the technical advantages of combinatorial auctions, as Kelly and Steinberg (2000) point out in practice we see few applications of this sort. The main implementation problems arise from the complexity of administering combinatorial auctions. Bidders have to evaluate offer combinations with enormous precision to take full advantage of synergies and economies of scale. Secondly, there is a large universe of offers that companies can make for subsets of goods to be auctioned, and this increases the size of the problem exponentially. Typically, the resulting problem is in theory NP-hard and in practice proves difficult to solve.

Andersson et al. (2000) compare the results from recent adjudication heuristics and traditional heuristics. They also compare these results with those from a mixed integer programming approach. In a similar study, Fujishima et al. (1999) compare a mathematical programming approach that ensures an optimal solution with a market-based heuristic method. Nisan (2000) studies the problem of representing the offer function. The author formalized multiple representation languages and analyzes strengths and weaknesses.

Baird (1984) describes an auction that attempts to reduce the levels of fish catches when the participants present offers corresponding to the price at which they would be willing to maintain their catch levels. Brewer and Plott (1996) study the application of combinatorial type auctions to the adjudication of railway line segments. Letchford (1996) describes an auction to hire school bus operators, which allows discounts for those bidding on more than one contract. In this case, the instance was relatively small: 201 individual and three group offers.

Progress in combinatorial algorithms and computer-processing capacity has contributed to the rebirth of combinatorial auctions. The consulting firm, Saitech-INC, offers a software called SBIDS for auctioning off packages of routes among transportation companies. The OptiBid™ system, from Logistics.com, has been used for transportation contract auctions for companies including Ford, Wal-Mart and K-Mart, worth US$5 billion through January 2000 (Vries and Vohra, forthcoming). As far as we know, there are not many more real world problems where a combinational auction has been successfully used.

In our case, companies participating in the JUNAEB auction are encouraged to present offers for sets of TUs, to take advantage of economies of scale. If the JUNAEB required companies to present separate offers for each TU, they would run the risk of 'exposure' (Rothkopf et al., 1997). If they offer a cheap price for each individual TU, thinking they will adjudicate the whole set, they may lose money if they adjudicate just one. However, if they offer higher prices for each TU, consistent with adjudicating just one, their chances of winning several bids decline.

## 3 The auction process

Chile's educational system consists of more than 14,000 schools grouped into 90 TUs, which in turn fall under the country's 13 administrative regions (see Fig. 1). Every year, the JUNAEB, JUNJI and INTEGRA auction off the food services for 30 TUs for the 3-year period.

The JUNAEB school meals program is divided into five subprograms: pre-school education, primary education, secondary education, vacations and households. Pre-school and primary education programs are compulsory. Pre-school education lasts 2 years, primary education 8, and secondary education 4. JUNJI has a subprogram for meals at child-care facilities, while INTEGRA has a subprogram for meals at infant-care facilities.

Each subprogram includes different meals, with different nutritional and caloric contents. For example, the high-school subprogram involves providing four types of rations, the RM-350 (breakfast, 350 cal), the RM-650 (lunch, 650 cal), the RM-800 (lunch, 800 cal) and the RM-1000 (dinner, 1000 cal). Each TU has demand for each type of ration in each subprogram during the auction period.

Companies also bid on 'extra' services, which may be included with regular products for the JUNAEB programs. Examples of these services include more frequent vegetables, steel trays and glasses made of glass instead of plastic.

The auction process involves six main steps that must be strictly followed in terms of order and timing. The first is registration of firms, followed by their classification. Companies next submit their bids, including a technical project and financial bids. The process ends with assigning the

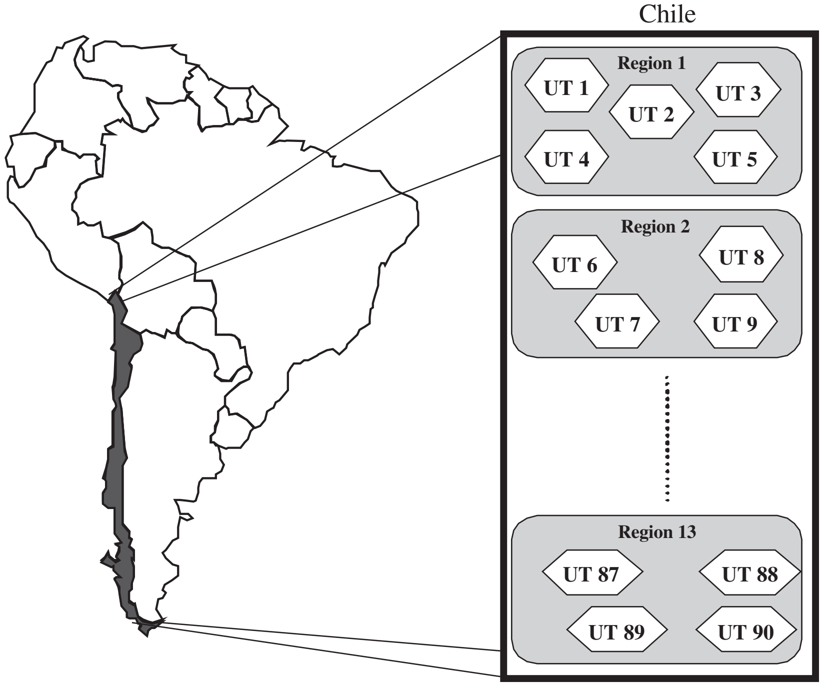


Fig. 1. Chile's regional divisions, which in turn are divided into territorial units (TUs), averaging seven TUs per region.

