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Talking up liquidity: insider trading and investor relations

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Abstract

Managements ("insiders") of many corporations, especially small or newly-public firms, invest considerable resources in investor relations. We develop a model to explore the incentives of insiders to undertake such costly investments. We point out that insiders may undertake such investments not necessarily to improve the share price, but to enhance the liquidity of their block of shares. This leads to a divergence of interest between insiders and dispersed outside shareholders regarding investor relations. Our model predicts that the demographics of insiders (e.g. liquidity needs, size of equity stakes) are important determinants of the extent of investor relations across firms.

JEL classification: G0; G3

Keywords: Investor relations; Information disclosure; Insider trading; Liquidity; Newly-public firms

1. Introduction

Investor relations are a set of activities that firms engage in with investors and analysts. Such activities include voluntary information disclosures, competition for analyst coverages, and interactions with investors for the purpose of expanding the shareholder base.[1] Investor relations are costly; voluntary disclosures involve costs in producing and disseminating the information, and both courting analysts and attracting institutional investors

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[1] See, for example, Brennan and Tamarowski (2000) for a history and overview of investor relations.

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require a lot of time and effort from top executives, particularly for small or newly-public firms.[2] Yet despite such costs, many corporations, especially small or newly-public firms, are heavily engaged in investor relations. The purpose of this paper is to explore the reason behind such costly investor relations activities.

There are a number of theories for why firms invest considerable resources in investor relations. Broadly speaking, these theories argue that investor relations can boost the mean level of a firm's stock price because certain key assumptions of (strictly) efficient markets may not always hold. The proposed mechanisms for the positive effects of investor relations on stock prices include correction of mis-valuations by costly selective disclosures (Verrecchia, 1983, 1990), limited investor familiarity with firms (Merton, 1987), increased efficiency in firms' real investment (Fishman and Hagerty, 1989), reduced information asymmetry and cost of capital (Diamond and Verrecchia, 1991), and incomplete information and short-selling restrictions (Trueman, 1996). In these models, investments in investor relations benefit all existing shareholders and there is typically unanimity among them regarding a firm's investor relations policy.

Existing empirical studies on investor relations, however, have generated some findings that cannot be easily explained by these theories. First, the empirical literature has not convincingly established a strong causal relationship between investor relations and the long-term mean level of stock prices; the results in Byrd et al. (1993) and Botosan (1997), for example, are mixed at best. Secondly, there is strong evidence that demographics of insiders may be important determinants of the extent of investor relations across firms; Richardson et al. (2001), for example, found that earnings-guidance (a form of investor relations) is more prominent for firms whose insiders sell stocks from their personal accounts after earnings announcements.

Motivated in part by the above empirical evidence, we propose in this paper an alternative role of investor relations. From our perspective, it is interesting to look at the insiders' incentives to undertake such costly investments since insiders typically have discretion over the investor relations policy. In contrast to existing theories, we point out that insiders may undertake such investments not necessarily to improve share price, but to enhance the liquidity of their own block of shares in case they have to sell their equity stakes for liquidity (such as life cycle or diversification) reasons. In fact, insiders may engage in investor relations even absent any positive effects on the share price. Furthermore, there may exist a divergence of interests between the insider and dispersed outside shareholders regarding the investor relations policy. While the costs of investor relations are shared by all shareholders, the insider benefits from increased liquidity disproportionally since dispersed shareholders—with their small holdings—care little about market liquidity.

[2] For many small and newly-public firms, most of the costs of investor relations is due to the time and attention spent by top executives who are normally pressured to provide direct access to stock analysts and institutional investors. Although we are not aware of any formal study that documents such costs, a few senior stock analysts whom we interviewed estimated that, for the average small and newly-public firm (with a market capitalization less than $1.5 billion) that decides to engage in investor relations with analysts and institutional investors, investor relations activities account for about 20–25% of its CEO's time and about 50% of its CFO's time. Given that the time and attention of top executives are especially important for such young firms, investor relations activities are indeed quite costly for these firms. (See also Nocera (1997) for more details on corporate competition for coverage by top analysts.)

The premise of our model is relevant for many corporations. More than ever, managements are rewarded with large equity stakes through vested option grants, which they need to then liquidate. Since insider trading laws limit the selling of these stakes to only certain narrow time periods each year, insiders frequently worry about market liquidity during these periods (Richardson et al., 2001; Kahl et al., 2003). Our model is especially relevant for small or newly-public firms, which comprise an increasingly larger part of the market. The managements (or insiders) of these firms tend to own a substantial fraction of the equity shares of their companies,[3] and their equity stakes typically represent a substantial portion of their total wealth. So, they especially want to liquidate their shares for various diversification or life cycle reasons. Even absent any lock-up restrictions, many insiders quickly discover that their holdings are highly illiquid. This market illiquidity is not simply a wide bid–ask spread but rather a lack of market depth in that large trades have a large price impact. It is very difficult to sell a large block of shares of many newly-public companies without a significant price discount.[4]

Our model features the following essential elements. There is an all-equity firm with an insider (founder) who has a large block of shares and control rights to decide on the firm's investor-relations policy. The insider and some outside traders receive signals about future firm value on which they can speculate. At the same time, the insider experiences a random liquidity shock and may need to sell a significant fraction of his stock holdings for purely non-informational reasons. Because traders are asymmetrically informed, trades will have price impact because of adverse selection.

The insider decides on the firm's investor-relations policy before receiving any signals about future firm value.[5] By investing in investor relations, the insider increases the precision of a public signal about the firm value, which reduces the information asymmetry between the insiders and the market, and increases market liquidity.[6] Investor relations are costly, with higher costs leading to higher precisions of the public signal about the firm value. These costs come out of firm value and are thus borne by all shareholders.

[3] For example, Field and Hanka (2000) found that for a sample of 464 non-venture financed firms that went public during 1988 and 1991, executives of these companies on average owned 33% of the total shares outstanding one year after the IPO. See also Barry et al. (1990) and Megginson and Weiss (1991).

[4] In some cases, the amount of stocks that an insider can sell *at any price* may be limited. The SEC Rule 144, for example, caps what an insider who hold restricted stocks at a public company can sell over a three-month period at the greater of one percent of the total number of shares outstanding or, if the firm is listed on a stock exchange or quoted on Nasdaq, the average weekly trading volume over the four weeks preceding the sale; see, for example, Kahl et al. (2003) and Osborne (1982) for more details. Given the relatively low volume for small or newly-public firms, it may take insiders at these firms years to sell even a good portion of their stakes.

[5] Decisions on some investor relations activities, such as certain non-recurring discretionary disclosures, may be made *after* the insider obtains some key information on the firm's business fundamentals. But given the competition in courting analysts and attracting institutional investors, managements of small or newly-public firms often need to commit to a long-term investor relations strategy before the future prospect of the firm is known.

[6] This assumption follows the approach in Diamond and Verrecchia (1991) and Boot and Thakor (2001). Empirically, there is good evidence that investor relations stimulate analyst coverage which in turn improves market liquidity. For instance, Brennan and Subrahmanyam (1995) find that large trades move prices much less in stocks with higher analyst coverage. For additional empirical evidence, see Lev (1992), Byrd et al. (1993), Botosan (1997), Healy et al. (1995), Brennan and Tamarowski (2000), Lang and Lundholm (2000).

In deciding on these investments, the insider faces the following tradeoff. On the one hand, the insider shares not only the costs of a more precise public signal but also loses potential benefits from informed trading. On the other hand, the improved market liquidity means that his liquidity trades will have less price impact. Because investor relations are costly, the insider will not always choose to make such investments. But for wide ranges of parameters of our model, we show that the benefits of improved market liquidity for his liquidity trades are substantial enough that he engages in investor relations.

In contrast, dispersed shareholders with few shares in the firm care little about market depth. In equilibrium, the insider may invest excessively (relative to the benchmark of maximizing share value) in liquidity-enhancing activities at the expense of the longer-run value of the firm. This arises precisely because of his large block of shares relative to the holdings of dispersed shareholders and the limited liquidity of the stock.

Our model has several empirical implications. Some are consistent with the two empirical facts mentioned earlier, and some are yet to be tested. First, we show that firms that are more heavily engaged in investor relations may not necessarily have higher long-term stock prices (or lower costs of capital), all else equal. This is broadly consistent with the findings in Byrd et al. (1993) and Botosan (1997). Secondly, our model predicts that demographics of insiders, such as their equity stakes and liquidation needs, are important determinants of the extent of investor relations across firms. One component of this prediction—that we ought to see more investor relations for firms in which insiders are on average net sellers, all else equal—is indeed consistent with the findings in Richardson et al. (2001).[7] Other components of this prediction (discussed in Section 5.5) are yet to be tested.

A broader message of our model is that agency considerations are important for understanding why managements focus on investor relations. More specifically, our model is reminiscent of agency theories which argue that managers, with substantial equity stakes, consider personal risk when making decisions that affect firm risk (Jensen and Mecking, 1976). For instance, management may engage in negative-NPV diversification strategies from the perspective of shareholders because such strategies diversify their high equity stakes.[8] The downside of managers having too much equity is also a theme of our model.[9]

More generally, our paper argues that agency problems potentially underlie any corporate decisions that affect the market liquidity of firms' equities, especially for small or newly-public firms. The investor relations policy is just one—albeit a highly representative one—of many corporate decisions that can affect the firm's stock liquidity. Agency

[7] An alternative explanation of such findings is based on short-term artificial inflation in stock prices due to accounting manipulations and investor irrationalities (Teoh et al., 1998a, 1998b; Lang and Lundholm, 2000). This theory is easily distinguishable from ours as it makes different assumptions on market rationality and have different implications on the relationship between investor relations and stock price levels.

[8] Amihud and Lev (1981), for example, suggested that risk reduction can be a managerial motive for conglomerate mergers.

[9] Another plausible agency story about investor relations is that managers engage in investor relations for career concerns. By talking to analysts and being visible, a manager may increase his value in external labor markets. This hypothesis is empirically distinguishable from ours in that managers with greater equity stakes ought to be less worried about career concerns. In our model, it is these high equity stakes that lead insiders to consider the liquidity of their shares possibly at the expense of adopting a value maximizing investor relations policy.

problems due to insiders' preference for equity liquidity can also be potentially important in our understanding of other corporate decisions such as real investment decisions over multiple projects that differ in their impact on the firm's stock liquidity.

Finally, we note that while we developed our intuition based on a model in which investor relations increase the precision of the public signal about the firm value, the main idea of the paper remains valid if we assume instead that investor relations only encourage trading by non-informational (noise), or even irrational, traders. We incorporate such a mechanism into our model as well.[10]

Our paper proceeds as follows. We discuss the related literature in Section 2. In Section 3, we develop the model. We solve for the equilibrium in Section 4. In Section 5, we derive the insider's optimal investor relations policy and discuss the empirical implications of the model. We discuss extensions to our model in Section 6, and conclude in Section 7. All proofs are in Appendix A.

2. Related literature

Most theories of investor relations explore how investor relations can boost the mean stock price when certain key assumptions for strict market efficiency do not hold. In these models, there is typically unanimity among shareholders on the optimal level of investments in investor relations. We now review a few strands of this literature.

