Turf Wars*

Helios Herrera†
SIPA
Columbia University

Michael M. Ting‡
Department of Political Science and SIPA
Columbia University

March 14, 2012

Abstract

Turf wars occur in environments where competition undermines the ability of agents to collaborate. We develop a model of competition and collaboration between government agencies. In the game, one agent possesses property rights over jurisdiction to an issue. This agent can decide whether to share jurisdiction with other agents. Sharing increases joint productivity but also increases competition over a fixed prize. Each agent's productivity is private information, and so the sharing decision is possibly a signal of the originating agent's productivity. We derive conditions under which agents will voluntarily share information, and also discuss possible remedies to failures of collaboration.

*Draft. We thank Scott Ashworth, Alessandra Casella, Wouter Dessein, seminar participants at Columbia University, and panel participants at the 2011 annual meetings of the Midwest Political Science Association, European Consortium for Political Research, and the Conference on Positive Political Economy at Academia Sinica for helpful comments.

†School of International and Public Affairs, 420 W 118th St., New York NY 10027 (hhh2108@columbia.edu).

‡Political Science Department, 420 W 118th St., New York NY 10027 (mmt2033@columbia.edu).

The "turf war" is one the most commonly recognized organizational pathologies. While there is no consensus on the definition of the term, accounts of the phenomenon typically possess several common elements. Agents, such as government bureau heads or corporate division managers, perceive themselves to be in competition with one another over resources, promotions, or publicity. Given the opportunity to pursue an important task or assignment, these agents will then attempt to exclude rivals from participation. Tactics might include withholding crucial information, or using decision-making rights to shunt rivals' activities into low-profile tasks. Importantly, principals wish to induce agents to collaborate, but have only limited abilities to enforce such behavior.

Unsurprisingly, turf battles are widely believed to have significant, negative effects on organizational performance. In his classic analysis of bureaucratic politics, Wilson (2000) devoted an entire chapter to describing the consequences of turf-motivated strategies. Moreover, examples involving some of the largest organizations and most significant pieces of legislation are not difficult to find. The following list illustrates five major instances of turf wars, as well as efforts to overcome them.

Medicare Payment Systems and Accountable Care Organizations. Most doctors in the U.S. operate either individually or out of small practices. In addition, insurers generally reimburse individual practitioners on a service-by-service (also known as "Fee for Service") basis. For example, the Medicare program, which covers Americans of age 65 and over, uses a schedule that consists of over 6,700 discrete services and associated fees. In addition to giving incentives for over-treatment, these arrangements often force patients to seek care in a piecemeal fashion, and can hinder collaboration between care providers by introducing disincentives to "lose" patients (Medicare Payment Advisory Commission 2008, Lee and Mongan 2009).[1] An emerging alternative arrangement is the accountable care organization (ACO), which would make networks of providers jointly responsible for all of a patient's medical needs. ACOs would accomplish this by giving incentives for practitioners to cooperate and to hold down total treatment costs. The 2010 Patient Protection and Affordable Care Act authorized the creation of ACOs for Medicare beneficiaries.

Emergency Management in New York City. The events of September 11, 2001 revealed widespread deficiencies in the ability of the city's agencies to share jurisdiction over emergency responses. The police and fire departments' activities on that day were often wastefully redundant, and the lack

[1]The Institute of Medicine, an arm of the National Academy of Sciences, has also extensively criticized fee for service reimbursement. One report stated that

> [T]he American health care system does not have well-organized pro- grams to provide the full complement of services needed by people with such chronic conditions as heart disease, cancer, diabetes, and asthma. Nor do we have mechanisms to coordinate the full range of services needed by those with multiple serious illnesses. And our current health system has only a rudimentary ability to collect and share patient information." (Institute of Medicine, 2001)

of interoperable radio equipment hampered even attempts at cooperation.[2] These shortcomings resulted in the 2004 announcement of the Citywide Incident Management System, which attempted to identify more clearly a lead agency for specific types of emergency events.

U.S. Military Branches. The National Security Act of 1947 established the basic structure of the modern U.S. national security bureaucracy. The law preserved the relative autonomy of the individual armed services, which in turn led to low levels of coordination between functionally similar units. In the Korean and Vietnam wars, the Navy and Air Force ran essentially independent air campaigns, and subsequent operations in Lebanon and Grenada in the early 1980s were marred by the services' inability to communicate. As Lederman (1999) documents, this performance record culminated in the 1986 Goldwater-Nichols Act. Among the reforms were the creation of Unified Combat Commands, which allowed local commanders to coordinate centrally the activities of all American forces operating in a given region.

U.S. Intelligence Reform. In its comprehensive analysis of the 9/11 attacks, the National Commission on Terrorist Attacks upon the United States (2004) prominently criticized the organization of U.S. intelligence gathering. The report (commonly known as the *9/11 Commission Report*) argued that with 14 competing intelligence agencies spread across several federal departments, officials would have difficulty aggregating information relevant to the disruption of attacks. It recommended the creation of a central office to coordinate intelligence gathering activities across these agencies. The position of Director of National Intelligence was officially created by the Intelligence Reform and Terrorism Prevention Act of 2004.

Drug Enforcement. Wilson (1978) discusses the U.S. Drug Enforcement Agency's (DEA) Geographical Drug Enforcement Program (G-DEP), which the agency used to allocate "buy money" for drug investigations. The money was allocated across DEA regions in the U.S. on the basis of previous arrests, with more significant arrests (e.g., of wholesalers) earning larger rewards. This, however, resulted in some perverse sharing incentives:

> Many drug distribution networks cut across regional lines. One organization may bring brown heroin from Mexico in to Detroit, where it is cut and then sent on to Boston or New York to be sold on the street. Six DEA regions have an interest in this case: region 15 headquartered in Mexico City, regions 11 (Dallas) and 14 (Los Angeles), with responsibilities along the Southwest border, region 6 based in Detroit, and regions 1 (Boston) and 2 (New York). If agent and regional directors believe they are rewarded for their stats, they will have an incentive to keep leads and informants to themselves

[2]See John Farmer, "Saving Our Lives and Protecting Their Turf." *New York Times*, May 15, 2005, and Christine Hauser, "Police and Fire Radios Are Talking to Each Other." *New York Times*, July 31, 2008.

> in order to take credit for a Mexican heroin case should it develop. A more appropriate strategy would be for such information to be shared so that an interregional case can be made. ... The perceived evaluation and reward system of the organization ... threatens to lessen the credit, and therefore (it is believed) the resources, available for a given region.

Collectively, G-DEP induced a bias toward capturing street-level offenders, even though most agents and outsiders would have preferred higher profile cases.[3] The DEA responded to these sharing issues by creating investigation-specific inter-regional task forces known as Central Tactical Units. However, these units would often only displace the turf issue, as regional offices were reluctant to share their best agents and the assignment of credit for a successful investigation could be difficult.

We develop a model of organizational turf wars. It is, to our knowledge, the first model to consider how turf wars arise, and how they might be controlled. As Posner (2005: 143n) notes, "The literature on turf wars is surprisingly limited, given their frequency and importance." Accordingly, the model is simple and attempts to capture only the essential elements of a turf battle. We view these to be as follows:

1. Joint production
2. Competition
3. Property rights

The first requirement is perhaps the most obvious: questions about responsibility over a task can only arise between agents who are capable of contributing to the production of a relevant outcome. Of course, if agents cared primarily about outcomes, then they would generally welcome the participation of other agents. Hence, the second requirement: agents must be in competition. The competition might be an explicit prize, for example a promotion in a rank-order tournament, or it may reflect the ability to undermine or sabotage the production of other agents. The intensity of the competition might emerge naturally from basic indivisibilities; *i.e.*, some prizes may only be awardable to a single "winner." Examples include gaining favorable media attention, or securing a

[3]Wilson mentions variations of this problem at several levels in the investigation and prosecution of drug law violators. U.S. Attorneys often had incentives to keep cases under their own control, instead of developing larger cases. DEA agents also had incentives to pursue low-level offenders because local police agencies tended not to share potential informants, and so had to develop their own pool. Finally, the Federal Bureau of Investigation also suffered from some of the same internal issues faced by the DEA.

new contract or a prestigious project assignment.[4]

The third requirement generates the unique setup of our model, which is that agents have *de facto* property rights over jurisdictions. We focus on the simplest version of such property rights, which is to assume that one agent, whom we designate the *originator*, can choose to share her jurisdiction with other agents, or go it alone to protect her turf. Originator status may arise from technology or statutory assignments of responsibility, or from a principal's inability to re-assign property rights.

Our model therefore considers a set of agents whom, if given jurisdiction, can make additive contributions to some collective output, or policy. Agents vary in their productivity in producing policy, and their productivity parameters are private information. Thus, a key problem will be that agents are uncertain over the benefits of a shared jurisdiction arrangement. Agents care about the overall level of policy production. Through their individual efforts, they also compete over a non-negative exogenous award from winning, labeled β. To keep the model simple, sharing always results in a higher level of output, and thus society or a welfare-maximizing principal would strictly prefer collaboration to autonomy.