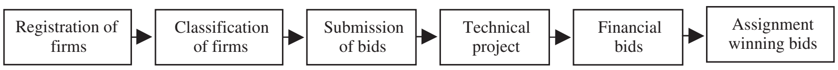


Fig. 2. Chronological chart of the auction process.

winning bids, using the mathematical model to evaluate different scenarios. In Fig. 2, we present a chronological chart of the main activities in this new auction process. We describe the first five steps in this section.

### 3.1 *Registration of firms*

The process begins with a call for expressions of interest and registration of interested firms. JUNAEB then evaluates the interested parties from a managerial, technical, legal and financial points of view, eliminates those who do not satisfy minimum standards, and assesses the capacities of the remaining.

### 3.2 *Classification of firms*

The firms qualifying as reliable are then classified according to two further aspects:

- *Financial and operating capacity*: firms are sorted into one of five categories, according to the maximum number of TUs they would be capable of serving, anywhere between one and eight TUs.
- JUNAEB also evaluates the past performance of companies providing food services. The result is a performance index based on multiple factors, such as surveys of the children, warehouse infrastructure, the cold chain, and others. This performance index is used in the optimization model objective function; so companies consider it very important. This evaluation is subject to pre-established rules and there is an appeals procedure if the company disagrees with its evaluation.

### *3.3 Submission of bids*

Following this, a public call to tender is made and the ground rules for the bidding process are put on sale. Potential concession holders (currently numbering about 35) then submit their bids in electronic format, on CDs. This facilitates company submissions and data input for the assignment model. Each CD receives a code and the bidder's name code key is deposited with a notary; so evaluators do not know who the winner is until they award the contract. Each firm presents a technical project for the meals service and different financial bids.

### *3.4 Technical project*

The technical project is based on requirements clearly established by the JUNAEB, which include:

- *Nutritional requirements*: Different meals must comply with their respective nutritional specifications.
- *Food structure*: Specifies the type of service (lunch, breakfast, etc.) and the frequency (or minimum and maximum presence) of certain foods, and the minimum variety required in the meals provided.
- *Inputs*: Specifies the minimum quality characteristics.
- *Operating conditions*: Specifies hygiene, supplies, food handling, supervision, etc.
- *Infrastructure*: Specifies furniture, equipment and crockery, etc.

The JUNAEB assesses whether each firm meets the requirements. Firms meeting these requirements remain in the bidding process, going on to compete on the basis of price and performance, through their respective financial bids.

### *3.5 Financial bids*

Each financial bid presented by a firm specifies the coverage, from one to eight TUs, depending on the upper limit permitted by the company's classification. As mentioned, firms can present as many bids as they wish.

Companies can raise the unit price if demand during contract execution is less than projected by the JUNAEB, affecting the structure of fixed and variable service costs. The auction includes prices for four demand tranches. The first is projected demand; the second operates if demand is

80% of projections; the third comes into force if demand is 60% of projections. The fourth comes into effect if demand is more than 4% above projections. The second and third tranches include surcharges on the unit fee while the fourth tranche involves a discount. Changes in demand can occur due to natural disasters, strikes or other unpredictable events. This differential price system means that companies do not have to assume risks that have nothing to do with their management.

If the different programs, rations, structures and tranches are considered, we can see that an offer could include about 200 different prices. The offer is accepted or rejected as a whole. If the offer is accepted, the JUNAEB can specify the products and services hired, from those included in the offer, dynamically during the 3-year duration of the contract. Accepting a firm's bid requires that the firm provide all meals services to the corresponding TUs.

Given the different services provided, there are 30 meal types, each defined by the calories to be provided and detailed nutritional requirements. For example, meal B350 is a 350-cal breakfast for primary school children, while meal M1000 is a 1000-cal lunch for secondary school children. For each, nutritionists have specified three food structures using different possible food combinations to meet the required number of calories. These food structures vary in quality, thereby enabling JUNAEB to price a variety of products, some of which are better than others. For example, for a 650-cal lunch structure specifies meat (beef, chicken, turkey, pork or mutton in different forms) ten times a month, while another specifies meat 14 times a month. Thus, each product consists of meal fully defined by the JUNAEB nutrition team, which establishes the number of calories, nutritional requirements and the frequency of required foods.

To evaluate each offer, we considered the different scenarios that can affect contract implementation, particularly demand tranches and food structures. Each TU has projected demand for the different ration types in each subprogram. Therefore, the number of rations and their unit value depends on demand and the food structure that we choose for the scenario.

Once these variables are defined we can evaluate, an offer by calculating the unit price for each service and multiplying it by demand for that scenario in the TUs included in the offer. The sum of these values corresponds to the value of the offer in that scenario.