One strand of the literature focuses on how investor relations can reduce the information asymmetry in the market, thereby improving a firm's stock price. In Diamond and Verrecchia (1991), stocks are illiquid because of an adverse-selection cost to trading. Since large investors invest less in illiquid stocks, this illiquidity tends to depress the stock price. Investor relations, whether it be the public disclosure of valuable information by the firm or attracting analysts who conduct costly research and disseminate information, level the playing field for all investors. Hence, investor relations improve a stock's liquidity, which tends to lead to a higher stock price.[11]

Another strand of the literature focuses on how investor relations, through information disclosures, can cause stock prices to reflect firms' fundamentals more accurately (without necessarily reducing the information asymmetry among traders), which leads to higher stock prices. Verrecchia (1983, 1990) shows that when disclosure is costly and credible, managers who believe that their firms are undervalued will voluntarily provide additional information to investors if the value of reduced undervaluation exceeds the costs of expanded disclosure. In Fishman and Hagerty (1989), disclosures can make a firm's stock price more efficient, which leads to increased efficiency in the firm's real investment decisions, and thus a higher stock price.

[10] In the Merton (1987) model of investors with limited awareness of available stocks, this mechanism could also potentially work, although he did not explore it.

[11] Another paper that makes a similar point is Brennan and Tamarowski (2000). They argue that investor relations level the playing field among investors. This lowers adverse-selection costs and leads to a lower bid–ask spread or other trading costs, which results in a higher stock price based on an argument along the lines of Amihud and Mendelson (1986).

A third strand of the literature focuses on how investor relations can boost stock prices when investors have limited awareness (of existing stocks) or incomplete information. In Merton (1987), risk-averse investors do not invest in stocks that they do not know anything about. Investor relations makes investors aware of a stock, thereby increasing the stock's investor base and the stock price through a risk sharing effect. Trueman (1996) offers a model in which the manager of a firm pursues investor relations so as to attract informed investors. Each additional informed investor who faces constraints on short sales can potentially increase the stock price, given that the market has incomplete information about the exact number of such informed investors who trade in the company's stock.

A fourth strand of the literature focuses on financial disclosures among multiple, competing firms, the externalities that arise in such competitions, and the appropriate disclosure regulations to adopt in the presence of such externalities. In Fishman and Hagerty (1989), investor relations involve disclosing information that lowers the fixed cost for outside traders to obtain valuable information about the firm. Firms, driven by the positive effects of a more efficient market price on the efficiency of real investment decisions and stock price levels, compete for analysts and informed traders. Such competition can lead firms to overinvest in costly disclosures than is socially optimal. In Admati and Pfleiderer (2000), firms' values are correlated and disclosures made by one firm are used by investors in valuing other firms. As such, there are also externalities associated with disclosure decisions in this context. They find that the equilibrium of a voluntary disclosure game is socially inefficient and regulation that requires a minimal precision level sometimes, but not always, improves welfare. Recently, Boot and Thakor (2001) address questions about what kind of information and how much of it firms should voluntarily disclose. They answer these questions by taking into account the effects of disclosures on the incentives of outsiders to gather information and incentives for financial innovation.

Our paper is the closest to the papers in the first strand of the literature in that information asymmetry among investors is the source of market illiquidity. Like these papers, we assume that the insider can reduce the information asymmetry in the market through investor relations. Unlike these papers, we emphasize the incentives of the insider to pursue investor relations. More specifically, the insider has an incentive to engage in costly investor relations even absent any beneficial effects of such activities on the mean level of the stock price. Hence, there is a potential divergence of interest between insiders and dispersed shareholders on the firm's investor relations policy. This economic mechanism for investor relations is new relative to existing theories.

3. The model

Our model has one all-equity firm and three dates: 0, 1, and 2.

At date 0, the shares of the firm are owned by two groups of investors: an insider who has control rights, and dispersed shareholders each of whom owns a tiny fraction of the firm. We will often refer to the insider as the management. The total fraction of the insider's holdings of the firm's equity at date 0 is $\theta \in (0, 1)$. We assume that θ is exogenously given.[12]

At date 2, the firm will be liquidated, and its intrinsic liquidation value, $\tilde{v}$, will be known to the public. Based on information available at date 0, $\tilde{v}$ has a normal distribution, with its mean and variance given by:

$$\mathbf{E}(\tilde{v}) = \bar{v}, \qquad \mathbf{var}(\tilde{v}) = \sigma_v^2. \tag{1}$$

No one has any additional signal about $\tilde{v}$ at date 0, and there is no trading at date 0. Trading will start at date 1.

3.1. Investor relations

We assume that the insider has to commit to an investor relations policy at date 0 before receiving any signals about $\tilde{v}$ at date 1. This assumption captures a realistic aspect of investor relations: it takes time for the management to convince analysts to initiate coverage on the company and to get investors interested in the company. In such instances, much of the future prospect of the firm is unknown to insiders when they invest in investor relations.

We model investor relations in the following parsimonious manner. We assume that there is a public signal produced between date 0 and date 1 given by

$$\tilde{s} = \tilde{v} + \tilde{e}, \tag{2}$$

where $\tilde{e}$ is independent of $\tilde{v}$ and has an unconditional mean and variance given by

$$\mathbf{E}(\tilde{e}) = 0, \qquad \mathbf{var}(\tilde{e}) = \sigma_e^2. \tag{3}$$

The precision of the public signal $1/\sigma_e^2$ depends on investments in investor relations made by the insider. By incurring costs C at date 0, the insider can increase the precision of the public signal:

$$\left\{ \frac{1}{\sigma_e^2} = f(C)\text{: } f' > 0,\ f'' \leqslant 0,\ f(0) = 0 \right\}. \tag{4}$$

This is a natural way to model the effects of investor relations because management, while courting analysts and institutional investors, is essentially committing to opening up many aspects of the firm's business for scrutiny. In this view, analysts generate and disseminate information about the firm and hence level the playing field among all market participants.[13] The cost C is simply netted from the liquidation value of the firm at date 2.

[12] Alternatively, we can think of θ as determined endogenously in the early growth stage of the firm (before date 0) when the insider, possibly the entrepreneur/founder, was allocated a block of shares for agency and liquidity reasons that apply specifically to the early growth phase of the firm. Endogenizing θ along this line would not change our results.

[13] Consistent with this premise, Brennan et al. (1993) found that the stock prices of firms with high analyst coverage respond more quickly to macroeconomic news. Hong et al. (2000) found that stock prices of firms with more analyst coverage also respond more quickly to firm specific information, particularly bad news. Dempsey (1989) finds that the more analysts who follow a firm, the less likely is the market to be surprised by the firm's quarterly earnings announcement. This means that when more analysts follow a firm, there is less potential for profitable informed trading ahead of earnings announcements; in other words, the more level is the playing field.

It is important to keep in mind that there are a number of other ways to model investor relations. In Section 6, we consider a popular alternative view that investor relations can increase the amount of noise trading in the stock without necessarily increasing precision of public information about the firm's value. It turns out that the results are quite similar.

3.2. Liquidity and speculative trades

At date 1, there is trading in the stock. Before the start of trading at date 1, every market participant gets to observe the public signal $\tilde{s}$. Since the level of C chosen by the insider at date 0 is known to all before the start of trading at date 1, market participants know the precision of the public signal. There are several sets of traders at date 1.

3.2.1. The insider

The insider has both information- and liquidity-based trading motives. He gets to observe $\tilde{v}$ before the start of trading at date 1. So he can speculate on his private information by selling his initial stake or by buying more shares. In addition, he would like to liquidate $\tilde{\theta}_L$ (measured as a fraction of the firm's total equities) for liquidity reasons completely unrelated to the information that he receives about the firm. The realized value of $\tilde{\theta}_L$ is known only to the insider immediately before date 1.

Let X_i denote the number of shares that the insider sells at date 1, with the remaining shares of the insider $\theta - X_i$ being liquidated at date 2, and let P_t denote the date-t share price. If the insider realizes a liquidity shock of $\tilde{\theta}_L$ and does not sell $X_i = \tilde{\theta}_L$, he suffers a penalty (utility loss) given by $-l(X_i - \tilde{\theta}_L)^2$, where $l \geqslant 0$ measures the cost of being away from an ideal liquidation level of $\tilde{\theta}_L$. Nonetheless, the insider may not want to sell exactly $\tilde{\theta}_L$ because of his private information $\tilde{v}$. Hence, the insider is both a speculator and a liquidity trader.

More specifically, the insider chooses X_i to maximize the expectation (conditional on the insider's information set at date 1) of the following sum:

$$-l\big(X_i - \tilde{\theta}_L\big)^2 + X_i \widetilde{P}_1 + (\theta - X_i)\widetilde{P}_2.$$

When $l = \infty$, the insider simply sets $X_i = \tilde{\theta}_L$, ignoring his private information. When l is finite, the insider may want to shade X_i away from $\tilde{\theta}_L$ to take into account his private information and to trade strategically so as to increase the expected combined proceeds from his trades in date 1 and date 2.

Furthermore, we assume that $\tilde{\theta}_L$ is independent of $\tilde{v}$ and $\tilde{e}$ and normally distributed with a mean given by

$$\mathbf{E}\big(\tilde{\theta}_L\big) \equiv \bar{\theta}_L = M\theta, \quad \text{where } 0 \leqslant M \leqslant 1. \tag{5}$$

Since $\bar{\theta}_L$ is assumed to be positive, the insider on average wants to sell a fraction of his initial stake in the firm. Also, we assume that the standard deviation of the liquidity shock is proportional to the insider's initial stake in the firm:

$$\mathbf{var}(\tilde{\theta}_L) \equiv \sigma_L^2 = K^2\theta^2, \quad \text{where } K \geqslant 0. \tag{6}$$

The four parameters l, M, K, and θ summarize the insider's liquidity trading needs. There is an intuitive interpretation for these liquidity needs, as well as our modeling choice for them above, in the context of newly-public firms. The insider, after lock-up expiration, may need to sell a portion of their initial holdings because of liquidity needs. Both the mean level and the uncertainty (i.e., standard deviation) of the ideal amount that the insider would like to sell at date 1 are proportional to his initial holding θ, as assumed in (5) and (6). These assumptions are made to capture the idea that the insider's liquidity needs, as well as uncertainty of such needs (in absolute amount), are likely to be higher if he has a larger initial holding θ.[14] Presumably it is because of borrowing constraints that the insider needs to sell his shares in order to finance his liquidity needs. The penalty parameter for not being able to sell the ideal number of shares, l, captures how binding these borrowing constraints are. If $l = 0$, even if he faces a liquidity shock, the insider does not have to sell his shares of the firm to finance it—presumably because he has external wealth or does not face any borrowing constraints. So, when we speak of the insider facing or experiencing liquidity needs, we are typically thinking of these four parameters as being non-zero. So the insider needs to sell a large fraction of his initial stake for non-informational reasons.

