TURNOUT AND POWER SHARING

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Abstract. Differences in electoral rules and/or legislative, executive or legal institutions across countries induce different mappings from election outcomes to distributions of power. We explore how these different mappings affect voters' participation in a democracy. Assuming heterogeneity in the cost of voting, the effect of such institutional differences on turnout depends on the distribution of voters' preferences for the parties: when the two parties have similar support, turnout is higher in a winner-take-all system than in a power sharing system; the result is reversed when one side has a larger base. The results obtained in the rational voter model are shown to continue to hold in other models of turnout such as ethical voter models and voter mobilization models. Finally, the theoretical comparative results are validated by an experiment, comparing the Levine and Palfrey (2007) results with new data on power sharing elections.

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1. Introduction

Voter participation is an essential component of democracy, and changes in the level of electoral participation may affect the political positioning of the competing parties and ultimately public policy. Yet the positive analysis of turnout is still far from established and many questions remain. This paper provides a comparative analysis, both theoretical and experimental, of fundamental causes of the variation in turnout based on differences in institutions for political power sharing.

Is it possible to characterize the influence of institutional systems on turnout, making sure, in addition, that such comparative results are robust to different modeling assumptions? In particular, does turnout depend in any identifiable way on the type of democratic regime, the electoral rules, legislative organization rules, or the degree of separation of powers?

Our idea to make headway on this topic is as follows: different institutional systems impact the mapping from election outcomes (henceforth *vote shares*) to the relative weight of different parties in decision making (henceforth *power shares*).[1] Hence we try to assess in a general way the role of institutions on electoral participation by characterizing how that vote-shares-to-power-shares mapping affects voters' incentives to vote and parties' campaign efforts. The role of individual institutions in determining electoral participation can then be separately evaluated by looking at their impact on that mapping. In sum, the degree of influence on policy for given electoral outcomes is the key exogenous variable for our analysis: we will often refer to this reduced form mapping simply as the "institutional system" ranging from a winner-take-all system to a fully proportional power sharing system as modelled, for example, in Lizzeri and Persico (2001).[2]

The results below also depend on another key parameter, namely the expected "winning margin" or "closeness" of the election. Before explaining how the vote-share to power-share

[1]The relative power of the majority party for a given election outcome varies with the degree of separation of powers, the organization of chambers, the assignment of committee chairmanships and institutional rules on agenda setting, allocation of veto powers, and obviously electoral rules. See Lijphart (1999) and Powell (2000) for a comprehensive analysis of the impact of political institutions on what they call degree of proportionality of influence, which is basically our vote-shares to power-shares mapping. Electoral rules determine the mapping from vote shares to seat shares in a legislature, whereas the other institutions determine the subsequent mapping from seat shares to power shares across parties.

[2]See also Persico and Sahuguet (2006) and Iaryczower and Mattozzi (2010).

mapping and the expected closeness of an election jointly determine turnout, we need to highlight the main modeling choices that we make in the paper.

Since there is no established or canonical model of electoral turnout, we will analyze the variation in electoral turnout in more than one model. The common assumptions to all of the models that we discuss are that (1) the distribution of citizens' preferences over the set of alternatives (candidates or parties or coalitions of parties) is common knowledge; (2) the only relevant decision by each citizen is whether to go to vote or not; (3) each voter is described in a two-dimensional type space, i.e. her preferred party and her cost of voting: the cost of voting for each citizen is drawn from a general continuous distribution with non negative support.

The main model that we consider in our analysis is the standard rational voter model (see e.g. Palfrey and Rosenthal (1985)) under population uncertainty[3], extending the analysis to the proportional influence or proportional power sharing system, i.e. the system in which power shares are expected to be proportional to vote shares. However, we will also analyze the same comparative questions using mobilization and ethical voting models,[4] and the robustness of the main comparative findings will finally be analyzed in the experimental laboratory.

Several recent theoretical papers have identified a neutrality result for equilibrium turnout models of winner-take-all elections, which we call *the full underdog compensation effect*: The theoretical claim is that in equilibrium pivotal voting models the expected vote shares of the two parties are *equal* independent of the distribution of partisan preferences in the population. Unfortunately, this result is not robust, as our paper demonstrates. Rather, it is an extremely fragile finding, based on special and empirically suspect technical assumptions about the distribution of costs in those two models. The property of full underdog compensation identified in earlier papers is due to either an assumption that the distribution of voting costs is degenerate (Goeree and Grosser 2007, Taylor and Yildirim 2010a), or is bounded below by a strictly positive minimum voting cost (Krasa and Polborn 2009), or is identical for voters from different parties.[5] It is, in fact, not a general property of pivotal voting models. In this paper we also

[3]Viewing the size of the electorate as a random variable (see Myerson 1998 and 2000) has the advantage of simplifying the computations without altering the incentives driving the results. Krishna and Morgan (2011) recently obtained important results in a model similar to ours, but with common values, in which population uncertainty is key.

[4]See Shachar and Nalebuff (1999), Coate and Conlin (2004) and Feddersen and Sandroni (2006).

[5]In contrast, the original Palfrey-Rosenthal (1985) model rules out the first two of these special cases, and explicitly allows for different distributions of voting costs for the two parties. Taylor and Yildirim (2010b) show that the neutrality result generally fails if the lower bound of two parties' voting costs is differernt.

show that the underdog compensation varies with the vote-share to power-share mapping: the underdog compensation is always larger in a proportional power sharing system.[6]

Following Ledyard (1984) and Palfrey and Rosenthal (1985) we assume a continuously differentiable distribution of voting costs, but unlike these other models, the support of the cost distributions of voters is the non negative real numbers. In our model we allow the support to go all the way to zero; i.e., the density of voting costs at zero is *not zero.* The technical details are more complicated, but the intuition is illustrated starkly in Palfrey and Rosenthal (1985). They assume that the support of voting costs not only includes zero, but also some negative voting costs. For simplicity, suppose all voters have costs drawn independently from some continuous distribution F on $[-\epsilon, \overline{c}]$ where ϵ can be any arbitrarily small positive number. Palfrey and Rosenthal's result is that in the limit only voters with non-positive voting costs turn out, and therefore the limiting turnout rate for each party is identical and equal to $F(0)$. *There is no underdog compensation at all: the party vote shares are precisely equal to their population shares!*[7] We show that much of this logic carries over to the limiting case when $\epsilon = 0$. Specifically, for all $\epsilon \geq 0$ the equal vote share result fails. Moreover, when $\epsilon = 0$ there is always a higher participation rate of the underdog party supporters than the supporters of the favorite to win: the party with higher ex-ante support is always expected to win, but by a smaller margin of victory than the ex-ante support advantage (e.g. the opinion polls) would predict.

Comparing turnout across systems boils down to comparing the individual benefits of voting across systems. In a proportional power sharing system the expected marginal benefit of a single vote is proportional to the marginal change in the vote share determined by that vote. Whereas in a winner take all system the marginal benefit of a vote is proportional to the probability of that vote being pivotal. Both marginal benefits obviously decrease as the

[6]This comparison has welfare consequences: the winner-take-all system yields higher expected total utility due to the lower underdog compensation. This welfare corollary cannot be established with models where the underdog compensation effect is full so all elections are tied in expectation. Even though we will devote some attention to this welfare discussion in the paper at some point, our main focus is the comparative analysis of turnout *per se*. For a rich analysis of welfare comparisons in a model similar to ours, see Kartal (2010).

[7]Coate, Conlin and Moro (2008) find no support for the full underdog compensation effect in their empirical study of referenda. They claim "The logic of pivotal-voter models implies that elections must be expected to be close even if there is a significant difference between the sizes of the groups or the intensity of their preferences." This paper shows that this is a false claim. In fact, except under very special and questionable assumptions, equilibrium pivotal voter models imply nothing of the kind.

number of voters increases. In large elections the comparison of turnout across systems hence depends on the asymptotic speed with which a larger population reduces the individual benefit of voting, i.e. the magnitude of the "size effect."[8] Quantitatively we show that in a proportional system the benefit of voting decreases asymptotically as $1/N$ when N, the expected size of the electorate, increases; whereas in a winner-take-all system such asymptotic speed is slower when the election is expected to be a tie and much faster otherwise. This fact determines the main conclusion, namely that turnout is higher in a proportional system when the election has a clear favorite party while a winner-take-all system induces higher turnout otherwise.[9]

In order to obtain analytical results, we use population uncertainty (see Myerson 1998 & 2000). Viewing the size of the electorate as a random variable has the advantage of simplifying the computations. Krishna and Morgan (2011) and McMurray (2010) recently obtained important results in a model similar to ours, but with common values, in which population uncertainty is key.[10] Population uncertainty does not alter the incentives driving the results: numerical computations we performed with fixed population sizes confirm that all our comparative results do not depend on the population uncertainty.

Even though the comparative analysis using the rational voter model is, we believe, well justified and widely used, our objective is not to argue in favor of the costly voting models over other models. To the contrary, given that our objective is to *convince* all readers about the validity of our comparative institutional results, without necessarily taking a methodological strong stand in favor of the rational voter model, we study the same questions using other well known approaches. In particular, the voter mobilization approach to turnout following the work by Morton (1987 & 1991), Uhlaner (1989), and Shachar and Nalebuff (1999), and the ethical voter models proposed by Feddersen and Sandroni (2006) are possible alternatives to the strategic voting model because they yield more plausible turnout point estimates if one assumes the distribution of voting costs is bounded below by zero. They also have some

[8]As in Levine and Palfrey (2007) the "size effect" measures how the benefit of voting and hence turnout decrease with the size of the population.

[9]We conduct the bulk of the analysis for the case of two parties, but we show in the appendix the robustness of all comparisons to changes in the number of parties: in a proportional power sharing system the order of magnitude of the size effect does not depend on the distribution of ex ante support of parties, nor does it depend on the number of parties present in the election. Hence the comparison with the winner take all system is also unaffected by the number of parties. We also show that in a proportional power sharing system turnout increases as the number of parties increases.

[10]See also Krishna and Morgan (2010) for a similar model with private and common values.

conceptual appeal, as they propose specific mechanisms through which voter turnout might be higher than in traditional pivotal voter models. More importantly for the present paper, however, is to investigate whether our results comparing electoral systems are robust to such extensions of the pivotal voter model.

We show that the alternative models we consider (in which voters cooperate) have the same qualitative properties of the rational model (in which voters do not cooperate).[11] The robustness of this comparative finding depends crucially on a feature common to all models: full underdog compensation does not occur, so an ex-ante uneven election always remains ex-post uneven.

Our results suggest the general point that any model of large elections featuring partial (or zero) underdog compensation effect, yields the robust prediction that winner take all system induces higher turnout when the citizens' support for the two main parties or party coalitions is very close, while more proportional systems induce higher turnout when one party has a larger ex-ante support. The intuition is that in the winner-take-all system when preferences are not evenly split the non full underdog compensation preserves the ex-ante leading party as the ex-post leading party in equilibrium, hence preserving a high expected winning margin, which discourages participation. In a more proportional power sharing system, a less competitive election (i.e. a higher expected winning margin) does not affect the incentives to vote as much.

If one moves away from the convenient population uncertainty world (convenient in terms of the possibility to obtain analytical results) it is easy to find the exact equilibrium conditions to be used for numerical computation of the predictions for any known number of voters. Hence it is possible to test the comparative results in the laboratory. Since Levine and Palfrey (2007) gave us already a preliminary set of data with winner-take-all rules, we adopted the same treatments even in the new proportional experiments, so that the data could be pooled together.

The experimental results confirm the theoretical predictions of the general model, as well as other predictions on the *competition effects* that came out specifically from the known-population model computations.

While there is some empirical evidence about the relationship between ex ante closeness and turnout (Blais), we are not aware of any empirical work focusing on the interaction effect

[11] An additional benefit of studying the same questions with these models of mobilization and ethical voting is that they are computationally simpler, and hence we can extend the analysis to *any* power sharing system, whereas in the benchmark strategic model only the two extremes can be compared.

of expected closeness and the degree of power sharing of the institutional system. Even though the mapping from vote shares to power shares is only partially determined by the electoral rules (vote shares to seat shares), some empirical work on electoral rules is definitely related to our topic: the empirical evidence on turnout in national elections (see e.g. Powell (1980, 1986), Crewe (1981), Jackman (1987) and Jackman and Miller (1995), Blais and Carthy (1990) and Franklin (1996)) all conclude that, everything else being equal, turnout is lower in plurality and majority elections than under Proportional Representation.[12] On the other hand, experimental evidence (see Schram and Sonnemans (1996)) suggests the opposite. We will show that these seemingly inconsistent findings are easily reconcilable, since the experimental design featured perfect symmetry in the ex-ante supports for the two parties, i.e. in the case in which we show that we should expect higher turnout under a winner take all system.[13]

The paper is organized as follows. Section 2 contains the complete analysis of a rational voter model of turnout, comparing the properties of proportional power sharing system and winner-take-all system. Section 3 contains the analysis of the ethical voter model and the mobilization model, where even intermediate proportionality levels can be considered, and where we confirm the robustness of our main comparative results across modeling choices. Section 4 will contain the experimental analysis, and section 5 will offer some concluding remarks and describe potential paths of future research. All proofs are in Appendix B, while Appendix A contains a welfare corollary and a simple extension of the main comparative result of the paper to the multiple-party case. A sample of the instructions from one of the experimental sessions is in Appendix C.

2. Rational Voter Turnout

Consider two parties, A and B, competing for power. Citizens have strict political preferences for one or the other, chosen exogenously by Nature. We denote by $q \in (0,1)$ the preference split, i.e. the chance that any citizen is assigned (by Nature) a preference for party A (thus $1-q$ is the expected fraction of citizens that prefer party B). Without loss of generality, we assume that $q \leq 1/2$, so that the A party is the underdog party (with smaller ex-ante support)

[12] The standard caveat is that cross sectional studies are not to be considered conclusive evidence, because of the small sample size and few data points, cultural and idiosyncratic characteristics that are difficult to control for, as emphasized in Acemoglu (2005).

[13] Similar experimental findings to ours can also be found in Kartal (2011), an experimental analysis developed independently.

and the B party is the leader party (with larger ex-ante support). The indirect utility for a citizen of preference type i, $i = A, B$, is increasing in the share of power that party i has. For normalization purposes, we let the utility from "full power to party i" equal 1 for type i citizens and 0 for the remaining citizens.[14]

Beside partisan preferences, the second dimension along which citizen differ from one another is their cost of voting: each citizen's cost of voting c is drawn from a distribution with infinitely differentiable pdf $f(c)$ over the support $c \in [0, \overline{c}]$, with $\overline{c} > 0$ (we denote the cdf as $F(c)$).[15] The cost of voting and the partisan preferences are two independent dimensions that determine the type of a voter.

For any vote share V obtained by party A, an institutional system γ determines power shares $P_\gamma^A(V) \in [0,1]$ and $P_\gamma^B(V) = 1 - P_\gamma^A(V)$. Given the above normalization, these are the reduced form "benefit" components of parties' (respectively, voters') utility functions that will determine the incentives to campaign (respectively, vote) in an institutional system. In this section we study the base model in which parties do not campaign nor attempt to coordinate or mobilize voters, hence turnout depends exclusively on voters' comparison between the policy benefits of voting for the preferred candidate and the opportunity costs of voting.

In terms of the size of the electorate, we find it convenient to assume that the population is finite but uncertain. There are n citizens who are able to vote at any given time, but such a number is uncertain and distributed as a Poisson distribution with mean N:

$$n \sim \frac{e^{-N}(N)^n}{n!}$$

Most analytical statements in the first part of the paper are made for a large enough population, namely they are true for every N above a given $\overline{N}$. However, we will easily establish very similar results for small elections via numerical computations.