Once offers have been evaluated, the adjudication process begins. The selection of winning offers is done by an adjudication committee formed by the ministries of finance, health care, education and the main executives within the JUNAEB.

## 4 Combinatorial analysis and model

The concession lasts 3 years and can face different scenarios. The methodology for assigning contracts consists of choosing robust solutions, that is, those that not only meet the technical and financial constraints, but also are among the best in all or in most of the scenarios that may occur during this period. The factors defining different scenarios are as follows:

(i) *Objective function*. We can optimize considering only costs and performances for JUNAEB, or considering parameters for the three institutions: JUNAEB, JUNJI and INTEGRA (two options).

(ii) *Company performance*. We can minimize auction costs or optimize a mixed function that also includes the company performance index (two options).
(iii) *Food structure*. We optimize different food structures, to examine the financial feasibility of improving service. Moreover, we consider different combinations of extra services, e.g. more fruit and vegetables (seven options).
(iv) *Demand tranches*. To evaluate the robustness of a solution, we must consider demand tranches that could affect contract operation for 3 years (four options).
(v) *Price bands*. Offers that are too cheap may cause companies to go broke and serious problems in replacing the service, as has often occurred. Because of this, the JUNAEB is inclined to eliminate the riskiest offers, thus defining two options, one involving the participation of the too cheap offers and the other with their elimination, *a priori*. To identify the too cheap offers, regional level statistics are used.
(vi) *Limit of companies per region*. The JUNAEB considers it appropriate that a minimum number of firms operate in a single region, to ensure that there are options nearby in case some company goes broke. Having too many companies per region is not a good idea either, because the administration becomes more costly to the JUNAEB. Because imposing this limitation involves a cost, scenarios with and without this restriction are evaluated (two options).
(vii) *Solution dispersion*. To keep companies working for JUNAEB in the long term and to generate more competition among firms, the board prefers to employ several companies. Because this involves a cost, optimization must consider conditions with and without a minimum number within the scenarios (two options).

We have considered 500–700 scenarios in evaluating auctions, from the maximum of 896 that would appear with all the combinations, given that the committee considered some unlikely. We built an integer linear programming model to find the optimal solution for each scenario. First we analyzed the constraints and then studied the objective function.

Let $K$ represent the set of companies participating in the auction, $I$ the set of TUs to auction, $J$ the set of financial bids, $R$ the set of the country's regions and $S$ the set of scenarios to be evaluated. Let $J$ be the set of financial bids, $J_k$ the bids from company $k$, $J^i$ the bids including the TU $i$, $J_{kr}$ the bids of company $k$ that include some TUs in region $r$. We call $u_j$ the set of TUs included in offer $j$, $n_j^s$ the number of rations provided by offer $j$ under contract in scenario $s$, and $c_j^s$ the cost of offer $j$ in scenario $s$. Let $cap(k)$ represent the maximum number of TUs in which company $k$ can obtain the concession, $N^s$ the total number of rations included in the auction within scenario $s$. Given that the number of rations and the unit price depend on the scenario, we will call the budget for scenario $s$, $budget(s)$. For a set $A$, its cardinality is denoted by $|A|$. The main decision variable is $X_j$ which is 1 if bid $j$ is selected and 0 otherwise.

### 4.1 Constraints

The following equation proposes that all TUs territorial units must be auctioned:

$$\sum_{j \in J^i} X_j \geqslant 1 \quad \text{for all } i \in I. \tag{1}$$

This is the structure of a classical set-covering problem, where the bids are the covering elements and the TUs the objects to be covered. Our case is known as a weighted set-covering problem because the objective function includes different weights for the decision variables.

The set-covering problem belongs to the NP-hard class the most difficult combinatorial problems. Fulkerson et al. (1974) present a set of difficult instances for the covering problem. Avis (1980) shows that for these instances a branch-and-bound algorithm requires exploring an exponential number of solutions. Is it not efficient to find approximate solutions. Chvatal (1979), based on work of Johnson (1974) and Lovasz (1975), designed a heuristic with guaranteed error bounded by the harmonic series $H(t)$ equal to $\sum_{i=1}^{t} i^{-1}$, where $t$ is the maximum number of elements covered by some covering object. Lund and Yannakakis (1993) demonstrated that a better approximation is unlikely to be found.

Chvatal's heuristic consists of selecting the covering element that satisfies the most constraints, not previously covered, per cost unit. That is, this is equivalent to the intuitive method of choosing the offer that covers more TUs per cost unit, after eliminating these TUs from the problem and repeating the procedure until completing a covering for the problem. Considering the maximum of eight TUs per offer, the theory indicates that this method could provide a solution that amounts to 2.72 times the optimal solution.

In the last auction in 2002, 30 TUs were adjudicated and more than 25,000 offers arrived, for a covering of 25,000 columns and 30 rows. What could give rise to concern about the algorithmic complexity is the number of offers and their constant growth over time, up from 4,500 offers in 1997. However, the number of TUs to be assigned remains 30 and this will not change in the short term.