In our approach, the insider's liquidity trading needs are specified exogenously. Previous studies by Bhattacharya and Spiegel (1991) and Glosten (1989), for example, have constructed models in which informed traders have both information-based and (endogenously modeled) hedging-based trading motives. In contrast, for the insider in our model, who is best thought of as an entrepreneur/founder of a young company, it is probably more reasonable to assume that his non-informational trading motive is due to his liquidity needs rather than his need to use his block holdings to hedge against his other holdings. In addition, the assumption of risk neutrality for all investors, which rules out hedging-based trading motives for technical reasons, allows us to focus on the impact of investor relations on market liquidity, as well as the associated divergence of interest between the insider and outside diverse shareholders, without introducing any effect on stock price levels. This helps distinguish our work from the existing literature on optimal investor relations, in which the focus has been on the impact of investor relations on firms' price levels.

3.2.2. Dispersed shareholders

The dispersed shareholders do not have any private information about $\tilde{v}$. They may experience liquidity needs at date 1. However, since their holdings are such a small fraction of the shares outstanding of the firm, we can, without loss of generality (as we will show below), assume that they do not face any liquidity needs at date 1, and hence do no trade at date 1.

3.2.3. Outside speculator and noise traders

Finally, other traders without stakes in the firm at date 0 may trade the shares of the firm at date 1. First, a risk-neutral speculator (who has conducted costly research about the firm) observes the realization of $\tilde{v}$ and then optimally speculates on his private information at date 1 by trading an amount equal to X_s.[15] Noise traders want to trade $\tilde{u}$ shares with

[14] If we think of such liquidity needs as due to shocks to consumption needs, then it is reasonable to assume that shocks to one's ideal consumption level should be related to one's market wealth.

mean and variance given by

$$\mathbf{E}(\tilde{u}) = 0, \qquad \mathbf{var}(\tilde{u}) = \sigma_u^2. \tag{7}$$

The noise trader demand $\tilde{u}$ is also normally distributed, and $\tilde{v}$, $\tilde{e}$, $\tilde{\theta}_L$, $\tilde{u}$ are mutually independent.

3.3. Price setting mechanism

For tractability, we will use Kyle (1985) to model the price setting at date 1.[16] Specifically, risk-neutral, competitive market makers set the price for the stock at date 1 based on the aggregate order flow $\widetilde{Y}$, which comprises of the market sales orders of the insider, $\widetilde{X}_i$, the speculator's buy order $\widetilde{X}_s$, and the noise trades $\tilde{u}$:

$$\widetilde{Y} = -\widetilde{X}_i + \widetilde{X}_s + \tilde{u}. \tag{8}$$

Note that from the perspective of the market makers, these orders are all random given the market maker's information. The price at date 1 is set in the following manner:

$$\widetilde{P}_1 = \mathrm{E}\big[\tilde{v} - C \bigm| \widetilde{Y}, \tilde{s}\big]. \tag{9}$$

In using this trading mechanism, we are implicitly assuming that the insider and other liquidity traders cannot credibly communicate to the market makers that particular trades are due to purely liquidity reasons. If the insider could somehow convince the market that his trades are due only to non-informational reasons, then his sale orders may not have any price impact and the insider may have little incentive to undertake investor relations.[17]

4. The security market equilibrium

In this section, we characterize the security market equilibrium for a fixed level of investment in investor relations at date 0 given by C. This investment C results in a fixed level of $1/\sigma_e^2$ at date 0 given by the functional form in Eq. (4). The market equilibrium proceeds given C. In Section 5, we derive the optimal level of investment C chosen by the insider.

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[15] Our results would be qualitatively similar even if we did not have a speculator in the model. See Section 6. In reality, there are many outsiders (e.g. mutual fund managers) that can obtain valuable private information about the firm. To enrich the model, and to ensure that our results are robust to the existence of informed speculators outside of the company, we include an outside speculator here.

[16] Of course, the actual trading environment for shareholders, especially for insiders, is very different from the stylized setup of the Kyle (1985) model. For example, the existence of multiple markets is important given that the insider can sell his block holdings in the so-called "upstairs" market through intermediated searches and negotiations. One can argue, however, that the Kyle model—with only one measure of liquidity for only one market—can still capture the basic economic effects of asymmetric-information-based illiquidities even under such realistic considerations; after all, the liquidity of the upstairs market is integrally related to the liquidity in the downstairs market.

[17] In the context of our model, if our market makers observed the market order of the insider and his realization of $\tilde{\theta}_L$, then there may be no price impact. We do not, however, analyze equilibria of this case.

The equilibrium prices at date 2 and date 0 are easy to calculate. Since C gets paid out of the terminal firm value, $\widetilde{P}_2 = \tilde{v} - C$. Moreover, since the level of C chosen is observable to all at date 0, and there is no information available at date 0 about $\tilde{v}$, it follows that[18]

$$P_0 = \mathbf{E}\big(\widetilde{P}_1\big) = \bar{v} - C. \tag{10}$$

The equilibrium at date 1 is described as follows in Theorem 1.

Theorem 1. *At date* 1, *there is a unique Bayesian–Nash linear equilibrium, which is characterized by the following properties.*

(i) *The noise traders combine to place a market order in the amount of* $\tilde{u}$.
(ii) *The speculator, after observing* $\tilde{v}$, *and thus* $\tilde{e}$ *and* $\widetilde{P}_2 = \tilde{v} - C$, *submits a market buy order in the amount of*

$$\widetilde{X}_s(\tilde{v}, \tilde{e}) = \frac{2l + \lambda}{\lambda(4l + 3\lambda)}\big[\tilde{v} - \mathbf{E}(\tilde{v} \mid \tilde{s})\big]. \tag{11}$$

(iii) *The insider, after observing* $\tilde{v}$, $\tilde{e}$, *and* $\tilde{\theta}_L$, *submits a market sales order in the amount of*

$$\widetilde{X}_i\big(\tilde{v}, \tilde{e}, \tilde{\theta}_L\big) = \frac{1}{4l + 3\lambda}\big[\mathbf{E}(\tilde{v} \mid \tilde{s}) - \tilde{v}\big] + \frac{l}{l + \lambda}\tilde{\theta}_L + \frac{l\lambda}{(l + \lambda)(2l + \lambda)}\bar{\theta}_L. \tag{12}$$

(iv) *Competitive, risk-neutral market makers set the price as*

$$\widetilde{P}_1 = \mathbf{E}(\tilde{v} \mid \tilde{s}) - C + \lambda\big[\tilde{u} + \widetilde{X}_s(\tilde{v}, \tilde{e}) - \big[\widetilde{X}_i\big(\tilde{v}, \tilde{e}, \tilde{\theta}_L\big) - \bar{\theta}_L\big]\big] - \frac{\lambda^2}{2l + \lambda}\bar{\theta}_L, \tag{13}$$

with the variable λ *given by*

$$\frac{\lambda^2(4l + 3\lambda)^2}{2(l + \lambda)(2l + \lambda)}\left[\sigma_u^2 + \left(\frac{l}{l + \lambda}\right)^2 \sigma_L^2\right] = \hat{\sigma}_v^2, \tag{14}$$

where

$$\hat{\sigma}_v^2 \equiv \frac{\sigma_v^2 \sigma_e^2}{\sigma_v^2 + \sigma_e^2} = \mathbf{var}(\tilde{v} \mid \tilde{s}). \tag{15}$$

Equation (14) has a unique strictly positive solution.

Consider the equilibrium price function given in Eq. (13). The first two terms, $\mathbf{E}(\tilde{v} \mid \tilde{s}) - C$, denote simply the publicly available forecast of the terminal firm value net the investment in investor relations C. The next term reflects the information on the firm's value contained in the unexpected order flow observed by the market makers. The final term,

$$-\frac{\lambda^2}{2l + \lambda}\bar{\theta}_L < 0,$$

[18] We assumed earlier that there is no trading at date 0, so the price P_0 calculated here can be thought of the true market value of the firm. Alternatively, we can assume that there is trading at date 0, but that insiders are not allowed to trade at date 0 (which can be thought of as the lock-up period).

may seem harder to understand at first glance. But it arises because market makers need to take account of the insider's *strategic* trading behavior. Given that the price set by market makers is inversely related to the net order flow, the insider always strategically reduce his sales order in order to boost the sales price at date 1. For example, even if both his information about the firm value and his optimal liquidity trading needs are the same as those expected by the market; i.e., $\tilde{v} = \mathrm{E}[\tilde{v} \mid \tilde{s}]$ and $\tilde{\theta}_L = \bar{\theta}_L$, the insider still sells $X_i < \bar{\theta}_L$ so as to increase the price for all of X_i shares at the expense of a small liquidity cost. The last term in (13) then reflects the market maker's rational expectation of the above strategic trading behavior by the insider. Note that this term is proportional to $\bar{\theta}_L$, the mean of the investor's liquidity-based sales at date 1, because the expected incentive for the insider's strategic trading is proportional to $\bar{\theta}_L$.

We consider a couple of special cases to further develop intuition for the equilibrium. First, consider the extreme case in which $l = 0$; i.e., there is no liquidity penalty. In this case, it is easy to see from Eqs. (11) and (12) that the speculator's optimal market buy order and the insider's optimal market sale order collapse to

$$\widetilde{X}_s(\tilde{v}, \tilde{e}) = -\widetilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) = \frac{1}{3\lambda}\big[\tilde{v} - \mathbf{E}(\tilde{v} \mid \tilde{s})\big]. \tag{16}$$

The equilibrium price given in (13) simplifies to

$$\widetilde{P}_1 = \mathbf{E}(\tilde{v} \mid \tilde{s}) + \lambda\big[\tilde{u} + \widetilde{X}_s(\tilde{v}, \tilde{e}) - \widetilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L)\big], \tag{17}$$

where

$$\lambda = \frac{\sqrt{2}}{3}\frac{\hat{\sigma}_v}{\sigma_u}. \tag{18}$$

This special case corresponds to the Kyle (1985) model with two identically-informed traders—the insider without any liquidity trading needs and the outside speculator—and noise traders.

Now consider the opposite extreme in which $l = \infty$; i.e., there is an infinite penalty for the insider to deviate from his ideal liquidation position $\tilde{\theta}_L$. In this limiting case, it is easy to see that the insider's optimal market sale order given in Eq. (12), of course, reduces to

$$\widetilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) = \tilde{\theta}_L. \tag{19}$$

In order words, the insider does not speculate on his private information. The speculator's market buy order given in Eq. (11) reduces to

$$\widetilde{X}_s(\tilde{v}, \tilde{e}) = \frac{1}{2\lambda}\big[\tilde{v} - \mathbf{E}(\tilde{v} \mid \tilde{s})\big]. \tag{20}$$

The equilibrium price given in (13) simplifies to

$$\widetilde{P}_1 = \mathbf{E}(\tilde{v} \mid \tilde{s}) + \lambda\big[\tilde{u} + \widetilde{X}_s(\tilde{v}, \tilde{e}) - \big[\widetilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) - \bar{\theta}_L\big]\big], \tag{21}$$

where

$$\lambda = \frac{\hat{\sigma}_v}{\sqrt{\sigma_u^2 + \sigma_L^2}}. \tag{22}$$

This special case corresponds to the Kyle (1985) model with one informed trader, the outside speculator, and noise traders—both the insider with liquidity trading needs only and outside noise traders. Notice that the speculator trades more aggressively on his private information (also observed by the insider) when $l = \infty$ than when $l = 0$. The reason, of course, is that when $l = \infty$, the insider does not speculate at all, leaving the speculator more room to aggressively exploit his private information.