It is worth noting here that the model implicitly assumes that a principal such as a legislature could not simply step in and force agents to collaborate. (Given the starting assumption that collaboration is good, this ability would trivialize the model.) The motivating logic behind centralizing re-organizations such as those proposed by the *9/11 Commission Report* is that agencies would be more easily induced to share if they were placed under one roof. While this approach has no doubt had its successes, such reforms have not been uniformly successful. One reason for this is that competition may be more pronounced within organizations than between them (*e.g.*, Posner 2005). In the U.S. Central Intelligence Agency, for example, the two main branches (operations and analytics) are historically fierce rivals (Gates 1987). Thus, we believe that collaboration is plausibly non-contractible, and will focus instead on more limited remedies for resolving turf wars.

We begin with a simple non-strategic example that illustrates some basic tradeoffs between collaboration and autonomy. Here, the productivity of each agent is exogenously fixed and known, and agents do not exert effort. The originator can address the problem by herself, or share it with the remaining agents. Sharing increases the capacities brought to bear on the problem, and thus always increases output. However, by sharing the originator reduces her probability of receiving β. The main comparative statics relation derived here concerns the complementarity of the agents'

[4] This assumption is often considered natural in the bureaucratic setting. For example, Downs (1966) argued that bureaucracies were in a constant state of competition, and in particular that "No bureau can survive unless it is continually able to demonstrate that its services are worthwhile to some group with influence over sufficient resources to keep it alive."

efforts. Sharing is "easier" where there is synergy in the agents' capabilities. Moreover, high synergies cause better originators to share, while low synergies cause worse originators to share. The former result is bad for an outside actor who cared about outcomes, since she would wish for worse originators to share. Finally, we show that there is no equilibrium where the originator always works and the partner always shares.

The full strategic model incorporates incomplete information about agent productivity, and also examines the non-originating agent's effort conditional upon receiving jurisdiction. In choosing whether to take on a partner, the originator must consider the partner's inference of the originator's ability. For example, if a low-productivity partner agent believes that a sharing originator is highly productive, then the partner will have less incentive to work, because she perceives little chance of winning β. We find that high-productivity originators are more likely to share, and that only high-productivity partners will work conditional upon sharing. High values of β also make the originator unwilling to share jurisdiction, but low values of β are not necessarily helpful, since they may provide insufficient incentive for a partner to work. By contrast, a high level of outcome motivation by the originator may both increase the incentives for the originator to share and also for the partner to work. Collaboration requires both sharing by the originator and working by the partner, and this is generally best achieved when both players have high outcome motivations and the lever of competition is moderate.

The model also suggests partial remedies for turf wars. While a hypothetical principal would not be able to mandate sharing, there are more moderate strategies that may help to increase collaboration. First, if the reward β is manipulable, then an originator might be induced to share. As the above discussion suggests, values of β that are neither too high nor too low may be most conducive to collaboration. Second, the policy motivation of agents might can be considered a form of control via appointments: under some conditions society might benefit from "zealots" who care more about policy, while under others it might prefer "careerists" who care more about promotion.[5] While the agent with the higher outcome motivation would be more willing to share, the less motivated agent may induce more collaboration as the originator, as the more motivated agent will be likelier to work as partner. Finally, a hypothetical principal may also benefit from the ability to manipulate the distribution of agent ability levels, or the cost of effort. Reducing the cost of effort or increasing the ability levels of originators may increase the set of originator types who would be willing to share or the set of partners who would be willing to work.

There have been very few theoretical or empirical attempts to address explicitly the idea of a turf

[5]For a more detailed treatment of this topic, see Gailmard and Patty (2007).

war.[6] Perhaps the most salient is King's (1994, 1997) analysis of the division of labor within the U.S. Congressional committee system. In this account, committee jurisdictions are often not established *ex ante* by statute, but rather grow from the successful acquisition of "common law" jurisdictions. Congressional parliamentarians are key players in the development of jurisdictions, since they decide which committees receive bills under consideration. This results in a very different conception of property rights than in the organizational setting considered in this paper: parliamentarians, instead of agents, decide who has jurisdiction, and jurisdictions are typically exclusive to one committee rather than shared.

The model developed here is related to two families of theoretical work. First, there is an extensive literature on multi-agent contracting models. One strand (*e.g.*, Holmström and Milgrom 1990, Itoh 1991, Marx and Squintani 2009) examines how principals may structure agent incentives to induce cooperative behavior. Of particular interest is Dessein, Garicano, and Gertner (2010), who examine the design of managerial incentives to capture synergies in multi-divisional firms. Another examines sabotage in rank-order tournaments (*e.g.*, Lazear 1989, Chen 2003). Although we do not model it here, agent sabotage might be one way in which turf battles are fought, especially when agents are are brought together on a task involuntarily.[7] These models focus on moral hazard problems, and feature a principal with access to a rich space of contracts, as opposed to the crude jurisdictional tools available to the originator.

A second family deals with information sharing. Like sabotage, the failure to reveal relevant information might in some contexts be an element of an agent's strategy to protect turf. Despite some significant differences in modeling approaches, a key question here is the extent to which players are willing to reveal their private information, even when they might be in competition. Okuno-Fujiwara, Postlewaite, and Suzumura (1990) develop a highly general two-stage model in which players decide non-cooperatively in the first stage whether to make a verifiable report of their information, and derive conditions for full revelation.[8]

Several models have pursued this idea in more specific strategic contexts. Austen-Smith and Riker (1987) study committee decision-making over a unidimensional policy space. In their model,

[6] A JSTOR article search of the term "turf" in the fields of economics, political science, and management yields 13 title hits and 22 abstract hits, none of which are associated with a formal model.

[7] In at least some environments, the effects of sabotage might be expected to be small. Konrad (2000) develops a somewhat different model of sabotage in a rent seeking (lobbying) game, and shows that the externalities of sabotage are dissipated when the number of lobbyists becomes large.

[8] Relatedly, a number of papers on information sharing in oligopoly competition also consider the question of when competitors might share (*e.g.*, Gal-Or 1986, Creane 1995, Raith 1996). A standard feature of these models is that firms are able to commit to sharing information (through a hard information report) prior to its revelation. This might be interpreted as a trade association. See also Modica (2010), who shows how competing firms might contribute to open source projects, and Baccara and Razin (2007), who develop a bargaining model in which innovators need to share ideas in order to develop them but worry that this process might cause those ideas to be stolen.

members acquire a private signal about a common uncertain parameter that affects everyone's payoffs, engage in a "debate" via simultaneous cheap talk, and then issue proposals and vote. In equilibrium, some legislators usually conceal their information, and final decisions might not reflect all revealed information. Stein (2008) models conversations among two competitors, each of whom have complementary "ideas" in alternating periods. Each player is willing to reveal her idea to the competitor will use it to form a better idea that will be passed back in turn. High levels of complementarity and skill then sustain informative conversation in equilibrium. Finally, Alonso, Dessein, and Matouschek (2008) study institutional centralization and decentralization, where two agents wish to choose locally ideal actions but also coordinate their actions. Under decentralization, agents can make cheap talk reports to each other and choose policy individually, while under centralization, reports go to a manager who chooses local policies. Their main result, which relates to the prevailing arguments about the desirability of centralization for inducing collaboration, is that decentralization is preferable when agents are relatively homogeneous.

The remainder of the paper is organized as follows. The next section describes the game theoretic model. Section 3 builds some intuition by developing the decision theoretic model. Section 4 then derives the equilibrium of the game theoretic model and the main result on leadership selection. Section 5 concludes.

1 Model

In the basic game, there are two agents, labeled A1 and A2. One of the agents (without loss of generality, A1) will have irrevocable jurisdiction over a task. We label A1 the originator. The originator's key decision is whether to share jurisdiction with A2.

Each agent Ai generates an output level $x_i \in [0, \Theta]$ $(\Theta > 0)$ for the task when she has jurisdiction over it. She cares about the sum of the agents' outputs, which we will call policy, and also about a prize $\beta \geq 0$ for achieving the highest output. This might represent career concerns, or a promotion. Thus each Ai receives the following utility from the task outcomes:

$$u_i = \begin{cases} m_i(x_1 + x_2) + \beta \mathbf{1}_{x_i > x_{-i}} & \text{if both agents have jurisdiction} \\ m_i x_i + \beta & \text{if only A}i\text{ has jurisdiction} \\ m_i x_{-i} & \text{if A}i\text{ does not have jurisdiction.} \end{cases} \tag{1}$$

Here, $m_i > 0$ is a measure of Ai's "policy" motivation, which will contrast usefully with her motivation to win.

Conditional upon Ai having jurisdiction, output x_i is determined by:

$$x_1 = \theta_1 \sim U[0, \Theta]$$

$$x_2 = \begin{cases} \theta_2 \sim U[0,\Theta] & \text{if } e = 1 \\ 0 & \text{if } e = 0. \end{cases}$$

The parameter $\theta_i \sim U[0,\Theta]$ represents Ai's productivity. Each θ_i is private information, though in the next section we examine a case in which it is common knowledge. The parameter e is A2's effort level. Zero effort is costless and results in zero output, while positive effort costs $k > 0$ and results in an output equal to productivity. Not exerting effort may be usefully interpreted as a refusal by A2 to accept shared jurisdiction. Note that A1 does not exert effort. We assume, in effect, that as originator she has already committed to participating the project (due perhaps to statutory requirements) and will therefore realize her full productivity. This assumption is relaxed in a subsequent extension.

The sequence of the game is as follows.

1. Nature draws the agents' productivity types.
2. A1 decides whether to extend jurisdiction to A2 (or equivalently, whether to exclude A2 from participation).
3. If given jurisdiction, A2 chooses effort.

