Shimer Meets the Production Based Asset Pricing Crowd: Insider-Outsider Labour Relations and Asset Returns*

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Abstract

Beginning with Shimer (2005) and Hall (2005), a recent branch of the business cycle literature has emphasized the role of wage rigidity in accounting for the statistical characteristics of key variables describing labour market activity over the business cycle; in particular, high vacancy and unemployment volatility and a high negative correlation between the two. In response we extend the Mortensen-Pissarides structure of period-by-period Nash wage bargaining to an environment where there is limited participation in the financial asset markets and labour force heterogeneity (insider-outsider labour relations). We show that a reasonable calibration of the resulting model accounts well not only for aggregate fluctuations in unemployment and vacancies but also for the observed wedge between variations at the intensive margin (hours per worker) and at the extensive margin (total hours) over the business cycle. The model also achieves a satisfactory resolution of the major financial asset puzzles; namely, a low risk-free rate, a high equity premium, and an upward sloping term structure. The key to these results is variation in distribution risk arising as an endogenous outcome of the wage bargaining.

Keywords: Nash bargaining; business cycles; equity premium puzzle; limited participation

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1 Introduction

A recent body of research (e.g. Hall (2005) and Shimer (2005)) argues that the conventional search model of employment dynamics due to Mortensen and Pissarides (MP henceforth) cannot account for the key cyclical movements in labour market activity. In particular, the high cyclical volatility of vacancies and unemployment as well as their highly negative correlation at business cycle frequencies are difficult to match within the standard business cycle context. The consensus on this problem is that the mechanism for wage determination within their framework, period-by-period Nash bargaining between firms and workers, induces too much volatility in wages. This excessively flexible movement in wages, in turn, dampens the cyclical movement in firms' incentive to hire and keeps vacancy volatility low. Other authors suggest that the introduction of wage rigidity will allow this framework to account for observed employment volatility. Indeed, the introduction of exogenously imposed wage stickiness makes it possible for the framework to be consistent with observed employment volatility[1].

In this paper, we address this issue from a different perspective: while we retain the contract structure of period-by-period Nash wage bargaining, we extend the MP model to an environment where the asset market is incomplete and perfect risk-sharing between capital owners and workers cannot necessarily guaranteed. In particular, we develop Nash wage bargaining between capitalists and workers in a macro model with two key features, namely, limited participation in the stock market and labour-force heterogeneity (insider-outsider labour relation).

The real business cycle model with single productivity shock and a capital-accumulation technology with adjustment costs is the foundation on which we build. There are two types of agents: insider-stockholders (capitalists) and outsider-nonstockholders (workers). Insider-stockholders have full access to financial markets, including the stock market and the bond market. In contrast, outsider-nonstockholders, who comprises the majority of households, do not participate in the stock market. They smooth their consumption intertemporally by trading in the risk-free bond market. Risk-free bonds are thus available to all households. The assumption of limited asset market participation is empirically appropriate. In fact, it is well documented that more than two-thirds of US households held no stock before the 1990s and that households in the top 20% owned more than 98% of stocks even during the 1990s when the participation rates had increased substantially (Mankiw and Zeldes (1991) and Poterba (2000)). In contrast to existing models with limited asset market participation, our model also allows for segmented labour markets due to labour-force heterogeneity. Insider-stockholders, as the name suggests, trade their labour services exclusively in the segmented labour market for insider-stockholders. The labour market for insider-stockholders is characterized by employment adjusting along the intensive margin. In other words, the labour income risk of the insider-stockholder entirely originates from fluctuations in hours worked, not in total employment: their jobs are secure, and they never leave the firm's

[1] For greater details, see Section 4.2.

employment. In contrast, the outsider's labour market is characterized by employment variation at the extensive margin as well as the variation of hours worked in the intensive margin. Another feature of this outsider's labour market arrangement is that firms representing insider-stockholders and outsider-nonstockholders Nash-bargain over wages within the MP framework. The assumption of labour-force heterogeneity also has empirical support; Kydland (1984) reports that the average standard deviation of annual hours rises substantially in going from the highly educated to the less well educated, while the average number of hours worked declines. In particular, a notable feature is that variations at the extensive margin are one of main contributors to these differences between the top education group (16.3%) and the bottom education group (32.4%).

What emerges in this environment is that the Nash wage bargaining between capital owners and workers makes it possible for capital owners to provide workers with partial income insurance against their labour income variations. This is manifest as the countercyclical variations in the bargaining power of workers, and will be called *distribution risk*. This countercyclical distribution risk dampens the variations of the Nash bargaining wage for workers (outsider-nonstockholders), who comprise the majority of the workforce, and consequently the variations of hours per worker are severely moderated. More specifically, a high-productivity realization coincides with the situations where distribution risk falls and (total) wage bills rise less than output in the short run. Conversely, a low-productivity realization coincides with the situations where distribution risk rises and wage bills fall less than output in the short run. The sluggish response of the wage income to output over the business cycle, called *the operating leverage effect or the fixed wage income effect*, creates the variable labour income share observed in the data. Capital owners (insider-stockholders) are prevented from smoothing their consumption due to frictional reallocations of capital stock and labour hours: namely, *a priori* specified costs of adjusting capital and low-amplitude labour hours. Capital owners then try to smooth their consumption by employment of outsider-nonstockholders; in other words, capital owners try to do so via more variations at the extensive margin. As a result, high-productivity shock realizations increase job vacancies and employment dramatically and vice versa. This sequence of events resolves the unemployment volatility puzzle raised by Shimer (2005). Indeed, our distribution risk is exactly the same as the Nash bargaining power shock Shimer cites without invoking the sources of it (Shimer, 2005). Hence, our model can be viewed as an attempt to provide a microfoundation for the Shimer's *ad hoc* Nash bargaining power shock; in other words, this will be a direct answer to Shimer's unanswered question: as stated with clarity in Shimer (2005), "*It seems plausible that a model with a combination of wages and labor productivity shocks could generate the observed behavior of unemployment, vacancies, and real wages... the answered question is what exactly a wage shock is.*"

In addition, the very same operating leverage effect plays a key role in generating a sizable equity premium: wage bills vary less than output, falling proportionately less in recessions and increasing less during expansions, and as a result, the risk of the firm's free cash flow and derived dividends tends to increase substantially. Shifting the risk of the wage income onto the residual payment to capitalists has the consequence of making capital fundamentally riskier. Hence, stocks are less attractive to investors and the equity premium rises.