The key parameter in Theorem 1 is λ, which measures the impact of trades on price or market illiquidity. In general, we will need to numerically solve Eq. (14) to yield λ as a function of the parameters l (the liquidity penalty), σ_L^2 (the variance of the liquidity shock), σ_v^2 (the variance of the firm terminal value), σ_u^2 (the variance of noise trades), and σ_e^2 (the noisiness of the public signal).

However, we can prove an intuitive result regarding λ without resorting to any numerical solutions. These results will be useful in understanding the optimal level of investments in investor relations, C, chosen by the insider.

Lemma 1. *The measure of market illiquidity, λ, is decreasing in the precision of the public signal, $1/\sigma_e^2$, and the variance of noise trades, σ_u^2.*

All else equal, when the public signal is more precise or the variance of noise trades is higher, there is less adverse selection in the market (from the perspective of the market makers) and λ is smaller. By undertaking investments of C in investor relations, the insider can increase $1/\sigma_e^2$ and thereby decrease the market illiquidity as measured by λ. It is to the determinants of these investments that we now turn.

5. Optimal investor relations policy

The basic premise of our paper is that absent any actions on the part of the firm, the illiquidity of its stocks, λ, is non-trivial. It is useful to note that this premise is especially applicable for small or newly-public firms. For very large and liquid firms, this premise is probably less reasonable. Nonetheless, with existing trading laws which force insiders to sell at very narrow windows, even insiders at large firms have to worry about the liquidity of their block of shares.

We will show that for insiders at these firms who face stringent liquidity needs, they have a strong incentive to undertake investments in investor relations to decrease λ. Even though it is the insider who ultimately determines C, it is useful to first consider the optimal C that dispersed shareholders prefer.

5.1. The interest of dispersed shareholders

Let x be the fraction of the company held by a dispersed shareholder. Here think of x as being close to zero. Now suppose that dispersed shareholders only liquidate their shares at date 2. Then the expected revenue for a given dispersed shareholder in this instance is simply

$$x(\bar{v} - C).$$

Of course, dispersed shareholders would prefer that $C = 0$ since they do not face any liquidity needs at date 1 and the cost comes out of the terminal firm value.

Now suppose that for some reason, dispersed shareholders want to liquidate their shares at date 1. Then the expected revenue in this instance for a given dispersed shareholder is

$$x(\bar{v} - C) - \lambda x^2. \tag{23}$$

Notice that the penalty for market illiquidity (the term involving λ) is proportional to the square of the fraction of shares of the firm held by the dispersed shareholder. Since x is close to zero, it follows that $x^2 \approx 0$ and so again the optimal choice for the dispersed shareholder is to choose $C = 0$. In other words, dispersed shareholders would prefer that no investments be made in investor relations. Their preferred solution coincides with the benchmark of maximizing firm value at date 2.

5.2. The interest of the insider

We next consider the insider's choice of C. By paying a cost C, the insider increases $1/\sigma_e^2$ according to the functional form given in (4). The insider chooses C to maximize at date 0 the expected profits given by

$$I(C) = \mathbf{E}\big[-l\big(\widetilde{X}_i - \tilde{\theta}_L\big)^2 + \widetilde{X}_i\big(\widetilde{P}_1 - \widetilde{P}_2\big) + \theta\widetilde{P}_2\big].$$

Using the solution given in Theorem 1, we can explicitly calculate $I(C)$. This is given in Lemma 2.

Lemma 2. *The insider's expected profits are given by*

$$I(C) = \theta(\bar{v} - C) + \frac{l + \lambda}{(4l + 3\lambda)^2}\hat{\sigma}_v^2 - \frac{l\lambda^2}{(2l + \lambda)^2}\bar{\theta}_L^2 - \frac{l\lambda}{l + \lambda}\sigma_L^2. \tag{24}$$

In Eq. (24), the first term, $\theta(\bar{v} - C)$, is simply the expected terminal value of the firm net of the cost of investor relations. The remaining three terms reflect the expected profit associated with the insider's informed trading and the expected utility of the insider's liquidity trades.

Broadly speaking, these four terms capture the following tradeoff of investor relations. In investing C in investor relations, the insider shares the cost of C with dispersed shareholders and loses potential speculative profits as investor relations result in disclosure of private information. On the other hand, investor relations lower the market illiquidity as measured by λ, which decreases the price impact of his liquidity trades. When his liquidity needs are substantial and markets are highly illiquid, the insider may engage in costly investor relations.

To get some intuition for this tradeoff, consider the special case of $l = 0$ in which the insider does not face a penalty for being far away from his ideal liquidating position $\tilde{\theta}_L$. In this instance, it is easy to see that $I(C)$ of Eq. (24) reduces to

$$I(C) = \theta(\bar{v} - C) + \frac{\lambda}{2}\sigma_u^2. \tag{25}$$

The optimal solution for this objective function is $C = 0$. By investing in investor relations, the insider shares part of the cost C. Also, in choosing a non-trivial C, λ will be lower and hence the second term involving σ_u^2 is also lower. This second term captures the speculative profits of the insider, which he would have to give up if he engaged in investor relations. Hence, he has no incentive to do so if he is not forced to liquidate his shares to finance his liquidity needs.

Next, consider the special case in which the initial stake of the insider is zero (i.e., $\theta = 0$), and $I(C)$ takes the form of

$$I(C) = \frac{\lambda^2}{2(2l+\lambda)}\sigma_u^2. \tag{26}$$

In this case, the insider does not have any liquidity trading needs, and has no incentive to invest in investor relations. The case in which $M = K = 0$ (i.e., the insider has no liquidity trading needs despite possible non-zero initial stake in the firm) is equivalent to the case of $\theta = 0$.

The above special cases allow for analytic solutions of the optimal costly effort in investor relations. In general, however, analytic tractability is not feasible, and we need to solve numerically for the equilibrium C by assuming a functional form for f (as defined in (4)). It turns out that for large parameter ranges, it is optimal for the insider to choose a non-trivial level of investments in investor relations. Below we will solve the model for a wide range of parameters.

5.3. Divergence of interests

Even without resorting to numerical calculations, we can prove a key result regarding the divergence of interests between the insider and dispersed shareholders regarding investor relations. In Theorem 2, we provide a sufficient condition for the insider to engage in costly investor relations.

Theorem 2. *Suppose the penalty coefficient $l > 0$ and the insider's holding $\theta > 0$. If*

$$f'(0) > G\bigl(\bar{v}, \sigma_v^2, \sigma_u^2, l, \theta, K, M\bigr) > 0, \tag{27}$$

where the function G is defined as

$$G\bigl(\bar{v}, \sigma_v^2, \sigma_u^2, l, \theta, K, M\bigr) \equiv \frac{\theta}{\sigma_v^4}\frac{1}{\frac{1}{A(l,\lambda_0,\sigma_u)}\Bigl[\frac{l^2}{(l+\lambda_0)^2}\sigma_L^2 + \frac{4l^2\lambda_0}{(2l+\lambda_0)^3}\bar{\theta}_L^2 + \frac{2l+3\lambda_0}{(4l+3\lambda_0)^3}\sigma_v^2\Bigr] - \frac{l+\lambda_0}{(4l+3\lambda_0)^2}} \tag{28}$$

and λ_0 is the equilibrium market illiquidity measure for the case of zero investor-relations investment (i.e., λ_0 is the solution of (14) for $C = 0$), then the insider invests $C > 0$. Since dispersed shareholders prefer that $C = 0$, there is an overinvestment in investor relations relative to the benchmark of maximizing firm value. In addition, there exist parameter values for which condition (27) holds.

It is not necessarily the case that the insider always chooses a non-trivial C if he has liquidity needs at date 1. Whether he does depends on a number of factors. Certainly, the benefits of liquidity need to be weighed against the costs of diverting firm resources and losing out on speculative profits. As Eq. (27) shows, this tradeoff also depends on how quickly the precision of the public signal increases with C. That is, it depends on the functional form of f. When the precision of the public signal does not respond sufficiently quickly to C, the insider may still optimally choose $C = 0$ even if he has extreme liquidity needs.

To better understand the nature of the divergence of interests pointed out in Theorem 2, we focus on a special case in which f is assumed to be linear in C:

$$f(C) = \alpha_e C, \tag{29}$$

and the insider faces an infinite penalty for deviating from his ideal liquidation position, i.e. $l = \infty$. While the case of $l = \infty$ is not very realistic, we can obtain closed form solutions for λ and C, which will serve to provide some insight into the nature of the equilibrium.

The results of this special case are summarized in Proposition 1.

Proposition 1. *Suppose the penalty coefficient $l = \infty$ and the signal precision $f(C) = \alpha_e C$. There is a unique solution for C, the investment in investor relations, given by*

$$C^* = \mathrm{Max}\left\{\frac{K^{4/3}\theta^{2/3}}{2(2\alpha_e)^{1/3}(\sigma_u^2 + K^2\theta^2)^{1/3}} - \frac{1}{\alpha_e\sigma_v^2},\ 0\right\}.$$

The equilibrium illiquidity measure λ is given by

$$\lambda^* = \mathrm{Min}\left\{\left[\frac{\theta}{2\alpha_e\sigma_L^2(\sigma_u^2 + \sigma_L^2)}\right]^{1/3},\ \frac{\sigma_v}{2\sqrt{\sigma_u^2 + \sigma_L^2}}\right\}.$$

Suppose condition (27) holds (i.e. when α_e, the sensitivity of signal precision to investment in investor relations, is sufficiently large), then it is easy to show that $C^* > 0$. In this instance, it is also easy to show that the chosen C^* is increasing in $K\theta$, the standard deviation of the insider's liquidity shock. Intuitively, when $K\theta$ is large, the insider's liquidity trading needs are more severe and so he invests more in investor relations. The equilibrium C^* does not depend on M in this special case because the insider will not trade strategically given the infinite penalty l for deviating from his optimal amount of sales, $\tilde{\theta}_L$.

It is important, however, to note that the comparative statics obtained in the above special case do not hold generally. When θ is large, the insider also owns a larger fraction of the firm and so ends up paying for a higher fraction of the costs C. In general, for a given functional form f, whether C^* increases or decreases in θ depends on the magnitude of l. In addition, C^* may also depend on M as strategic trading by the insider is important for $l < \infty$. To gain a better understanding of these general cases with $0 < l < \infty$, we need to numerically solve for equilibria of our problem.

5.4. Numerical solution

We now solve our model numerically. The basic result we want to demonstrate is that when the insider's liquidity trading needs are not severe, he has little incentive to invest in investor relations. If his liquidity trading needs are severe, however, then he spends a non-trivial fraction of the firm's resources on investor relations.

Within our model, the insider's liquidity trading needs are collectively represented by the expectation and standard deviation of his ideal liquidation amount, $M\theta$ and $K\theta$ respectively, as well as the penalty coefficient from deviating from his ideal liquidation amount, l. Our goal is therefore to solve the model for a range of values for parameters θ, l, M, and K, while fixing all the other parameters at some reasonable numbers. When l is large *and* either $M\theta$ or $K\theta$ is large, his liquidity needs are severe, and we should expect to find high equilibrium levels of investment in investor relations, C. On the other hand, if l is small, *or* if both $M\theta$ and $K\theta$ are small, then his liquidity needs are not severe, and we expect to find low equilibrium C.