An initial combinatorial analysis indicates that any solution selects at most 30 offers, the number of TUs. Therefore, the number of solutions is limited by the number of subsets of offers involving at most 30 elements, which is a polynomial $O(|J|^{30})$. This observation means that strictly speaking our covering problem with a fixed number of TUs is not NP-hard.

A more finely tuned combinatorial analysis shows that the limited number of companies, about 35, also simplified our covering problem. Let us consider the simplified problem of assigning $a$ TUs among $b$ companies, with no capacity or other type of restriction. The number of different solutions is $b^a$, given that TUs are distinguishable and each can be assigned to any of $b$ companies. This result, where the number of offers does not explicitly appear, is very interesting because it is consistent with our experience: we went from 4,500 to 25,000 offers, but the complexity did not rise significantly. In the same period, the number of TUs and companies remained relatively constant.

Companies' financial capacity is reflected by the following constraint,

$$\sum_{j \in J_k} |u_j| \cdot X_j \leqslant cap(k) \quad \text{for all } k \in K. \tag{2}$$

Each of these restrictions corresponds to a knapsack problem, so altogether these are known as multi-knapsack. A cover for company $k$ is a subset of offers from $J_k$, (which we call $cov_k$) such that the TUs included in these offers exceed $cap(k)$ but meets the limit if we eliminate any of its offers. Therefore, the following cover constraint is valid for the multi-knapsack,

$$\sum_{j \in cov_k} X_j \leqslant |cov_k| - 1 \quad \text{for all } k \in K.$$

The literature reports cover inequalities as an effective way of reinforcing formulation (Crowder et al., 1983). We implement a variant that we call cover-clique inequality, as follows. Let $T_k$ represent the set of offers from company $k$ that includes a number of TUs greater than half its capacity. Then, for all $j1, j2$ in $T_k$ we see that a cover inequality is $X_{j1} + X_{j2} \leqslant 1$. These constraints form a clique, a complete graph for the maximum stable set problem, which can more strongly be expressed as:

$$\sum_{j \in T_k} X_j \leqslant 1 \quad \text{for all } k \in K. \tag{2a}$$

Cover-clique inequalities were very useful for resolving difficult instances within the auction. We note that cover-clique inequalities are better cuts than cover inequalities alone.

To apply a minimum and maximum number of companies per region, we have to define the following variable $Y_{kr}$ that takes the value 1 if company $k$ serves some TU in region $r$ and 0 otherwise. Let $mnfr(r)$ be the minimum number of companies in region $r$ required by the JUNAEB and $mxfr(r)$, the maximum. The following equation expresses this constraint:

$$mnfr(r) \leqslant \sum_{k \in K} Y_{kr} \leqslant mxfr(r) \quad \text{for all } r \in R. \tag{3a}$$

We need to relate $Y$ with $X$ variables. In the first place, we must require that the variable $Y_{kr}$ is worth zero when there are no active offers from company $k$ in region $r$. This equation would be:

$$Y_{kr} \leqslant \sum_{j \in J_{kr}} X_j \quad \text{for all } k \in K, \text{ for all } r \in R. \tag{3b}$$

Secondly, it must be required that if an offer from company $k$ including some TU in region $r$ is selected, the respective $Y$ variable is worth one. The following equation imposes this:

$$\sum_{j \in J_{kr}} X_j \leqslant |J_{kr}| \cdot Y_{kr} \quad \text{for all } k \in K, \text{for all } r \in R. \tag{3c}$$

Inequality (3c) appears in problems involving the uncapacitated facility location. An equivalent formulation (see Balakrishnan et al., 1989), would be:

$$X_j \leqslant Y_{kr} \quad \text{for all } k \in K, \text{ for all } r \in R, \text{ for all } j \in J_{kr}. \tag{3d}$$

We refer to formulation (3c) as 'compact', while formulation (3d) will be considered 'extended'. The linear relaxation of the extended formulation is better than that for the compact one, in the sense that it will tend to be more whole. Inequality (3c) amounts to $Y_{kr}$ being more than or equal to the average for the $X$ variables involved, which in general is a small fraction because although there are many of these variables only a few are active in the solution.

Moreover, inequality (3d) means that $Y_{kr}$ is greater than or equal to the maximum of $X$ variables involved, which clearly provides a larger value for $Y_{kr}$ if any of the $X$ variables is activated. When the upper bound $mxfr(r)$ is active, the compact formulation is very poor and possible convergence in a branch-and-bound method will be very slow. In this case, it seems reasonable to use the extended formulation. Similarly, if $mxfr(r)$ is not an active limit, it seems better to use the compact formulation, because the linear relaxation is significantly faster to solve and the reinforcement provided by (3d) is unnecessary.