The natural solution concept for this game is perfect Bayesian equilibrium. As the subsequent derivations show, there is essentially a unique equilibrium. Formally, strategies consist of A1's sharing choice $s \in \{share, noshare\}$ and A2's effort level $e \in \{0,1\}$ conditional upon having jurisdiction. As a tie-breaking rule, we assume that A1 will share when indifferent. This reflects the prospect that sharing jurisdiction probably involves non-trivial administrative costs. Finally, A2 has posterior beliefs over θ_1 given A1's sharing decision, denoted $\rho_1 : \{share, noshare\} \to \mathcal{P}$, where $\mathcal{P}$ is the set of measurable subsets of $[0,\Theta]$. These beliefs are consistent with Bayes' rule along the equilibrium path. An out of equilibrium information set can only be encountered if A1 shares when she was not expected to do so. In these cases, we simply assume that A2's beliefs are concentrated on $\theta_1 = \Theta$. Although the equilibrium does not require such extreme beliefs, this designates the type that would benefit most from sharing, since she would win with probability one.[9]

2 Example: Complete Information, No Effort

To illustrate some intuitions, we begin with a decision-theoretic version of the model. Agents cannot exert any effort, and all productivity parameters are common knowledge. We do generalize the

[9] This refinement is similar to the Banks and Sobel (1987) D1 requirement.

model in two ways. First, we allow $n \geq 2$ agents, each with corresponding productivity $\theta_i \in (0,1)$, where $\sum \theta_i = 1$. To do this in a parsimonious way we simplify the calculation of the winner of β by letting Ai's probability of victory be θ_i when all agents have jurisdiction. Second, we use the more general CES production function for determining output.

We assume initially that A1's sharing choice remains binary, in that she either shares with all other agents or none. The expected utility for Ai, given sharing by A1, is:

$$u_i = m_i \left(\sum \theta_i^\rho\right)^{1/\rho} + \beta\theta_i.$$

Here $\rho > 0$ is the substitutability/complementarity of abilities. For example, if $\rho = 1$ then abilities are perfect substitutes, for $\rho < 1$ there are synergies in working together (the total ability is larger than the sum of the abilities), and conversely for $\rho > 1$.

The utilities for sharing and not sharing are then:

$$\begin{aligned} u_1^S &= m_1 \left(\sum_i^n \theta_i^\rho\right)^{1/\rho} + \beta\theta_1 \\ u_1^N &= m_1\theta_1 + \beta \end{aligned}$$

Sharing happens if and only if:

$$m_1 \left(\left(\sum_{i=1}^n \theta_i^\rho\right)^{1/\rho} - \theta_1\right) > \beta\left(1 - \theta_1\right)$$

It will then be convenient to express the net gain from sharing and the condition for sharing as:

$$A(\theta_1) = \frac{\left(\sum_{i=1}^n \theta_i^\rho\right)^{1/\rho} - \theta_1}{1 - \theta_1} > \frac{\beta}{m_1}$$

We are mainly interested in seeing how the originator's productivity affects the propensity to share. The results depend on how changes in θ_1 affect θ_i for $i \neq 1$. One simple way to do this is to assign non-negative linear "weights" to each player's technological parameter, of the following form:

$$\theta_i = \pi_i(1 - \theta_1).$$

It is straightforward to derive each π_i. Note that $\theta_1 = 1 - \theta_i/\pi_i$ for each $i \neq 1$, which implies that θ_i/π_i is constant for all $i \neq 1$. Furthermore, to ensure that $\sum_i \theta_i = 1$, the weights π_i must satisfy $\sum_{i \neq j} \pi_i = 1$. This implies that for each $i \neq 1$:

$$\sum_{k \neq j} \pi_k = \sum_{k \neq j} \frac{\theta_k}{\theta_i}\pi_i = 1,$$

and therefore we have the following unique weights for each agent and originator pair:

$$\pi_i = \frac{\theta_i}{\sum_{k\neq 1}\theta_k}.$$

This is simply agent i's relative weight among the set of non-originators. These weights imply that as θ_1 increases, the remaining θ_i's must all shrink in proportion with their relative size.

The first result presents the basic comparative statics of the model.

Proposition 1 Conditions for Sharing, No Effort

(i) For any $\theta_1 \in [0,1)$, $\rho \lesseqgtr 1 \Longrightarrow \frac{dA}{d\theta_1} \gtreqless 0$.

(ii) For any $\theta_1 \in (0,1)$, $\rho \lesseqgtr 1 \Longrightarrow A(\theta_1) \gtreqless 1$, and:

$$A(0) = \begin{cases} 1 & \text{if } n = 2 \\ \lesseqgtr 1 \text{ if } \rho \gtreqless 1 & \text{if } n > 2. \end{cases}$$

(iii) $\frac{dA}{d\rho} < 0$.

Part (i) of the result shows how synergies matter for the originator's sharing decision. For $\rho < 1$ and β/m sufficiently high, there is a "cutoff" $\overline{\theta}_1$ such that the originator shares if $\theta_1 > \overline{\theta}_1$. From a welfare perspective this is a bad result, as it would be better for lower productivity originators to share. For $\rho > 1$ and β/m sufficiently low, this is reversed: there is another cutoff $\underline{\theta}_1$ such that sharing occurs if $\theta_1 < \underline{\theta}_1$. Note that the cutoff depends on θ_1, because $A(\cdot)$ depends on which player is the originator. Interestingly, in the linear case ($\rho = 1$) the propensity to share does not depend on θ_1, and therefore depends only on the agents' relative policy motivation.

Part (ii) shows how the critical value for β/m depends on synergies. For $n > 2$, there are values of β/m near 1 for which sharing is either optimal for all θ ($\rho < 1$) or not optimal for all θ ($\rho > 1$). Part (iii) simply establishes the intuitive result that the benefit of sharing is decreasing in ρ (*i.e.*, increasing in synergy). Thus, the greater the synergy, the more inclined the originator will be to share.

We can also consider what would happen if the originator could choose the set of agents she shares with. There are two cases. In the first, A1 can choose only one additional agent, or partner, to share the task. In the second, A1 decides how many partners to take on, though all partners are identical in capability.

In the first case, since the originator can only choose one partner, the probability of victory conditional upon sharing with Ak is $\theta_1/(\theta_1+\theta_k)$. A1 would be willing to add agent Ak if:

$$\begin{aligned} m_1\left(\theta_1^\rho + \theta_k^\rho\right)^{1/\rho} + \beta\frac{\theta_1}{\theta_1+\theta_k} &> m_1\theta_1 + \beta \\ \left(\theta_1^\rho + \theta_k^\rho\right)^{1/\rho} - \theta_1 &> \frac{\theta_k}{\theta_1+\theta_k}\left(\frac{\beta}{m_1}\right). \end{aligned}$$

Note that this expression is identical to that of the two-agent case when $\theta_1 + \theta_k = 1$. There are two effects of increasing θ_k: increasing the probability of a successful outcome, and decreasing the probability of winning the award.

Rearranging, the preceding expression can be rewritten:

$$A(\theta_1, \theta_k) = (\theta_1 + \theta_k) \left[\left(\left(\frac{\theta_1}{\theta_k} \right)^{\rho} + 1 \right)^{1/\rho} - \frac{\theta_1}{\theta_k} \right] > \frac{\beta}{m_1}. \tag{2}$$

This expression implies that holding θ_1/θ_k constant, sharing becomes harder as $\theta_1 + \theta_k$ shrinks. This happens because the contribution of sharing toward the collective outcome becomes smaller, while the probability of "winning" remains the same. Likewise, holding $\theta_1 + \theta_k$ constant, sharing becomes easier (harder) as θ_1 increases if $\rho < (>)\ 1$.

The next result characterizes the originator's choice of a single partner. Part (i) considers the strength of the originator. When $\rho < 1$ (*i.e.*, there are synergies), stronger originators will be more inclined to share with a given partner. The results are weaker when $\rho > 1$ but stronger originators will often do better in a partnership than weaker originators. Part (ii) considers the more interesting question of whom an originator would choose. Often stronger partners are preferred. Interestingly, when $\rho > 1$ the originator prefers stronger partners; *i.e.*, the expected loss in the victory bonus is offset by the fact that more capable partners are "better." This is also true when $\rho < 1$ and potential partners are stronger than the originator. The relationship may be reversed if potential partners are weaker than the originator.

Comment 1 Optimal Partner

For potential partner Ak:

(i) If $\rho < 1$, $\frac{\partial A}{\partial \theta_1} > 0$. If $\rho > 1$, $\frac{\partial A}{\partial \theta_1} > 0$ if $\theta_1 \le \theta_k$ and ρ sufficiently large; $\lim_{\rho \to \infty} \frac{\partial A}{\partial \theta_1} = 0$ if $\theta_1 > \theta_k$.

(ii) If $\rho > 1$, $\frac{\partial A}{\partial \theta_k} > 0$. If $\rho < 1$, $\frac{\partial A}{\partial \theta_k} > 0$ if $\theta_1 \le \theta_k$; $\lim_{\rho \to \infty} \frac{\partial A}{\partial \theta_k} < 0$ if $\theta_1 > \theta_k$.