The main contribution of this paper is to propose a new economic mechanism that overcomes the unemployment volatility puzzle emphasized by Shimer and Hall; moreover, the very same mechanism achieves a satisfactory resolution of the major financial asset puzzles. We show that the (short-run) operating leverage effect is a key factor in reproducing the pronounced fluctuations of unemployment over the business cycle. Specifically, we take into account Nash wage bargaining in the environment where the asset market is incomplete, i.e., there is limited participation in the financial asset markets and labour force heterogeneity (insider-outsider labour relations). What emerges from this consideration is that the fully endogenized Nash bargaining power shock, called distribution risk, plays the key role in generating the operating leverage effect in our context. We then show that a reasonable calibration of the resulting model, which takes into account the stylized financial statistics, accounts well not only for aggregate fluctuations in unemployment and vacancies but also for the observed wedge between variations at the intensive margin (hours per worker) and at the extensive margin (total hours) over the business cycle. In turn, the model also achieves a satisfactory replication of the major financial stylized facts: a low risk-free rate, an upward-sloping real term structure, and a substantial equity premium. In contrast to existing macro-asset pricing models, the model is unique in the sense that without compromising the overall performance on the financial front, it can reproduce the full range of labour market stylized facts.

The rest of the paper is structured as follows. Section 2 presents the basic ingredients of the model. In Section 3, we present the basis for its calibration and numerical solution. Section 4 examines the quantitative performance of our model and shows that the model can account well for the basic features of the US data, including employment dynamics and financial statistics. In Section 4, we also discuss how the operating leverage effects work in our model framework and address some robustness checks of our model. Section 5 concludes the paper. The appendix collects the log-normal formulae for the calculations of asset prices and a model with full participation in financial markets, which will be compared with the baseline model.

2 The Model

Consider a discrete-time infinite horizon economy with two distinct infinitely lived agents, "insider-stockholders" and "outsider-nonstockholders." We postulate a continuum of "insider-stockholders" distributed on a measurable set of Lebesgue measure μ_s and a continuum of "outsider-nonstockholders" is indexed on a measurable set of Lebesgue measure 1.

2.1 Insider-stockholder

Following Guvenen (2003, 2005) the insider-stockholder, endowed with one unit of time, supplies labour services to the (representative) firm and trades securities– both equity claims to the firm's net income stream, and a one-period risk-free real bond. What distinguishes our model from the Guvenen model, however, is that the insider-stockholder trades his labour services exclusively in the segmented labour market for insider-stockholders. This market is characterized by employment adjusting along the intensive margin only; i.e., the labour income risk of the insider-stockholder entirely originates from fluctuations in hours worked, not in total employment. This environment implies that the firm and workers have an ongoing relationship and that wages are not allocational, as Barro (1997) points out. The environment also can be viewed as Lucas' (1978) span of control or Rosen's (1982) hierarchy, where workers are assigned to managerial, production, and non-market tasks based on their comparative advantage.

Given his information set Ω_t^s, the representative insider-stockholder s maximizes his lifetime expected utility as given by:

$$V^s(\Omega_0^s) = \max_{\{h_t^s, c_t^s, e_{t+1}^s, b_{t+1}^s\}} E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t^s - X_t, h_t^s)] \tag{1}$$

s.t.

$$c_t^s + p_t^e e_{t+1}^s + p_t^f b_{t+1}^s \leq w_t^s h_t^s + (p_t^e + d_t) e_t^s + p_t^f b_t^s \tag{2}$$

In problem (1), u denotes his period utility function, c_t^s his period t consumption, and h_t^s his period t labour hours supplied, respectively. X_t is the exogenous habit stock, evolving according to

$$X_t = \xi X_{t-1} + (1-\xi)\chi \bar{c}_{t-1}^s$$

where χ is the habit parameter of the insider-stockholder group and $\xi = 0$. Here, $\bar{c}_{t-1}^s$ denotes the average consumption level of the whole insider-stockholder group in the previous period:

$$\bar{c}_{t-1}^s \equiv \frac{1}{\mu_s} \int c_{t-1}^s d\varkappa$$

where $\varkappa$ stands for the measure of insider-stockholders. d_t denotes the period t dividend payment by the firm to its stockholders; e_t^s and b_t^s, respectively, his period t stock and bond holdings; and the corresponding period t prices of these securities are p_t^e and p_t^f. w_t^s is the insider-stockholder's wage rate, exogenous from his perspective. Accordingly, $E_t^s \equiv E(\cdot \mid \Omega_t^s)$ denotes the expectation operator conditional on his information set Ω_t^s, and β is the economy-wide subjective discount factor.

We adopt a variation of GHH preference for the insider-stockholder:

$$u(c_t^s - X_t, h_t^s) = u(c_t^s - X_t - H(h_t^s))$$

where $H(\cdot)$ is his disutility of labour hours. This specification of the period utility function combines the standard GHH preference with a special form of external habit formation or "catching up with the Joneses" (see Abel (1990)). Neglecting the lagged average consumption level of the whole insider-stockholder group ($\chi = 0$), the preference function specified above is reduced to the standard GHH preference widely employed in the investment-shock literature (Greenwood, Hercowitz, and Huffman (1988)). It is known that the class of GHH preferences has an extremely weak short-run wealth effect on the labour supply. In fact, the Hicksian wealth effect of the real wage increase on hours worked is zero for this class of preferences.[2] Knowledge of this fact helps to define the representative insider-stockholder correctly; otherwise, the representative insider-stockholder will decrease his labour supply in response to a positive productivity shock because of the short-run wealth effect.