To impose a minimum number of companies to be assigned through the auction, we need to define the $Z_k$ variable, which is 1 if company $k$ presents a winning offer and 0 otherwise. Let *mnfa* be the minimum number of companies presenting a winning bid. The constraint imposing the limit is:

$$mnfa \leqslant \sum_{k \in K} Z_k. \tag{4a}$$

To relate $Z$ and $Y$ variables, we impose the following constraint, similar to (3b):

$$Z_k \leqslant \sum_{r \in R} Y_{kr} \quad \text{for all}\, k \in K. \tag{4b}$$

Finally, we require variables to be binary,

$$X,\, Y,\, Z \in \{0,\, 1\}. \tag{5}$$

In the 2002 auction, 33 companies participated, presenting 25,000 offers for 30 TUs in seven of Chile's regions. The model had 25,264 binary variables. The compact formulation had 573 constraints, considering formulation (3a) as two constraints and without considering cover-clique inequalities or budget restrictions, which are added below. The extended formulation contains 25,342 constraints of which 25,000 are class (3d), which reveals the difference in size among formulations. The model contains three known combinatorial structures, each NP-hard. These are set-covering, multi-knapsack and uncapacitated facility location. The resulting model is also NP-hard given that it contains as a special case the set-covering problem, when bounds *mnfr*(*r*) and *mnfa* are 0, and bounds *cap*(*k*), *mxfr*(*r*) are infinite.

### 4.2 Objective function

First we minimize the cost of scenario $s$, which we call $f_0^s$. Therefore,

$$f_0^s = \min \sum_{j \in J} c_j^s \cdot X_j.$$

Then we seek an objective function that includes companies' past performance in applying contracts, as a powerful long-term sign to encourage companies to ensure they get good service evaluations. In this case, we add the budget constraint,

$$\sum_{j \in J} c_j^s \cdot X_j \leqslant budget(s). \tag{6}$$

Let $r_k$ represent the performance index for company $k$. One possibility is to maximize the arithmetic mean of the ration performance index, which we call $f_1^s$, which would be equal to:

$$f_1^s = \frac{1}{N^s} \cdot \max \sum_{k \in K} \sum_{j \in J_k} r_k \cdot n_j^s \cdot X_j.$$

This function presents a serious problem, which is that it assumes that the lack of quality in one region is offset in a linear fashion by excess quality in another. The following example illustrates that this criterion is not very appropriate in this application. Suppose we hold an auction including regions 1 and 2, involving 100 rations each. Company A's performance index is 200 and it offers \$45 for region 1. Company B's performance index is 800 and it offers \$45 for region 2. Company Q's performance index is 450 and it offers \$92 for regions 1 and 2. The institution's budget is \$95. If we apply $f_1^s$ we would end up selecting companies A and B in regions 1 and 2, respectively. Evidently, region 1 would receive service from a poorer performer and region 2 from a better performing company.

Another alternative would be to maximize $f_2^s$, the geometric mean of the rations assigned, shown as follows:

$$f_2^s = \max \left( \prod_{k \in K} \prod_{j \in J_k} r_k^{n_j^s \cdot X_j} \right)^{\frac{1}{N^s}}.$$

If we take the logarithm, we obtain the equivalent problem:

$$f_2^s = \frac{1}{N^s} \cdot \max \sum_{k \in K} \sum_{j \in J_k} n_j^s \cdot \log(r_k) \cdot X_j$$

If we apply $f_2^s$ to the previous example, the solution would be to give regions 1 and 2 to company Q. This function, however, presents some practical problems. First, professionals who are not engineers or mathematicians, such as nutritionists, lawyers, physicians or teachers, find it difficult to understand this. Another problem is that the cost does not explicitly appear in the objective function, as we note in the following sections.

In search of a balance in this sense, we propose the following objective function: $f_3^s$ and present here:

$$f_3^s = \min \sum_{k \in K} \sum_{j \in J_k} \frac{c_j^s}{r_k} \cdot X_j.$$

We implement $f_3^s$ for two reasons. First, the cost of the offer, unlike in the previous case, forms part of the objective function. Professionals with no understanding of OR can interpret the cost as unimportant when it does not explicitly appear in the objective function. It can be very difficult for them to interpret the role of the budget constraint plays on equilibrium. In these cases, we believe there is a chance that companies will underestimate the importance of cost of this process and will tend to raise their prices. Second, it is easy for any professional to interpret the effect of the performance index on the selection process: if the allocation falls within the budgeted amount, a company with a performance index that is $i$% better than another's may be $i$% more expensive and the offers will be equivalent. Obviously, an active budgetary constraint favors the cheapest offer. Moreover, the function $f_3^s$ also tends to exclude very low indices. In the previous example, $f_3^s$ selects company Q, as does $f_2^s$.

## 5 Solution and implementation

The process of evaluating offers and adjudication cannot take more than one week, for practical and legal reasons. The analysis consists of evaluating the offers according to the scenario and then finding the optimal solution for each scenario. It is essential to do this in no more than 24 h, because normally during the week unexpected events occur and some evaluation parameters change, such as the budget or the number of rations. If we consider 700 scenarios, we see that the resolution for each should take 2 min on average. The critical element is to find the optimal solution, given that evaluating offers. Preparing reports and comparison tables are automated and very quick.

To solve these instances efficiently we first apply a simple algorithm with a 1-min time limit. This consists of resolving the compact formulation with no additional cuts. Several scenarios are solved on the first step. We used CPLEX with the branch-and-bound method to solve, in its different version according to the auction year.

To solve the remaining scenarios, we use a more sophisticated version of the method, adding cover-clique inequalities, also with a time limit (2 min). For the scenarios that remain unsolved we again make the algorithm more sophisticated, using the extended formulation and a new time limit (5 min). At this stage, all scenarios may end up solved or a couple may remain pending.