5.4.1. *Parameter choices*

We now specify values, or ranges of values, for all of our model parameters. While we do not think that such a stylized model can accurately capture the market illiquidity faced by insiders, it is still important for us to make such parameter choices as reasonable as possible. Given that the focus of our paper is on small, newly-public firms, we should have such firms in mind when we judge the reasonableness of our parameter choices.

First, we choose ranges of values for θ, K, M, and l—parameters over which we need to conduct numerical comparative statics analyses. For insider holding θ for such firms, we think that 20% is a reasonable number. So, we will consider solutions for θ ranging from 0 to 0.4. For M and K, we assume that a "typical" insider liquidates 50% of his stake of θ on average at date 1 (i.e. set $M = 0.5$) and faces a liquidity shock with a standard deviation equal to 40% of his stake (i.e. set $K = 0.4$). So, at the base case of $\theta = 0.2$, this "typical" insider on average liquidates $\bar{\theta}_L = 0.5(0.2) = 0.1$ with a standard deviation of $\sigma_L = 0.4(0.2) = 0.08$. We also solve our model for M and K in the range between 0 and 0.8.

It is not obvious how to calibrate l for insiders at these firms, so we have a degree of freedom here in deciding what is a sensible value for l. We will provide solutions for l from 0 to 1000. In case we need to fix l while varying other parameters, we choose $l = 500$.

Secondly, we choose values for all other model parameters. We set the mean terminal value of the firm, $\bar{v}$, to be 100 with a standard deviation, σ_v, to be 50. The standard deviation of the noise trades σ_u is set to be 0.02. One justification for this number is that σ_u is one-fourth of σ_L for the case in which $\theta = 0.2$. So we are essentially assuming that the amount of noise trades is small compared to the liquidity trades of the insider. We think this is a sensible way to capture the illiquidity of the insider's shares since for small, newly-public firms, the float of the firm is small in comparison to the insider's holdings.

Furthermore, we assume that $1/\sigma_e^2$ is linear in C with a slope of α_e as given in Eqs. (4) and (29). We choose α_e such that when the insider spends 1% of the firm's value on investor relations, half of the private information is revealed to the public; i.e., $\hat{\sigma}_v^2/\sigma_v^2 = 0.5$ when $C = 1$ (see (15) for the definition of $\hat{\sigma}_v$). This condition gives us an $\alpha_e = 0.0004$.

5.4.2. *Solution method*

The numerical solution approach for our problem is simple. Solving for the optimal C is a non-linear optimization problem (of maximizing $I(C)$ in (24)) of a single variable C over

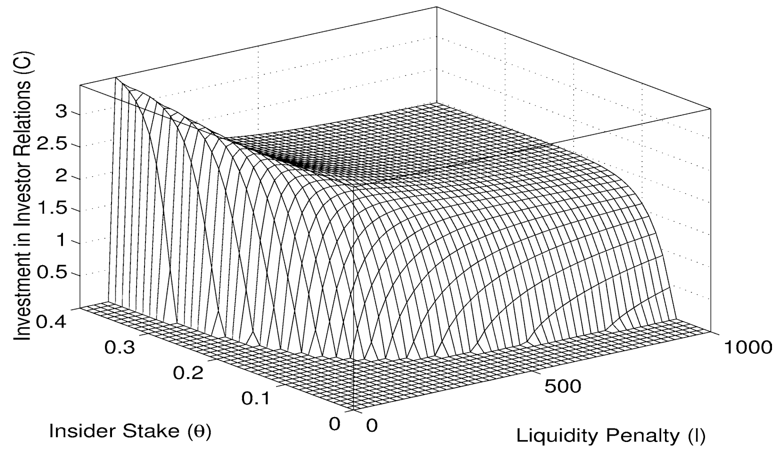

Fig. 1. The investment policy of the firm is shown for various values of the insider stake θ and the liquidity penalty l. The other parameters are set as follows: $\bar{v} = 100$, $\sigma_v = 50$, $M = 0.5$, $K = 0.4$, $\sigma_u = 0.02$, $\alpha_e = 0.0004$.

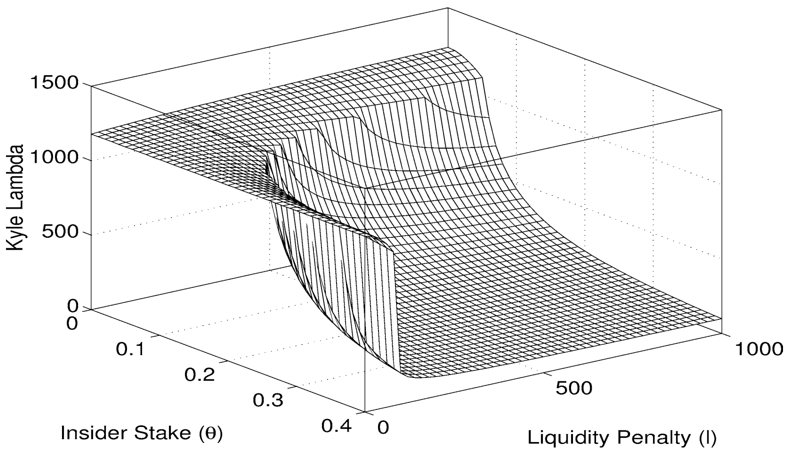

Fig. 2. The equilibrium market illiquidity of the firm's stocks (Kyle lambda) is shown for various values of the insider stake θ and the liquidity penalty l. The other parameters are set as follows: $\bar{v} = 100$, $\sigma_v = 50$, $M = 0.5$, $K = 0.4$, $\sigma_u = 0.02$, $\alpha_e = 0.0004$.

a bounded support, given each set of model parameters. It is easy to show that the optimal C cannot exceed the average value of the firm $\bar{v}$. Hence, we are looking for a solution between 0 and $\bar{v}$. It is then straightforward to identify the global optimum numerically for different sets of parameters.

5.4.3. Numerical results

We present results from our numerical comparative statics analyses as follows. Figures 1 and 2 show, respectively, the level of investment in investor relations (C) and the accompanying equilibrium measure of market illiquidity (λ) for various values of parame-

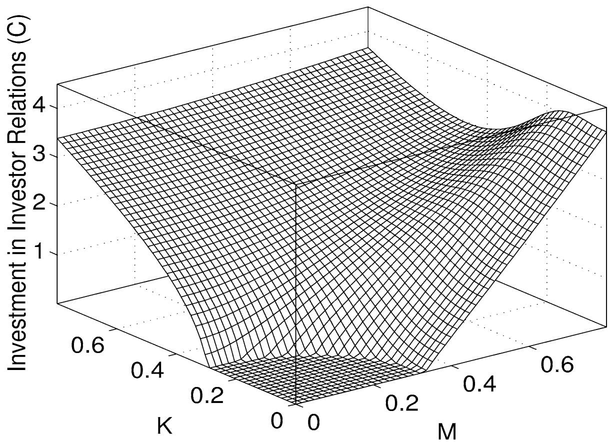

Fig. 3. The investment policy of the firm is shown for various values of K and M, the standard deviation and the mean, respectively, of the insider's ideal liquidation amount, as a fraction of his stake θ. The other parameters are set as follows: $\theta = 0.2$, $l = 500$, $\bar{v} = 100$, $\sigma_v = 50$, $\sigma_u = 0.02$, $\alpha_e = 0.0004$.

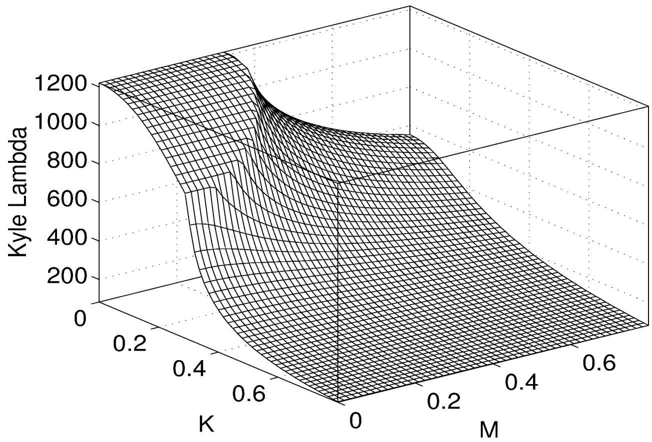

Fig. 4. The equilibrium market illiquidity of the firm's stocks (Kyle lambda) is shown for various values of K and M, distribution parameters for the insider's liquidity shocks. The other parameters are set as follows: $\theta = 0.2$, $l = 500$, $\bar{v} = 100$, $\sigma_v = 50$, $\sigma_u = 0.02$, $\alpha_e = 0.0004$.

ters θ and l, while holding K and M fixed. Figures 3 and 4 show, respectively, the level of investment in investor relations (C) and the accompanying equilibrium measure of market illiquidity (λ) for various values of parameters K and M, while holding θ and l fixed.

First, consider Fig. 1. As discussed before, when *either* the insider stake (θ) *or* the liquidity penalty (l) is very small, the insider's liquidity needs are not severe, and we intuitively expect that the optimal C should be zero or very small. Indeed, the figure indicates that when $\theta < 0.05$ *or* $l < 80$, there is no investment in investor relations. On the other hand, when θ and l are *both* large enough, the optimal C is positive, as we expect intuitively because the insider's liquidity needs are more severe in these cases.

Note that this does not mean that C is increasing in θ or l in general. In Fig. 1, if we fix $\theta = 0.3$ and look at how C changes as l goes from 100 to 1000, for example, we find that as l gets larger, the insider actually reduces the level of investments in investor relations. This might seem surprising because, all else equal, as l gets larger, the insider faces more stringent liquidity needs. Hence one might expect him to invest more to reduce market illiquidity. However, in equilibrium, market makers realize that for a larger l, the insider is less aggressive in speculating on his private information. So all else equal (including any given C), the market maker is more confident that trades are more likely to be liquidity driven and that there is less adverse selection in the market. So the market is more liquid when l is large and the insider needs to invest less. The exact relationship between C and l depends on these two offsetting effects.

Notice also that in the parameter space where $C > 0$, C may increase or decrease in θ. On the one hand, the larger is θ, the more severe are the insider's liquidity trading needs, and the more likely he is to choose a higher C. On the other hand, as θ increases, the insider owns a larger fraction of the firm and therefore shares more of the costs of investor relations. All else equal, this means he is less likely to invest for a large θ. The lack of monotonicity of C with respect to θ is due to the tradeoff between these two offsetting effects.