Citizens have to choose to vote for party A, party B, or abstain. If a share α of A types vote for A and a share β of B types vote for B, the expected turnout T is

$$T = q\alpha + (1-q)\beta$$

[14]This normalization will allow us to match party utility and voters's utilities in a simple way under all the institutional systems that will be considered.

[15]One could allow for the support to include negative voting costs. This trivially implies a zero compensation effect, as explained in the introduction.

We look for a Bayesian equilibrium in which all voters of type A with a cost below a threshold c_α vote for type A and voters of type B with a cost below c_β vote for B. So on aggregate, type A citizens vote for A with chance $\alpha = F(c_\alpha)$ and type B citizens vote for B with chance $\beta = F(c_\beta)$.

In any equilibrium strategy profile (α, β), the expected marginal benefit of voting, B_γ, must be equal to the cutoff cost of voting (indifference condition for the citizen with the highest cost among the equilibrium voters). Hence the equilibrium conditions can be written as

$$B_\gamma^A(\alpha,\beta) = F^{-1}(\alpha), \qquad B_\gamma^B(\alpha,\beta) = F^{-1}(\beta)$$

We compare the above equilibrium conditions in two systems which differ on the benefit side: a winner-take-all system ($\gamma = M$) and a proportional power sharing system ($\gamma = P$).[16]

2.1. **Winner take all system ($\gamma = M$).** In the M system the expected marginal benefit of voting B_M^A is the chance of being pivotal for a type A citizen, namely

$$B_M^A = \sum_{k=0}^{\infty} \left(\frac{e^{-qN\alpha} (Nq\alpha)^k}{k!} \right) \left(\frac{e^{-(1-q)N\beta} ((1-q)N\beta)^k}{k!} \right) \frac{1}{2} \left(1 + \frac{(1-q)N\beta}{k+1} \right)$$

namely the chance that an A citizen by voting either makes a tie and wins the coin toss or breaks a tie where it would have lost the coin toss. Likewise, for the type B citizens we have

$$B_M^B = \sum_{k=0}^{\infty} \left(\frac{e^{-qN\alpha} (Nq\alpha)^k}{k!} \right) \left(\frac{e^{-(1-q)N\beta} ((1-q)N\beta)^k}{k!} \right) \frac{1}{2} \left(1 + \frac{qN\alpha}{k+1} \right)$$

Equating the benefit side to the cost side we obtain a system of two equations in (α, β) (the M system henceforth). We now show that asymptotically turnout for each party is zero as a percentage of the population, but is infinite in absolute numbers. Moreover, the ratio of turnouts for each party remains finite.

Lemma 1. *Any equilibrium solution (α_N, β_N) to the M system (if it exists) has the following three properties*

$$\lim_{N\to\infty} \alpha_N = \lim_{N\to\infty} \beta_N = 0, \qquad \lim_{N\to\infty} N\alpha_N = \lim_{N\to\infty} N\beta_N = \infty, \qquad \lim_{N\to\infty} \frac{\alpha_N}{\beta_N} \in (0,\infty)$$

[16] Recall that the interpretation is not restricted to electoral rules, as explained in the introduction. Two countries with the same electoral rule can have very different mappings from electoral outcomes to power shares, and this is the summary or reduced form variable that we are interested in and that affects turnout.

The above lemma allows us to use some approximations to show existence and uniqueness of an equilibrium for N large and also the following characterization results.[17]

Lemma 2. *There exists an equilibrium* (α, β) *in the M system. For uniqueness it suffices that* F *is weakly concave. The equilibrium has the following properties:*

- Size effect:
$$\frac{dT_M}{dN} < 0$$
- Partial underdog compensation effect:
$$q < 1/2 \quad \Longrightarrow \quad \alpha > \beta, \quad q\alpha < (1-q)\beta$$

The size effect shows how the benefit of voting declines for larger electorates, although we will show that the rate of decline depends crucially on whether the parties do or do not have the same support ex-ante. The partial underdog compensation shows that the party with less supporters has higher relative expected turnout but lower expected turnout overall. We discuss all these effects in the following section.

2.2. **Discussion of the M System.** The partial underdog compensation arises from the following simple equilibrium relationship between the turnout rates for the two parties (see Appendix)

$$q\alpha \left(F^{-1}(\alpha)\right)^2 = (1-q)\beta \left(F^{-1}(\beta)\right)^2 \tag{1}$$

Since for heterogeneous costs $F^{-1}(\alpha)$ is increasing, then $q < 1/2$ implies an underdog compensation (i.e. $\alpha > \beta$) that must be partial (i.e. $q\alpha < (1-q)\beta$). As a consequence, we have a balanced election with a 50% expectation of victory from each side *only* when $q = 1/2$. With homogeneous costs the result would be different: homogeneous costs mean that $F^{-1}(\alpha) = c = F^{-1}(\beta)$, which implies $q\alpha = (1-q)\beta$, i.e. full underdog compensation and a 50% chance of victory regardless of the ex-ante preference split q.

To understand why the heterogeneity of the cost distribution is so important, assume for instance that $q = 1/3$ so that the leader party has double the ex-ante support than the underdog party. To have an election with a 50-50 chance of victory (i.e. $q\alpha = (1-q)\beta$), the underdog party would have to turn out twice as much as the leader party. We claim that the latter cannot happen unless citizens have homogenous costs. Suppose not; then on the benefit side,

[17] We thank John Morgan for pointing out the importance of proving this non trivial lemma for the approximation results and proofs that will follow.

in a strategy profile with an ex-ante even outcome, the gross benefit of voting is the same across all voters (as they all individually face the same even environment). On the cost side, since the underdog party has to turn out more, then we must have $\alpha = F(c_\alpha) > \beta = F(c_\beta)$. With heterogenous cost this means that the equilibrium cost thresholds would have to be different $c_\alpha > c_\beta$ which, in turn, implies that the cost thresholds cannot both be equal to the benefit. In other words, the underdog supporters cannot fully rebalance the election because turning out in a higher proportion means that types with a higher cost would have to turn out as well. To have an equilibrium with full underdog compensation (same benefit) we must have $c_\alpha = c_\beta$ (same cost), which happens when F is constant so costs are homogeneous.

Conversely, as the costs become equal, the equilibrium must exhibit full underdog compensation. The intuition is as follows. Suppose, to the contrary, that with homogenous costs a pure strategy equilibrium with partial underdog compensation existed so the ex-ante underdog is expected to lose the election. With such a strategy profile, a supporter of the underdog party who is abstaining, by deviating and going to vote would bring the election closer to a tie, hence he would have a higher benefit than the benefit of his fellow supporters of the underdog party that were voting according to that strategy profile, a contradiction.

Assuming heterogenous costs determines more appealing features. First of all the underdog compensation being just partial is what guarantees that the party with more ex-ante support is the more likely winner of the election. This natural outcome is corroborated by observed winning margins. Second, on the normative side, having the election result be determined by a coin toss as in the homogenous cost full underdog compensation case is clearly unappealing from a welfare perspective.[18]

The distinction between homogenous and heterogeneous costs and hence between full and partial underdog compensation is also key for turnout predictions. The different equilibria with different cost assumptions, namely a 50-50 outcome versus a non 50-50 outcome, imply very different overall turnout numbers in large elections. In fact, the benefit of voting and hence the

[18] The fact that the 50-50 benchmark result is pervasive in the literature prompted the question of whether it is of any use to have people vote at all as the preferences of the electorate are not reflected in the outcome. See e.g. Borgers (2004) and Krasa and Polborn (2010). A different line of work that tries to avoid the full compensation undesirable outcome assumes that the preference split q remains unknown to voters: if so, then the compensation effect which rebalances the election and lowers welfare cannot be triggered properly. Hence opinion polls, which reduce uncertainty about q, may be welfare reducing. See Goeree and Grosser (2007) and Taylor and Yilidirm (2009).

turnout are proportional to

$$B_M \sim \frac{e^{-\left(\sqrt{q\alpha}-\sqrt{(1-q)\beta}\right)^2 N}}{\sqrt{N}}$$

In the homogenous cost case, in which $q\alpha = (1-q)\beta$, this implies that turnout declines at the rate $N^{-1/2}$. In the heterogeneous cost case, where $q\alpha \neq (1-q)\beta$ unless $q = 1/2$, turnout declines at an exponential rate for $q \neq 1/2$[19] and declines at the algebraic rate $N^{-1/2}$ when $q = 1/2$.[20]

Even though the nature of our work is positive, we want to conclude this discussion of the M system with a simple welfare corollary:

Corollary 3. *Asymptotically, for the population N going to infinity, neither subsidies nor penalties for voters can improve total expected utility in the M system.*

This could be easily shown by adapting the proof of proposition 5 in Krasa and Polborn (2009), since their model is similar to our model of the M system but with a positive voting cost lower bound $\underline{c} > 0$. They show that in the limit the optimal subsidy to voters converges to $\underline{c}$. Thus, when one considers the same model but with zero as lower bound $\underline{c} = 0$, the optimal subsidy in the limit must be zero.[21] Intuitively, on the one hand introducing a subsidy is unnecessary since asymptotically the party with larger ex-ante support always wins the election in any case. On the other hand, introducing a penalty for voting would bring us back the inefficient lower bound $\underline{c} > 0$ in the voting cost distribution.

2.3. **Proportional Power Sharing System ($\gamma = P$).** With proportional power sharing (P system) the share of power is proportional to the vote share obtained in the election. So if

[19]For $q \neq 1/2$ the argument of the exponential function diverges to $-\infty$ (see Lemma 1 and Proposition 7).

[20]Chamberlain and Rothschild (1981) obtain a similar result on rates of convergence in a model in which two candidates receive votes as binomial random variables. They assume no abstention, so the number of votes can be seen as flips of identical coins with a certain bias q. They show that if you toss an even number n of coins, the chance of obtaining the same number of heads and tails (the chance of a tie) drops asymptotically like $N^{-1/2}$ when the coins are unbiased ($q = 1/2$) and exponentially if the coins are biased ($q < 1/2$).

[21]Krasa and Polborn (2010) obtain the inefficient full compensation result with a non degenerate cost distribution because its support $[\underline{c}, \overline{c}]$ is bounded away from zero. Hence, unlike what we obtain in Lemma 1, only a finite number of voters will go to vote even when the population N grows unboundedly large. Asymptotically their model is isomorphic to a homogenous cost model with cost $\underline{c} > 0$.

(a,b) are the absolute numbers of votes for each party, the power of parties A and B would be respectively $\left(\frac{a}{a+b}, \frac{b}{a+b}\right)$.[22]

The expected marginal benefit of voting B_P^i for party i is the expected increase in the vote share for the preferred party induced by a single vote, namely

$$B_P^A = \sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\left(\frac{e^{-qN\alpha}(qN\alpha)^a}{a!}\right)\left(\frac{e^{-(1-q)N\beta}((1-q)N\beta)^b}{b!}\right)\left(\frac{a+1}{a+b+1}-\frac{a}{a+b}\right)\right)$$

$$B_P^B = \sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\left(\left(\frac{e^{-qN\alpha}(qN\alpha)^a}{a!}\right)\left(\frac{e^{-(1-q)N\beta}((1-q)N\beta)^b}{b!}\right)\left(\frac{b+1}{a+b+1}-\frac{b}{a+b}\right)\right)$$

In this case, unlike in the M system, we have double summations because an A supporter, for instance, has an impact on the electoral outcome not only in the event of a tied election ($a=b$ and $a=b-1$), but also in all the other cases $a \neq b$. In the P system voters always have some impact on the electoral outcome albeit very small, whereas in the M system voters have a large impact in the very small chance event that $a=b$ and zero impact otherwise. A non obvious quantitative question is to compare how the expected impacts of a voter in the M and in the P systems decline with the electorate size N. Luckily, after some manipulation the double summations above can be expressed in a simple form.

Lemma 4. *The marginal benefit of voting in the P system has the closed form*

$$(2) \qquad \begin{aligned} B_P^A &= \frac{(1-q)\beta}{NT^2} - \left(\frac{((1-q)\beta)^2-(q\alpha)^2+(1-q)\beta\frac{1}{N}}{2T^2}\right)e^{-NT} \\ B_P^B &= \frac{q\alpha}{NT^2} + \left(\frac{((1-q)\beta)^2-(q\alpha)^2-q\alpha\frac{1}{N}}{2T^2}\right)e^{-NT} \end{aligned}$$

Similarly to what we obtained for the M model, we now show that asymptotically turnout for each party is zero as a percentage of the population, but is infinite in absolute numbers, moreover the party turnout ratio stays finite.

Lemma 5. *Any solution (α_N, β_N) (if it exists) to the P system has the following three properties*

$$\lim_{N\to\infty}\alpha_N = \lim_{N\to\infty}\beta_N = 0, \qquad \lim_{N\to\infty}N\alpha_N = \lim_{N\to\infty}N\beta_N = \infty, \qquad \lim_{N\to\infty}\frac{\alpha_N}{\beta_N} \in (0,\infty)$$

[22]We assume that if nobody votes, power is shared equally, namely

$$\frac{a}{a+b} = \frac{b}{a+b} = \frac{1}{2} \quad \text{for} \quad a=b=0$$

As in the M system, the above lemma allows us to use some approximations to show existence and uniqueness of an equilibrium for N large and also the following characterization results.

Lemma 6. *In the P system there is always a unique equilibrium (α, β). The equilibrium has the following properties:*

- Size effect:

$$\frac{dT_P}{dN} < 0$$

- Partial underdog compensation effect:

$$q < 1/2 \quad \Longrightarrow \quad \alpha > \beta, \quad q\alpha < (1-q)\beta$$

The relation describing quantitatively the underdog compensation under the P system is

$$q\alpha F^{-1}(\alpha) = (1-q)\beta F^{-1}(\beta) \tag{3}$$

which is slightly different from equation (1) describing the underdog compensation under the M system.

2.4. **Main Comparison.** The size effect and the underdog compensation effect, though qualitatively similar, are quantitatively different across the two institutional systems. We now turn to the implications of these differences and to the comparison of turnout incentives across systems. Turnout is larger in a proportional power sharing system when there is a favorite party, while it is higher in a winner take all system if the election is even.

Proposition 7. .

- *Comparative turnout: for any $q \in (0,1)$, $\exists \bar{N}_q$ such that for $N > \bar{N}_q$*

$$\begin{aligned} T_M &> T_P \qquad \text{for } q = 1/2 \\ T_P &> T_M \qquad \text{for } q \neq 1/2 \end{aligned}$$

- *Comparative underdog compensation:*

$$\frac{1-q}{q} = \left(\frac{\alpha_P}{\beta_P}\right)^{n+1} = \left(\frac{\alpha_M}{\beta_M}\right)^{2n+1}$$

where $n \geq 1$ is the lowest integer for which $\frac{d^n F^{-1}}{dx^n}|_{x=0} \in (0, \infty)$.