Another key to this strategy is that it is possible to parallelize the work on several different computers. On some occasions, we have worked on four computers at the same time. The two or three most difficult instances that eventually remain unsolved are dealt with on individual computers. We have worked with an optimality tolerance of 0.01%, given the amounts involved. However, assigning a suboptimum solution can involve enormous costs in terms of the process image. Although the difference in the objective function is limited to 0.01%, companies' participation in alternative solutions may end up very different. One company, for example, may in the best case adjudicate many rations and in the approximate solution none or very few. This would be unacceptable and unfair, damaging the credibility of the process. Because of this, we solve to optimality all scenarios with any probability of being chosen.

The empirical evidence indicates that it is best to start with the compact formulation because its linear relaxation is solved so much more quickly than the extended formulation or that with the cover-clique inequalities, although the solution of the compact formulation tends to be more fractional. The compact formulation is very efficient for resolving easy instances. Moreover, some instances take longer to solve with the sophisticated algorithm, while the simple algorithm may solve it right away. Bixby (2002) reports similar results.

Table 1
Resolution time (seconds) for 10 random instances using different versions of CPLEX

| Problem | CPLEX Version | | |
|---|---|---|---|
| | 8.0.1 | 3.0 | 2.2 |
| 0008.sav.gz | 25.86 | 204.89 | 202.09 |
| 0016.sav.gz | 19.98 | 72.67 | 65.63 |
| 0076.sav.gz | 13.06 | 15.78 | 150.63 |
| 0078.sav.gz | 15.15 | 28.11 | 44.61 |
| 0079.sav.gz | 213.48 | 534.00 | 3897.33 |
| 0142.sav.gz | 66.69 | 173.56 | 161.32 |
| 0143.sav.gz | 223.00 | 2347.30 | 15240.11 |
| 0144.sav.gz | 132.26 | 306.22 | 1994.62 |
| 0207.sav.gz | 245.59 | 550.45 | 5991.84 |
| 0208.sav.gz | 71.11 | 613.78 | 3004.83 |
| Geometric mean speedup | | 3.38 | 11.51 |

One issue to consider is how computers and optimization programs have evolved, in our case CPLEX, from 1997 to 2002. In 1997, we used CPLEX 2.2 and 3.0 on a 300-MHz PC Pentium II. In 2002, we ran CPLEX 8.0 on a 1.8-GHz Pentium 4 computer.

Many techniques that we describe to solve the model have actually been included in the latest versions of CPLEX. In particular, (3d) constraints, which are an extension of (3c) constraints and cover inequalities. Older versions of CPLEX automatically carried out some of these improvements, but very inefficiently and it was better to turn off this feature of the software to achieve better results. Bixby (2002) reports similar experiences when he experimented with 10 random instances using new and old CPLEX versions on a 667 MHz Alpha station. Resolution times reported in Table 1 clearly show that solution times have improved with the inclusion of techniques similar to those that we are reporting. As a reference, the most difficult problem in the 2002 auction took 20 min to solve using CPLEX 8.0 and a 1.8-GHz Pentium 4.

```
Num. Prob: 1        Instance: as1b0e1

PARAMETERS
 ----------
 1. Food Structure: Alt. 1 Standard
 2. Demand Level:   80-100 %
 3. Objective Function: JUNAEB
 4. Firm's Performance Rating: NOT CONSIDERED
 5. Predetermined Minimum Price: NO LIMIT
 6. Minimum Number of Firms: NO LIMIT
 7. Number of Firms per Region: LIMITED

 ADDITIONAL FEATURES
 --------------------
 DEFAULT

 COSTS
 -----
 Total JUNAEB+JUNJI+INTEGRA=   5785.48 mn (Under Budget:  503.13 mn)
 Total JUNAEB=                4278.16 mn (Under Budget:  434.31 mn)
 Total JUNJI =                 933.38 mn (Under Budget:   88.42 mn)
 Total INTEGRA=                573.94 mn (Under Budget:   -19.6 mn)
 Total Accepted bids:       6

 Firms per Region

 Region Firms LowBnd UpBnd
 ---------------------------
    8      2     1     3
    9      3     1     5
   10      2     1     5
 ---------------------------

OPTIMAL SOLUTION  (Managerial Index Average = 652.1)

Firm  Bid  #TU     Total   JUNAEB    JUNJI    INTEGRA  TU
                   ChP mn  ChP mn    ChP mn   ChP mn
 -----------------------------------------------------------------
 765  1090  3      702.03   507.90   89.70   104.43   902   903   904
 765  1401  1     1551.43  1245.60  149.40   156.43   906
 969   109  3      119.59    78.56   10.50    30.53  1001  1002  1004
 673   734  2      282.46   245.46   21.60    15.40   801   802
 753  3465  3     2425.10  1636.70  583.75   204.65   905  1004  1005
 753  3508  2      704.87   563.94   78.43    62.50   803   804
 -----------------------------------------------------------------
   Total          5785.48  4278.16  933.38   573.94
```

Fig. 3. Sample model solution report.