In the second case, the originator can choose t agents to partner with, but all agents are identical ($\theta_i = \theta$). It will be convenient to count the originator as a "partner." Thus, the originator's objective is:

$$L(t) := m \left(t \left(\frac{1}{n} \right)^{\rho} \right)^{1/\rho} + \frac{\beta}{t}. \tag{3}$$

where $t \in \{1, \ldots, n\}$.

This objective is considerably simpler than that with heterogeneous agents, and so it is straightforward to derive the following result.

Comment 2 Optimal Number of Homogeneous Partners

If $\frac{\beta}{m_1} \geq \frac{n^{1/\rho}-1}{n-1}$, then $t^* = 1$; otherwise, $t^* = n$.

As intuition would suggest, when the bonus from victory is relatively important, then there is less sharing. Somewhat more interestingly, there is never an interior solution, and so the originator will either share with one agent or all agents.

3 Main Results

We now discuss the game with effort and incomplete information. We begin by considering the subgame following A1's choice to share, where agent A2 has posterior beliefs about A1's productivity that are uniformly distributed on the interval $[\underline{\theta}, \overline{\theta}]$. While this is not a fully general treatment of A2's possible beliefs, we will subsequently argue that A1's sharing strategies are monotonic and therefore must take this form in equilibrium.

A2 works if:

$$m_2(E[\theta_1 \mid share] + \theta_2) - k + \beta\omega(\underline{\theta}, \overline{\theta}, \theta_2, e) > m_2 E[\theta_1 \mid share]. \tag{4}$$

Here $\omega(\underline{\theta}, \overline{\theta}, \theta_2, e)$ represents the probability that A2 wins. In the uniform case we have:

$$\omega(\underline{\theta}, \overline{\theta}, \theta_2, 1) = \begin{cases} 0 & \text{if } \theta_2 < \underline{\theta} \\ \frac{\theta_2 - \underline{\theta}}{\overline{\theta} - \underline{\theta}} & \text{if } \theta_2 \in [\underline{\theta}, \overline{\theta}] \\ 1 & \text{if } \theta_2 > \overline{\theta}, \end{cases}$$

Expression (4) simplifies to:

$$m_2\theta_2 + \beta\omega(\underline{\theta}, \overline{\theta}, \theta_2, e) > k.$$

As intuition would suggest, working becomes more attractive to A2 as m_2, θ_2 and β increase, and less attractive as k increases. The expression implies that for any $[\underline{\theta}, \overline{\theta}]$, there is a unique type $\tilde{\theta}_2$ such that higher productivity types work and lower productivity types shirk. This type is given by:

$$\tilde{\theta}_2 = \begin{cases} \frac{k}{m_2} & \text{if } \underline{\theta} \geq \frac{k}{m_2} \\ \min\left\{\Theta, \frac{k(\overline{\theta} - \underline{\theta}) + \beta\underline{\theta}}{m_2(\overline{\theta} - \underline{\theta}) + \beta}\right\} & \text{if } \underline{\theta} < \frac{k}{m_2} \end{cases} \tag{5}$$

The first line of this expression gives a corner case where A2 is indifferent between working and not even though she expects to lose β with certainty. The second gives the interior case where the indifferent A2 expects to win with some probability. Because this set of types necessarily hits a corner at Θ, this threshold must be weakly increasing in $\underline{\theta}$. Thus, contracting the set of sharing originator types (*i.e.*, increasing $\underline{\theta}$) will not expand the set of partner types willing to work.

Given $\tilde{\theta}_2$, the probability that A2 will work given $[\underline{\theta}, \overline{\theta}]$ is:

$$\phi(\underline{\theta}, \overline{\theta}) = 1 - \frac{\tilde{\theta}_2}{\Theta}. \tag{6}$$

And given θ_1, the probability that A2 wins conditional upon working is:

$$\xi(\theta_1) = \begin{cases} 1 & \text{if } \theta_1 \leq \tilde{\theta}_2 \\ \frac{\Theta - \theta_1}{\Theta - \tilde{\theta}_2} & \text{if } \theta_1 \geq \tilde{\theta}_2. \end{cases} \tag{7}$$

Note that if $\theta_1 < \tilde{\theta}_2$, then the probability of A1 winning is zero.

We can now characterize A1's sharing decision. Suppose that in equilibrium, the set of sharing originators is $[\underline{\theta}, \overline{\theta}]$. Then A1 shares if:

$$m_1 \left(\theta_1 + \phi(\underline{\theta}, \overline{\theta}) \frac{\tilde{\theta}_2 + \Theta}{2} \right) + (1 - \phi(\underline{\theta}, \overline{\theta}) \xi(\theta_1)) \beta \geq m_1 \theta_1 + \beta \tag{8}$$

Simplifying, A1 shares if $\phi(\underline{\theta}, \overline{\theta}) = 0$ or when $\phi(\underline{\theta}, \overline{\theta}) > 0$ and:

$$\frac{m_1(\tilde{\theta}_2 + \Theta)}{2} - \xi(\theta_1)\beta \geq 0 \tag{9}$$

There are two notable facts about this expression. First, it is independent of $\phi(\cdot)$ when $\phi(\cdot) \neq 0$. Second, it does not depend on the functional form of $\xi(\cdot)$; *i.e.*, it applies to any measurable sharing strategy on $[0, \Theta]$. This allows us to establish an important monotonicity feature of sharing in equilibrium. If any type θ_1' shares in equilibrium, then all higher types will also be willing to share. To see why, suppose that some type $\theta_1'' > \theta_1'$ does not share. By sharing out of equilibrium, she would not affect A2's strategy, and so would only affect A2's chances of winning. Thus, type θ_1'' would automatically satisfy (9) if type θ_1' does.

The immediate consequence of the monotonicity of sharing incentives is that when sharing occurs, the set of sharing types is simply $(\underline{\theta}, \Theta]$. The final step in deriving the equilibrium is then to characterize $\underline{\theta}$. At an interior solution, this is the (unique) value of θ_1 that satisfies (9) with equality. Higher values of $\underline{\theta}$ imply lower levels of sharing. There are also corner solutions, where either no types or all types will share. Our main result characterizes these cases.

Proposition 2 Conditions for Sharing

(i) (Full Sharing) A1 shares for all θ_1 if:

$$\beta \leq \max \left\{ k - m_2\Theta, \frac{\Theta(m_1/2 - m_2) + \sqrt{\Theta^2(m_2 - m_1/2)^2 + 2m_1\Theta(k + m_2\Theta)}}{2} \right\}.$$

(ii) (No Sharing) If the conditions of part (i) are not met, then A1 will not share for any θ_1 if $\Theta < \min\{k/m_2, \beta/m_1\}$.

(iii) (Mixed Sharing) If the conditions of parts (i) and (ii) are not met, then the set of sharing types is $[\theta_1^*, \Theta]$, where:

$$\theta_1^* = \begin{cases} \frac{(m_1-2m_2)\beta\Theta+m_1(k+m_2\Theta)\Theta-2\beta^2}{m_1(k-\beta)+m_2(m_1\Theta-2\beta)} & \text{if } \frac{k}{m_2} > \Theta, \text{ or } \frac{k}{m_2} \leq \Theta, \frac{m_1}{2}\left(\frac{k}{m_2}+\Theta\right) > \beta \\ \Theta - \frac{m_1(\Theta^2-(k/m_2)^2)}{2\beta} & \text{otherwise.} \end{cases}$$

Proposition 2 provides some basic intuitions about the effects of turf battles on sharing. Part (i) shows that "full" sharing — *i.e.*, sharing by all types of originators — occurs when β is sufficiently low. This generally accords with the intuition that high levels of competition will inhibit sharing. Full sharing also occurs as k increases, since A1 is trivially willing to share when A2 is deterred from working. Part (ii) then addresses the case where no originators will share. Here, as intuition would suggest, high levels of competition and effort cost (β and k) will expand the set of parameters for which no sharing will result. Finally, part (iii) addresses the cases where the pooling conditions of parts (i) and (ii) are not met. The key result here is that high productivity types will share while low productivity types will not.

A key comparative statics result concerns the policy motivations of the two players, m_1 and m_2. Roughly speaking, higher policy motivations by the originator (*i.e*, higher m_1) make the conditions for sharing easier to satisfy, and thus lead to "more" sharing. Increasing m_2 has more ambiguous effects. Increasing m_2 reduces the range of parameters under which A1 does not share. However, the range of parameters under which full sharing occurs can actually contract for high values of m_2, which reflects the increasing competitive threat of A2. The following comment summarizes some of these results.

Comment 3 Comparative Statics on m_1, m_2

(i) (Full Sharing) The set of parameters under which A1 shares is larger for higher values of m_1, for m_1 sufficiently large.

(ii) (No Sharing) The set of parameters under which A1 does not share is smaller for higher values of m_1 and m_2.

(iii) (Mixed Sharing) θ_1^* is decreasing in m_1.

While Proposition 2 describes the originator's decision to share, it does not characterize the incentives for players to *collaborate*; *i.e.*, for A1 to share and for A2 to work. Even under full sharing, a set of sufficiently low types of A2 (characterized by $\tilde{\theta}_2$ in expression (5)) will not work.

For A2 productivity types near 0, the policy gains from working and the probability of victory are both low, and hence not worth the cost of effort. Thus, except in the limiting case where full sharing occurs and $k \to 0$, there is no equilibrium in which A2 always works. Expression (5) also shows that when m_2 is sufficiently high, policy motivations alone will induce some types to work even when they expect to lose with certainty.