Regarding the comparative underdog compensation, we have already explained in section 2.2 that with heterogeneous costs full compensation is impossible in equilibrium, and a similar explanation holds for the proportional power sharing system. In both systems the underdog compensation is partial: the ex-ante favorite party obtains the majority of the votes in a large election, but the underdog party has a higher turnout of its supporters. The above proposition shows that the underdog compensation is larger in the P system, namely

$$q < 1/2 \quad \Rightarrow \quad \alpha_P/\beta_P > \alpha_M/\beta_M > 1 \tag{4}$$

Compared to the P system, in the M system minority voters are always more discouraged to vote relative to majority voters. This result could also be stated as a higher *relative winning margin* in the M system than in the P for any given preference split q, where the relative winning margin W is defined as[23]

$$W := \frac{|q\alpha - (1-q)\,\beta|}{T}$$

Regarding turnout, the intuition behind the turnout result relies on how fast the marginal benefit of voting decreases in the two models as the electorate gets larger. The M system has two asymptotic regimes: it decreases exponentially for $q \neq 1/2$ and for $q = \frac{1}{2}$ it decreases at the algebraic rate of $N^{-1/2}$. Since we have only partial underdog compensation, then for any $q \neq 1/2$ the majority party is always the more likely side to win. Hence the chance of a tied election, which is what drives rational voters to turn out, is much smaller than in the case $q = 1/2$ for any population size N.[24]

The benefit from voting in the P system drops asymptotically at the intermediate rate of N^{-1}. This rate is independent of q as in the power sharing system the event that a voter is pivotal or the chance of a tied election have no special relevance.

[23]This result, while apparent when the M and P systems are solved, is not trivial ex-ante as there are two competing effects: in the M system, while minority voters α are discouraged to vote, also (and for the same reason) majority voters β are. So, it is not obvious that α/β should be smaller in the M system than in the P system (where neither effect is present).

[24]The two rates of convergence derived above do not depend on the (Poisson) population uncertainty in this model. For instance, Herrera and Martinelli (2006) analyze a majority rule election without population uncertainty. They introduce aggregate uncertainty in a different way, which allows to obtain a closed form for the chance of being pivotal, namely $\frac{(a+b)!}{2^{a+b+1}a!b!}$. As it can be seen using Stirling's approximation, that marginal benefit for large a and b has exactly the square root decline on the diagonal ($a = b$) and the exponential decline off the diagonal $(a = \omega b,\ \omega \neq 1)$.

It is perhaps now intuitive that a winner take all system, unlike a proportional power sharing one, should have two quite different rates of convergence regimes (although as we explained this is not the case with a degenerate cost distribution). Be that as it may, only an explicit computation could determine that the rate of convergence in the P system is quantitatively *in between* the two rates of convergence in the M system: $N^{-1} \in \left(N^{-1/2}, e^{-N}\right)$.

In order to illustrate the comparison in terms of turnout as well as underdog compensation effects, we now turn to a numerical example.

2.5. **Example.** Consider the cost distribution family ($z > 0$): $F(c) = c^{1/z}$ with $c \in [0,1]$.

This example yields an explicit solution for the P system, i.e.

$$\alpha_P = \left(\frac{1}{N} \frac{(1-q)\, q^{\frac{1}{z+1}} (1-q)^{\frac{1}{z+1}}}{\left(q(1-q)^{\frac{1}{z+1}} + (1-q)\, q^{\frac{1}{z+1}}\right)^2} \right)^{\frac{1}{z+1}} \qquad \beta_P = \left(\frac{1}{N} \frac{q}{1-q} \frac{(1-q)\, q^{\frac{1}{z+1}} (1-q)^{\frac{1}{z+1}}}{\left(q(1-q)^{\frac{1}{z+1}} + (1-q)\, q^{\frac{1}{z+1}}\right)^2} \right)^{\frac{1}{z+1}}$$

The M system equilibrium has no closed form solution, namely (α_M, β_M) jointly solve

$$\beta_M = \left(\frac{q}{1-q}\right)^{\frac{1}{2z+1}} \alpha_M, \qquad \alpha_M^z = \frac{e^{-N\left(\sqrt{(1-q)\beta_M} - \sqrt{q\alpha_M}\right)^2}}{\sqrt{N}} \left(\frac{\sqrt{q\alpha_M} + \sqrt{(1-q)\,\beta_M}}{4\sqrt{\pi}\,(q(1-q)\,\alpha_M \beta_M)^{1/4}} \right)$$

Setting $N = 3000$ and $z = 5$, the numerical solutions to the M system yield a clear illustration of the comparative result of proposition 7. In the picture below we compare, as the preference split q varies, the turnout T in the M system (continuous line) and in the P system (dashed line).

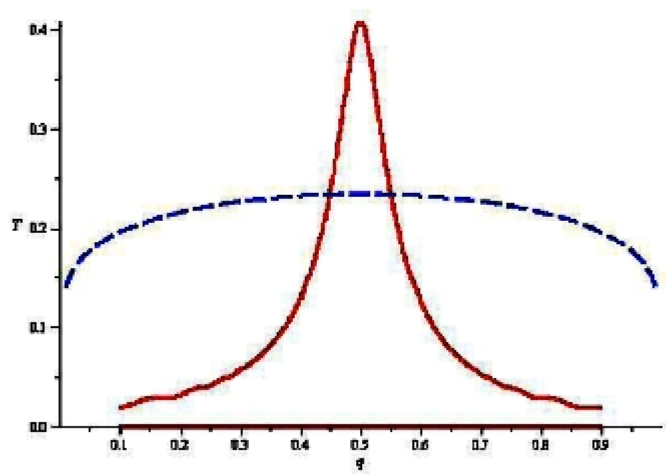

Figure 1: Turnout as a function of q in the M (continuous) and P (dashed) models ($z = 5$, $N = 3000$).

When one party (e.g. party B) has the ex-ante advantage over the other party (A), we have a higher turnout in the P system. Numerically, for instance when $q = 1/3$, we have

$q = 1/3$	α	β	α/β	W	T
P	24.8%	22%	1.27	27.8%	23%
M	7.1%	6.7%	1.06	30.9%	6.8%

Note also in both the M and the P systems the presence of the underdog compensation ($\alpha > \beta$) which is partial ($q\alpha < (1-q)\,\beta$). Moreover, note the higher underdog compensation α/β in the P system and consequently the higher relative winning margin W in the M system.

Whereas when the election is close and no party has an ex ante advantage, i.e. $q = 1/2$, turnout T in the M system surpasses the turnout in the P system

$q = 1/2$	α	β	T
P	23.5%	23.5%	23.5%
M	40.9%	40.9%	40.9%

Note for different q's the much larger variability of turnout numbers T in the M system when compared to the P system.

To compare the underdog compensations in general, the picture below illustrates how the ratio α/β varies with q in the P system (dashed line) and in the M system (continuous line). Contrast these decreasing curves with the steeper one that is obtained in the M system under homogeneous cost (dotted line) when there is full underdog compensation, the election is expected to be tied and the winning margin is zero regardless of the initial preference split. In sum, this example illustrates how the underdog compensation is higher in a proportional power sharing system, while the turnout is lower in a proportional power sharing system only when the distribution of party supporters is symmetric.

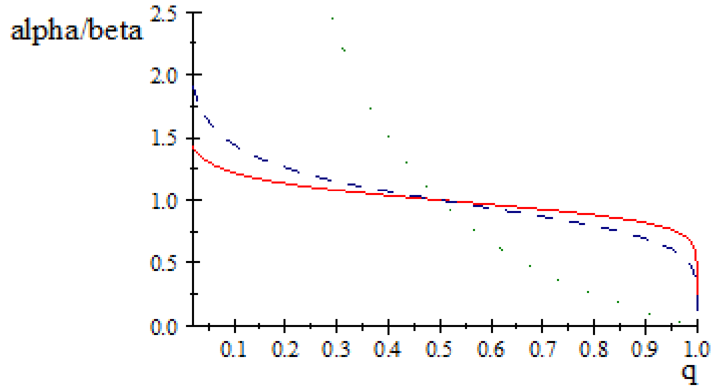

Figure 2: Underdog compensation α/β as a function of q in the P (dashed), M (continuous) and M with homogenous cost (dotted) models ($z = 5$).

3. Experimental Analysis

Even though the point estimates of turnout when using the costly voting rational model are much lower than real turnout levels in elections, as is well known, there is no reason to believe that the comparative predictions of the rational model shouldn't be of guidance. To verify that indeed the comparative results of the paper correspond to actual voting behavior, we bring the model presented in section 2 to the laboratory. In laboratory elections we can have only a finite number of voters, but the model is easily adapted to this case. The equilibrium conditions for our laboratory implementation of the model, with finite electorates and no population uncertainty[25], are given below. It is straightforward to exactly characterize symmetric Bayesian equilibrium for these finite environments, and comparative statics that are similar to the Poisson model can be computed directly from these exact equilibrium solutions.

In what follows, let N_A denote the number of voters with a preference for party A and $N_B = N - N_A$ denote the number of voters with a preference for party B, and assume without loss of generality that $N_A \leq N_B$. As before, a symmetric equilibrium is characterized by two cutoff levels, one for each party, c^α and c^β, with corresponding expected turnout levels equal to $\alpha = F(c_\alpha)$ and $\beta = F(c_\beta)$. The equilibrium conditions are slightly different for the M and P systems, and these are derived next.

[25]The reason to consider *known* population size is that the analytical computations with the Poisson game approach apply only to the limiting case of very large electorates, which is not feasible in the laboratory.

3.0.1. *Equilibrium conditions for M.* Given expected turnout rates in the two parties, α and β the expected marginal benefit of voting for a party A citizen is equal to:

$$B_M^A = \frac{1}{2}\left[\begin{array}{c} \sum_{k=0}^{N_A-1}\binom{N_A-1}{k}\binom{N_B}{k}\alpha^k(1-\alpha)^{N_A-1-k}\beta^k(1-\beta)^{N_B-k}+ \\ \sum_{k=0}^{N_A-1}\binom{N_A-1}{k}\binom{N_B}{k+1}\alpha^k(1-\alpha)^{N_A-1-k}\beta^{k+1}(1-\beta)^{N_B-1-k} \end{array}\right]$$

$$B_M^B = \frac{1}{2}\left[\begin{array}{c} \sum_{k=0}^{\min\{N_A,N_B-1\}}\binom{N_A}{k}\binom{N_B-1}{k}\alpha^k(1-\alpha)^{N_A-k}\beta^k(1-\beta)^{N_B-1-k}+ \\ \sum_{k=0}^{N_A-1}\binom{N_A}{k+1}\binom{N_B-1}{k}\alpha^{k+1}(1-\alpha)^{N_A-1-k}\beta^k(1-\beta)^{N_B-1-k} \end{array}\right]$$

where $\frac{1}{2}$ is the value of creating or breaking a tie. In each expression, the first summation is the probability of your vote breaking a tie, and the second summation is the probability of your vote creating a tie, given turnout rates α and β. The equilibrium conditions for c_M^α and c_M^β are given by:

$$\begin{aligned} c_M^\alpha &= B_M^A \\ c_M^\beta &= B_M^B \end{aligned}$$

3.0.2. *Equilibrium conditions for P.* Given expected turnout rates in the two parties, α and β the expected marginal benefit of voting for a party A citizen is equal to:

$$B_P^A = \sum_{j=0}^{N_A-1}\sum_{k=0}^{N_B}\left[\frac{j+1}{j+1+k} - \frac{j}{j+k}\right]\binom{N_A-1}{j}\binom{N_B}{k}\alpha^j(1-\alpha)^{N_A-1-j}\beta^k(1-\beta)^{N_B-k}$$

$$B_P^B = \sum_{j=0}^{N_B-1}\sum_{k=0}^{N_A}\left[\frac{j+1}{j+1+k} - \frac{j}{j+k}\right]\binom{N_B-1}{j}\binom{N_A}{k}\beta^j(1-\beta)^{N_B-1-j}\alpha^k(1-\alpha)^{N_A-k}$$

Where the first term in the summation is the increase in vote share and the second term is the probability of the vote share being equal to $\frac{j}{j+k}$ without your vote, given turnout rates α and β.[26] The equilibrium condition for c_P^α and c_P^β are given by:

$$\begin{aligned} c_P^\alpha &= B_P^A \\ c_P^\beta &= B_P^B \end{aligned}$$

[26]By convention, we denote $\frac{j}{j+k} = .5$ if $j = k = 0$.

3.0.3. *Experimental design and parameters.* All our electorates in the experiment have exactly $N = 9$ voters, with three different N_A treatments: $N_A = 2, 3, 4$. We consider two different distributions of voter costs. In our *low cost* (or, equivalently, *high benefit*) elections c_i is uniformly distributed on the interval [0,.3]. In our *high cost* (or, equivalently, *low benefit*) elections c_i is uniformly distributed on the interval [0,.55]. Table 1 below gives the symmetric equilibrium expected turnout levels (by party and total turnout) for each treatment, rounded to two decimal places.

N_A	N_B	c_{max}	Rule	α^*	β^*	$\frac{\alpha^* N_A + \beta^* N_B}{N}$
4	5	.3	M	.60	72	.67
3	6	.3	M	.51	.52	.52
2	7	.3	M	.45	.40	.41
4	5	.55	M	.46	.45	.46
3	6	.55	M	.41	.37	.39
2	7	.55	M	.38	.30	.32
4	5	.3	P	.48	.43	.45
3	6	.3	P	.55	.39	.45
2	7	.3	P	.67	.36	.43
4	5	.55	P	.35	.31	.33
3	6	.55	P	.40	.29	.32
2	7	.55	P	.48	.26	.31

Table 1. Equilibrium turnout rates by treatment.

There are five main theoretical hypotheses comparing turnout in the M and P voting systems in the elections we study. We state these below:

- H1 For the larger party, turnout is higher in M than in P.
- H2 Total expected turnout is higher under M than under P.
- H3 The competition effect is reversed for the smaller party in the proportional vote system. That is, for the smaller party, turnout decreases as their share of the electorate increases. Under M, the usual competition effect applies to both parties: elections that are expected to be closer lead to higher turnout.[27]
- H4 The competition effect on total expected turnout is negligible in P elections.

[27] In the general model with population uncertainty we were not able to obtain general results on the competition effects, whereas the numerical analysis of the known population case allows for these additional predictions.

H5 In all P elections we study, there is an underdog effect. There is an underdog effect in all M elections, except for reverse underdog effects in the low cost 5-4 and 6-3 M elections.

In the experimental section we will return to these five predictions of the known population model. But, before that, let us evaluate the robustness of the main comparative predictions by looking at other voting models.

3.1. **Procedures.** A total of 153 subjects participated in 1700 elections across 17 sessions. Each session consisted of two parts with 50 nine-voter elections in each part. The parameters were the same in all elections within a part, but in each session exactly one parameter was changed between part I and part II. In all sessions the same voting rule (M or P) was used in all 100 elections. For all of the treatments except for the 7-2 elections, the distribution of voting costs were the same for all 100 elections. Half of these sessions were conducted with part I having 5-4 elections and part II having 6-3 elections. The other half of the sessions reversed the order so the 6-3 elections were in part I and the 5-4 elections in part II. For the 7-2 elections, half the elections in a session were conducted with $c_{\max} = 55$ and half with $c_{\max} = 30$, in both orders. Subjects were informed of the exact parameters (N_A, N_B, $C_{\max}$ and the voting rule) at the beginning of each part. Before each election, each subject was randomly assigned to either group A or group B and assigned a voting cost, drawn independently from the uniform distribution between 0 and $c_{\max}$, in integer increments. Therefore, each subject gained experience as a member of the majority and minority party in both parts of the session. Instructions were read aloud so everyone could hear, and Powerpoint slides were projected in front of the room to help explain the rules. After the instructions were read, subjects were walked through two practice rounds and then were required to correctly answer all the questions on a computerized comprehension quiz before the experiment began. After the first 50 rounds, a very short set of new instructions were read aloud to explain the change of parameters.