Moreover, this class of GHH preferences implies that the marginal rate of substitution between consumption and labour supply depends only on the labour supply. That is, the labour supply is determined independently of intertemporal consumption-savings choice and thus the effect of intertemporal consumption substitution on the labour supply is completely eliminated. Indeed, the marginal rate of substitution between consumption and labour supply in this model economy reads as:

$$-\frac{u_h(c_t^s - X_t, h_t^s)}{u_c(c_t^s - X_t, h_t^s)} = H'(h_t^s). \tag{3}$$

Conditional upon his information set Ω_t^s, the recursive formulation of the insider-stockholder's problem is represented as:

$$V^s(\Omega_t^s) = \max_{\{c_t^s, h_t^s, e_{t+1}^s, b_{t+1}^s\}} \left[\begin{array}{c} u(c_t^s - X_t, h_t^s) \\ +\lambda_t^s [w_t^s h_t^s + (p_t^e + d_t) e_t^s + p_t^f b_t^s - c_t^s - p_t^e e_{t+1}^s - p_t^f b_{t+1}^s] \\ +\beta E(V^s(\Omega_{t+1}^s) \mid \Omega_t^s) \end{array} \right] \tag{4}$$

where λ_t^s is the Lagrange multiplier associated with the insider-stockholder's budget constraint (2).

The solution to the above recursive problem (4) is characterized by

$$w_t^s = H'(h_t^s) \tag{5}$$

$$p_t^e = \beta E_t[\Lambda_{t,t+1}^s (p_{t+1}^e + d_{t+1})] \tag{6}$$

[2] For more detail, see Jaimorvich and Rebelo (2006).

$$p_t^f = \beta E(\Lambda_{t,t+1}^s \mid \Omega_t^s)] \tag{7}$$

where $\Lambda_{t,t+1}^s$ is the insider-stockholder's IMRS.

2.2 Outsider-nonstockholder

We postulate that a continuum of infinitely-lived outsider-nonstockholders, uniformly distributed on a set of Lebesgue measure 1, supply labour services via a Nash bargaining wage contract in their segmented labour market (specified later). These agents differ from insider-stockholders in their investment opportunity sets, job opportunity sets and consumption-smoothing motives. First, the outsider-nonstockholder group is restricted from participating in the equity market, although they can freely trade one-period risk-free bonds. This limited participation creates an asymmetry in consumption-smoothing opportunities; outsider-nonstockholders have to rely exclusively on the bond market, whereas insider-stockholders have the additional tool of adjusting their physical capital holdings in response to shocks. Second, we adopt heterogeneity in the preference specification (Hornstein and Uhlig (1999)) for the baseline model: capital owners (insider-stockholders) are subject to a "habit formation" feature while workers (outsider-nonstockholders) are not. As Hornstein and Uhlig (2000) suggests, this can be viewed as modelling the result of self-selection: agents who easily get used to a high consumption level, i.e. have habit formation preferences, may be more likely to build up a large capital stock over time than agents who do not. In Section 4, we show that the habit formation feature of capitalists has a negligible effect on the relatively volatile behavior of labour market activity over the business cycles. In other words, our pronounced operating leverage effect is independent of the habit formation of capitalists. However, the habit formation still plays an important role in replicating the stylized financial statistics. In particular, the incorporation of the habit formation makes the aggregate EIS implied from the model consistent with Hall's empirical findings: Hall (1988) estimates the aggregate EIS close to zero. Indeed, the aggregate EIS in our model economy is 0.0307. This low EIS seems to be consistent with an upward (real) term structure. The same intuition is found in Binsbergen et al. (2008); in their estimated DSGE model with fully specified Epstein-Zin preferences, they find that a low elasticity of intertemporal substitution (around 0.06) is estimated from upward-sloping (nominal) yield curve data and macro data. We discuss the implied EIS in the separate section. The third distinction is that outsider-nonstockholders trade their labour services exclusively in the segmented labour market for outsider-stockholders. Unlike the insider's labour market, the outsider's labour market is characterized by the variation of employment at the extensive margin as well as the variation of hours worked in the intensive margin. Another feature of this labour market arrangement is that firms and outsider-nonstockholders bargain over wages with search and matching friction. Since the model allows for heterogeneous agents, this bargaining wage contracting is modified to suit the environment where the workers (outsider-nonstockholder) bargain over wages with the capital owners (stockholders). The resulting Nash bargaining wage is a hybrid of the standard Nash bargaining wage in the representative agent model and the risk-sharing contract proposed by Danthine and Donaldson (2002). The modified bargaining wage nevertheless is offered on a period-by-period basis. This additional labour income risk due to the variation at the extensive margin and the contractual nature of this bargaining wage makes it difficult for stockholders, who have a strong consumption-smoothing motive, to smooth their consumption.

Following Merz (1995), each outsider-nonstockholder is viewed as a large extended family which contains a continuum of family members uniformly distributed on a set of Lebesgue measure 1. Each family consists of outsiders, employed and unemloyed, who pool their financial and labour incomes before choosing per-capita consumption and (risk-free) asset holdings. Accordingly, given its information set Ω_0^n, the representative outsider-bondholder solves[3] :

$$V^n(\Omega_0^n) = \max_{\{h_t^n, c_t^n, b_{t+1}^n\}} E_0 \sum_{t=0}^{\infty} \beta^t [v(c_t^n - n_t L(h_t^n))] \quad (8)$$

s.t.

$$c_t^n + p_t^f b_{t+1}^n \leq w_t^n h_t^n n_t + b(1-n_t) + b_t^n + T_t. \quad (9)$$

In the above problem, $v(\cdot)$ denotes his period utility function, $L(\cdot)$ is his disutility of labour function, and h_t^n is his period t labour hours supplied; c_t^n and b_t^n are his period t consumption and his period t bond holdings, respectively; w_t^n is the outsider-nonstockholder's wage determined through the contracting process in the labour market for outsider-nonstockholders; b represents unemployment benefits and T_t is lump sum transfers from the government. Similarly, $E_t \equiv E(\ \cdot \mid \Omega_t^n)$ is the expectation operator conditional on his information set Ω_t^n.