In addition, it is interesting to note from Fig. 1 that the optimal investor relations policy does not change smoothly with changes in θ or l. For instance, fix $l = 100$ and consider how C changes as θ increases from 0 to 0.4. Roughly, there is no investment for θ less than 0.3. Then as θ rises above this level, the optimal C jumps to 2.5, i.e. 2.5% of the mean firm value of 100. This is due to the non-linear nature of the insider's optimization problem (of maximizing $I(C)$ in (24)). Technically, the non-linearity of the problem, driven by the above offsetting effects, create two local maxima over the variable C, one at $C = 0$ and the other at some $C > 0$. As θ increases gradually, the strictly positive local maximum gradually overtakes the local maximum at $C = 0$ and becomes the global maximum, resulting in the jump in the (globally) optimal investment level C for the insider. We have experimented with other parameter ranges and find that this feature is robust and does not represent an anomaly.

Despite the local non-monotonicity discussed above, the broad pattern of Fig. 1 shows that the insider indeed chooses to invest more in investor relations when his liquidity needs are more severe, as represented by large θ and l.

For completeness, we also show the equilibrium λ's in Fig. 2 that result from the insider's optimal investments shown in Fig. 1. For θ and l small, $C = 0$ and hence the equilibrium illiquidity λ is large. As one would expect, when $C > 0$, there is a substantial and precipitous drop in λ.

Next, consider Fig. 3. As discussed before, on the one hand, for fixed θ and l, when *both* K and M are very small, the insider's liquidity needs are not severe, and we intuitively expect that the optimal C should be zero or very small. Indeed, the figure indicates that for small K and M, there is no investment in investor relations. On the other hand, if the fixed θ and l are both sufficiently high (as is indeed the case for our chosen $\theta = 0.2$ and $l = 500$), when *either* K *or* M is large enough, the optimal C is positive, as we expect intuitively because the insider's liquidity needs are more severe in these cases.

Moreover, all else equal, the insider's optimal C increases with M over all parameter ranges in Fig. 3. This is reasonable because, as we mentioned while discussing Theorem 1, the insider will strategically reduce his sales order to boost the price in his favor *given* any pricing mechanism by market makers. The market makers, however, would rationally take such insider strategic behavior into account in setting prices. In equilibrium, the insider is actually penalized because of his inability to pre-commit to not trade strategically. The larger is M (the average liquidation amount as a fraction of the fixed insider stake θ), the larger the incentive for the insider to trade strategically, the larger the penalty that the insider will incur in equilibrium, and the more likely that he will invest more in investment relations in order to boost the stock's liquidity and lower the penalty for his strategic trading.[19]

In contrast, the insider's optimal C does not always increase with the parameter K. In Fig. 3, if we fix $M = 0.8$ and look at how C changes as K goes from 0 to 0.8, for example, we find that as K gets larger, the insider's choice of C may increase or decrease. This lack of monotonicity reflects the tradeoff of two offsetting effects. All else equal, as K increases, on the one hand, the insider liquidity shock at date 1 is more severe, and he is more likely to increase his investment in investor relations in order to enhance the stock's liquidity; on the other hand, the larger liquidity shock for the insider leads to more noise trading and less adverse selection in the market, and the insider is then less likely to increase his investment in investor relations in order to further enhance market liquidity.

Despite the local non-monotonicity of C over K, the broad pattern of Fig. 3 shows that the insider indeed chooses to invest more in investor relations when his liquidity needs are more severe, as represented by large M and K.

For completeness, we also show the equilibrium λ's in Fig. 4 that result from the insider's optimal investments shown in Fig. 3. For small K or small M, $C = 0$ and hence the equilibrium illiquidity λ is large. As one would expect, when $C > 0$, there is a substantial drop in λ.

5.5. Empirical implications

Since our model is highly stylized, we do want to guard against overinterpreting the various implications of our model. However, two sets of empirical implications are novel to our model. Some of these implications are consistent with existing empirical evidence, and some are yet to be tested.

First, our model showed that costly investments in investor relations by a firm may *not necessarily* be accompanied by an increase in the firm's long-term stock price (or a decrease in its cost of capital). Insiders may engage in costly investor relations in order to boost the liquidity for their own block of shares, but such increased liquidity (e.g., more market depth for large block of shares) may not be valued much by the marginal investors

[19] In fact, this comparative statics is so intuitive that one expects that an analytic proof for it might exist, even though we do not have an analytic solution for the optimal C. Indeed, we can show that this comparative statics result holds locally *if* the global optimal solution C for $I(C)$ (in (24)) is achieved at the same local interior maximum. But given the highly non-linear nature of the optimization problem, it is difficult to show that the global optimal solutions—even those interior solutions with $C > 0$—always come from the same local maximum.

of the firm's shares. So empirically we may possibly observe very small or no effect on long-term stock prices even though firms make significant investment in investor relations. This is broadly consistent with the evidence of intensive investor relations by small or newly-public firms despite lack of strong empirical support for any long-term price level effect (Byrd et al., 1993; Botosan, 1997).

Secondly, our model also generate a set of predictions relating the intensity of investor relations across firms to certain demographic features of insiders. These predictions are testable. There are a number of different measures of investor relations. One often used measure is earnings-guidance or earnings-management as used in Richardson et al. (2001). Other proxies include presentations to analyst societies (Byrd et al., 1993) and conference calls (Tasker, 1998). In addition, demographic information on insiders such as their trading activities, shares owned and other personal characteristics such as their wealth are available. Hence, it is feasible to relate the intensity of investor relations in the cross-section to insider demographics.

Perhaps the most robust of our model's implications is that, all else equal, firms in which insiders are large net sellers of their shares ought to have a greater intensity of investor relations. One can think of the size of these sales as one of the measures of the insider's liquidity needs. In our model, the average net sales by the insider, based on (12), is

$$\mathrm{E}_0\big[\widetilde{X}_i\big(\tilde{v}, \tilde{e}, \tilde{\theta}_L\big)\big] = M\theta/(2l + \lambda). \tag{30}$$

For example, all else equal (including θ), as M increases, the insider's average net sales increases (because the equilibrium λ decreases as shown by Fig. 4), and the insider's optimal investment in investor relations C increases as well, as Fig. 3 shows. More generally, the insider's average net sales in (30) are proportionally related to θ and M, and inversely related to the equilibrium λ. Given that the broad patterns of Figs. 1–4 show that the equilibrium investment level C is also positively related to θ and M and inversely related to λ, our model indeed predicts that firms in which insiders are large net sellers of their shares should make more investment in investor relations. This prediction is indeed supported by Richardson et al. (2001) who found that earnings-guidance (a form of investor relations) is more prominent for firms whose insiders sell stocks from their personal accounts after earnings announcements.

Our model also generates some new predictions that are yet to be tested. The insider's average net sales amount is not the only measure of his liquidity needs. Other measures include his liquidity shock, $K\theta$, and the penalty for not meeting his ideal liquidation amount, l. Since it is difficult to measure $K\theta$ empirically, we can use θ to capture some of the liquidity shock effects. The broad pattern in Fig. 1 shows that a firm with large insider ownership ought to be more likely to engage in investor relations than one without. Also, to the extent that we can proxy for the liquidity penalty l by using an insider's outside wealth, our model predicts that a firm with insiders who have little outside wealth is more likely to engage in investor relations than other firms, all else equal.

6. Further discussions and extensions

In this section, we discuss some of the important elements of our model and consider some extensions.

6.1. Investor relations increases the variance of noise trades

An alternative and more cynical way to model investor relations is to assume that by incurring costs C, the insider can increase the variance of noise trades σ_u^2. In this view, analyst coverage stimulates lots of noise trading in a stock. One can think of these noise traders as individual investors who watch television shows in which analysts tout stocks and end up buying stocks based on what analysts recommend on these shows.

More specifically, assume that the insider can increase the date-1 variance of noise trades by incurring costs, C, at date 0 according to the following functional form:

$$\left\{\sigma_u^2 = g(C)\colon\ g' > 0,\ g'' \leqslant 0,\ g(0) = \underline{\sigma}_u^2\right\}. \tag{31}$$

The equilibrium given in Theorem 1 still holds. The objective function of the insider is the same as in Lemma 2 except that $\lambda(C)$ depends on C indirectly through $\sigma_u^2(C)$ as opposed to $1/\sigma_e^2$. From Lemma 1, we know that as C increases, λ decreases. The optimal choice for the insider has to be solved numerically.

Without calculating the exact level of C, we can prove the following theorem, which is the analog to Theorem 2.

Theorem 3. *Assume that investments in investor relations* (C) *increases the variance of noise trades* (σ_u^2). *If*

$$g'(0) > \frac{\theta A(l,\lambda_0,\underline{\sigma}_u)}{\frac{l^2}{(l+\lambda_0)^2}\sigma_L^2 + \frac{4l^2\lambda_0}{(2l+\lambda_0)^3}\bar{\theta}_L^2 + \frac{2l+3\lambda_0}{(4l+3\lambda_0)^3}\hat{\sigma}_v^2}\,\frac{2(l+\lambda_0)(2l+\lambda_0)}{\lambda_0^2(4l+3\lambda_0)^2}, \tag{32}$$

where $\lambda_0 = \lambda(C=0)$ *and*

$$A(l,\lambda,\sigma_u) \equiv \frac{\lambda(4l+3\lambda)(16l^3+36l^2\lambda+27l\lambda^2+6\lambda^3)}{2(l+\lambda)^2(2l+\lambda)^2}\sigma_u^2 + \frac{l^3\lambda(4l+3\lambda)(16l^2+20l\lambda+7\lambda^2)}{2(l+\lambda)^4(2l+\lambda)^2}\sigma_L^2, \tag{33}$$

then there is overinvestment in investor relations relative to the benchmark of maximizing firm value. That is, the insider invests $C > 0$, *whereas dispersed shareholders prefer* $C = 0$.

Unlike the case in which investor relations meant disclosure of private information, the insider no longer loses speculative profits from investor relations. In fact, because the insider has private information which he can speculate on at date 1, he will also want to undertake a non-trivial investment in investor relations to boost noise trades so as to camouflage his trades even absent any liquidity needs. Hence, modeling investor relations in this manner only makes it easier for us to obtain our results.

6.2. Equilibrium without an outside speculator

Intuitively, one might suspect that the existence of outside speculators might be driving all of our results since it is easier for the insider to disclose valuable private information because the insider is not the sole beneficiary of this information. Our results do not depend qualitatively on whether there is an outside speculator who has the same information as the insider. To see that this is the case, we report the key results of our model assuming that there is no outside speculator.

In following theorem, we describe the equilibrium without an outside speculator.

Theorem 4. *Assume that there is no outside speculator. At date* 1, *there is a unique Bayesian–Nash linear equilibrium, which is characterized by the following properties.*

(i) *The noise traders combine to place a market order in the amount of* $\tilde{u}$.
(ii) *The insider, after observing* $\tilde{v}$, $\tilde{e}$, *and* $\tilde{\theta}_L$, *submits a market sales order in the amount of*

$$\widetilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) = \frac{1}{2(l+\lambda)}\big[\mathbf{E}(\tilde{v} \mid \tilde{s}) - \tilde{v}\big] + \frac{l}{l+\lambda}\tilde{\theta}_L + \frac{l\lambda}{(l+\lambda)(2l+\lambda)}\bar{\theta}_L. \tag{34}$$

(iii) *Competitive, risk-neutral market makers set the price as*

$$\widetilde{P}_1 = \mathbf{E}(\tilde{v} \mid \tilde{s}) - C + \lambda\big[\tilde{u} - \big[\widetilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L) - \bar{\theta}_L\big]\big] - \frac{\lambda^2}{2l+\lambda}\bar{\theta}_L, \tag{35}$$

where λ *is given by a positive solution for*

$$\frac{4\lambda(l+\lambda)^2}{2l+\lambda}\left[\sigma_u^2 + \left(\frac{l}{l+\lambda}\right)^2\sigma_L^2\right] = \hat{\sigma}_v^2. \tag{36}$$

Equation (36) has a unique strictly positive solution.