The wording in the instructions was written so as to induce as neutral an environment as possible.[28] There was no mention of voting or winning or losing or costs. The labels were abstract. The smaller group was referred to the alpha group (A) and the larger group was referred to as the beta group (B). Individuals were asked in each round to choose X or Y. For the M treatment, if more members of $A(B)$ chose X than members of $B(A)$ chose X, then each member of $A(B)$ received 100 and each member of group $B(A)$ received 0. In case of a tie, each member of each group received the expected value of a fair coin toss, 50. For the

[28]A sample of the instructions from one of the sessions is in Appendix E.

P treatment, each voter received a share of 100 proportional to the number of voters in their party that chose X compared to the number of voters in the other party that chose X. The voting cost was implemented as an opportunity cost and was referred to as a "Y bonus". It was added to a player's earnings if that player chose Y instead of X. If a player chose X, that player did not receive their Y bonus in that election. Y-bonuses were randomly redrawn in every election, independently for each subject, and subjects were only told their own Y bonus. Bonuses were integer valued and took on values from 0 to 30 in the low cost treatment and 0 to 55 in the high cost treatment. Payoffs were denominated in points that were converted to US dollars at a pre-announced rate.[29] Each subject earned the sum of their earnings across all elections. All decisions took place through computers, using the Multistage experimental software program.[30] Subjects were registered students at Caltech.[31] Each session lasted about forty five minutes and subjects earned between eleven and seventeen dollars, in addition to a fixed payment for showing up on time.

[29]Each point was equal to $.01.

[30]http://multistage.ssel.caltech.edu

[31]Data for the high cost M 5-4 and 6-3 elections are from an earlier study with UCLA students as subjects (Levine and Palfrey 2007), which also used the Multistage software and the same protocol.

3.2. **Experimental Results.** Table 2 summarizes the observed turnout rates by treatment. The table reports turnout by party and also total turnout for each experimental treatment. The last three columns give the equilibrium turnout levels. The table also reports standard errors clustered at the individual voter level.

N_A	N_B	c_{max}	Rule	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{T}$	α^*	β^*	T^*
4	5	.3	M	0.622 (.042)	0.636 (.056)	0.630 (.048)	.60	.72	.67
3	6	.3	M	0.513 (.050)	0.520 (.073)	0.52 (.063)	.51	.52	.52
2	7	.3	M	0.490 (.063)	0.360 (.071)	0.39 (.060)	.45	.40	.41
4	5	.55	M	0.479 (.024)	0.451 (.034)	0.464 (.020)	.46	.45	.46
3	6	.55	M	0.436 (.021)	0.398 (.031)	0.411 (.019)	.41	.37	.39
2	7	.55	M	0.330 (.046)	0.284 (.037)	0.294 (.028)	.38	.30	.32
4	5	.3	P	0.547 (.025)	0.486 (.020)	0.51 (.016)	.48	.43	.45
3	6	.3	P	0.547 (.054)	0.465 (.061)	0.49 (.042)	.55	.39	.45
2	7	.3	P	0.600 (.040)	0.421 (.049)	0.461 (.041)	.67	.36	.43
4	5	.55	P	0.362 (.026)	0.370 (.039)	0.367 (.020)	.35	.31	.33
3	6	.55	P	0.477 (.037)	0.305 (.033)	0.362 (.027)	.40	.29	.32
2	7	.55	P	0.515 (.029)	0.320 (.037)	0.363 (.032)	.48	.26	.31

Table 2. Observed turnout rates. Standard errors in parenthesis.

Using these turnout data, we turn to the five hypotheses generated by the theoretical equilibrium turnout levels. Recall that there are five main theoretical predictions from the pivotal voter model about differences between turnout in the M and P voting systems in the elections we study. We go through each of these briefly below.

- H1 **For the larger party, turnout is higher in M than P.** We find support for this hypothesis except for the extreme landslide elections ($N_A = 2$) where turnout rates are slightly higher in P than M. Thus the theory is supported in four out of six paired comparisons.
- H2 **Total turnout is higher under M than under P.** We find support for this hypothesis except for the extreme landslide elections ($N_A = 2$) where turnout rates are slightly higher in P than M. However, the low cost elections are the one exception where turnout is predicted to be higher in P than M. Thus the theory is supported in five out of six paired comparisons.

H3 **The competition effect is reversed for the smaller party under P. That is, for the smaller party, turnout decreases as their share of the electorate increases. Under P for the majority party, as well as under M, the usual competition effect applies to both parties: elections that are expected to be closer lead to higher turnout.** We measure the competition effect as the difference in turnout between the 5-4 and 6-3 elections and the difference in turnout between the 6-3 and 7-2 elections. The sign is correctly predicted in all cases (sixteen out of sixteen comparisons).

H4 **The competition effect on total expected turnout is larger in the M elections than the P elections.** This is exactly what we find in the data. The sign is correctly predicted in all four cases (four out of four comparisons). The competition effect differences are reported in Table 3. For example, for the low cost elections ($c_{\max} = .33$) turnout declines by 38% under the M rule, (from 0.63 to 0.39) and only 10% under the P rule (0.51 to 0.46) in toss-up elections (5-4) compared to the landslide elections (7-2). In the high cost elections ($c_{\max} = .33$) the contrast is even sharper: 37% decline under the M rule, (from 0.46 to 0.29) and less than 3% decline under the P rule (0.37 to 0.36).

Comparison	$c_{\max}$	M	P
$\widehat{T}_{5/4} - \widehat{T}_{6/3}$	0.30	0.11	0.02
$\widehat{T}_{6/3} - \widehat{T}_{7/2}$	0.30	0.13	0.03
$\widehat{T}_{5/4} - \widehat{T}_{7/2}$	0.30	0.24	0.05
$\widehat{T}_{5/4} - \widehat{T}_{6/3}$	0.55	0.04	0.01
$\widehat{T}_{6/3} - \widehat{T}_{7/2}$	0.55	0.13	0.00
$\widehat{T}_{5/4} - \widehat{T}_{7/2}$	0.55	0.17	0.01

Table 3. Competition Effect M vs. P

H5 **In all P elections we study, there is an underdog effect. There is an underdog effect in all M elections, except for the predicted reverse underdog effects in the low cost 5-4 and 6-3 M elections.** Thus, all of our underdog hypotheses have support in the data. We find that in all P elections there is an underdog effect, with one exception where the difference is less than one percentage point ($\widehat{\alpha} = .362$, $\widehat{\beta} = .370$). That one exception is the 5-4 high cost treatment, where theory predicts the smallest effect (less than four percentage points). In the M elections, all predicted underdog and reverse underdog effects are observed in the data. (eleven of twelve comparisons)

Thus, the comparative statics are correctly predicted in 40 out of 44 paired comparisons, with many of these differences significant at the 5% or 10% level using individual-level clustered standard errors. To illustrate how close the equilibrium turnout rates are to the equilibrium turnout rates, Figure 1 presents a scatter plot of the observed vs. equilibrium turnout rates. A perfect fit of the data to the theory would have all the points lined up along the 45% degree line. A simple OLS regression of the observed turnout on equilibrium turnout, using the 36 points in the graph gives a slope of .815, an intercept of .097 and an R-squared equal to .871. The theoretical model slightly underestimates turnout when the model prediction is below 50% and over-estimates turnout when the model prediction is over 50%, consistent with the findings of Levine and Palfrey (2007) on their much larger data set for plurality elections.

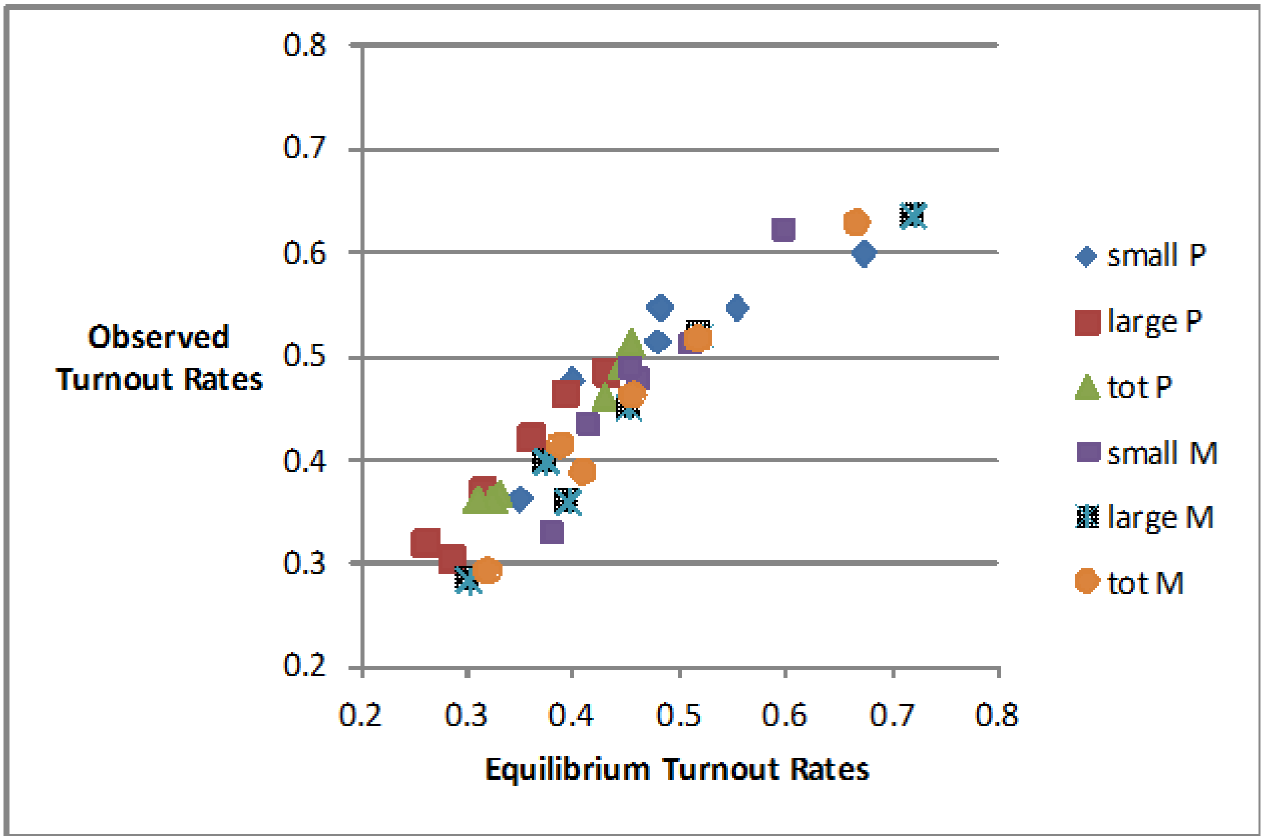

Figure 3: Scatter plot of observed vs. equilibrium turnout rates.

4. Robustness and Extensions

The pivotal voter model analyzed above assumes a high degree of rationality on the part of voters, and the equilibrium nature of the model also assumes that voters have rational expectations about the turnout decisions of other voters and the probability of being pivotal. While the experimental findings are largely supportive of the model, we believe it is important to dig deeper into the question of whether the main theoretical results in the paper are robust to changes in the basic assumptions of the model. In this section, we consider two directions of robustness: relaxing the full rationality assumption; relaxing the assumption that voters are selfish and make their voting decisions independently.

4.1. **Relaxing the rationality assumption.**

4.1.1. *Quantal Response Equilibrium.* The first assumption we relax is that voters exactly best respond in equilibrium. The leading model of "soft" optimization in game theory (i.e., players do not always choose exact best responses) is quantal response equilibrium (QRE). The standard QRE model[32] posits that voters choose probabilitistically, with choice probabilities increasing continuously in expected payoffs, as, for example in probabilistic voting models. That is, actions with higher expected payoffs are more likely to be chosen than actions with lower expected utility, the "best" action is not necessarily chosen with probability one, as it would be in a Nash equilibrium. McKelvey and Palfrey (1995) show that QRE can be represented as a Bayesian Nash equilibrium where players receive privately observed i.i.d. full-support payoff disturbances, in addition to the payoffs of the game itself. The model has been widely used to analyze experimental game data and explains a wide range of systematic deviations from Nash equilibrium.

In the voting game, adding these independent disturbances turns out to be equivalent to changing the cost distribution to have full support on the real line. As explained earlier, this leads to a zero underdog compensation effect if there are an infinite number of voters, since only voters with non-positive voting costs will turnout in the limit; with full support this implies the same positive fraction of voters will turn out from both parties. In a finite electorate, underdog compensation effect can be solved computationally. The main theoretical effects of the Nash equilibrium model for large electorates therefore apply to the QRE model.

As in Levine and Palfrey (2007), it is a straightforward exercise to fit our data to the logit QRE model. We have done so and this tightens up the correlation between predicted and actual turnout values. To a first approximation, the effect of QRE on predicted turnout levels is to bring them all closer to .5 (See Goeree and Holt 2005). The improvement however is modest. Appendix 2 gives the fitted turnout rates and graphs them against the data, analogously to Figure 2 above.

[32] See Goeree et al. (2005)

4.1.2. *Irrational expectations.* Past experimental work on turnout (Duffy and Tavits 2010) and related participation games (Palfrey and Rosenthal 1991) suggest that voters misperceive the probability of being pivotal. Thus, rather than having rational expectations about the behavior of other voters, they have irrational expectations.[33] This can easily be adapted to the model in many ways. For example, we may posit that voters *overestimate* their pivot probabilities, which is equivalent to overestimating the benefits from voting. This will simply change the equilibrium cutpoints, but will lead to more rather than less voting compared with the Nash equilibrium (or less if they underestimate pivot probabilities). However, as long as these perceived pivot probabilities are exactly zero and have a similar effect on voters from both parties, all the main results will continue to go through. In particular, there would still be a partial compensation effect and the it would be different in M systems than P systems.

4.1.3. *Lack of Strategic Sophistication.* Recent theoretical models have been developed to analyze strategic situations where the players of the game are "unsophisticated" in a strategic sense. The leading models are based on "levels of strategic thinking". Level 0 players are usually modeled as being completely random. Thus, in the context of the turnout model, any level-0 voter would turn out with probability 1/2 independently of her voting cost. Level 1 players make optimal voting decisions as a function of their voting cost, assuming all other voters are level 0. Level k voters make optimal voting decisions as a function of their voting cost, assuming all other voters are level k-1 or possibly a mixture of lower levels. Thus, these are models of heterogeneous irrational expectations that take a highly structured form. While it is beyond the scope of this paper to work out the fine details of a specific level k model for the turnout game, it is easy to see that there will never be a full underdog compensation effect if there are any level 0 players[34], or if the model is combined with quantal response behavior.

4.2. **Non-selfish voting models.**

4.2.1. *Citizen Duty Models.* The first and simplest model of non-selfish considerations in the turnout decision is the citizen duty model proposed by Riker and Ordeshook (). According to this model, failing to vote produces a utility loss due to a guilty conscience for being a poor citizen. It is trivial to see that adding this term (with or without private information and heterogeneity) simply changes the distribution of voting costs, as in the QRE model. For

[33] Alternatively, they have rational expectations about the other voters's behavior, but have beliefs about the probability of being pivotal that are different from the equilibrium pivot probability.

[34] This is true for other specifications of level 0 players as well.

example in it's simplest form, it would just shift voting costs down by the citizen duty term, D.