Again, we adopt a special form of GHH preference for the representative outsider-nonstockholder's period utility. Conditional upon his information set Ω_t^n, the recursive formulation of the insider-stockholder's problem is represented as:

$$V^n(\Omega_t^n) = \max_{\{c_t^n, b_{t+1}^n, h_t^n\}} \left[\begin{array}{c} v(c_t^n - n_t L(h_t^n)) \\ +\lambda_t^n(b_t^n + w_t^n h_t^n n_t + b(1-n_t) - p_t^f b_{t+1}^n - c_t^n) \\ +\beta E(V^n(\Omega_{t+1}^n) \mid \Omega_t^n)] \end{array} \right] \quad (10)$$

where λ_t^n is the Lagrange multiplier associated with the outsider-nonstockholder's budget constraint (9).

[3] More "structual" form of the contemporaneous utility is to introduce search effort per worker seeking employment:

$$v(c_t^n - n_t L(h_t^n) - (1-n_t)L(e))$$

where e is search effort. However, empirical studies show that search effort is negligible. Therefore, without loss of generality, we assume that $L(0) = 0$.

The solution to the above recursive problem (10) is characterized by

$$v_c(c_t^n - n_t L(h_t^n)) = \lambda_t^n \tag{11}$$

$$w_t^n = L'(h_t^n) \tag{12}$$

$$p_t^f = \beta E(\frac{v_c(c_{t+1}^n - n_{t+1} L(h_{t+1}^n)))}{v_c(c_t^n - n_t L(h_t^n))} \mid \Omega_t^n)]. \tag{13}$$

Note that outsider-nonstockholders' hours are supplied under the condition that the (hourly) wage equals the marginal rate of substitution of consumption for leisure.

2.3 Search in the labour market for outsiders (outsider-nonstockholders)

There is one infinitely lived representative firm that behaves competitively.[4] The firm hires n_t outsider-bondholders from the outsider's labour market in period t. The firm also posts ν_t vacancies in order to attract new outsiders for its period $t+1$ production. The total number of unemployed outsiders who search for a job in period t, u_t, is given by:

$$u_t \equiv 1 - n_t.$$

Based on the Mortensen and Pissarides search theory, we assume that the following matching technology exists in the labour market for outsiders:

$$M(\nu_t, 1 - n_t) = \sigma_m \nu_t^{\sigma} (1 - n_t)^{1-\sigma}$$

where σ_m is a parameter, and $m_t \equiv M(\nu_t, 1 - n_t)$ represents "matches," the number of new hired outsiders.

The probability that the firm fills a vacancy in period t, q_t, is given by

$$q_t = \frac{M(\nu_t, 1 - n_t)}{v_t} = \frac{m_t}{v_t}.$$

The probability that a searching outsider finds a job, s_t, is given by

$$s_t = \frac{M(\nu_t, 1 - n_t)}{1 - n_t} = \frac{m_t}{u_t}.$$

[4] Equivalently, it can be assumed that there is a continuum of infinitely lived identical competitive firms distributed on the unit interval $[0, 1]$.

q_t and s_t are exogenous from the perspectives of the firm and outsiders, respectively.

Employment relationships between the firm and outsiders may be severed for exogenous reasons in each period t: this is represented as the the probability of separation ρ. The specification that the job separation rate is constant and the job finding probability should be endogenous is consistent with Hall (2005) and Shimer (2005); they report that while the job finding probability is indeed cyclical, the separation rate is substantially less so. Outsiders losing a job in period t (ρn_t outsiders) are not allowed to search until the next period. Therefore,

$$u_{t+1} = \rho n_t + (1 - n_t) - m_t = 1 - n_{t+1}.$$

2.4 Firm

Each period, the firm produces output y_t according to the following aggregate production function:

$$y_t = f(k_t, h_t^s, h_t^n n_t) z_t$$

where z_t, k_t, h_t^s, and $h_t^n n_t$ denote, respectively, aggregate productivity shock, capital stock in period t, labour (hours) supplied by the insider-stockholder, and the total labour hours supplied by the outsider-bondholder. n_t represents the number of outsiders employed from the matching labour market for outsiders at the end of period $t-1$ and h_t^n is outsiders' labour hours demanded in the search labour market. n_t evolves according to the following law of motion:

$$n_{t+1} = (1 - \rho) n_t + q_t \nu_t.$$

Each period, ρn_t separates exogenously from the firm's employment pools, n_t and is augmented by posting vacancies ν_t and hiring new outsiders $q_t \nu_t = m_t$. The firm owns the (physical) capital stock, k_t. Each period the capital stock depreciates at the rate of δ and is augmented by new investment i_t.

Two costs of adjusting the firm's capital stock and the employment size of outsider-nonstockholders are next introduced. Merz and Yashiv (2007) report that the simultaneous introduction of these two adjustment costs empirically affects the market value of the firm; ignoring either cost does not match with their empirical evidence.

Capital adjustment costs have a long tradition in the investment theory literature. Such costs form a wedge between the shadow price of capital installed within the firm and the price of an additional unit of capital. We replace the standard capital-accumulation technology with the specification employed in Jermann (1998):

$$k_{t+1} = (1 - \delta) k_t + G(\frac{i_t}{k_t}) k_t$$

where the adjustment cost function $G(\cdot)$ is given by

$$G(\frac{i_t}{k_t}) = \frac{a_1}{1-\frac{1}{\xi}}(\frac{i_t}{k_t})^{1-\frac{1}{\xi}} + a_2$$

and a_1 and a_2 are chosen so that $G(\delta) = \delta$, and $G'(\delta) = 1$. With these identifications, the elasticity parameter $\xi \equiv -\frac{1}{G''(\delta)\delta} > 0$ is independent of the determination of the model's steady-state equilibrium, i.e. the steady state is not affected by the positive value ξ; $\xi = \infty$ corresponds to the benchmark case of no adjustment costs. This specification enables Tobin's q to vary by differentiating between the (shadow) prices of the installed capital and the new investment good prices.