Proof. Available upon request.

We next provide the sufficient condition for when the insider will overinvest in investor relations.

Theorem 5. *Suppose the penalty coefficient* $l > 0$ *and the insider's holding* $\theta > 0$. *If*

$$f'(0) > \frac{\theta}{\sigma_v^4}\left[\frac{\frac{l^2}{(l+\lambda_0)^2}\sigma_L^2 + \frac{4l^2\lambda_0}{(2l+\lambda_0)^3}\bar{\theta}_L^2 + \frac{1}{4(l+\lambda_0)^2}\sigma_v^2}{\frac{8(l+\lambda_0)(l^2+3l\lambda_0+\lambda_0^2)}{(2l+\lambda_0)^2}\sigma_u^2 + \frac{8l^3}{(2l+\lambda_0)^2}\sigma_L^2} - \frac{1}{4(l+\lambda_0)}\right]^{-1} > 0, \tag{37}$$

where $\lambda_0 = \lambda(C = 0)$, *then the insider invests* $C > 0$. *Since dispersed shareholders prefer that* $C = 0$, *there is an over-investment in investor relations relative to the benchmark of maximizing firm value.*

Proof. Available upon request.

We have numerically solved for the optimal investor relations policy. The results are similar to those described in Section 5.4. There are a large range of parameters for which $C > 0$. We omit this discussion for brevity.

7. Conclusion

In this paper, we develop a model to study the incentives of insiders in small and newly-public firms to undertake costly investor relations. We find that insiders have a strong incentive to allocate resources to enhance the liquidity of their own block of stocks because of potential liquidity needs. In contrast, dispersed shareholders care little about market illiquidity because of their relatively small holdings. This leads to a divergence of interest on investor-relations policies for such firms. The main testable implications of our model is that the demographics of insiders (e.g. liquidity needs, size of equity stakes) are potentially important determinants of the extent of investor relations across firms.

More generally, our paper points out an agency problem driven by insiders' preference for liquidity of their block of shares. While we focus on the implications of such an agency problem on corporate investment in investor relations, it would be interesting to study the effect of such an agency problem on other corporate decisions. We leave this for future research.

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Appendix A. Proofs

Proof of Theorem 1. We look for a Nash equilibrium in which both the insider and the speculator take each other's strategy as given and optimize. The noise traders submit their random market order. The market makers set the price to be the expected value of the time-2 price of the firm conditioning on the information they have at $t = 1$.

At $t = 1$, the speculator observes $\tilde{v}$ and $\tilde{e}$, and the insider observes $\tilde{v}$, $\tilde{e}$, and $\tilde{\theta}_L$. Let $\widetilde{X}_s(\tilde{v}, \tilde{e})$ denote the speculator's equilibrium market buy order, and let $\widetilde{X}_i(\tilde{v}, \tilde{e}, \tilde{\theta}_L)$ denote the insider's equilibrium market sell order.

The market maker observes $\tilde{s} \equiv \tilde{v} + \tilde{e}$ and $\omega \equiv \tilde{u} + \widetilde{X}_s - \widetilde{X}_i$. He sets the time-1 price to be

$$\widetilde{P}_1(\tilde{s}, \tilde{\omega}) = \mathbf{E}\big[\tilde{v} - C \mid \tilde{s}, \tilde{\omega}\big]. \tag{A.1}$$

We conjecture that the result of this price setting procedure leads to the following linear functional form for the price:

$$\begin{aligned}\widetilde{P}_1(\tilde{s},\tilde{\omega}) &= \gamma+\alpha\tilde{s}+\lambda\tilde{\omega}\\ &= \gamma+\alpha(\tilde{v}+\tilde{e})+\lambda\big[\tilde{u}+\widetilde{X}_s(\tilde{v},\tilde{e})-\widetilde{X}_i\big(\tilde{v},\tilde{e},\tilde{\theta}_L\big)\big],\end{aligned}\tag{A.2}$$

and we will later confirm this conjecture.

Under the linear price rule and given the insider's strategy, the speculator's strategy $\widetilde{X}_s(\tilde{v},\tilde{e})$ should solve

$$\max_x \mathbf{E}_{\tilde{v},\tilde{e}}\big[x\big[(\tilde{v}-C)-\widetilde{P}_1\big(\tilde{v}+\tilde{e},\tilde{u}+x-\widetilde{X}_i\big(\tilde{v},\tilde{e},\tilde{\theta}_L\big)\big)\big]\big],$$

where $\mathbf{E}_{\tilde{v},\tilde{e}}$ denotes taking the expectation conditioning on observing $\tilde{v}$ and $\tilde{e}$. Solving this problem yields

$$\widetilde{X}_s(\tilde{v},\tilde{e})=\frac{1}{2\lambda}\big[(\tilde{v}-C)-\big[\gamma+\alpha(\tilde{v}+\tilde{e})-\lambda\mathbf{E}_{\tilde{v},\tilde{e}}\big[\widetilde{X}_i\big(\tilde{v},\tilde{e},\tilde{\theta}_L\big)\big]\big]\big].\tag{A.3}$$

For the insider, under the linear price rule and given the speculator's strategy, his market sell order $\widetilde{X}_i(\tilde{v},\tilde{e},\tilde{\theta}_L)$ should solve

$$\max_x \mathbf{E}_{\tilde{v},\tilde{e},\tilde{\theta}_L}\big[-l\big(x-\tilde{\theta}_L\big)^2+x\big[\widetilde{P}_1\big(\tilde{v}+\tilde{e},\tilde{u}+\widetilde{X}_s(\tilde{v},\tilde{e})-x\big)-(\tilde{v}-C)\big]+\theta(\tilde{v}-C)\big]$$

which yields

$$\widetilde{X}_i\big(\tilde{v},\tilde{e},\tilde{\theta}_L\big)=\frac{1}{2(l+\lambda)}\big[2l\tilde{\theta}_L+\big[\gamma+\alpha(\tilde{v}+\tilde{e})+\lambda\widetilde{X}_s(\tilde{v},\tilde{e})\big]-(\tilde{v}-C)\big].\tag{A.4}$$

Solving (A.3) and (A.4) simultaneously gives the solution of the Nash equilibrium:

$$\widetilde{X}_s(\tilde{v},\tilde{e})=\frac{2l+\lambda}{\lambda(4l+3\lambda)}\big[(\tilde{v}-C)-\big(\gamma+\alpha(\tilde{v}+\tilde{e})\big)\big]+\frac{2l\lambda}{\lambda(4l+3\lambda)}\bar{\theta}_L,\tag{A.5}$$

$$\begin{aligned}\widetilde{X}_i\big(\tilde{v},\tilde{e},\tilde{\theta}_L\big) &= \frac{1}{4l+3\lambda}\big[\big(\gamma+\alpha(\tilde{v}+\tilde{e})\big)-(\tilde{v}-C)\big]+\frac{l}{l+\lambda}\tilde{\theta}_L\\ &\quad+\frac{l\lambda}{(l+\lambda)(4l+3\lambda)}\bar{\theta}_L.\end{aligned}\tag{A.6}$$

Given these strategies by the speculator and the insider, the market maker sets price $\widetilde{P}_1$ according to (A.1). From (A.5) and (A.6), we have

$$\begin{aligned}\tilde{u}+\widetilde{X}_s-\widetilde{X}_i &= \tilde{u}+\frac{2(l+\lambda)}{\lambda(4l+3\lambda)}\big[(\tilde{v}-C)-\big[\gamma+\alpha(\tilde{v}+\tilde{e})\big]\big]-\frac{l}{l+\lambda}\tilde{\theta}_L\\ &\quad+\frac{l(2l+\lambda)}{(l+\lambda)(4l+3\lambda)}\bar{\theta}_L.\end{aligned}$$

So the market maker should set the price to be

$$\begin{aligned}\widetilde{P}_1 &= \mathbf{E}\big[\tilde{v}-C\mid\tilde{v}+\tilde{e},\tilde{u}+\widetilde{X}_s-\widetilde{X}_i\big]\\ &= \mathbf{E}\left[\tilde{v}\mid\tilde{v}+\tilde{e},\tilde{v}+\frac{\lambda(4l+3\lambda)}{2(l+\lambda)}\tilde{u}-\frac{l\lambda(4l+3\lambda)}{2(l+\lambda)^2}\big(\tilde{\theta}_L-\bar{\theta}_L\big)\right]-C\end{aligned}$$

$$= \frac{\bar{v}}{\sigma_v^2} + \frac{\tilde{v}+\tilde{e}}{\sigma_e^2} + \frac{1}{\Sigma^2}\Bigg[\tilde{v} + \frac{\lambda(4l+3\lambda)}{2(l+\lambda)}\tilde{u} - \frac{l\lambda(4l+3\lambda)}{2(l+\lambda)^2}\big(\tilde{\theta}_L - \bar{\theta}_L\big)\Bigg]\Bigg/\left(\frac{1}{\sigma_v^2} + \frac{1}{\sigma_e^2} + \frac{1}{\Sigma^2}\right) - C, \tag{A.7}$$

where

$$\Sigma^2 \equiv \left[\frac{\lambda(4l+3\lambda)}{2(l+\lambda)}\right]^2 \sigma_u^2 + \left[\frac{l\lambda(4l+3\lambda)}{2(l+\lambda)^2}\right]^2 \sigma_L^2. \tag{A.8}$$

With some algebra, one can show that the price function in (A.7) is indeed of the linear form conjectured in (A.2). By comparing the two equations and matching the linearity coefficients, we obtain:

$$\gamma = -C + \frac{\hat{\sigma}_v^2}{\sigma_v^2}\bar{v} + \frac{2l\lambda}{2l+\lambda}\bar{\theta}_L, \tag{A.9}$$

$$\alpha = \frac{\hat{\sigma}_v^2}{\sigma_e^2}, \tag{A.10}$$

where

$$\hat{\sigma}_v^2 \equiv \mathbf{var}(\tilde{v} \mid \tilde{v}+\tilde{e}) = \frac{\sigma_v^2\sigma_e^2}{\sigma_v^2+\sigma_e^2},$$

and λ is given by

$$\frac{\lambda^2(4l+3\lambda)^2}{2(l+\lambda)(2l+\lambda)}\left[\sigma_u^2 + \left(\frac{l}{l+\lambda}\right)^2 \sigma_L^2\right] = \hat{\sigma}_v^2. \tag{A.11}$$

With the linear pricing function solved, and using

$$\mathbf{E}(\tilde{v} \mid \tilde{v}+\tilde{e}) = \frac{\hat{\sigma}_v^2}{\sigma_v^2}\bar{v} + \frac{\hat{\sigma}_v^2}{\sigma_e^2}(\tilde{v}+\tilde{e}),$$

we show that the equilibrium trading strategies for the speculator and the insider are:

$$\widetilde{X}_s(\tilde{v},\tilde{e}) = \frac{2l+\lambda}{\lambda(4l+3\lambda)}\big[\tilde{v} - \mathbf{E}(\tilde{v} \mid \tilde{s})\big],$$

$$\widetilde{X}_i\big(\tilde{v},\tilde{e},\tilde{\theta}_L\big) = \frac{1}{4l+3\lambda}\big[\mathbf{E}(\tilde{v} \mid \tilde{s}) - \tilde{v}\big] + \frac{l}{l+\lambda}\tilde{\theta}_L + \frac{l\lambda}{(l+\lambda)(2l+\lambda)}\bar{\theta}_L,$$

and the market maker's pricing function can be rewritten as

$$\widetilde{P}_1 = \mathbf{E}(\tilde{v} \mid \tilde{s}) - C + \lambda\big[\tilde{u} + \widetilde{X}_s(\tilde{v},\tilde{e}) - \big[\widetilde{X}_i\big(\tilde{v},\tilde{e},\tilde{\theta}_L\big) - \bar{\theta}_L\big]\big] - \frac{\lambda^2}{2l+\lambda}\bar{\theta}_L.$$

So the conjectured equilibrium is verified.