4.2.2. *Group Mobilization Models.* Even though we believe that the analysis conducted so far provides per se many new insights, we want to extend the analysis to other turnout models. We want to show that our main comparative results are robust, they hold in any model of turnout. The rational voter model features the well known free riding problem among voters, which makes turnout in a large election be typically small. Since in large elections the turnout is not always small, the free riding problem seems to be overcome in some way. Social scientists differ on how to interpret the fact that this collective action problem is by-passed in an election. Regardless of how this might happen, it is important to know whether the turnout comparisons across electoral systems depend on the presence of this free riding problem. We want to know if our results hold also when the positive externality of voting among supporters of the same party is somehow internalized, and so regardless of the size of the population turnout is high. Moreover, we can study the mapping from vote shares to power shares not only when it is at the two extremes of proportional power sharing and winner take all, but also when it is intermediate, while only the comparison of the two extremes was feasible for us to do in the rational voter model analyzed so far.

We first turn to a group mobilization model a la Shachar and Nalebuff (1999) where parties' campaign efforts and spending are able to mobilize and coordinate citizens to go vote. In that model, each group can "purchase" turnout of its party members by engaging in costly get-out-the-vote efforts. Thus, parties trade off mobilization costs for higher expected vote shares, taking as given the mobilization choice of the other party. In this, there is zero underdog compensation in large electorates, as in the pivotal voter model with negative costs, the QRE model, and the citizen duty models.

4.2.3. *Ethical Voter Models.* The ethical voter model of Feddersen and Sandroni (2006)) assumes that citizens are "rule utilitarian" so they act as one. This involves an equilibrium between two party-planners on each side A and B. In this solution each planner looks at the total benefit from the outcome of the election considering the total cost of voting incurred by the supporters of his side, taking the other planner's turnout strategy as given.. The logic of the ethical voter models is similar to the group mobilization models. They are almost identical on the benefit side but differ slightly on the cost side. In the case of the ethical voter model, one gets almost exactly the same kind of partial compensation as in the pivotal voter model with non-negative costs. Details are given in Appendix C.

5. Concluding Remarks

For any distributions of partisan preferences and voting costs, we have shown that turnout (of rational voters as well as of ethical voters and of mobilized voters) depends on the degree of proportionality of influence in the institutional system in a clear way: higher turnout in a winner-take-all system than in a proportional power sharing system when the population is evenly split in terms of partisan preferences, and vice versa when one party's position has a clear majority of support.

In any considered model and in any considered power sharing system, partial (or zero) underdog compensation occurs, which guarantees that the ex-ante favorite party obtains in expectation the higher vote share in the election. Hence, in a winner-take-all system it is quite clear that underdog cannot win, which greatly discourages the contest, but with power sharing there is no absolute winner and some competition remains.

The theoretical results are robust to wide range of alternative assumptions about the voting game and about the rationality of the voters. The common feature of all the various models considered in this paper is that with heterogeneity of voting costs full underdog compensation is not possible, and hence the majority party is expected to maintain a considerable advantage and winning margin in the election. The small probability of victory for the minority, i.e. the low competitiveness of the electoral race, depresses significantly the incentives to turn out in the winner take all system. Whereas in a power sharing system the incentives to vote or to mobilize voters are affected to a much lesser extent by the competitiveness or the expected closeness of the electoral race.

Our prediction that for the larger party, turnout is higher in a winner-take-all system than in a proportional power sharing system was confirmed by the experimental analysis, as well as most of the other predictions concerning differences in competition and underdog effects: in particular, it is interesting that the competition effect is reversed for the smaller party in the proportional system. That is, for the smaller party, turnout decreases as their share of the electorate increases. With a winner-take-all system, the usual competition effect applies to both parties: elections that are expected to be closer lead to higher turnout. The prediction that competition effect on total expected turnout is negligible in a proportional system also found strong support.

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Appendix A

Welfare corollary. Even though we are primarily interested in the comparative positive analysis of turnout across institutional systems, the results obtained using the Poisson game approach allow us to establish a welfare comparison between a winner-take-all system and a proportional power sharing system.[35]

>From a welfare perspective, in a world of heterogeneous costs where full compensation effects cannot hold, the winner-take-all system should intuitively dominate.[36]

Using total utility as reasonable welfare criterion, it is in line with standard practice to assess one system to be better than another if for every realization of the parameter q the total

[35] For a broader discussion of welfare across electoral systems with different costly voting models see also Kartal (2010).

[36] Of course this intuition is model-specific, and as soon as one either changes the model or adds considerations like fairness, representation, and so forth, the comparison is not robust. That is why we focused primarily on positive analysis of turnout, where robustness has been possible to achieve.

sum of expected benefits minus total voting costs is higher in such a system. The following corollary establishes the welfare comparison between systems for N large.

Corollary 8. *For $q \neq 1/2$, the winner-take-all system (M) yields higher welfare than a proportional power sharing system (P).*

To see how this follows from our results, consider first the gross benefit side. For any $q \neq 1/2$ partial underdog compensation in the M system ensures that the side with the ex-ante majority support always wins the election. Hence, for $q < 1/2$ (wlog) the total expected benefit is $(1 - q)$ as with N large that fraction of citizens obtains the normalized benefit of 1 and the remaining people get zero. In the P system the power is shared between the two sides, so the gross benefit is

$$q\left(\frac{q\alpha}{T}\right) + (1-q)\left(\frac{(1-q)\,\beta}{T}\right)$$

which for $q < 1/2$ is strictly less than $(1 - q)$.

Having established the comparison in terms of benefits, we just need to add that for any $q \neq 1/2$ the total cost due to voter participation in the M system is lower than in the P system by proposition 7.

For $q = 1/2$ the voting costs are higher in the M system due to higher participation, but the benefits are also marginally higher in the M system for any given realization of preferences in which one party has the majority, so welfare is ambiguous.

Extension to many parties. All the analysis in the paper is conducted by altering the mapping from vote shares to power shares but keeping, for simplicity, the two-party assumption. However, the comparative results in terms of turnout *do not seem to* depend on the number of parties under the P system. We can explicitly compute the equilibrium for any number of parties in the P system.[37] This allows us to obtain a simple comparative statics result within the proportional power sharing system: turnout increases in the number of parties.[38]

To keep notation simple, we illustrate the three party case. Define

$$A := \alpha q_A N, \quad B := \beta q_B N, \quad C := \gamma q_C N, \quad \text{with: } q_A + q_B + q_C = 1$$

The marginal benefit[39] for party A is

$$B_P^A = \sum_{a=0}^{\infty}\sum_{b=0}^{\infty}\sum_{c=0}^{\infty}\left(\frac{e^{-A}A^a}{a!}\right)\left(\frac{e^{-B}B^b}{b!}\right)\left(\frac{e^{-C}C^c}{c!}\right)\left(\frac{a+1}{a+b+c+1}-\frac{a}{a+b+c}\right)$$

Lemma 9. *The marginal benefit has the closed form*

$$B_P^A = \left(1-\frac{A}{A+B+C}\right)\frac{1-e^{-(A+B+C)}}{A+B+C} + \left(\frac{A}{A+B+C}-\frac{1}{3}\right)e^{-(A+B+C)}$$

By symmetry the expressions B_P^B and B_P^C for parties B or C are straightforward.

For any number of parties the following comparative statics result holds.

Proposition 10. .

- *The comparison between turnout in the P system and the M system continues to hold even when there are multiple parties in the P system.*

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[37] This is certainly true under sincere voting. However, we do not think strategic voting in proportional power sharing systems would change things: a single vote for the preferred party always increases marginally the vote share for that party even if that party has low support. In general, this would also depend on the distribution of the intensity of preferences for the parties. In any case, the comparison of turnout incentives across systems in large elections depends exclusively on the relative speed of convergence across systems. As we show, the (intermediate) N^{-1} speed of convergence with power sharing does not depend on the number of parties present nor, by the same argument, on the (possibly fewer) number of parties voters decide to concentrate their vote on if they were voting strategically, regardless of intensity of preferences or other cardinality considerations.

[38] The extension to multiple parties presented here could be useful especially for future research, because it could help to open a bit the reduced form proportionality of influence parameter. With many parties the reduced form linear mapping from vote shares to power shares a la Lizzeri and Persico (2001) can be explicitly obtained from a standard post election legislative bargaining model of alternating offers a la Baron and Ferejohn (1989): Snyder, Ting and Ansolabehere (2005) analyze the conditions under which the expected power shares are proportional to the vote shares.

[39] Assume again that if nobody votes, power is shared equally, namely

$$\frac{a}{a+b+c} = 1/3 \quad \text{for } a=b=c=0$$

- *If parties are symmetric, turnout in the P system increases as the number of parties increases.*

The fact that the turnout comparison result remains unchanged is due to the quantitative fact that, regardless of the number of parties involved in the election, the marginal benefit of voting in the P system still declines asymptotically at the intermediate rate $1/N$, as it was the case for the P system with two parties.

Within the $1/N$ order of magnitude of the size effect, turnout increases when there are more symmetric parties. This is consistent with the fact that smaller parties obtain a higher turnout in the P system. The intuition for the latter follows from the following two observations. First, fixing the number of votes z for all other parties, the vote share increase for party A is

$$\left(\frac{a+1}{a+z+1}-\frac{a}{a+z}\right)=\left(\frac{a^2+a}{z}+2a+1+z\right)^{-1}$$

which is larger for smaller values of the random variable a, i.e. the number of votes for party A. Second, for a given a, in the marginal benefit B_P^A (see (2) and expressions above) a smaller party (i.e. a party with a smaller q_A) assigns larger probability weight $\left(\frac{e^{-A}A^a}{a!}\right)$ to small values of a.

Appendix B: Proofs

Proof of Lemma. 1 We first show that

$$\lim_{N\to\infty}\alpha_N=\lim_{N\to\infty}\beta_N=0$$

Define the modified Bessel functions of the first kind, see Abramowitz and Stegun (1965), as

$$I_0(z):=\sum_{k=0}^{\infty}\frac{\left(\frac{z}{2}\right)^k}{k!}\frac{\left(\frac{z}{2}\right)^k}{k!},\qquad I_1(z):=\sum_{k=0}^{\infty}\frac{\left(\frac{z}{2}\right)^k}{k!}\frac{\left(\frac{z}{2}\right)^{k+1}}{(k+1)!}$$

Defining

$$x:=qN\alpha,\quad y:=(1-q)N\beta,\quad z:=2\sqrt{xy}$$

then the benefits of voting $\left(B_M^A,B_M^B\right)$ can be written as

$$B_M^A=\frac{1}{2}\sum_{k=0}^{\infty}\left(\frac{e^{-x}x^k}{k!}\right)\left(\frac{e^{-y}y^k}{k!}\right)\left(1+\frac{y}{k+1}\right)=\frac{e^{-x}e^{-y}}{2}\left(I_0(2\sqrt{xy})+\sqrt{\frac{y}{x}}I_1(2\sqrt{xy})\right)$$

$$B_M^B=\frac{1}{2}\sum_{k=0}^{\infty}\left(\frac{e^{-x}x^k}{k!}\right)\left(\frac{e^{-y}y^k}{k!}\right)\left(1+\frac{x}{k+1}\right)=\frac{e^{-x}e^{-y}}{2}\left(I_0(2\sqrt{xy})+\sqrt{\frac{x}{y}}I_1(2\sqrt{xy})\right)$$

For large z the modified Bessel functions are asymptotically equivalent and approximate to, see Abramowitz and Stegun (1965)[40]

$$I_0(z) \simeq I_1(z) \simeq \frac{e^z}{2\pi z}$$

For any exogenously fixed $(\alpha, \beta) \in (0,1]^2$ x and y go to infinity as N goes to infinity, so we can approximate the benefits of voting for large N as

$$B_M^A \simeq e^{-x-y+2\sqrt{xy}} \frac{\sqrt{x}+\sqrt{y}}{4\sqrt{\pi}\sqrt{\sqrt{xy}}} \frac{1}{\sqrt{x}}, \qquad B_M^A \simeq e^{-x-y+2\sqrt{xy}} \frac{\sqrt{x}+\sqrt{y}}{4\sqrt{\pi}\sqrt{\sqrt{xy}}} \frac{1}{\sqrt{y}}$$

As a consequence for any given $(\alpha, \beta) \in (0,1]^2$ the benefits of voting vanish as N grows, namely

$$\lim_{N\to\infty} B_M^A(\alpha,\beta) = 0, \qquad \lim_{N\to\infty} B_M^B(\alpha,\beta) = 0$$

Now consider (α, β) as endogenous, i.e. solutions to the system

$$B_M^A(\alpha,\beta) = F^{-1}(\alpha), \quad B_M^B(\alpha,\beta) = F^{-1}(\beta)$$

Since F and F^{-1} are increasing and continuous with $F(0) = 0$, then $B_M^A(\alpha,\beta) = F^{-1}(\alpha)$ implies $\lim_{N\to\infty} \alpha_N = 0$. Likewise, we have $\lim_{N\to\infty} \beta_N = 0$.

Next, we show that

$$\lim_{N\to\infty} N\alpha_N = \lim_{N\to\infty} N\beta_N = \infty, \qquad \lim_{N\to\infty} \frac{\alpha_N}{\beta_N} \in (0,\infty)$$

Suppose $\lim_{N\to\infty} N\alpha_N < \infty$ and $\lim_{N\to\infty} N\beta_N < \infty$, then

$$\lim_{N\to\infty} B_M^A(\alpha_N, \beta_N) > 0$$

and any solution to $B_M^A(\alpha,\beta) = F^{-1}(\alpha)$ would imply $\lim_{N\to\infty} \alpha_N > 0$, which contradicts $\lim_{N\to\infty} \alpha_N = 0$.

Suppose $\lim_{N\to\infty} N\alpha_N = \infty$ and $\lim_{N\to\infty} N\beta_N < \infty$, then $\lim_{N\to\infty} \frac{\alpha_N}{\beta_N} = \infty$ which implies (using a Taylor expansion of F^{-1} on the numerator and the denominator around zero) that $\lim_{N\to\infty} \frac{F^{-1}(\alpha_N)}{F^{-1}(\beta_N)} = \infty$.

For all N we have

$$\frac{B_M^A(\alpha_N,\beta_N)}{B_M^B(\alpha_N,\beta_N)} = \frac{F^{-1}(\alpha_N)}{F^{-1}(\beta_N)}$$

Taking the limit on one side we have

$$L := \lim_{N\to\infty} \frac{B_M^A(\alpha_N,\beta_N)}{B_M^B(\alpha_N,\beta_N)} = \lim_{\frac{x}{y}\to\infty} \frac{I_0\left(2\sqrt{xy}\right) + I_1\left(2\sqrt{xy}\right)\sqrt{\frac{y}{x}}}{I_0\left(2\sqrt{xy}\right) + I_1\left(2\sqrt{xy}\right)\sqrt{\frac{x}{y}}} \leq 1$$

[40] $X(z) \simeq Y(z)$ means that $\lim_{z\to\infty} \frac{X(z)}{Y(z)} = 1$.

In fact, $L \leq 1$ if $\lim_{\frac{x}{y}\to\infty} \frac{I_1(2\sqrt{xy})}{I_0(2\sqrt{xy})} = 0$ and $L = 0$ if $\lim_{\frac{x}{y}\to\infty} \frac{I_1(2\sqrt{xy})}{I_0(2\sqrt{xy})} \in (0, +\infty]$. So we have a contradiction as $L \leq 1$ cannot be equal to $\lim_{N\to\infty} \frac{F^{-1}(\alpha_N)}{F^{-1}(\beta_N)} = \infty$. The same argument shows that it cannot be the case that $\lim_{N\to\infty} N\alpha_N < \infty$ and $\lim_{N\to\infty} N\beta_N = \infty$.