The incomplete information model shares one obvious similarity with the no-effort model of the previous section. Higher types share, and higher outcome motivations by the originator generally encourage more sharing. However, the results contrast sharply with the linear production technology ($\rho = 1$) case of the no-effort model. The incentive for the originator to share did not change with θ_1 in the no-effort model, while here there are interior solutions where only sufficiently high types share. In this respect, the incomplete information model resembles the "synergies" case ($\rho < 1$) case of the no-effort model. As with that case, this equilibrium has some negative welfare implications, since a hypothetical principal or society would prefer that lower types share.

Example. To illustrate some of the comparative statics of the model, consider the following example, illustrated in Figure 1. We fix $k = 1$ and $\Theta = 2$ and vary m_1, m_2 and β. The figure shows that increasing m_1 (*i.e.*, moving from the left to the right panel of Figure 1) expands the set of parameter values under which full sharing occurs. It also contracts the set of parameters under which no sharing occurs. Increasing m_2 reduces full sharing because of the competitive threat from A2. Finally, as Proposition 2 suggests, high values of β are associated with lower sharing.

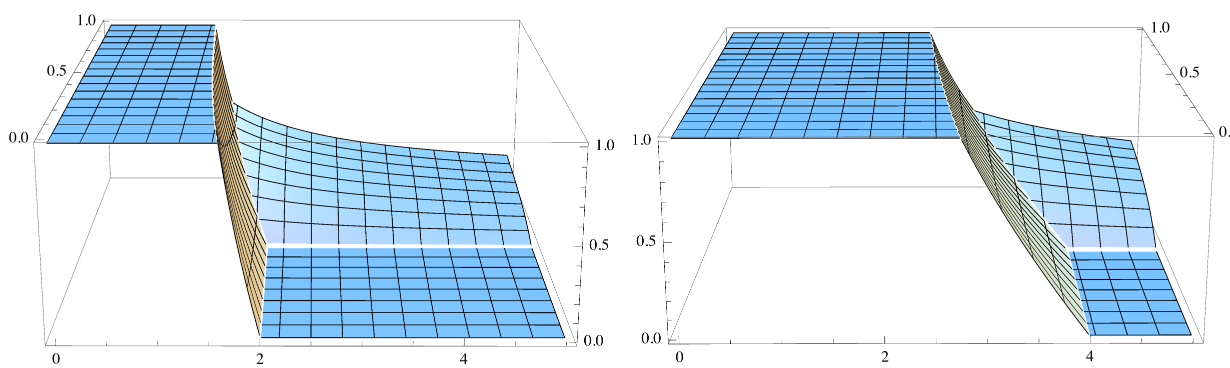

Figure 1: *Sharing Probabilities.* Let $k = 1$ and $\Theta = 2$. On the left, $m_1 = 1$, and on the right, $m_1 = 2$. Both plots depict the *ex ante* probability of sharing as a function of β (horizontal axis) and m_2 (upper axis).

Figure 2 plots A2's effort response given that A1 shares as well as the probability of *collaboration*, where A1 shares and A2 works. In terms of organizational productivity, collaboration is the key result of interest. The parametric assumptions are identical to those of the left side of Figure 1. The figure shows clearly that full sharing is often unlikely to improve organizational output: for low values of m_2 and β, only the top end of the θ_2 distribution will work. Expected effort is higher for higher values of m_2 and β, though of course for sufficiently high values of β, A1 becomes disinclined to share. Generally, the relationship between collaboration and β is not monotonic, and collaboration requires both highly motivated agents as well as moderate levels of competition.

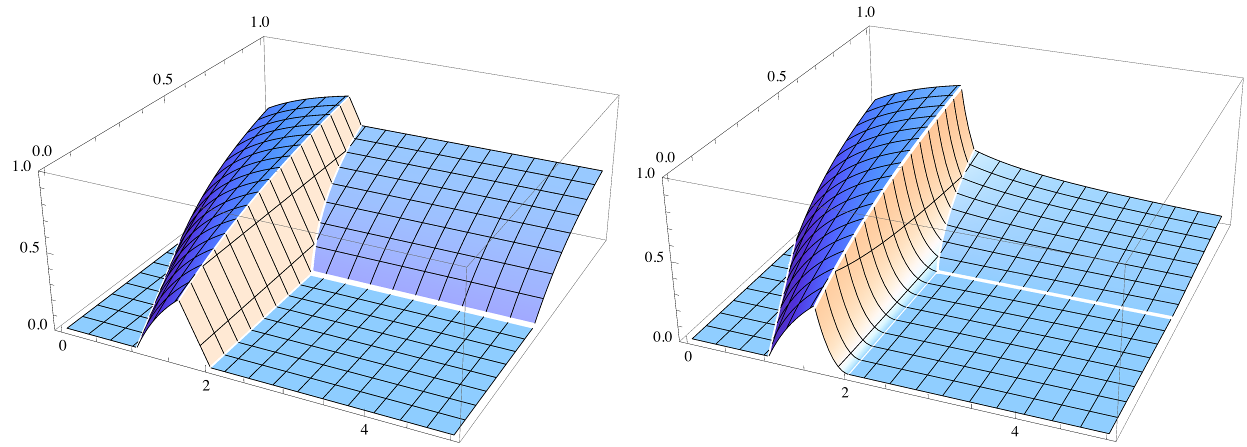

Figure 2: *Working and Collaboration Probabilities.* Let $k = 1$, $\Theta = 2$ and $m_1 = 1$, as in the left-hand side of Figure 1. The left-side plot depicts the probability that A2 works conditional upon sharing as a function of β (lower axis) and m_2 (upper axis). Note that for high values of m_2 and β, A2's effort choice is out of equilibrium. The right-side plots depicts the probability of collaboration, or the *ex ante* probability that A1 shares and A2 works

3.1 Remedies

Since a principal cannot simply mandate sharing, the model suggests three ways in which the rate of sharing might be improved. First, an originator might be in a position to control the size of the reward β. Proposition 2 characterizes the dependency of sharing on β. Roughly speaking, lower values of β encourage sharing, but also affect A2's incentives to work. As Figure 2 illustrates, the set of A2 types that would work is non-monotonic in β. The probability of collaboration is therefore maximized at a moderate value of β; in this example, the maximum β that permits full sharing by A1.

Second, a principal might be able to control, through appointment powers, m_i for each of the agents. One way to do this is by choosing which agent will become the originator, given m_1 and m_2. Higher values of m_1 generally encourage sharing, and so one approach is to designate the agent with the higher m_i as originator. This logic has its limits, however, since reducing the policy motivation of the partner will reduce her incentive to work and in turn discourage collaboration. If m_1 and m_2 are both sufficiently high to induce full sharing, then the probability of collaboration might be higher if the less motivated agent is the originator, since the more motivated agent is more likely to work. Naturally, a principal would do strictly better if she could simply choose the values of m_i for both agents. Appointing agents with high policy motivation can both encourage sharing and increase expected effort. In a political environment, this might correspond to using a political appointee, though perhaps at the cost of a loss of expertise relative to using a career civil servant.

Finally, a hypothetical principal may also consider manipulating other parameters of the model. Increasing the value of Θ, for example, reduces the range of parameters under which no sharing occurs and expands those under which full sharing occurs. This manipulation might correspond to increasing the salary of the agent, which would draw in higher-ability types. Somewhat perversely, however, increasing the potential ability of originators may have a negative effect on the willingness of partners to work. And while it would probably be difficult to achieve under most circumstances, reducing the cost of effort would unambiguously expand the set of partner types who would be willing to work, while sometimes also having beneficial effects on sharing as well.

3.2 Referrals and Originator Effort

Our basic model assumes that the originator does not need to exert effort to achieve her productivity level θ_1, and also that the originator cannot relinquish jurisdiction over the task. In some contexts, it is appropriate to relax both assumptions. An originator who is not legally bound to invest in productive capacity may decide to channel its resources elsewhere. Further, given the lack of productive capacity for a task, the originator may decide to "refer" a task to another agent (Garicano and Santo 2004).

Here we consider a game in which the originator initially chooses whether to exert effort. Each agent i's effort choice is denoted e_i, and her cost parameter is denoted k_i. For both i, output x_i is zero unless $e_i = 1$, and her productivity parameter θ_i is known to i when e_i is chosen. Following her effort choice the originator can choose to retain jurisdiction, share jurisdiction, or refer to task to the partner agent. A referral implies that none of the originator's productive capacity is used, although the originator continues to derive utility from the partner's effort and resulting productivity. We assume that neither agent receives β when both do not exert effort.

It is straightforward to show that in the extended game, originator effort and referrals are mutually exclusive. If A1 does not exert effort, then she can receive a positive payoff only by referring the case to A2. Similarly, if A1 exerts effort, then she cannot in equilibrium refer the case to A2, since she would have done strictly better by saving her effort and referring. Thus, the basic game analyzed above is identical to the game following the decision by A1 to exert effort, with the exception that the set of types choosing to share may be affected by the effort decision.