Finally, we prove the existence and uniqueness of such an equilibrium. To do so, we need to show that Eq. (A.11) has a unique solution. Define the left-hand side of (A.11) as

$$B(\lambda) \equiv \frac{\lambda^2(4l+3\lambda)^2}{2(l+\lambda)(2l+\lambda)}\left[\sigma_u^2 + \left(\frac{l}{l+\lambda}\right)^2 \sigma_L^2\right]. \tag{A.12}$$

Since $B(0) = 0$, $B(\infty) = \infty$, and

$$B'(\lambda) \equiv A(l,\lambda,\sigma_u) = \frac{\lambda(4l+3\lambda)(16l^3+36l^2\lambda+27l\lambda^2+6\lambda^3)}{2(l+\lambda)^2(2l+\lambda)^2}\sigma_u^2$$
$$+ \frac{l^3\lambda(4l+3\lambda)(16l^2+20l\lambda+7\lambda^2)}{2(l+\lambda)^4(2l+\lambda)^2}\sigma_L^2 \tag{A.13}$$
$$> 0, \tag{A.14}$$

Eq. (A.11) indeed has a unique strictly positive solution. So Theorem 1 is proved. □

Proof of Lemma 1. For given l and σ_L, taking the total differential of Eq. (14), we have

$$A(l,\lambda,\sigma_u)\,\mathrm{d}\lambda + \frac{\lambda^2(4l+3\lambda)^2}{2(l+\lambda)(2l+\lambda)}\,\mathrm{d}\sigma_u^2 = \mathrm{d}\hat{\sigma}_v^2, \tag{A.15}$$

from which we have

$$\mathrm{d}\lambda = \frac{1}{A(l,\lambda,\sigma_u)}\left[-\frac{\lambda^2(4l+3\lambda)^2}{2(l+\lambda)(2l+\lambda)}\,\mathrm{d}\sigma_u^2 + \mathrm{d}\hat{\sigma}_v^2\right], \tag{A.16}$$

which shows that λ is decreasing in $1/\sigma_e^2$ and σ_u^2. □

Proof of Lemma 2. From Theorem 1, we have

$$\widetilde{X}_i - \tilde{\theta}_L = \frac{1}{4l+3\lambda}\big[\mathbf{E}(\tilde{v} \mid \tilde{s}) - \tilde{v}\big] - \frac{\lambda}{l+\lambda}\big(\tilde{\theta}_L - \bar{\theta}_L\big) - \frac{\lambda}{2l+\lambda}\bar{\theta}_L,$$

and

$$\mathbf{E}\big[\big(\widetilde{X}_i - \tilde{\theta}_L\big)^2\big] = \frac{1}{(4l+3\lambda)^2}\hat{\sigma}_v^2 + \frac{\lambda^2}{(l+\lambda)^2}\sigma_L^2 + \frac{\lambda^2}{(2l+\lambda)^2}\bar{\theta}_L^2. \tag{A.17}$$

We also have

$$\widetilde{P}_1 - \widetilde{P}_2 = \frac{2l+\lambda}{4l+3\lambda}\big[\mathbf{E}(\tilde{v} \mid \tilde{s}) - \tilde{v}\big] + \lambda\tilde{u} - \frac{l\lambda}{l+\lambda}(\tilde{\theta}_L - \bar{\theta}_L),$$

and

$$\mathbf{E}\big[\widetilde{X}_i(P_1 - P_2)\big] = \frac{2l+\lambda}{(4l+3\lambda)^2}\hat{\sigma}_v^2 - \frac{l^2\lambda}{(l+\lambda)^2}\sigma_L^2. \tag{A.18}$$

From (A.17) and (A.18), we finally have

$$I(C) = \mathbf{E}\big[-l\big(\widetilde{X}_i - \tilde{\theta}_L\big)^2 + \widetilde{X}_i\big(\widetilde{P}_1 - \widetilde{P}_2\big) + \theta\widetilde{P}_2\big]$$
$$= \theta(\bar{v} - C) + \frac{l+\lambda}{(4l+3\lambda)^2}\hat{\sigma}_v^2 - \frac{l\lambda^2}{(2l+\lambda)^2}\bar{\theta}_L^2 - \frac{l\lambda}{l+\lambda}\sigma_L^2. \qquad \square$$

Proof of Theorem 2. Define

$$G(\bar{v}, \sigma_v^2, \sigma_u^2, l, \theta, K, M) \equiv \frac{\theta}{\sigma_v^4}\left[\frac{1}{A(l,\lambda_0,\sigma_u)}\left[\frac{l^2}{(l+\lambda_0)^2}\sigma_L^2 + \frac{4l^2\lambda_0}{(2l+\lambda_0)^3}\bar{\theta}_L^2 + \frac{2l+3\lambda_0}{(4l+3\lambda_0)^3}\sigma_v^2\right] - \frac{l+\lambda_0}{(4l+3\lambda_0)^2}\right]^{-1}, \tag{A.19}$$

where λ_0 is the solution of (14) for $C=0$, and $A(l,\lambda,\sigma_u)$ is defined in (A.13). We have

$$I'(0) = -\theta - \left[\frac{l^2}{(l+\lambda_0)^2}\sigma_L^2 + \frac{4l^2\lambda_0}{(2l+\lambda_0)^3}\bar{\theta}_L^2 + \frac{2l+3\lambda_0}{(4l+3\lambda_0)^3}\sigma_v^2\right]\frac{\mathrm{d}\lambda}{\mathrm{d}C}\bigg|_{C=0} - \sigma_v^4\frac{l+\lambda_0}{(4l+3\lambda_0)^2}f'(0). \tag{A.20}$$

Using (A.16), we have

$$\frac{\mathrm{d}\lambda}{\mathrm{d}C}\bigg|_{C=0} = -\frac{\sigma_v^4}{A(l,\lambda_0,\sigma_u)}f'(0). \tag{A.21}$$

Putting (A.21) back into (A.20), we have

$$I'(0) = \theta\left[\frac{f'(0)}{G(\bar{v}, \sigma_v^2, \sigma_u^2, l, \theta, K, M)} - 1\right].$$

So if the technical condition specified in the theorem holds, then $I'(0) > 0$. This means that the optimal choice of C for the insider must be positive.

Numerical calculations in Section 5.4 confirm that there indeed exist parameter values that satisfy condition (27). □

Proof of Proposition 1. For the case of $l=\infty$, we have, from (14), $\lambda = \hat{\sigma}_v/2\sqrt{\sigma_u^2+\sigma_L^2}$, and

$$I(C) = \theta(\bar{v}-C) - \lambda\sigma_L^2 = \theta(\bar{v}-C) - \frac{\sigma_L^2}{2\sqrt{\sigma_u^2+\sigma_L^2}}\frac{1}{\sqrt{1/\sigma_v^2+\alpha_e C}}.$$

Taking derivatives, we have

$$I'(C) = -\theta + \frac{\sigma_L^2}{4\sqrt{\sigma_u^2+\sigma_L^2}}\frac{\alpha_e}{(1/\sigma_v^2+\alpha_e C)^{3/2}}, \tag{A.22}$$

$$I''(C) = -\frac{3\sigma_L^2}{8\sqrt{\sigma_u^2+\sigma_L^2}}\frac{\alpha_e^2}{(1/\sigma_v^2+\alpha_e C)^{5/2}} < 0. \tag{A.23}$$

From (A.23) and the constraint that $C^* \geqslant 0$, we know that the optimal cost C^* is given by the maximum of zero and the solution to $I'(C)=0$. Solving $I'(C)=0$ and applying the constraint of $C \geqslant 0$, we have

$$C^* = \text{Max}\left\{\frac{K^{4/3}\theta^{2/3}}{2^{4/3}\alpha_e^{1/3}(\sigma_u^2 + K^2\theta^2)^{1/3}} - \frac{1}{\alpha_e \sigma_v^2},\ 0\right\},$$

and, given that $\mathrm{d}\lambda/\mathrm{d}C < 0$ (from (A.16)), the equilibrium λ is given by

$$\lambda^* = \text{Min}\left\{\left[\frac{\theta}{2\alpha_e \sigma_L^2(\sigma_u^2 + \sigma_L^2)}\right]^{1/3},\ \frac{\sigma_v}{2\sqrt{\sigma_u^2 + \sigma_L^2}}\right\}. \qquad \square$$

Proof of Theorem 3. From (A.16), we have

$$\left.\frac{\mathrm{d}\lambda}{\mathrm{d}C}\right|_{C=0} = -\frac{1}{A(l, \lambda_0, \underline{\sigma}_u)} \frac{\lambda^2(4l + 3\lambda)^2}{2(l + \lambda)(2l + \lambda)} g'(0), \tag{A.24}$$

from which we have

$$\begin{aligned} I'(0) &= -\theta - \left[\frac{l^2}{(l + \lambda_0)^2}\sigma_L^2 + \frac{4l^2\lambda_0}{(2l + \lambda_0)^3}\bar{\theta}_L^2 + \frac{2l + 3\lambda_0}{(4l + 3\lambda_0)^3}\sigma_v^2\right] \left.\frac{\mathrm{d}\lambda}{\mathrm{d}C}\right|_{C=0} \\ &= -\theta + \left[\frac{l^2}{(l + \lambda_0)^2}\sigma_L^2 + \frac{4l^2\lambda_0}{(2l + \lambda_0)^3}\bar{\theta}_L^2 + \frac{2l + 3\lambda_0}{(4l + 3\lambda_0)^3}\sigma_v^2\right] \\ &\quad \times \frac{1}{A(l, \lambda_0, \underline{\sigma}_u)} \frac{\lambda^2(4l + 3\lambda)^2}{2(l + \lambda)(2l + \lambda)} g'(0). \end{aligned}$$

So if (32) holds, then $I'(0) > 0$, which means that the optimal choice of C for the insider must be positive. $\square$

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