Second, we introduce a cost of adjusting employment. These costs influence the rate at which firms add new workers to their existing labour forces. We replace the standard assumption of fixed costs of posting a vacancy with quadratic labour adjustment costs, as in Gertler and Trigari (2006). Defining the hiring rate x_t as the ratio of new hires $q_t\nu_t$ to the existing workforce of outsider-nonstockholders, the quadratic adjustment costs of the employment size of outsider-nonstockholders is given by

$$\frac{\kappa}{2}x_t^2 n_t$$

where $x_t \equiv \frac{q_t\nu_t}{n_t} = \frac{\text{new hires}}{\text{existing workforce}} \equiv$ hiring rate and κ is a constant vacancy cost.

The (financial) capital structure of the representative firm consists of one perfectly divisible equity share and one-period risk-free bonds: the firm is not only equity-financed but also financed by the issuance of corporate bonds at price p_t^f. The total supply of corporate bonds is constant over time and equals a fraction φ of the average capital stock owned by the firm as in Danthine and Donaldson (2002). In each period, the firm makes net interest payments $(\varphi\bar{k} - p_t^f\varphi\bar{k})$ to bondholders. As Danthine and Donaldson (2002) demonstrates, the celebrated Modigliani-Miller theorem still holds true in our framework; that is, the existence of leverage has no effect on real allocations[5].

The firm's decision problem is to maximize its pre-dividend stock market value $d_t + p_t^e$ on a period-by-period basis given its information set Ω_t^f:

$$\max_{\{i_t, h_t^s, x_t\}} d_t + p_t^e \equiv d_t + E(\beta\Lambda_{t,t+1}^s(p_{t+1}^e + d_{t+1}) \mid \Omega_t^f) \tag{14}$$

[5] We can verify this property by solving the model without leverage. The fundamental reason behind this neutral Modigliani-Miller outcome is that the firm's crucial intertemporal decisons are all in accord with the intertemporal marginal rate of substitution of capitalists; the agency problem between firm owners and managers is negligible in this enviroment, i.e. there is no corporate governance problem. It turns out that the absence of corporate governance problem is important to derive Nash wage bargaining between capitalists and workers.

$$
\begin{aligned}
\text{s.t. } d_t &\equiv f(k_t, h_t^s, h_t^n n_t)z_t - i_t - w_t^s h_t^s - w_t^n h_t^n n_t - \frac{\kappa}{2}x_t^2 n_t - \varphi\bar{k} + p_t^f \varphi\bar{k} \\
k_{t+1} &= (1-\delta)k_t + G(\frac{i_t}{k_t})k_t \\
n_{t+1} &= (1-\rho)n_t + q_t \nu_t
\end{aligned}
$$

In the above problem, $\Lambda^s_{t,t+1}$ is the marginal rate of substitution of the insider-stockholders, w_t^s is their competitive wage and w_t^n is the Nash bargaining wage for outsider-nonstockholders (specified later).

Letting $V^f(\Omega_t^f) \equiv d_t + p_t^s$, the recursive representation of the firm's problem is written as:

$$
V^f(\Omega_t^f) = d_t + \beta E(\Lambda^s_{t,t+1} V^f(\Omega^f_{t+1}) \mid \Omega_t^f).
$$

The necessary and sufficient first-order condition for the firm's optimal investment decision is given by:

$$
i_t : \quad (-1) + \beta E(\Lambda^s_{t,t+1} V^f_{k_{t+1}} \mid \Omega_t^f)\frac{\partial k_{t+1}}{\partial i_t} = 0.
$$

By the envelope theorem,

$$
k_t : \frac{\partial V^f(\Omega_t^f)}{\partial k_t} = f_1(k_t, h_t^s, h_t^n n_t)z_t + \beta E(\Lambda^s_{t,t+1} V^f_{k_{t+1}} \mid \Omega_t^f)\frac{\partial k_{t+1}}{\partial k_t} = 0.
$$

The Euler equation is represented as:

$$
1 = \beta E(\Lambda^s_{t,t+1} G'(\frac{i_t}{k_t})[f_1(k_{t+1}, h^s_{t+1}, h^n_{t+1} n_{t+1})z_{t+1} + \frac{(1-\delta) + G(\frac{i_{t+1}}{k_{t+1}})}{G'(\frac{i_{t+1}}{k_{t+1}})} - \frac{i_{t+1}}{k_{t+1}}] \mid \Omega_t^f). \tag{15}
$$

The first-order condition for the firm 's optimal hiring decision of insiders is given by

$$
h_t^s : w_t^s = f_2(k_t, h_t^s, h_t^n n_t)z_t, \tag{16}
$$

while the first-order condition for the firm's optimal hiring decision of outsiders is given by

$$
x_t : \kappa x_t = \beta E_t \Lambda^s_{t,t+1} J_{t+1} \tag{17}
$$

where $J_t \equiv \frac{\partial V^f(\Omega_t)}{\partial n_t}$ is the firm's shadow value of one added outsider.

2.5 Nash bargaining

In this section, we introduce Nash wage bargaining between the firm and the outsider-nonstockholders. In this environment, there exists a wedge between capital owners' intertemporal marginal rate of substitution (IMRS) and workers' IMRS: the firm is the representative of capital owners (insider-nonstockholders), not workers. Nevertheless, we show that the Nash wage bargaining solution can be constructed in a tractable way. In other words, the firm's matching surplus and the outsider-nonstockholder's employment value and unemployment value can be defined in terms of current consumption so as to make them consistent with the firm's shadow value of one added worker and the outsider-nonstockholder's value of one employed worker, respectively. What emerges from this representation of Nash bargaining problem in terms of current consumption is a tractable form of Nash bargaining wage to nest as the special case the standard Nash bargaining wage in the representative agent model.