Table 2
Robustness of solutions

| $N$ | Food structure | Demand level | Objective function | Rating | Minimum price | Total min. firms | Min. firm per region | T2 (%) | T3 (%) | A1 standard (%) | A1 plus_4 (%) | A1 plus_2 (%) | A1 plus_3 (%) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | A1prepr1 | T1 | F1 | D0 | C1 | E0 | R1 | 0.0 | 0.0 | 1.4 | 1.0 | 0.4 | 2.0 |
| 2 | A1prepr1 | T1 | F1 | D0 | C1 | E1 | R1 | 0.0 | 0.0 | 1.4 | 1.0 | 0.4 | 2.0 |
| 3 | A1prepr1 | T1 | F1 | D1 | C1 | E0 | R1 | 0.0 | 0.0 | 1.4 | 1.0 | 0.4 | 2.0 |
| 4 | A1prepr1 | T1 | F1 | D1 | C1 | E1 | R1 | 0.0 | 0.0 | 1.4 | 1.0 | 0.4 | 2.0 |
| 5 | A1prepr2 | T1 | F1 | D0 | C1 | E0 | R1 | 3.8 | 6.1 | 7.5 | 8.5 | 7.0 | 7.9 |
| 6 | A1prepr2 | T1 | F1 | D0 | C1 | E1 | R1 | 3.8 | 6.1 | 7.5 | 8.5 | 7.0 | 7.9 |
| 7 | A1prepr2 | T1 | F1 | D1 | C1 | E0 | R1 | 0.6 | 1.2 | 2.0 | 2.1 | 1.1 | 2.6 |
| 8 | A1prepr2 | T1 | F1 | D1 | C1 | E1 | R1 | 0.6 | 1.2 | 2.0 | 2.1 | 1.1 | 2.6 |
| 9 | A2standa | T1 | F1 | D0 | C1 | E0 | R1 | 0.0 | 0.0 | 1.4 | 1.0 | 0.4 | 2.0 |
| 10 | A2standa | T1 | F1 | D0 | C1 | E1 | R1 | 0.0 | 0.0 | 1.4 | 1.0 | 0.4 | 2.0 |
| 11 | A2standa | T1 | F1 | D1 | C1 | E0 | R1 | 0.0 | 0.0 | 1.4 | 1.0 | 0.4 | 2.0 |
| 12 | A2standa | T1 | F1 | D1 | C1 | E1 | R1 | 0.0 | 0.0 | 1.4 | 1.0 | 0.4 | 2.0 |
| 13 | A2prepr1 | T1 | F1 | D0 | C1 | E0 | R1 | 0.0 | 0.0 | 1.4 | 1.0 | 0.4 | 2.0 |

For the optimal solution: indicates optimal deviation if applied to another scenario.

Figure 3 provides an example of how the optimal solution for a given scenario is reported. The report includes the parameters that defined the scenario, followed by a cost report and the respective deficits or surpluses in the budgets of the three institutions. Finally, details of the accepted offers appear, indicating the cost of each institution and the TUs involved. Other useful information that appears is the number of companies per region and the arithmetic average of the companies' performance index for this solution.

Table 2 is very useful because it makes it possible to examine the behavior of the optimal solution for a scenario if we apply it in other related scenarios. Each line indicates the parameters that define the scenario, which we will refer to as the baseline scenario. A related scenario has the same parameters as the baseline scenario, but for one. It then indicates how this optimal solution would shift if applied to a related scenario. For example, the baseline one includes the plus service R1. If we change to a related scenario that considers the plus service R2, the optimal solution of the baseline scenario would be 0.4% worse than the value of the optimal solution of the related scenario. This way we can evaluate how robust a solution is in different scenarios.

## 6 Conclusions

This adjudication method has been used from 1997 to 2002, with the exception of 1998, when there was no auction. The empirical results point to a significant improvement in this process where the OR tool is central to ensuring competence, transparency and fairness.

This new auction method overcame initial mistrust and today is accepted and valued by the JUNAEB and bidding firms, who see a transparent process that allows them to compete fairly. Since the implantation of this methodology, pressures on the adjudication committee have been

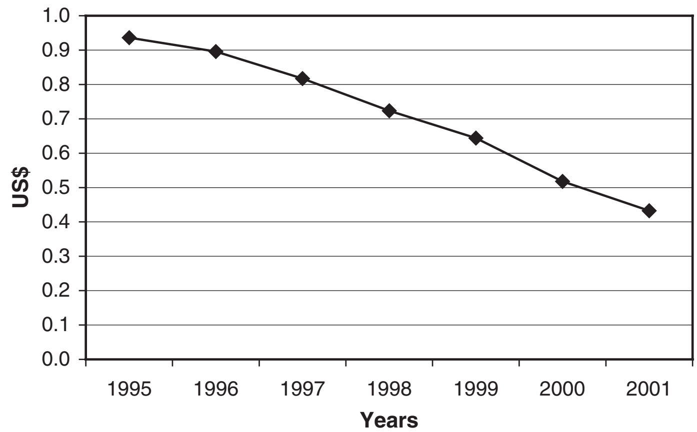


Fig. 4. Changing prices for the 700-cal ration.