The above arguments also imply that we cannot have either

$$\lim_{N\to\infty} \frac{\alpha_N}{\beta_N} = 0, \qquad \lim_{N\to\infty} \frac{\alpha_N}{\beta_N} = \infty$$

□

Proof of Proposition 2. For N large, since $\lim_{N\to\infty} N\alpha_N = \lim_{N\to\infty} N\beta_N = \infty$ we can use the asymptotic expression for the modified Bessel functions, so the system becomes

$$\begin{aligned} B_M^A &\simeq \frac{e^{-N(h-g)^2}}{\sqrt{N}} \frac{g+h}{4\sqrt{\pi}\sqrt{hg}} \frac{1}{g} = F^{-1}(\alpha) \\ B_M^B &\simeq \frac{e^{-N(h-g)^2}}{\sqrt{N}} \frac{g+h}{4\sqrt{\pi}\sqrt{hg}} \frac{1}{h} = F^{-1}(\beta) \end{aligned}$$

where we defined

$$g := \sqrt{q\alpha}, \quad h := \sqrt{(1-q)\,\beta_M(\alpha)}$$

The above system yields

$$\sqrt{q\alpha} F^{-1}(\alpha) = \sqrt{(1-q)\,\beta} F^{-1}(\beta)$$

Since the function $\sqrt{\alpha} F^{-1}(\alpha)$ is increasing we can define the function

$$\beta := \beta_M(\alpha)$$

where $\beta_M : [0,1] \longrightarrow [0,1]$ is an increasing and differentiable function with $\beta_M(0) = 0$. The system is reduced to a single equation

$$B_M^A(\alpha, \beta_M(\alpha)) = F^{-1}(\alpha),$$

We now show existence of a solution to the above equation by showing that the two continuous functions on either side must cross at least once.

Assume wlog $q < 1/2$. We have

$$\alpha \in (0,1] \quad \Longrightarrow \quad g < h$$

and for any fixed N, we have

$$\lim_{\alpha\to 0} \frac{e^{-N(h-g)^2}}{\sqrt{N}} \frac{g+h}{4\sqrt{\pi}\sqrt{hg}} \frac{1}{g} > \lim_{\alpha\to 0} \frac{e^{-N(h-g)^2}}{\sqrt{N}} \frac{2}{4\sqrt{\pi}\sqrt{hg}} = \infty$$

For $\alpha = 1$ we have $h > g = \sqrt{q}$, so for all N above a certain value we have

$$\frac{e^{-N(h-g)^2}}{\sqrt{N}}\left(\frac{g+h}{4\sqrt{\pi}\sqrt{gh}}\frac{1}{g}\right) < 1$$

which proves existence of a solution, because $F^{-1}(\alpha)$ is increasing and $F^{-1}(1) = 1$.

For uniqueness we need to show that the B_M^A is decreasing in α, namely that the following quantity is negative

$$\frac{d}{dg}\left(\frac{e^{-N(h-g)^2}}{\sqrt{N}}\frac{g+h}{4\sqrt{\pi}g\sqrt{hb}}\right) = \frac{e^{-N(h-g)^2}}{\sqrt{N}}\left(-2N(h-g)\frac{d(h-g)}{dh}\frac{g+h}{4\sqrt{\pi}g\sqrt{gh}} + \frac{d}{dh}\left(\frac{g+h}{4\sqrt{\pi}g\sqrt{gh}}\right)\right)$$

For large N this derivative will be negative if and only if

$$\frac{d(h-g)}{da} = \frac{\sqrt{1-q}}{\sqrt{q}}\frac{d\beta'}{d\alpha'} - 1 > 0$$

where we defined

$$\begin{aligned} \beta' &: = \sqrt{\beta}, \quad \alpha' := \sqrt{\alpha} \\ G(\alpha') &: = \alpha' F^{-1}\left((\alpha')^2\right) = \sqrt{\alpha}F^{-1}(\alpha) \end{aligned}$$

we have

$$\left(\sqrt{1-q}\right)G(\beta') = (\sqrt{q})\,G(\alpha') \quad \Longrightarrow \quad \frac{\sqrt{1-q}}{\sqrt{q}}\frac{d\beta'}{d\alpha'} = \frac{G'(\alpha')}{G'(\beta')}$$

So we need G' to be increasing

$$G'(\alpha') = \frac{d}{d\alpha}\left(\sqrt{\alpha}F^{-1}(\alpha)\right)\frac{d\alpha}{d\alpha'} = 2\frac{d}{d\alpha}\left(\alpha F^{-1}(\alpha)\right)$$

so it suffices for $\alpha F^{-1}(\alpha)$ to be weakly convex, so it suffices to have $F(\alpha)$ weakly concave.

As for the size effect, note that the marginal benefit side B_P^A decreases with N for all α while the cost side remains unchanged. Hence by the implicit function theorem as we increase N we have lower α which implies lower β and in turn lower turnout, formally

$$\begin{aligned} 0 &= \frac{d\left(B_M^A - F^{-1}\right)}{d\alpha}\frac{d\alpha}{dN} + \frac{d\left(B_M^A - F^{-1}\right)}{dN} \\ \frac{d\alpha}{dN} &= -\frac{\frac{dB_M^A}{dN}}{\frac{d\left(B_M^A - F^{-1}\right)}{d\alpha}} < 0 \quad \Longrightarrow \quad \frac{d\beta}{dN} < 0 \Longrightarrow \frac{dT_M}{dN} < 0 \end{aligned}$$

The underdog compensation is a consequence of F^{-1} being increasing, namely

$$\begin{aligned} q\alpha\left(F^{-1}(\alpha)\right)^2 &= (1-q)\,\beta\left(F^{-1}(\beta)\right)^2 \\ q &< 1/2 \quad \Longleftrightarrow \quad \alpha > \beta, \ \ q\alpha < (1-q)\,\beta \end{aligned}$$

□

Proof of Lemma 4. For given (α, β) call the expected number of voters for each party $R := qN\alpha$, $S := (1-q)N\beta$, we have

$$B_P^A = e^{-R-S} \sum_{a=0}^{\infty} \sum_{b=0}^{\infty} \left(\frac{R^a}{a!}\right) \left(\frac{S^b}{b!}\right) \left(\frac{a+1}{a+b+1} - \frac{a}{a+b}\right)$$

By differentiating and integrating the summands and inverting the series and integral operators we have

$$\begin{aligned} \sum_{b=0}^{\infty} \frac{S^b}{b!} \frac{a}{a+b} &= \frac{a}{S^a} \sum_{b=0}^{\infty} \int_0^S \frac{d}{dr} \left(\frac{1}{b!} \frac{r^{a+b}}{a+b}\right) dr = \\ &= \frac{a}{S^a} \int_0^S \sum_{b=0}^{\infty} \left(\frac{1}{b!} r^{a+b-1}\right) dr = \begin{cases} \frac{a}{S^a} \int_0^S r^{a-1} e^r dr & \text{for } a \geq 1 \\ 1/2 & \text{for } a = 0 \end{cases} \end{aligned}$$

and

$$\sum_{b=0}^{\infty} \frac{S^b}{b!} \frac{a+1}{a+b+1} = \frac{a+1}{S^{a+1}} \int_0^S r^a e^r dr$$

By inverting the series and integral operators again in the series over a, we have

$$\begin{aligned} B_P^A &= e^{-R-S} \left(\sum_{a=0}^{\infty} \frac{R^a}{a!} \left(\frac{a+1}{S^{a+1}} \int_0^S r^a e^r dr \right) - \sum_{a=1}^{\infty} \frac{R^a}{a!} \left(\frac{a}{S^a} \int_0^S r^{a-1} e^r dr \right) - \frac{1}{2} \right) \\ &= e^{-R-S} \left(\int_0^S \left(\begin{array}{c} \frac{1}{S} \left(\sum_{a=0}^{\infty} \frac{\left(\frac{R}{S} r\right)^a}{a!} + \sum_{a=1}^{\infty} \frac{\left(\frac{R}{S} r\right)^a}{(a-1)!} \right) \\ -\frac{R}{S} \sum_{a=1}^{\infty} \frac{\left(\frac{R}{S} r\right)^{a-1}}{(a-1)!} \end{array} \right) e^r dr - \frac{1}{2} \right) \\ &= e^{-R-S} \left(\frac{1}{S^2} \int_0^S e^{\left(1+\frac{R}{S}\right) r} (S - RS + Rr)\, dr - \frac{1}{2} \right) \\ &= \frac{S}{(R+S)^2} - \frac{e^{-(R+S)}}{(R+S)^2} \frac{S^2 - R^2 + S}{2} \end{aligned}$$

and by symmetry

$$B_P^B(R, S) = B_P^A(S, R)$$

□

Proof of Lemma 5. We first show that

$$\lim_{N \to \infty} \alpha_N = \lim_{N \to \infty} \beta_N = 0$$

For any fixed $\alpha > 0$ and $\beta > 0$, by inspection of the closed form expression (2) we see that $\lim_{N\to\infty} B_P^A(\alpha,\beta) = \lim_{N\to\infty} B_P^B(\alpha,\beta) = 0$, so the same argument obtained in Lemma (1) for the M system applies.

Next, we show that

$$\lim_{N\to\infty} N\alpha_N = \lim_{N\to\infty} N\beta_N = \infty, \qquad \lim_{N\to\infty} \frac{\alpha_N}{\beta_N} \in (0,\infty)$$

Summing the two P system equations we have

$$\frac{1}{NT}\left(1 - \frac{e^{-NT}}{2}\right) = F^{-1}(\alpha) + F^{-1}(\beta)$$

Since the RHS goes to zero the LHS will too, which means that NT must go to infinity so we cannot have both $\lim_{N\to\infty} N\alpha_N < \infty$ and $\lim_{N\to\infty} N\beta_N < \infty$. For N large, since the exponential terms e^{-NT} in (2) vanish faster than the hyperbolic terms, the system approximates to

$$\frac{(1-q)\beta}{NT^2} = F^{-1}(\alpha), \qquad \frac{q\alpha}{NT^2} = F^{-1}(\beta) \tag{5}$$

Suppose $\lim_{N\to\infty} N\alpha_N = \infty$ and $\lim_{N\to\infty} N\beta_N < \infty$, then $\lim_{N\to\infty} \frac{\alpha_N}{\beta_N} = \infty$ which implies (using a Taylor expansion of F^{-1} on the numerator and the denominator around zero) that $\lim_{N\to\infty} \frac{F^{-1}(\alpha_N)}{F^{-1}(\beta_N)} = \infty$. From (5) we have

$$\frac{1-q}{q}\frac{\beta_N}{\alpha_N} = \frac{F^{-1}(\alpha_N)}{F^{-1}(\beta_N)}$$

so we reach a contradiction as the above equality cannot hold as $N \to \infty$. The same argument shows that it cannot be the case that $\lim_{N\to\infty} N\alpha_N < \infty$ and $\lim_{N\to\infty} N\beta_N = \infty$.

The above arguments also imply that we cannot have either

$$\lim_{N\to\infty} \frac{\alpha_N}{\beta_N} = 0, \qquad \lim_{N\to\infty} \frac{\alpha_N}{\beta_N} = \infty$$

□

Proof of Proposition 6. The approximated system (5) yields

$$\begin{aligned} q\alpha F^{-1}(\alpha) &= (1-q)\beta F^{-1}(\beta) \\ q &< 1/2 \iff \alpha > \beta \end{aligned}$$

Since the function $\alpha F^{-1}(\alpha)$ is increasing we can define

$$\beta := \beta_P(\alpha)$$

where $\beta_P(\alpha) : [0,1] \longrightarrow [0,1]$ is an increasing differentiable function with $\beta_P(0) = 0$. We now reduced the P system to one equation

$$B_P^A := \frac{(1-q)\,\beta_P(\alpha)}{NT^2} = F^{-1}(\alpha)$$

which we now show has one and only one solution.

The cost side $F^{-1}(\alpha)$ is increasing from 0 to 1. Uniqueness comes from the fact that the benefit side decreases in α as its derivative is proportional to

$$\begin{aligned} \frac{\partial B_P^A}{\partial \alpha} &\propto \left[\beta_P'(\alpha)\left(q\alpha + (1-q)\,\beta_P(\alpha)\right) - 2\beta_P(\alpha)\left(q + (1-q)\,\beta_P'(\alpha)\right)\right] \\ &= -\left[\left((1-q)\,\beta - q\alpha\right)\beta_P'(\alpha) + 2q\beta_P(\alpha)\right] < 0 \end{aligned}$$

as

$$\alpha > \beta \quad \Longrightarrow \quad q\alpha < q\alpha \frac{F^{-1}(\alpha)}{F^{-1}(\beta)} = (1-q)\,\beta$$

Existence comes form the fact that for α approaching zero the benefit diverges as for any fixed N we have

$$\lim_{\alpha \to 0} \frac{1}{N} \frac{(1-q)\,\beta_P(\alpha)}{\left(q\alpha + (1-q)\,\beta_P(\alpha)\right)^2} > \lim_{\alpha \to 0} \frac{1}{N} \frac{(1-q)}{\alpha} \frac{\beta_P}{\alpha} = \infty$$

because

$$\lim_{\alpha \to 0} \frac{\beta_P}{\alpha} = \lim_{\alpha \to 0} \frac{q}{1-q} \frac{F^{-1}(\alpha)}{F^{-1}(\beta_P)} > \frac{q}{1-q} > 0$$

and for $\alpha = 1$ we have eventually (i.e. for all N above a certain value),

$$\frac{1}{N}\left(\frac{(1-q)\,\beta_P(1)}{\left(q + (1-q)\,\beta_P(1)\right)^2}\right) < F^{-1}(1) = 1$$

Hence a unique solution $(\alpha_P, \beta_P(\alpha_P))$ exists for the equilibrium problem.

The proofs for the size effect and the underdog compensation effect are analogous to the ones obtained in the M system. □

Proof of Proposition 7. First, we compare turnouts. Assuming the cost side $F^{-1}(\alpha)$ is the same in the two systems, it suffices to show that the benefit sides of the equations determining the equilibrium α are ranked.