It is also straightforward to derive some simple bounds that characterize A1's effort choice. First, if $m_1\theta_1 + \beta > k_1$, then A1 can cover her cost by simply retaining jurisdiction, and therefore works. Second, if $m_1\theta_1 + m_2\Theta/2 + \beta < k_1$, then A1 cannot expect any benefit from working. Let $\hat{\theta}_1^*$ denote the value of θ_1 at which A1 is indifferent between working and referral. These bounds then imply that the threshold for working satisfies:

$$\hat{\theta}_1^* \in \left(\frac{k_1 - m_2\Theta/2 - \beta}{m_1}, \frac{k_1 - \beta}{m_1}\right).$$

To see how this threshold affects A1's strategy, observe that in the basic game, her equilibrium payoff was weakly increasing in θ_1, for otherwise a higher type A1 could simply adopt the strategy of a lower type and achieve at least the latter's payoff. Because the payoff to a referral is constant in θ_1, referrals occur if and only if $\theta_1 < \hat{\theta}_1^*$.

The need to work and the ability to refer may also affect A1's sharing strategy. Consider two extreme cases. In the first, $m_1\theta_1^* + \beta > k_1$, and so type θ_1^* is willing to work and $\hat{\theta}_1^* < \theta_1^*$. The derivation of the set of originator types willing to share remains identical to that of the basic game. Consequently, none of the results about sharing and collaboration change. In the second, $m_1\theta_1^* + m_2\Theta/2 + \beta < k_1$, and so type θ_1^* is unwilling to work and $\hat{\theta}_1^* > \theta_1^*$. Since any originator willing to work will (by virtue of having productivity higher than θ_1^*) be willing to share, the set of sharing originator types is truncated to $[\hat{\theta}_1^*, \Theta]$.

Generally, the effect of originator effort on sharing and overall production will depend on the location of $\hat{\theta}_1^*$. When $\hat{\theta}_1^* < \theta_1^*$, sharing and collaboration probabilities are unchanged. If referrals occur in equilibrium, then overall production actually *improves* if A2 is willing to work (i.e., $m_2\theta_2 + \beta > k_2$) and $\theta_2 > \theta_1$. But when $\hat{\theta}_1^* > \theta_1^*$, sharing is strictly reduced. The removal of the lowest sharing originator types in turn reduces the set of A2 types willing to work, thus curtailing collaboration as well. As with the previous case, some improvement is possible under referrals when A2 is both willing to work and more productive than A1.

4 Discussion

This model attempts to formalize the popular notion of a turf war. As we argue, the minimal necessary components of a turf war are joint production, competition, and property rights over jurisdiction. From this starting point, our model produces a unique equilibrium in which high-productivity originators share and high-productivity partners exert effort. One implication is that turf battles hurt most when collaboration is most needed; that is, when originators are low productivity. The model also generates several non-obvious implications for how organizational systems might be designed. Perhaps most prominently, it shows that the reward from competition, β, can both help and hurt collaboration.

We view these three basic elements as necessary and sufficient conditions for turf wars, other factors may also be relevant. We do not consider agent bargaining or politicking, nor do we look at the role of oversight, as suggested by Wilson (2000), though oversight might be considered part of the determination of winners. Another important question concerns the way in which effort is aggregated. In our decision theoretic example, sharing can make collective outcomes worse by reducing the individual contribution to production, while in our game theoretic model, effort and ability are simply additive. If the model had rapidly diminishing returns to collaborative effort, then a principal might instead want agencies not to share, and to operate autonomously instead.

In addition, we believe that there the model could easily be turned on other interesting questions. The most important is probably to consider other simple steps a principal could take to induce better performance. For example, she could use a performance cutoff below which β is not awarded to any agent. Such a mechanism might correspond to an organization's implicit threat to fill a higher position by appointing an outsider rather than promoting from within. This might induce cooperation by effectively punishing for a failed project under autarchy, but it may also run the risk of reducing incentives to work for certain types of agents.

5 Proofs

Proof of Proposition 1. (i) Letting $k_1 = \theta_1/(1-\theta_1)$, this gives us:

$$\begin{aligned} A(\theta_1) &= \frac{\left(\theta_1^\rho + \sum_{i\neq 1}(\pi_i(1-\theta_1))^\rho\right)^{1/\rho} - \theta_1}{1-\theta_1} \\ &= \left(k_1^\rho + \sum_{i\neq 1}(\pi_i)^\rho\right)^{1/\rho} - k_1. \end{aligned}$$

$A(\theta_1)$ has the same sign as $\frac{dA}{dk_1}$. Taking the derivative with respect to k_1:

$$\begin{aligned} \frac{dA}{dk_1} &= \left(k_1^\rho + \sum_{i\neq 1}(\pi_i)^\rho\right)^{\frac{1-\rho}{\rho}} k_1^{\rho-1} - 1 \\ &= \left(1 + k_1^{-\rho}\sum_{i\neq 1}(\pi_i)^\rho\right)^{\frac{1-\rho}{\rho}} - 1. \end{aligned}$$

This is positive (zero) (negative) if:

$$\frac{1-\rho}{\rho}\ln\left(1 + k_1^{-\rho}\sum_{i\neq 1}(\pi_i)^\rho\right) > (=)(<)\ 0.$$

Since the term in parentheses is greater than 1, $\frac{dA}{dk_1} > (=)(<)\ 0$ if $\rho < (=)(<)\ 1$.

(ii) Note first that:

$$A(0) = \left(\sum_{i\neq 1}(\pi_i)^\rho\right)^{1/\rho}.$$

For $n = 2$, there is only one term in the summation, and $\pi_i = 1$, so $A(0) = 1$. For $n > 2$, since $\sum_{i\neq 1}\pi_i = 1$, it is clear that $A(0) > (=)(<)\ 1$ if $\rho < (=)(>)\ 1$. Finally, since $A'(\theta_1) > (=)(<)\ 0$ for $\rho < (=)(>)\ 1$, we have $A(\theta_1) > (=)(<)\ 1$ when $\rho < (=)(>)\ 1$ for $\theta_1 > 0$.

(iii) Let $\tilde{A}(\rho) = \left(k_1^\rho + \sum_{i\neq 1}(\pi_i)^\rho\right)^{1/\rho}$, which has the same derivative as $A(\cdot)$ with respect to ρ. Differentiating $\tilde{A}(\theta_1)$ with respect to ρ, we obtain:

$$\frac{1}{\tilde{A}}\frac{d\tilde{A}}{d\rho} = \frac{1}{\rho}\frac{k_1^\rho \ln k_1 + \sum_{i\neq 1}(\pi_i)^\rho \ln \pi_i}{k_1^\rho + \sum_{i\neq 1}(\pi_i)^\rho} - \frac{1}{\rho^2}\ln\left(k_1^\rho + \sum_{i\neq 1}(\pi_i)^\rho\right)$$

Observe that $\tilde{A}(\rho) > 0$ for all ρ, and therefore $\frac{d\tilde{A}}{d\rho}$ is negative if:

$$\frac{k_1^\rho \ln k_1 + \sum_{i\neq 1}(\pi_i)^\rho \ln \pi_i}{k_1^\rho + \sum_{i\neq 1}(\pi_i)^\rho} < \frac{1}{\rho}\ln\left(k_1^\rho + \sum_{i\neq 1}(\pi_i)^\rho\right)$$

Now since $\pi_i > 0$ for all i and $\ln \pi_i < 0$, the left-hand side of the above expression is strictly less than $\ln k_1$. And since $\pi_i > 0$ for all i, the right-hand side is strictly greater than $\ln k_1$, thus establishing the result.

Proof of Comment 1. (i) As derived in the proof of Proposition 1, the bracketed expression in (2) is positive and increasing in θ_1 if $\rho < 1$. It follows that $A(\theta_1, \theta_k)$ is increasing in θ_1 if $\rho \leq 1$.

For $\rho > 1$, we differentiate with respect to θ_1, and so $\frac{\partial A}{\partial \theta_1} < (>)\ 0$ if:

$$(\theta_1 + \theta_k)\left[\left(\left(\frac{\theta_1}{\theta_k}\right)^{\rho} + 1\right)^{1/\rho - 1}\left(\frac{\theta_1}{\theta_k}\right)^{\rho-1}\frac{1}{\theta_k} - \frac{1}{\theta_k}\right] + \left(\left(\frac{\theta_1}{\theta_k}\right)^{\rho} + 1\right)^{1/\rho} - \frac{\theta_1}{\theta_k} \quad < (>) \quad 0$$

$$\left[\left(\left(\frac{\theta_1}{\theta_k}\right)^{\rho} + 1\right)^{1/\rho - 1}\left(\frac{\theta_1}{\theta_k}\right)^{\rho} - \frac{\theta_1}{\theta_k}\right] + \left[\left(\left(\frac{\theta_1}{\theta_k}\right)^{\rho} + 1\right)^{1/\rho - 1}\left(\frac{\theta_1}{\theta_k}\right)^{\rho-1} - 1\right] + \left(\left(\frac{\theta_1}{\theta_k}\right)^{\rho} + 1\right)^{1/\rho} - \frac{\theta_1}{\theta_k} \quad < (>) \quad 0.$$

Following the proof of Proposition 1, both bracketed expressions are negative for $\rho > 1$, while $\left(\left(\frac{\theta_1}{\theta_k}\right)^{\rho} + 1\right)^{1/\rho} - \frac{\theta_1}{\theta_k}$ is positive. Letting $k = \theta_1/\theta_k$ and simplifying, we have $\frac{\partial A}{\partial \theta_1} < (>)\ 0$ if:

$$\frac{1}{(k^{\rho}+1)^{1-1/\rho}} - \frac{2k+1}{2k^{\rho} + k^{\rho-1} + 1} < (>)\ 0.$$

There are two cases. First, suppose $k \leq 1$ (i.e., $\theta_1 < \theta_k$). Since $(k^{\rho}+1)^{1/\rho-1} \leq 1$, it follows that $\frac{dA}{d\theta_1} < 0$ if: $2k^{\rho} + k^{\rho-1} < 2k$, or $k^{\rho-1} < 2k/(2k+1)$. This is clearly satisfied for ρ sufficiently large. Second, suppose $k > 1$. The limits of both fractions in the above expression as $\rho \to \infty$ are clearly 0. Thus $\lim_{\rho\to\infty} \frac{\partial A}{\partial \theta_1} = 0$.