Firm's shadow value Presuming that the firm's decision variables are chosen optimally, the firm's pre-dividend stock market value $V^f(\Omega_t^f) \equiv V_t^f \equiv d_t + p_t^e$ can be represented recursively as follows:

$$\begin{aligned} V_t^f &= d_t + p_t^e \\ &= d_t + \beta E(\Lambda_{t,t+1}^s(p_{t+1}^e + d_{t+1}) \mid \Omega_t^f) \\ &= d_t + \beta E(\Lambda_{t,t+1}^s V_{t+1}^f \mid \Omega_t^f) \end{aligned}$$

Let $J_t \equiv \frac{\partial F_t}{\partial n_t}$ be the per-capita value to the firm of hiring one outsider in period t:

$$J_t = MPL_{nt} - w_t^n h_t^n + \frac{\kappa}{2} x_t^2 + (1-\rho)\beta E_t \Lambda_{t,t+1}^s J_{t+1}$$

where $MPL_{nt} \equiv \frac{\partial y_t}{\partial n_t} = h_t^n f_3(k_t, h_t^s, h_t^n n_t) z_t$ is defined as "extensive marginal product of outsiders' labour."[6]

The first-order condition for vacancy posting equates the marginal cost of adding an outsider with discounted marginal benefit:

$$\kappa x_t = \beta E_t \Lambda_{t,t+1}^s J_{t+1}. \tag{18}$$

Note that condition (18) is identical to the firm's optimal hiring decision of outsiders (17).

Using the definition of J_t, we have the following equivalent optimality condition:

$$\kappa x_t = \beta E_t \Lambda_{t,t+1} [h_{t+1}^n f_3(k_{t+1}, h_{t+1}^s, h_{t+1}^n n_{t+1}) z_{t+1} - w_{t+1}^n h_{t+1}^n + \frac{\kappa}{2} x_{t+1}^2 + (1-\rho)\kappa x_{t+1}].$$

Distribution risk Before defining the employment value and unemployment value to the outsider, we introduce the ratio between the insider-stockholder's marginal utility and the outsider-nonstockholder's

[6] In the matching labour market for outsiders, we distinguish between the "extensive marginal product of outsiders' labour" and the "intensively marginal product of insiders' labour." Similarly, intensively marginal product of labour, $MPL_{h^n t}$, is defined as $\frac{\partial y_t}{\partial h_t^n} = n_t z_t f_3(k_t, h_t^s \cdot 1, h_t^n \cdot n_t)$.

marginal utility:

$$\phi_t \equiv \frac{u_c((c_t^s - \chi c_{t-1}^s - H(h_t^s))}{v_c(c_t^n - n_t L(h_t^n))} = \frac{\lambda_t^s}{\lambda_t^n}. \tag{19}$$

The ratio ϕ_t is viewed as the extent to which the risk-sharing between insider-stockholders (capital owners) and outsider-nonstockholders is efficient. Some remarks are in order. First, if ϕ_t is constant across time and in all states, the relation (19) coincides with the efficient risk-sharing condition. Second, suppose that ϕ_t is time-varying. A larger ϕ_t is evidence of a greater share of aggregate income to workers and a smaller ϕ_t suggests a greater share to capital owners (shareholders). In particular, suppose that ϕ_t is constant in all states but time-varying. Then the relation (19) is reduced to the optimality condition of the Boldrin-Horvath type *optimal* contract (1995)[7]. Lastly, suppose that ϕ_t is time-varying and countercyclical over the business cycle. This countercyclicality means that when a high-productivity state is realized, a smaller ϕ_t is realized and insider-stockholders (capital owners) reap most of the benefits from that productivity; in comparison, when a low-productivity state is realized, a greater share of aggregate income goes to outsider-nonstockholders, i.e. the normally low payment to capital owners is further reduced by labour's priority claim on output. Indeed, the countercyclicality of ϕ_t captures the idea that the shares of income going to labour and capital are not equally risky. Empirically, labour's share is much less risky than the share going to capital; labour's claim on output is largely fixed and negotiated prior to the actual realization of the output. In an earlier paper, Danthine and Donaldson (2002) posit that the observed variations in factor income shares are the result of exogenous changes in this ratio ϕ_t which they refer to as *distribution risk* (hereafter we call the ratio ϕ_t distribution risk). This risk is assumed to be uninsurable. They view ϕ_t as capturing the relative bargaining power of the two parties at the time the contract is negotiated. The assumed countercyclicality of this distribution risk guarantees that labour's share is much less risky than the share going to capital. In comparison, our endogenous distribution risk measure is very different. We make any *a priori* assumption neither about the cyclicality of distribution risk nor about the source of this risk; rather, distribution risk in this economy is generated in equilibrium; our economy features one source of uncertainty resulting from systemic risk (the economy-wide productivity shock). It turns out that our distribution risk is indeed *countercyclical* over the business cycle. Second, our Nash bargaining wage contract between capitalists (insider-stockholders) and labourers (outsider-nonstockholders) precisely identifies distribution risk ϕ_t with the balance of bargaining power between capitalists and labourers. As a result, we provide a structural specification of the source of distribution risk.

Outsider-nonstockholder's shadow value The present discounted value to an outsider of employment in terms of current consumption in period t, W_t is defined recursively as

[7] Here the *optimal* contract is not necessarily optimal in the Pareto sense.

$$W_t = w_t^n h_t^n + (1-\rho)\beta E_t \Lambda_{t,t+1}^n W_{t+1} + \rho\beta E_t \Lambda_{t,t+1}^n U_{t+1}$$

where $\Lambda_{t,t+1}^n \equiv \frac{\lambda_{t+1}^n}{\lambda_t^n}$ is the outsider-nonstockholder's IMRS.

We recursively define U_t as the present discounted value to an outsider of unemployment in terms of current consumption in period t:

$$U_t = L(h_t^n) + b + s_t\beta E_t \Lambda_{t,t+1}^n W_{t+1} + (1-s_t)\beta E_t \Lambda_{t,t+1}^n U_{t+1}.$$

Thus, the value of being unemployed depends upon the outsider's current disutility of supplying hours $L(h_t^n)$ (measured in units of final good consumption) , his unemployment benefits b, and the likelihood of his being employed plus being unemployed next period; an unemployed outsider has a chance of finding a new job, s_t.