For any $q \neq 1/2$ we need to show that eventually (i.e. for any N above a given $\overline{N}$) we have

$$B_M^A(\alpha, \beta_M(\alpha)) < B_P^A(\alpha, \beta_P(\alpha)), \qquad \text{for all } \alpha \in (0,1]$$

namely

$$e^{-N\left(\sqrt{q\alpha}-\sqrt{(1-q)\beta_M}\right)^2}\sqrt{N} < \frac{(1-q)\,\beta_P}{\left(q\alpha+(1-q)\,\beta_P\right)^2}\left(\frac{\sqrt{q\alpha}+\sqrt{(1-q)\,\beta_M}}{4\sqrt{\pi}\left(q\,(1-q)\,\alpha\beta_M\right)^{1/4}}\frac{1}{\sqrt{q\alpha}}\right)^{-1}$$

which is satisfied as LHS above converges to zero, whereas the RHS is a positive constant for all $\alpha \in (0,1]$ because

$$\begin{aligned} \alpha &\in (0,1] &\Longrightarrow&\quad \beta_P \in (0,1],\quad \beta_M \in (0,1] \\ q &\neq 1/2 &\Longrightarrow&\quad \sqrt{q\alpha} \neq \sqrt{(1-q)\,\beta_M\,(\alpha)} \end{aligned}$$

Hence, for any eventually we have

$$q \neq 1/2 \quad\Longrightarrow\quad \alpha_M < \alpha_P$$

The symmetry property $\beta\,(q) = \alpha\,(1-q)$ (which holds in both the M and P systems) implies

$$q \neq 1/2 \quad\Longrightarrow\quad \beta_M < \beta_P$$

hence

$$q \neq 1/2 \quad\Longrightarrow\quad T_M < T_P$$

For $q = 1/2$ we have $\alpha = \beta$ in both P and M systems. We need to show that eventually

$$B_M^A > B_P^A, \qquad \alpha \in (0,1]$$

namely

$$\frac{1}{\sqrt{N}}\left(\frac{2\sqrt{q\alpha}}{4\sqrt{\pi}}\right)\frac{1}{q\alpha} > \frac{1}{N}\left(\frac{q\alpha}{2\,(2q\alpha)^2}\right)$$

Rearranging we have

$$\sqrt{N}\left(\frac{1}{2\sqrt{\pi}\sqrt{q\alpha}}\right) > \left(\frac{1}{8q\alpha}\right)$$

which is satisfied as the RHS is a positive constant and the LHS increases to infinity. Hence

$$q = 1/2 \quad\Longrightarrow\quad \alpha_M > \alpha_P \quad\Longrightarrow\quad T_M > T_P$$

Next, we compare underdog compensation effects. Given that for the M system we have

$$q\alpha_M\left(F^{-1}\left(\alpha_M\right)\right)^2 = (1-q)\,\beta_M\left(F^{-1}\left(\beta_M\right)\right)^2$$

and for the P system we have

$$q\alpha_P\left(F^{-1}\left(\alpha_P\right)\right) = (1-q)\,\beta_P\left(F^{-1}\left(\beta_P\right)\right)$$

then

$$\frac{1-q}{q} = \left(\frac{\alpha_P}{\beta_P}\right)^2 \left(\frac{\frac{F^{-1}(\alpha_P)}{\alpha_P}}{\frac{F^{-1}(\beta_P)}{\beta_P}}\right) = \left(\frac{\alpha_M}{\beta_M}\right)^3 \left(\frac{\frac{F^{-1}(\alpha_M)}{\alpha_M}}{\frac{F^{-1}(\beta_M)}{\beta_M}}\right)^2$$

By definition of derivative at zero we have

$$\frac{dF^{-1}}{dx}|_{x=0} = \lim_{x\to 0} \frac{F^{-1}(x)}{x} \in (0,\infty)$$

For N large, α and β converge to zero both in the M and in the P system so

$$\lim_{N\to\infty} \left(\frac{\frac{F^{-1}(\alpha)}{\alpha}}{\frac{F^{-1}(\beta)}{\beta}}\right) = 1$$

and the result follows. If $\frac{dF^{-1}}{dx}|_{x=0} \in \{0,\infty\}$ then the above limit is indeterminate and the result need not be true. If the function F^{-1} is infinitely differentiable and n is the lowest integer for which

$$\frac{d^nF^{-1}}{dx^n}|_{x=0} \in (0,\infty)$$

then by iterating the procedure we have

$$\lim_{N\to\infty} \left(\frac{\frac{d^{n-1}F^{-1}(\alpha)}{d\alpha^{n-1}}}{\frac{d^{n-1}F^{-1}(\beta)}{d\beta^{n-1}}}\right) = 1$$

so the underdog compensation comparison generalizes to

$$\frac{1-q}{q} = \left(\frac{\alpha_P}{\beta_P}\right)^{n+1} = \left(\frac{\alpha_M}{\beta_M}\right)^{2n+1}$$

□

Proof of Lemma 9. Express the following series by differentiating and integrating the summands and inverting the series and integral operators

$$\begin{aligned}\sum_{b=0}^{\infty} \frac{B^b}{b!}\frac{a}{a+b+c} &= \frac{a}{B^{a+c}}\sum_{b=0}^{\infty}\int_0^B \frac{d}{dr}\left(\frac{1}{b!}\frac{r^{a+b+c}}{a+b+c}\right)dr \\ &= \frac{a}{B^{a+c}}\int_0^B\sum_{b=0}^{\infty}\left(\frac{1}{b!}r^{a+b+c-1}\right)dr = \begin{cases}\frac{a}{B^{a+c}}\int_0^B r^{a+c-1}e^r dr & \text{for } a\geq 1\\ 1/3 & \text{for } a=c=0\end{cases}\end{aligned}$$

and likewise

$$\sum_{b=0}^{\infty}\frac{B^b}{b!}\frac{a+1}{a+b+1} = \frac{a+1}{B^{a+c+1}}\int_0^B r^{a+c}e^r dr$$

We compute the marginal benefit for party A by inverting the series and integral operators again over the series over a.

$$\begin{aligned} B_P^A &= e^{-(A+B+C)}\left(\sum_{c=0}^{\infty}\frac{C^c}{c!}\left(\begin{array}{c}\sum_{a=0}^{\infty}\frac{A^a}{a!}\left(\frac{a+1}{B^{a+c+1}}\int_0^B r^{a+c}e^r dr\right)\\ -\sum_{a=1}^{\infty}\frac{A^a}{a!}\left(\frac{a}{B^{a+c}}\int_0^B r^{a+c-1}e^r dr\right)\end{array}\right)-\frac{1}{3}\right)\\ &= e^{-(A+B+C)}\left(\sum_{c=0}^{\infty}\frac{C^c}{c!}\left(\begin{array}{c}\int_0^B \frac{r^c}{B^{c+1}}\left((Ar/B)\,e^{(Ar/B)}+e^{(Ar/B)}\right)e^r dr\\ -\int_0^B \frac{r^{c-1}}{B^c}\left((Ar/B)\,e^{(Ar/B)}\right)e^r dr\end{array}\right)-\frac{1}{3}\right)\end{aligned}$$

Inverting the series and integral operators again over the series over c.

$$\begin{aligned} B_P^A &= e^{-(A+B+C)}\left(\begin{array}{c}\int_0^B\left((Ar/B)\,e^{(Ar/B)}+e^{(Ar/B)}\right)\left(\sum_{c=0}^{\infty}\frac{C^c}{c!}\frac{r^c}{B^{c+1}}\right)e^r dr\\ -\int_0^B\left((Ar/B)\,e^{(Ar/B)}\right)\left(\sum_{c=0}^{\infty}\frac{C^c}{c!}\frac{r^{c-1}}{B^c}\right)e^r dr-\frac{1}{3}\end{array}\right)\\ &= e^{-(A+B+C)}\left(\int_0^B\left(\left(\frac{A}{B}re^{\frac{A+B+C}{B}r}+e^{\frac{A+B+C}{B}r}\right)\frac{1}{B}-\frac{A}{B}e^{\frac{A+B+C}{B}r}\right)dr-\frac{1}{3}\right)\end{aligned}$$

Computing the integral and simplifying, we have

$$\begin{aligned} B_P^A &= e^{-(A+B+C)}\left(\left(\begin{array}{c}\left(AB\left(\frac{1-e^{A+B+C}}{(A+B+C)^2}+\frac{e^{A+B+C}}{A+B+C}\right)+\left(B\frac{e^{A+B+C}-1}{A+B+C}\right)\right)\frac{1}{B}\\ -\frac{A}{B}\left(B\frac{e^{A+B+C}-1}{A+B+C}\right)\end{array}\right)-\frac{1}{3}\right)\\ &= \left(1-\frac{A}{A+B+C}\right)\frac{1-e^{-(A+B+C)}}{A+B+C}+\left(\frac{A}{A+B+C}-\frac{1}{3}\right)e^{-(A+B+C)}\end{aligned}$$

□

Proof of Proposition 10. A similar calculation gives the analogous result for r parties:

$$B_P^A(r)=\left(\begin{array}{c}\left(1-\frac{A}{A+B+C+...+r}\right)\frac{1-e^{-(A+B+C+...+r)}}{A+B+C+...+r}\\ +\left(\frac{A}{A+B+C+...+r}-\frac{1}{r}\right)e^{-(A+B+C+...+r)}\end{array}\right)$$

For large enough N, B_P^A approximates to

$$\begin{aligned} B_P^A &\simeq \left(1-\frac{A}{A+B+C+...+r}\right)\frac{1}{A+B+C+...+r}\\ &= \left(\frac{\beta q_B+\gamma q_C+..}{(\alpha q_A+\beta q_B+\gamma q_C+..)^2}\right)\frac{1}{N}\end{aligned}$$

so the benefit still decreases as N^{-1}, which implies a higher turnout than in M except in the case when the two parties in M have the same ex-ante support: $q=1/2$.

For r parties with equal ex-ante support we have

$$q_A=q_B=q_C=...=q_r=1/r \quad\Longrightarrow\quad \alpha=\beta=\gamma=...$$

the first order condition for a party becomes

$$\left(1-\frac{1}{r}\right)\frac{1-e^{-\alpha_r N}}{\alpha_r N}\approx\left(1-\frac{1}{r}\right)\frac{1}{\alpha_r N}=F^{-1}\left(\alpha_r\right)$$

so the turnout for that party α_r increases in r. Overall turnout increases too as in this symmetric case we have.

$$T_r=\alpha_r$$

□

Appendix C: Group Mobilization and Ethical Voting

As before, the population is a continuum of measure one, divided into q A supporters and $(1-q)$ B supporters. For any voting cost thresholds (c_α, c_β), i.e., given that the voter participation for each side is $(\alpha = F(c_\alpha), \beta = F(c_\beta))$, turnout is again $T = q\alpha + (1-q)\beta$. We assume F is weakly concave.[41]

5.1. **Cost Side for Mobilization Model.** A mobilization model assumes that more campaign spending by a party brings more votes for the party according to an exogenous technology. In major elections, candidates and parties engage in hugely expensive get-out-the-vote drives. Empirical evidence suggests that these drives are effective. We consider a very simple version of group mobilization. We assume the cost for a party of mobilizing to the polls all his supporters with voting cost below c is $l(c)$, where $c \in [0, \overline{c}]$ and l is increasing, convex and twice differentiable. We also assume it is infinitely costly for a party to turn out all its supporters: $l(\overline{c}) = \infty$.

5.2. **Benefit Side under any Power Sharing Rule.** The expected vote shares for party A and B are

$$V=\frac{q\alpha}{T},\qquad 1-V=\frac{(1-q)\,\beta}{T}$$

The expected power share as a function of the vote share is the standard "contest success function"[42]

$$P_\gamma^A(V)=\frac{V^\gamma}{V^\gamma+(1-V)^\gamma},\qquad P_\gamma^B(V)=\frac{(1-V)^\gamma}{V^\gamma+(1-V)^\gamma}$$

[41] The same condition was needed to have uniqueness of a solution in the rational voter M-model.

[42] See for instance Hirshleifer (1989), among others. When nobody votes ($\alpha=\beta=0$) assume equal shares ($V=1/2$).

Below we illustrate the power share P_γ^A as a function of the vote share V for various power sharing parameters γ, namely: $\gamma = 1$ (i.e. the P system, dashed line), $\gamma = 5$ (i.e. approaching the M system, continuous line), and $\gamma \to \infty$ (i.e. a pure M system, dotted line).

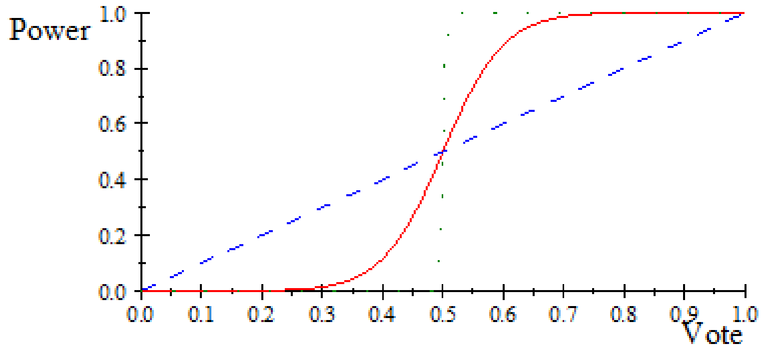

Figure 4: Power Sharing Functions in the P (dashed), approaching the M system (continuous) and pure M system (dotted).

The marginal benefits with respect to (c_α, c_β) are respectively

$$\frac{\gamma}{V(1-V)} \frac{\left(\frac{V}{1-V}\right)^\gamma}{\left[1+\left(\frac{V}{1-V}\right)^\gamma\right]^2} \left(\frac{(1-q)\beta}{T^2}\right) qf(c_\alpha), \quad \frac{\gamma}{V(1-V)} \frac{\left(\frac{V}{1-V}\right)^\gamma}{\left[1+\left(\frac{V}{1-V}\right)^\gamma\right]^2} \left(\frac{q\alpha}{T^2}\right)(1-q) f(c_\beta)$$

5.3. **First Order Conditions and Underdog Compensation Effects.** For the ethical voter models we have as first order conditions

$$\begin{aligned} \frac{\gamma}{V(1-V)} \frac{\left(\frac{V}{1-V}\right)^\gamma}{\left[1+\left(\frac{V}{1-V}\right)^\gamma\right]^2} \left(\frac{(1-q)\beta}{T^2}\right) qf(c_\alpha) &= qc_\alpha f(c_\alpha) \\ \frac{\gamma}{V(1-V)} \frac{\left(\frac{V}{1-V}\right)^\gamma}{\left[1+\left(\frac{V}{1-V}\right)^\gamma\right]^2} \left(\frac{q\alpha}{T^2}\right)(1-q) f(c_\beta) &= (1-q) c_\beta f(c_\beta) \end{aligned}$$

which gives the condition

$$q\alpha F^{-1}(\alpha) = (1-q)\beta F^{-1}(\beta)$$

The above is a partial underdog compensation condition which happens to be the same as the partial underdog compensation condition (3) obtained in the P system of the rational voter model.

For the mobilization model we have

$$\frac{\gamma}{V(1-V)}\frac{\left(\frac{V}{1-V}\right)^{\gamma}}{\left[1+\left(\frac{V}{1-V}\right)^{\gamma}\right]^{2}}\left(\frac{(1-q)\beta}{T^{2}}\right)qf(c_{\alpha}) = l'(c_{\alpha})$$
$$\frac{\gamma}{V(1-V)}\frac{\left(\frac{V}{1-V}\right)^{\gamma}}{\left[1+\left(\frac{V}{1-V}\right)^{\gamma}\right]^{2}}\left(\frac{q\alpha}{T^{2}}\right)(1-q)f(c_{\beta}) = l'(c_{\beta})$$

which yields the following *zero underdog compensation* condition

$$\frac{\alpha l'(c_{\alpha})}{f(c_{\alpha})}=\frac{\beta l'(c_{\beta})}{f(c_{\beta})} \implies T=\alpha=\beta, \quad c_{\alpha}=c_{\beta}$$

that is, both parties turn out the same proportion of their supporters.[43]

5.4. **Solution to the Mobilization Model.** The mobilization model is reduced to one equation in one unknown, equating marginal benefit (MB) and marginal cost

$$MB=\gamma\frac{\left(\frac{V}{1-V}\right)^{\gamma}}{\left[1+\left(\frac{V}{1-V}\right)^{\gamma}\right]^{2}}=\gamma\frac{\left(\frac{q}{1-q}\right)^{\gamma}}{\left[1+\left(\frac{q}{1-q}\right)^{\gamma}\right]^{2}}=G(\alpha) \tag{6}$$

where

$$G(\alpha):=\alpha\frac{l'(c_{\alpha})}{f(c_{\alpha})}$$

is increasing in α. The solution is hence unique and it exists because $l(\bar{c})=\infty$. The solution has the following properties:

(1) Turnout $T=\alpha$ increases when the marginal benefit (MB) increases;

(2) As γ goes to infinity (M model) the marginal benefit goes to infinity when $q=1/2$ and goes to zero otherwise;

(3) When $\gamma=1$ (P model) the marginal benefit $\frac{\left(\frac{q}{1-q}\right)}{\left[1+\left(\frac{q}{1-q}\right)\right]^{2}}$ is positive for all $q\in(0,1)$ and peaks but stays finite at $q=1/2$.