(ii) As derived in the proof of Proposition 1, the bracketed expression in (2) is positive and increasing in θ_k if $\rho > 1$. It follows that $A(\theta_1, \theta_k)$ is increasing in θ_k if $\rho \geq 1$.

For $\rho < 1$, we differentiate with respect to θ_k and substitute $k = \theta_1/\theta_k$:

$$\begin{aligned}\frac{\partial A}{\partial \theta_k} &= (\theta_1 + \theta_k)\left[-\left(\left(\frac{\theta_1}{\theta_k}\right)^{\rho} + 1\right)^{1/\rho - 1}\left(\frac{\theta_1^{\rho}}{\theta_k^{\rho+1}}\right) + \frac{\theta_1}{\theta_k^2}\right] + \left(\left(\frac{\theta_1}{\theta_k}\right)^{\rho} + 1\right)^{1/\rho} - \frac{\theta_1}{\theta_k} \\ &= k^2 - (k^{\rho}+1)^{1/\rho-1}\, k^{\rho}(k+1) + (k^{\rho}+1)^{1/\rho}.\end{aligned}$$

Thus $\frac{\partial A}{\partial \theta_k} > (<)\ 0$ if:

$$k^2 > (<)\ (k^{\rho}+1)^{1/\rho}\left[\frac{k^{\rho}(k+1)}{k^{\rho}+1} - 1\right].$$

There are two cases. First, suppose $k \leq 1$. Then $\frac{\partial A}{\partial \theta_k} > 0$ if $k^{\rho}(k+1) \leq k^{\rho}+1$, or $k^{\rho+1} \leq 1$, which holds for any $\rho > 0$. Second, suppose $k > 1$. Since $\lim_{\rho\to\infty}(k^{\rho}+1)^{1/\rho} = k$, we have $\lim_{\rho\to\infty} \frac{\partial A}{\partial \theta_k} < 0$

if $\lim_{\rho\to\infty} \frac{k^\rho(k+1)}{k^\rho+1} > k+1$. Applying L'Hopital's Rule yields $\lim_{\rho\to\infty} \frac{k^\rho(k+1)}{k^\rho+1} = \frac{(k+1)k^\rho \ln k + k^\rho}{k^\rho \ln k} = k+1+\frac{1}{\ln k}$, thus proving the result.

Proof of Comment 2. We first give the condition under which A1 prefers 1 partner to n partners. This is the case if:

$$\begin{aligned} \frac{m_1}{n} + \beta &\geq m_1 n^{1/\rho - 1} + \frac{\beta}{n} \\ \frac{\beta}{m_1} &\geq \frac{n^{1/\rho} - 1}{n-1}. \end{aligned} \tag{10}$$

Next we derive conditions under which A1's objective is either strictly increasing or decreasing. Differentiating the objective $L(t)$ (see (3)) yields $L'(t) = m_1 t^{1/\rho-1}/(n\rho) - \beta t^{-2}$. This implies that $L'(1) = m_1/(n\rho) - \beta$, which is positive if $\beta/m_1 < 1/(n\rho)$. Note also that $L'(t) > 0$ for all $t \in [1, n]$ if $m_1 t^{1/\rho+1}/(n\rho) - \beta > 0$. Since this expression is increasing in t, it follows that $L'(t) > 0$ for all $t \in [1, n]$ iff $L'(1) > 0$. Likewise, $L'(n) < 0$ if $\beta/m > n^{1/\rho}/\rho$, and $L'(t) < 0$ for all $t \in [1, n]$ iff $L'(n) < 0$.

Thus, there can only be a corner solution (10) if $\beta/m \notin [1/(n\rho), n^{1/\rho}/\rho]$. Moreover, for β/m_1 in this interval, $L'(1) < 0$ and $L'(n) > 0$. By the continuity of $L(\cdot)$, this implies that there cannot be a local interior maximum if there is a unique t for which $L'(t) = 0$. Manipulating $L'(t)$, we have $L'(t) = 0$ if:

$$t^{1/\rho+1} = \frac{\beta\rho n}{m_1},$$

which implies a unique t. Hence the maximum must be at a corner, which again implies that the solution is characterized by condition (10).

Proof of Proposition 2. We characterize θ_1^* by solving (where possible) for a type $\underline{\theta}$ that is indifferent between sharing and not sharing, given that all types $[\underline{\theta}, \Theta]$ share.

(i) We first show the result for $\beta \leq k - m_2\Theta$. Let $\psi(\underline{\theta})$ denote the interior value of $\tilde{\theta}_2$ from (5), holding $\overline{\theta} = \Theta$. Note that $\psi(x) = x$ only for $x = \Theta$ and $x = k/m_2$. Differentiating $\psi(\cdot)$, we obtain:

$$\frac{d\psi}{d\underline{\theta}} \leq 0 \quad \Leftrightarrow \quad (\beta - k)[m_2(\Theta - \underline{\theta}) + \beta] + m_2[k(\Theta - \underline{\theta}) + \beta\underline{\theta}] \leq 0.$$

Simplifying this expression yields $\beta \leq k - m_2\Theta$. Since $\psi(\Theta) = \Theta$, this condition implies $\psi(\underline{\theta}) \geq \Theta$ for all $\underline{\theta} \in [0, \Theta]$. Expressions (5) and (6) then imply that $\tilde{\theta}_2 = \Theta$ and $\phi(\underline{\theta}, \Theta) = 0$ for all $\underline{\theta} \in [0, \Theta]$, respectively. Finally, applying (8), A1 is indifferent between sharing and not sharing, and therefore shares when $\beta \leq k - m_2\Theta$.

Now consider the case where $\beta > k - m_2\Theta$. We begin by establishing some features of the sharing condition (9) when evaluated at some $\theta_1 = \underline{\theta}$. Define $\sigma(\underline{\theta}) = m_1(\tilde{\theta}_2 + \Theta)/2 - \xi(\underline{\theta})\beta$ as the left-hand side of (9), evaluated at $\theta_1 = \underline{\theta}$. First, it is easily verified that $\sigma(\underline{\theta})$ is continuous on $[0, \Theta]$. Second, we show that $\sigma(\underline{\theta})$ is strictly increasing on $[0, \Theta]$. Since $\beta > k - m_2\Theta$, $\psi(\underline{\theta})$ is strictly increasing and $\tilde{\theta}_2$ is weakly increasing. Thus $\sigma(\underline{\theta})$ is strictly increasing if $\tilde{\theta}_2$ is non-decreasing and $\xi(\underline{\theta})$ is non-increasing, with one relationship strict.

There are two subcases. First, suppose that $k/m_2 \leq \Theta$. Observe that for $\underline{\theta} \leq k/m_2$, $\tilde{\theta}_2 = \psi(\underline{\theta}) \geq \underline{\theta}$, and when $\underline{\theta} > k/m_2$, $\tilde{\theta}_2 = k/m_2 < \underline{\theta}$. Thus by (7), when $\underline{\theta} \leq k/m_2$, $\xi(\underline{\theta}) = 1$ (*i.e.*, A2 always wins conditional upon working) and $\sigma(\underline{\theta})$ is strictly increasing. When $\underline{\theta} > k/m_2$, $\xi(\underline{\theta}) = (\Theta - \underline{\theta})/(\Theta - k/m_2)$ is obviously strictly decreasing in $\underline{\theta}$, so $\sigma(\underline{\theta})$ is again strictly increasing. Second, suppose that $k/m_2 > \Theta$. Now for all $\underline{\theta}$, $\tilde{\theta}_2 = \psi(\underline{\theta}) \geq \underline{\theta}$; thus $\xi(\underline{\theta}) = 1$ and $\sigma(\underline{\theta})$ is stricly increasing.

Full sharing then occurs in equilibrium if and only if $\sigma(0) \geq 0$.[10] Note that $\xi(0) = 1$, so the condition for full sharing becomes:

$$\begin{aligned} \frac{m_1}{2}\left(\frac{k\Theta}{m_2\Theta + \beta} + \Theta\right) - \beta &\geq 0 \\ \frac{m_1}{2}[k\Theta + \Theta(m_2\Theta + \beta)] - \beta(m_2\Theta + \beta) &> 0 \qquad (11) \\ \frac{m_1}{2}(k\Theta + m_2\Theta^2) + \left(\frac{m_1}{2} - m_2\right)\Theta\beta - \beta^2 &\geq 0. \qquad (12) \end{aligned}$$

It is easily shown that the last expression has only one positive root, and so all types share if:

$$\beta \leq \frac{\Theta(m_1/2 - m_2) + \sqrt{\Theta^2(m_2 - m_1/2)^2 + 2m_1\Theta(k + m_2\Theta)}}{2}.$$

(ii) Since $\sigma(\underline{\theta})$ is increasing, A1 does not share if $\lim_{\underline{\theta}\to\Theta} \sigma(\underline{\theta}) < 0$. There are two cases. First, suppose that $k/m_2 \leq \Theta$. Note that $\tilde{\theta}_2 = k/m_2$ at $\tilde{\theta} = \Theta$ and $\lim_{\underline{\theta}\to\Theta} \xi(\underline{\theta}) = 0$, so the condition for no sharing becomes:

$$\frac{m_1}{2}\left(\frac{k}{m_2} + \Theta\right) < 0. \qquad (13)$$

This condition is obviously impossible to satisfy, and so $\lim_{\underline{\theta}\to\Theta} \sigma(\underline{\theta}) \geq 0$.