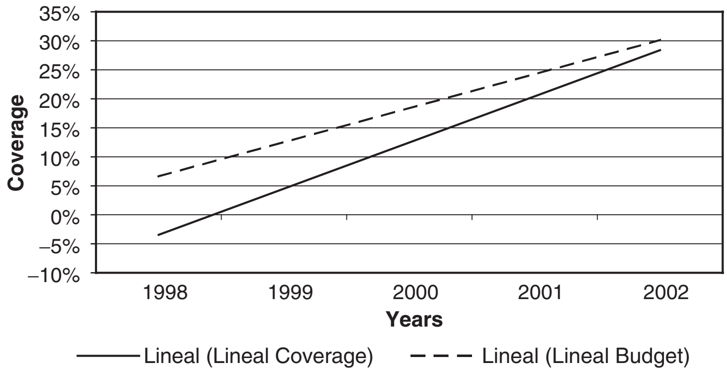


Fig. 5. Rising trends in budget and coverage percentages.

eliminated. The evaluation is so objective that the main disputes between companies and the JUNAEB focus on objective aspects such as the company's performance index. Even some catering firms, which initially opposed the system, have come to understand its benefits and now support the model.

As an early result, the JUNAEB reports annual savings of some US$40 million, which have been invested in improving coverage and ration quality. This amount appears comparing auctions in 1995 and 1999, which involved auctions for the same TUs, under the old and new methods (Epstein and Catalán, 1999, Epstein et al., 2002). As an example, Fig. 4 shows changes in the price for a 700-cal ration, the most typical within the system.

Moreover, this change made it possible to expand service coverage, making cost-cutting doubly significant, because the newly incorporated schools are those involving more difficult access and therefore higher costs. Figure 5 shows the trends in rising budgets and coverage, where this positive effect is apparent.

Food quality has also improved significantly. Breakfast ingredients changed from one serving of milk per week in 1995 to milk everyday; from bread twice a week to three times; and cereals were introduced. Lunch, the main meal also improved significantly, as is widely recognized by Ministry of Health and University authorities. Fresh salads rose from four to 14 every 20 days, meat from four servings to seven, fish remained of the same frequency but changed from canned to freshly caught. Fresh fruit frequency raised from six times a month to eight times a month. All these improvements involved real extra costs, but these were more than offset by savings from higher efficiency.

A key question deals with children's perception of the service, as they are the real customers. On a range of acceptability from one to seven, where one is the worst and seven the best rating, surveys indicate that ratings were between 5.1 and 6.2 in 1997, between 5.5 and 6.7 in 2000, and between 6.0 and 6.7 in 2001, a clear sign that companies have made an effort to improve their performance indices, a key variable within their management and adjudication of future contracts.

It is also important to analyze the changes in companies over these years. To improve productivity, they have invested in infrastructure, including the cold chain, air conditioning, stainless steel utensils, new kitchen utensils, water heaters and others. Employees have seen their wages rise 45% during this period. Even more surprising, perhaps, is that the return on capital rose from 28% in 1995 to 38% in 1999. The return on sales rose from 3.2% to 4.9% in the same period. We associate these positive effects with improved productivity.

The JUNAEB has received widespread recognition among public institutions for its optimal distribution of public resources among the poor. Indeed, 91% of these benefits go straight to students living below the poverty line.

## 7 OR in development

The objective of OR in development is to illustrate how the OR community actively participates in solving relevant problems in developing countries, particularly making management of public and private institutions more efficient. A scientific and efficient management of our institutions is our way to improve the quality of life of the people.

In developing countries such as ours, the low-income families are not able to educate and feed their children adequately. The role of the state is to support these children through efficient and focalized social policies. The school meal program is of crucial importance, probably the most important benefit for low-income family children. Many parents send their children to school motivated by the quality and quantity of the food they receive at school. Hence, the meal program not only improves the nutrition of these children, but also reduces school absenteeism. However, resources in developing countries are very limited, so it is vital for the state to assign them optimally. Money saved in this way means more and better nutrition for poor children and also better education.

Overall, we can say that this system offers a very good example of how an OR application can improve social policies. In this case, the system improved both competition and efficiency, and with them significantly boosted quality without adding costs.

We thank IFORS and OR in development for giving the researchers the opportunity to learn from valuable experiences from other countries, adding a lot of value to our work. The visibility that this application has had in our country because of OR in development has helped the whole

OR community in Chile to be better understood by public officials of the possibilities that our scientific approach can make.

The JUNAEB and the system today enjoy widespread national confidence and support. In a letter, then Minister of Education, Mariana Aylwin, wrote: 'The results from implementing this system were dramatic. The new procedure completely changed the character of the auction process.' The Food and Agriculture Organization is using Chile's JUNAEB as a model for school programs in other developing countries.

Transparency and competence are two master keys to achieving better productivity and more economic growth. As the JUNAEB experience reveals, the beneficiaries of greater productivity are not simply a couple of abstract macroeconomic indicators, but people of flesh and blood. How many free lunches still remain hidden within other public service or private company operations?

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