The picture below shows the marginal benefit as a function of the closeness of the election q for $\gamma=1$ (i.e. the P system, dashed line), and for $\gamma=5$ (i.e. approximating the M system, continuous line).

[43] We need to assume F weakly concave (as in the rational voter M model) to guarantee the LHS expressions above are increasing in their argument.

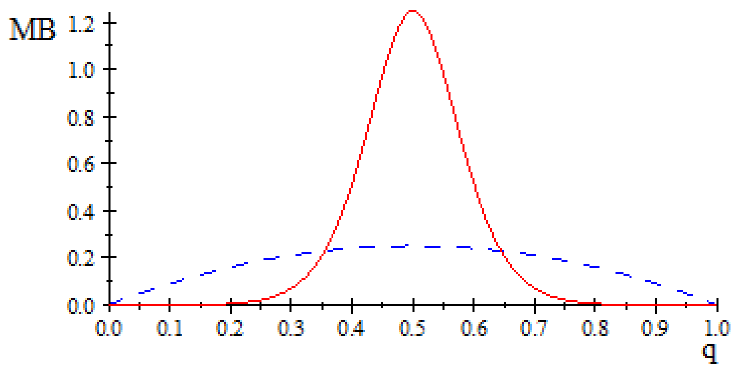

Figure 5: Marginal benefit as a function of q in the P system (dashed) and approaching the M system (continuous).

Turnout $T = \alpha$ can be obtained by a simple rescaling, namely by inverting (6) (i.e.: $T = G^{-1}(MB)$) which preserves the qualitative features of the picture above. Hence, the turnout comparison across systems is analogous to what we already obtained in the rational voter model (see Figure 1).

5.5. **Cost Side for Ethical Voter Model.** The ethical voter model assumes that citizens are "rule utilitarian" so they act as one. This means that we have to find a party-planner solution on each side A and B. In this solution each planner looks at the total benefit from the outcome of the election considering the total cost of voting incurred by the supporters of his side.[44] The cost of turning out the voters for the social planner on side A is the total cost born by all the citizens on side A that vote, namely

$$C(c_\alpha) := q \int_0^{c_\alpha} cf(c)\, dc$$

The citizens with cost below the planner-chosen cost threshold c_α vote because ethical voter models assume citizens get an exogenous benefit D (larger than their private voting cost $c \leq c_\alpha$) for "doing their part" in following the optimal rule established by the planner.

5.6. **Solution to the Ethical Voter Model.** The solution for the ethical voter model is more complicated, as the underdog compensation is strictly partial (not zero), so $\alpha \neq \beta$ and

[44]We assume "collectivism", so the planner on each side, A and B, only looks at the total cost of voting of the voters on his side. The results would not changed if we assumed "altruism" as in Feddersen and Sandroni (2006): each planner takes into account the cost of voting of all citizens that vote regardless of their side.

we maintain the two equations in two unknowns, that is

$$q\alpha F^{-1}(\alpha) = (1-q)\,\beta F^{-1}(\beta) = \gamma \frac{\left(\frac{q\alpha}{(1-q)\beta}\right)^{\gamma}}{\left[1+\left(\frac{q\alpha}{(1-q)\beta}\right)^{\gamma}\right]^{2}}$$

However, given that the underdog compensation is not full the comparative statics is similar to the case of zero compensation obtained in the mobilization model. Namely if a solution (α, β) exists,[45] then α and β, and hence T, increase when the marginal benefit increases. Taking limits, as γ goes to infinity (M model) the marginal benefit on the RHS goes to infinity when $q = 1/2$ and to zero otherwise. When $\gamma = 1$ (P model) the marginal benefit $\frac{\left(\frac{q}{1-q}\right)}{\left[1+\left(\frac{q}{1-q}\right)\right]^{2}}$ is positive for all $q \in (0, 1)$ and peaks but stays finite at $q = 1/2$.

Note that if the underdog compensation were full (which happens for instance with homogeneous costs) the marginal benefit becomes

$$MB = \gamma \frac{\left(\frac{q\alpha}{(1-q)\beta}\right)^{\gamma}}{\left[1+\left(\frac{q\alpha}{(1-q)\beta}\right)^{\gamma}\right]^{2}} = \frac{\gamma}{4}$$

so the result would be different: regardless of the initial preference split q, turnout and MB would increase with the intensity of the contest γ. As explained, the rational voter model with homogenous cost gives an equivalent result.

6. Appendix D: Results from QRE estimation

Table E1 displays the estimated logit QRE turnout rates. The estimated value of $\widehat{\lambda}$ is 7 for the the M data and 17 for the P data. There is essentially no change in the estimated QRE turnout rates if $\widehat{\lambda}$ is constrained to be equal in both treatments.

[45] Coate and Conlin (2004) and Feddersen and Sandroni (2006) provide specific conditions on the voting cost distributions that guarantee existence.

N_A	N_B	$c_{\max}$	Rule	$\widehat{\alpha}$	$\widehat{\beta}$	$\widehat{T}$	$\alpha^*_{\widehat{\lambda}}$	$\beta^*_{\widehat{\lambda}}$	$T^*_{\widehat{\lambda}}$
4	5	.3	M	0.622 (.042)	0.636 (.056)	0.630 (.048)	0.61	0.65	0.63
3	6	.3	M	0.513 (.050)	0.520 (.073)	0.52 (.063)	0.51	0.52	0.52
2	7	.3	M	0.490 (.063)	0.360 (.071)	0.39 (.060)	0.44	0.42	0.42
4	5	.55	M	0.479 (.024)	0.451 (.034)	0.464 (.020)	0.48	0.45	0.46
3	6	.55	M	0.436 (.021)	0.398 (.031)	0.411 (.019)	0.44	0.40	0.41
2	7	.55	M	0.330 (.046)	0.284 (.037)	0.294 (.028)	0.33	0.28	0.29
4	5	.3	P	0.547 (.025)	0.486 (.020)	0.51 (.016)	0.48	0.44	0.46
3	6	.3	P	0.547 (.054)	0.465 (.061)	0.49 (.042)	0.55	0.40	0.45
2	7	.3	P	0.600 (.040)	0.421 (.049)	0.461 (.041)	0.65	0.37	0.43
4	5	.55	P	0.362 (.026)	0.370 (.039)	0.367 (.020)	0.35	0.32	0.33
3	6	.55	P	0.477 (.037)	0.305 (.033)	0.362 (.027)	0.40	0.29	0.33
2	7	.55	P	0.515 (.029)	0.320 (.037)	0.363 (.032)	0.48	0.27	0.32

Table 4. Observed turnout rates. Clustered standard errors in parenthesis.

The scatter plot of the QRE turnout rates against the observed turnout rates is given below. Note that the slope has increased from 0.82 to 0.89, the constant term has decreased from 0.10 to 0.07 and the R^2 has increased from .87 to .91.

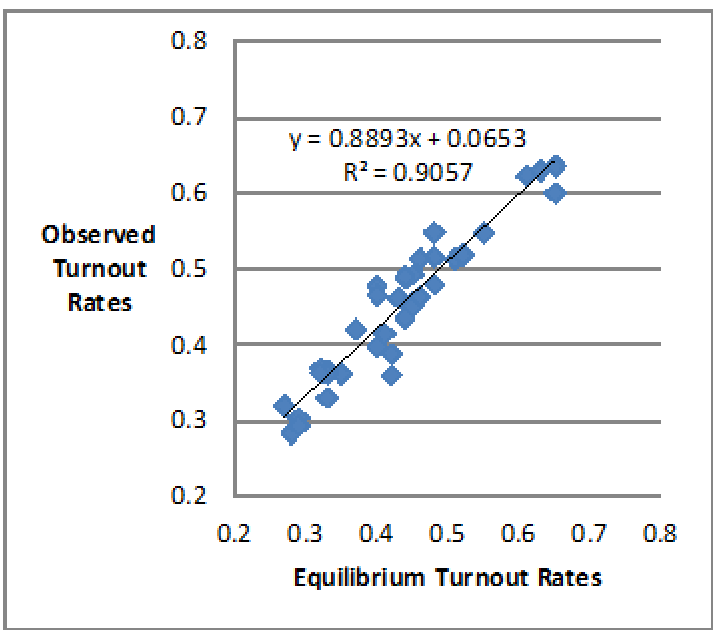

Figure 6: Turnout Rates

7. Appendix E: Online Supplementary Material

EXPERIMENT INSTRUCTIONS

Thank you for agreeing to participate in this decision making experiment. During the experiment we require your complete, undistracted attention, and ask that you follow instructions carefully. You may not open other applications on your computer, chat with other students, or engage in other distracting activities, such as using your phone, reading books, etc. You will be paid for your participation in cash, at the end of the experiment. Different participants may earn different amounts. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance. The entire experiment will take place through computer terminals, and all interaction between you will take place through the computers. It is important that you not talk or in any way try to communicate with other participants during the experiments. During the instruction period, you will be given a complete description of the experiment and will be shown how to use the computers. If you have any questions during the instruction period, raise your hand and your question will be answered out loud so everyone can hear. If you have any questions after the experiment has begun, raise your hand, and an experimenter will come and assist you.

We will begin with a brief practice session to help familiarize you with the computer interface. The practice rounds will be followed by 2 different paid sessions. Each paid session will consist of 50 rounds. At the end of the last paid session, you will be paid the sum of what you have earned in all rounds of the two paid sessions, plus the show-up fee of $5.00. Everyone will be paid in private and you are under no obligation to tell others how much you earned. Your earnings during the experiment are denominated in POINTS. Your DOLLAR earnings are determined by multiplying your earnings in POINTS by a conversion rate. In this experiment, the conversion rate is 0.002, meaning that 100 POINTS is worth 20 cents.

We will now go through two practice rounds to explain the rules for the first part of the experiment, and will explain the screen display. During the practice rounds, please do not hit any keys until I tell you, and when you are prompted by the computer to enter information, please wait for me to tell you exactly what to enter. You are not paid for these practice rounds.

[AUTHENTICATE CLIENTS]

Please pull out your dividers. Please double click on the icon on your desktop that says MULTISTAGE CLIENT. When the computer prompts you for your name, type your First and Last name. Then click SUBMIT and wait for further instructions.

SCREEN 1 (user interface)

[Point out while reading the following.]

You now see the first screen of the experiment on your computer. It should look similar to this screen. Please do not do anything with your mouse yet, until I have finished explaining the screen. [POINT TO PPT SLIDE DISPLAYED ON SCREEN IN FRONT OF ROOM]

Here are the instructions for the first part of the experiment. At the top of the screen will be your id number. Each of you has been assigned to one of two groups, called the ALPHA GROUP and the BETA GROUP. The ALPHA group always has 2 members and the BETA group always has 7 members. The screen informs you which group you will be in and reminds you how many members are in each group.

Each of you will be asked to choose either "X" or "Y" by clicking on a button with the mouse. Please wait and don't do anything yet.

The sample display in front of the room shows you what the screen looks like for a member of the Alpha group. The screen also tells you what your "Y bonus" is. This is an extra bonus you earn if you choose Y instead of X, independent of what other participants choose.

Your earnings are computed in the following way. It is very important that you understand this, so please listen carefully.

SCREEN 2

[Point while reading.] First suppose you choose X. To compute your earnings, we compare the number of members of your group choosing "X" to the number of members of the other group choosing X. Your payoff is 105 if the number of members in your group choosing X is greater than the number of members of the other group who choose X. Your payoff is 55 if the number of members in your group choosing X is equal to the number of members of the other group who choose X. Your payoff is 5 if the number of members in your group choosing X is fewer than the number of members of the other group who choose X.

Your earnings are computed slightly differently if you choose Y. Specifically, in addition to the above earnings (either 105, 55, or 5) you also earn your Y bonus. This payoff information is displayed in a table on your screen.

The amount of each participant's Y-bonus is assigned completely randomly by the computer at the beginning of each round and is shown in the second line down from the top of the screen. Y-bonuses are assigned separately for each participant, so different participants will typically have different Y-bonuses. What you see up the front is just an example of one participant's Y-bonus. In any given round you will have an equal chance of being assigned any Y-bonus between 0 and 30 points. Your Y-bonus in each round will not depend on your Y-bonus or decisions in previous rounds, or on the Y-bonuses and decisions of other participants. While you will be told your own Y-bonus in each round before making a decision, you will never be told the Y-bonuses of other participants. You will only know that each of the other participants has a Y-bonus that is some number between 0 and 30.

At this time, if your ID number is even, please click on row label Y; if your ID number is odd, please click on the row label X. Once everyone has made their selection, the results from this first practice round are displayed on your screen. It will look like

SCREEN 3

if your choice was X, and

SCREEN 4

if your choice was Y.

This completes the first practice round, and you now see a screen like this. The bottom of the screen contains a history panel. This panel will be updated to reflect the history of all previous rounds. [go over columns of history screen]

At the beginning of every new round you will be randomly re-assigned to new groups, and will have the opportunity to choose between "X" and "Y." In other words, you will not necessarily be in the same group during each round. You will also be randomly reassigned a new Y-bonus at the beginning of each round.

We will now go to a second practice round. When this practice round is over, an online quiz will appear on your screen. Everyone must answer all the questions correctly before we can proceed to the paid rounds. Does anyone have any question?

Please take note of your new group assignment, alpha or beta, since the group assignments are shuffled randomly between each round. Also, please take note of your new Y-bonus, which has been randomly redrawn between the values of 0 and 30.

[SLIDE 4]

GO TO NEXT MATCH

Please make your decision now by clicking on the row label X or Y.

A quiz is now displayed on your screen. Please read each question carefully and select the correct answer. Once everyone has answered all the questions correctly, you may all go on to the second page of the quiz. After everyone has correctly answered the second page of questions, we will begin the first paid session. If you have any questions as you are completing the quiz, please feel free to raise your hand and I will go to your workstation to answer your question.

The first paid session will follow the same instructions as the practice session. There will be a total of 50 rounds in the first paid session. Let me summarize those instructions before we start.

[Go over summary slide.] Are there any questions before we begin the first paid session? [Answer questions.] Please begin. There will be 50 rounds, and then you will receive new instructions. (Play rounds 1 – 50) The first session is now over.

SESSION 2

We will now begin session 2.

[SLIDE 5]

The second paid session will be slightly different from the first session. Let me summarize those rules before we start. Please listen carefully. The rules are the same as before with only one exception. In each round of this session, you will have an equal chance of being assigned a Y-bonus between 0 and 55 points. Again, our Y-bonus in each round will not depend on your Y-bonus or decisions in previous rounds, or on the Y-bonuses and decisions of other participants. While you will be told your own Y-bonus in each round before making a decision, you will never be told the Y-bonuses of the other participants. You will only know that each of the other participants has a Y-bonus that is some number between 0 and 55. You may choose "X" or "Y".

There will be 50 rounds in this second session. After each round, group assignments will be randomly reshuffled and everyone will be reassigned a new Y-bonus. Therefore, some rounds you will be in the Alpha group and other rounds you will be in the Beta group. In either case, everyone is told which group they are in and what their private Y-bonus is, before making a choice of X or Y.

Are there any questions before we begin the second paid session? (no quiz) Please Begin. (Play rounds 1 – 50)

Session 2 is now over. Please record your total earnings in dollars for the experiment on your record sheet. After you have recorded your earnings, click the 'ok' button. We cannot pay anyone until everyone has recorded their earnings AND clicked the ok button. Please remain seated and you will be called up one by one according to your ID number to have your recorded earnings amount checked against our own record. Please wait patiently and do not talk or use the computers.