The outsider-nonstockholder's matching shadow value in terms of final good consumption, S_t^n, is then defined as the difference between the employment value and the unemployment value:

$$\begin{aligned} S_t^n &\equiv W_t - U_t \\ &= (w_t^n h_t^n - L(h_t^n) - b) + (1-\rho-s_t)\beta E_t \Lambda_{t,t+1}^n S_{t+1}^n. \end{aligned} \tag{20}$$

Indeed, the value S_t^n can be derived from the marginal benefit of a outsider-nonstockholder family from having an additional family member in employment. The recursive representation of the outsider-nonstockholder's problem is:

$$\begin{aligned} V_t^n &\equiv V^n(\Omega_t^n) = \max_{\{b_{t+1}^n, h_t^n\}} \begin{bmatrix} v(c_t^n - n_t L(h_t^n) - (1-n_t)L(0)) \\ +\lambda_t^n (w_t^n h_t^n n_t + (1-n_t)b + b_t^n - p_t^f b_{t+1}^n - c_t^n) \\ +\beta E(V^n(\Omega_{t+1}^n) \mid \Omega_t^n) \end{bmatrix} \\ &\quad \text{s.t.} \\ n_{t+1} &= (1-\rho)n_t + s_t(1-n_t). \end{aligned} \tag{21}$$

The marginal benefit of one hired worker, $V_{nt}^n \equiv \frac{\partial V_t^n}{\partial n_t}$, can be obtained by applying the Envelope theorem to representation (21):

$$\frac{\partial V_t^n}{\partial n_t} = w_t^n h_t^n \lambda_t^n - (L(h_t^n) + b)\lambda_t^n + \beta E_t \frac{\partial V_{t+1}^n}{\partial n_{t+1}} \frac{\partial n_{t+1}}{\partial n_t}$$

where $\frac{\partial n_{t+1}}{\partial n_t} = (1-\rho-s_t)$.

Define the outsider-nonstockholder's shadow value to the firm of one hired worker, S_t^n, as

$$\begin{aligned} S_t^n &\equiv \frac{1}{\lambda_t^n}\frac{\partial V_t^n}{\partial n_t} \\ &= (w_t^n h_t^n - L(h_t^n) - b) + (1-\rho-s_t)\beta E_t \frac{\lambda_{t+1}^n}{\lambda_t^n} S_{t+1}^n. \end{aligned}$$

It follows immediately that the above shadow value, S_t^n, exactly coincides with the outsider-nonstockholder's matching shadow value (20).

Bargaining wage contract Before detailing the Nash bargaining wage contract between insider-stockholders and outsider-nonstockholders, several remarks are in order. Note that the firm's intertemporal decisions are all in accord with the intertemporal marginal rate of substitution of capitalists; the agency problem between firm owners and managers is non-existent in this environment. In an environment without corporate governance problems, firm's matching surplus can be identified with the marginal benefit to the representative shareholder of adding one worker (outsider-nonstockholder). In other words, the firm's matching surplus,V_{nt}^s, can be formulated as:

$$V_{nt}^s \equiv \frac{\partial V_t^s}{\partial n_t}$$

where $V_t^s \equiv V^s(\Omega_t^s)$ is the value function of insider-stockholders.

As shown in the previous section, outsider-worker's matching surplus, V_{nt}^n, can be readily identified with the marginal benefit of one hired worker:

$$V_{nt}^n \equiv \frac{\partial V_t^n}{\partial n_t}.$$

Identifying each matching surplus with its marginal benefit is appropriate in the situation where two heterogeneous agents with different attitudes toward risk bargain over wage. Indeed, the existing game theory literature holds that the division of a joint bargaining surplus can be significantly affected by heterogeneity in the agents' risk aversion coefficients[8]. Therefore, we argue that the matching surplus in this environment should be defined in terms of marginal benefits to capture the nontrivial effect of risk aversion on bargaining.

Based on previous reasoning, the Nash wage bargaining problem between capitalists and workers can be formulated in the following way:

$$\max_{w_t^n} (V_{nt}^s)^{1-\eta} \cdot (V_{nt}^n)^{\eta} \tag{22}$$

where η is the bargaining power parameter of the outsider-nonstockholder group viewed as exogenously

[8] For greater detail, see Roth and Rothblum (1982).

given.

The optimization above takes into account that in each period, outsider's hours worked is set according to the following condition:

$$MRS^n_{c,l} = w^n_t \tag{23}$$

where $MRS^n_{c,l}$ represents the outsider-nonstockholder's marginal rate of substitution for leisure.

The advantage of this condition (23) is that the determination of hours worked is independent of any definition of the joint matching surplus corresponding to the Nash wage bargaining. The more popular specification, namely, the efficient bargaining contract, requires that

$$h^n_t \in \arg\max M_t \equiv V^s_{nt} + V^n_{nt}$$

when the joint matching surplus, M_t, is defined as $V^s_{nt} + V^n_{nt}$. As Nash (1950) showed, however, the joint matching surplus M_t can be any (convex and compact) subset of the sum $V^s_{nt} + V^n_{nt}$, i.e. $M_t \subset V^s_{nt} + V^n_{nt}$. Hence we argue that the condition (23) does not lose much generality.

This mechanism for determining outsider's hours worked, proposed by Christiano et al (2007), can be viewed as capturing the idea that outsiders are monopolistic suppliers of labour hours as favored by New Keynesian models. Alternatively, the same specification can be viewed as describing the situation where a generic agency problem between the firm and workers cannot be ignored; the firm cannot completely induce workers' efforts (hours worked) since hours worked are in the nature of "hidden action." Indeed, condition (23) is strengthened by the observed fact that the hours worked per employee is rarely the object of bargaining agreements.