Second, suppose that $k/m_2 > \Theta$. Now $\tilde{\theta}_2 = \Theta$ at $\tilde{\theta} = \Theta$ and $\lim_{\underline{\theta}\to\Theta} \xi(\underline{\theta}) = 1$, so the condition for no sharing becomes:

$$\Theta m_1 < \beta. \qquad (14)$$

(iii) There are two cases. First, suppose that $k/m_2 \leq \Theta$. The analogous case in part (ii) implies that there exists some interior θ_1^* such that $\sigma(\underline{\theta}) = 0$.

[10]Full sharing also occurs if $\phi(0, \Theta) = 0$, but this occurs if and only if $\beta \leq k - m_2\Theta$, as detailed by the first part of the proof.

Since θ_1^* depends on $\tilde{\theta}_2$ and $\tilde{\theta}_2$ is at a corner at $\underline{\theta} = k/m_2$, we consider conditions under which θ_1^* will lie on either side of k/m_2. As $\sigma(\underline{\theta})$ is increasing, $\theta_1^* < k/m_2$ iff $\sigma(k/m_2) > 0$, or:

$$\frac{m_1}{2}\left(\frac{k}{m_2} + \Theta\right) > \beta. \tag{15}$$

If (15) holds, then since $\theta_1^* < k/m_2$ implies $\tilde{\theta}_2 < k/m_2$, θ_1^* is characterized by substituting $\psi(\underline{\theta})$ and $\xi(\underline{\theta}) = 1$ into (9):

$$\frac{m_1}{2}\left[\frac{k(\Theta - \underline{\theta}) + \beta\underline{\theta}}{m_2(\Theta - \underline{\theta}) + \beta} + \Theta\right] - \beta = 0. \tag{16}$$

The solution to (16) is then easily derived:

$$\theta_1^* = \frac{(m_1 - 2m_2)\beta\Theta + m_1(k + m_2\Theta)\Theta - 2\beta^2}{m_1(k - \beta) + m_2(m_1\Theta - 2\beta)} \tag{17}$$

If (15) does not hold, then since $\theta_1^* \geq k/m_2$ implies $\tilde{\theta}_2 = k/m_2$, θ_1^* is characterized by substituting $\tilde{\theta}_2$ and $\xi(\underline{\theta}) = (\Theta - \underline{\theta})/(\Theta - k/m_2)$ into (9):

$$\frac{m_1}{2}\left[\frac{k}{m_2} + \Theta\right] - \frac{\Theta - \underline{\theta}}{\Theta - k/m_2}\beta = 0. \tag{18}$$

The solution to (18) is then easily derived:

$$\theta_1^* = \Theta - \frac{m_1(\Theta^2 - (k/m_2)^2)}{2\beta}$$

Second, suppose that $k/m_2 > \Theta$. When $\Theta m_1 \geq \beta$, we have $\sigma(\Theta) \geq 0$ and $\tilde{\theta}_2 = \psi(\underline{\theta}) > \underline{\theta}$ for all $\underline{\theta} \in [0, \Theta)$. Thus, $\tilde{\theta}_2$ is interior for all values of $\underline{\theta}$ and θ_1^* is characterized by the the value of $\underline{\theta}$ such that $\sigma(\underline{\theta}) = 0$, as given by (17).

Observe finally that given θ_1^*, according to (8) no type higher (resp., lower) than θ_1^* can strictly benefit from not sharing (resp., sharing); thus each type of A1 will play the specified sharing strategy.

Summarizing all of the derivations yields the claimed result.

Proof of Comment 3 (i) The result follows straightforwardly from differentiating the upper bound in Proposition 2(i) that depends on m_1.

(ii) This follows straightforwardly from Proposition 2(ii).

(iii) To show that θ^* is decreasing m_1, observe that $\sigma(\underline{\theta})$, which is the left-hand side of the sharing condition (9) when evaluated at $\theta_1 = \underline{\theta}$, is increasing in m_1. Thus if $\sigma(\theta') = 0$ at some m_1', then $\sigma(\theta') > 0$ for $m_1'' > m_1'$. From the proof of Proposition 2(i), $\sigma(\underline{\theta})$ in increasing in $\underline{\theta}$. We conclude that θ' must be greater than the solution to (16) when $m_1 = m_1''$.

6 References

Alonso, Ricardo, Wouter Dessein, and Niko Matouschek. 2008. "When Does Coordination Require Centralization?" *American Economic Review* 98(1): 145-179.

Austen-Smith, David, and William Riker. 1987. "Asymmetric Information and the Coherence of Legislation." *American Political Science Review* 81(3): 897-918.

Baccara, Mariagiovanna, and Ronny Razin. 2007. "Bargaining on New Ideas: Rent Distribution and Stability of Innovative Firms." *Journal of European Economic Association* 5(6): 1095-1129.

Banks, Jeffrey S., and Joel Sobel. 1987. "Equilibrium Selection in Signaling Games." *Econometrica* 55(3): 647-661.

Chen, Kong-Pin. 2003. "Sabotage in Promotion Tournaments." *Journal of Law, Economics, and Organization* 19(1): 119-140.

Creane, Anthony. 1995. "Endogenous Learning, Learning by Doing and Information Sharing." *International Economic Review* 36(4): 985-1002.

Dessein, Wouter, Luis Garicano and Robert Gertner. 2010. "Organizing for Synergies." *American Economic Journal: Microeconomics*, forthcoming.

Downs, Anthony. 1966. *Inside Bureaucracy.* Boston: Little, Brown.

Gailmard, Sean, and John W. Patty. 2007. "Slackers and Zealots: Civil Service, Policy Discretion and Bureaucratic Expertise." *American Journal of Political Science* 51(4): 873-889.

Gal-Or, Esther. 1985. "Information Sharing in Oligopoly." *Econometrica* 53(2): 329-343.

Garicano, Luis, and Tano Santos. 2004. "Referrals." *American Economic Review* 94(3): 499-525.

Gates, Robert M. 1987. "The CIA and Foreign Policy." *Foreign Affairs* 66(2): 215-230.

Holmström, Bengt, and Paul Milgrom. 1990. "Regulating Trade among Agents." *Journal of Institutional and Theoretical Economics* 146(1): 85-105.

Institute of Medicine. 2001. *Crossing the Quality Chasm: A New Health System for the 21st Century.* Washington, DC: National Academy Press.

Itoh, Hideshi. 1991. "Incentives to Help in Multiagent Situations." *Econometrica* 59(3): 611-636.

King, David C. 1994. "The Nature of Congressional Committee Jurisdictions." *American Political Science Review* 88(1): 48-62.

King, David C. 1997. *Turf Wars: How Congressional Committees Claim Jurisdiction.* Chicago: University of Chicago Press.

Konrad, Kai A. 2000. "Sabotage in Rent-Seeking Contests." *Journal of Law, Economics, and Organization* 16(1): 155-165.

Lazear, Edward P. 1989. "Pay Equality and Industrial Politics." *Journal of Political Economy* 97(3): 561-580.

Lederman, Gordon. 1999. *Reorganizing the Joint Chiefs of Staff.* Westport, CT: Greenwood Press.

Lee, Thomas H., and James J. Mongan. 2009. *Chaos and Organization in Health Care.* Cambridge, MA: MIT Press.

Marx, Leslie M., and Francesco Squintani. 2009. "Individual Accountability in Teams." *Journal of Economic Behavior & Organization* 72(1): 260-273.

Modica, Salvatore. 2010. "Open Source without Free Riding." Unpublished manuscript, University of Palermo.

National Commission on Terrorist Attacks upon the United States. (Philip Zelikow, Executive Director; Bonnie D. Jenkins, Counsel; Ernest R. May, Senior Advisor). 2004. *The 9/11 Commission Report.* New York: W.W. Norton & Company.

Okuno-Fujiwara, Masahiro, Andrew Postlewaite, and Kotaru Suzumura. 1990. "Strategic Information Revelation." *Review of Economic Studies* 57(1): 25-47.

Posner, Richard. 2005. *Preventing Surprise Attacks: Intelligence Reform in the Wake of 9/11.* Stanford, CA: Rowman and Littlefield.

Raith, Michael. 1996. "A General Model of Information Sharing in Oligopoly." *Journal of Economic Theory* 71(1): 260-288.

Stein, Jeremy C. 2008. "Conversations among Competitors." *American Economic Review* 98(5): 2150-2162.

United States Medicare Payment Advisory Commission. 2008. *Report to Congress: Reforming the Delivery System.*

Wilson, James Q. 1978. *The Investigators.* New York: Basic Books.

Wilson, James Q. 2000. *Bureaucracy.* (2nd ed.) New York: Basic Books.