The wage w^n_t chosen by the bargaining problem (22) must satisfy the optimality condition[9]:

$$\eta V^s_{nt} = (1-\eta) V^n_{nt}. \tag{24}$$

Furthermore, the condition (24) can be rewritten as:

$$\eta \lambda^s_t J_t = (1-\eta)\lambda^n_t (W_t - U_t). \tag{25}$$

Condition (25) can be derived from the observation that $V^s_{nt} = \lambda^s_t J_t$ and $V^n_{nt} = \lambda^n_t (W_t - U_t)$. A standard calculation based on the condition (25) guarantees that the Nash bargaining wage between two heterogeneous groups is given by

$$w^n_t = \frac{(1-\eta)\frac{1}{\phi_t}}{(1-\eta)\frac{1}{\phi_t} + \eta} \frac{[L(h^n_t) + b - F^n_t]}{h^n_t} + \frac{\eta}{(1-\eta)\frac{1}{\phi_t} + \eta} \frac{[h^n_t f_3(k_t, h^s_t, h^n_t n_t) z_t + \frac{\kappa}{2} x^2_t + F^s_t]}{h^n_t} \tag{26}$$

[9] This condition is called the *constant surplus sharing rule*.

where $F_t^n \equiv \beta(1-\rho-s_t)E_t\frac{\lambda_{t+1}^n}{\lambda_t^n}(W_{t+1}-U_{t+1})$ and $F_t^s \equiv \beta(1-\rho)E_t\frac{\lambda_{t+1}^s}{\lambda_t^s}J_{t+1}$ denote, respectively, the future expected net present values from employment to the outsider and to the firm.

Letting $\eta_t \equiv \frac{\eta}{(1-\eta)\frac{1}{\phi_t}+\eta}$, the solution (26) can be rewritten as:

$$w_t^n = (1-\eta_t)\frac{[L(h_t^n)+b-F_t^n]}{h_t^n} + \eta_t\frac{[h_t^n f_3(k_t, h_t^s, h_t^n n_t)z_t + \frac{\kappa}{2}x_t^2 + F_t^s]}{h_t^n}. \tag{27}$$

From (27), η_t now plays the role of the Nash bargaining power. This wage nests the standard bargaining wage under the representative agent regime as the special case. In the case of the representative-agent construct, ϕ_t is equal to 1, so that the solution (27) is reduced to the standard Nash bargaining solution ($\eta_t = \eta$). Note also that the representative agent model always allows for the full access to the financial market, i.e. $\phi_t = 1$. This highlights the significant role of limited asset market participation in generating variable distribution risk ϕ_t.

More interestingly, it can be shown that up to a first-order approximation,

$$\hat{\eta}_t = (\text{constant}) \cdot \hat{\phi}_t.$$

In other words, the distribution risk can be identified with a Nash bargaining power shock up to a first-order approximation, and later it can be shown that this distribution risk is countercyclical over the business cycle. Indeed, the countercyclicality of our distribution risk will play the key role in generating the unemployment fluctuations over the business cycle: the countercyclicality of the distribution risk creates excessively smooth wages that induce the fixed wage income effect (the operating leverage effect), thus encouraging the observed volatility of the key indicators of interest in the labour market. So our distribution risk is exactly the same as the Nash bargaining power shock Shimer took into account without invoking its source (Shimer, 2005). In fact, we provide a microfoundation for the Shimer's *ad hoc* Nash bargaining power shock. Note that the only exogenous driving force in our economy is aggregate productivity shock and the very aggregate productivity shock induces the countercyclicality of our distribution risk. This will be a direct answer to Shimer's unanswered question, as stated with clarity in Shimer (2005): "*It seems plausible that a model with a combination of wages and labor productivity shocks could generate the observed behavior of unemployment, vacancies, and real wages... the answered question is what exactly a wage shock is*". Our model is exactly what Shimer seeks and it provides micro foundation for the exogenous distribution risk assumed in Danthine and Donaldson (2002).

2.6 Equilibrium

In this economy, market clearing requires that for all t,

$$
\begin{aligned}
e_t &= \int e_t^s d\varkappa = 1, \\
\phi\bar{k} &= \int b_t^s d\varkappa + \int b_t^n d\omega, \\
c_t &= \int c_t^s d\varkappa + \int c_t^n d\omega, \\
y_t &= c_t + i_t + \frac{\kappa}{2} {x_t}^2 n_t,
\end{aligned}
$$

where $\varkappa$ and ω respectively stand for the measure of insider-stockholders and the measure of outsider-nonstockholders. Lump sum transfers are taxed to balance the government budget constraint:

$$
T_t + (1 - n_t)b = 0.
$$

We define the equilibrium as follows:

Definition 1 *Under the above market-clearing conditions, a decentralized stationary recursive equilibrium is defined as: a set of decision rules* $\{c_t^s(\cdot), c_t^n(\cdot); h_t^s(\cdot), h_t^n(\cdot); e_{t+1}(\cdot); i_t(\cdot), h_t(\cdot); \nu_t(\cdot)\}$ *and a set of wage and price functions* $\{w_t^s(\cdot), w_t^n(\cdot); p_t^e(\cdot), p_t^f, d_t(\cdot)\}$ *given the information set of aggregate states* $\Omega = \{k_t, n_t; \lambda_t\}$ *such that (i)* $\{c_t^s(\cdot), h_t^s(\cdot); e_{t+1}(\cdot), b_{t+1}^s(\cdot)\}$ *solves the intertemporal problem (1) given the information set* Ω_t^s *(ii)*$\{c_t^n(\cdot), h_t^n(\cdot); b_{t+1}^n\}$ *solves the outsider-nonstockholder's intertemporal problem (8) given his information set* Ω_t^n *(iii)*$\{w_t^n(\cdot)\}$ *satisfies the optimality condition (25) (iv)* $\{i_t(\cdot), x_t(\cdot)\}$ *solves the firm's intertemporal problem given the information set* Ω^f *(14) (vi)* $w_t^s(\cdot)$ *satisfies the condition (16) (vii)*$\{p_t^e(\cdot), d_t(\cdot)\}$ *satisfies the Lucas asset pricing equations (6), while* $\{ p_t^f(\cdot)\}$ *satisfies the equations (7) and (13) (ix) The economy follows two laws of motion:* $k_{t+1} = (1-\delta)k_t + G(\frac{i_t}{k_t})k_t$ *and* $n_{t+1} = (1-\rho)n_t + q_t v_t$.

2.7 Asset Pricing

Under the decentralized stationary recursive equilibrium defined in Section 3.8, it is possible to define and compute financial variables. Using the dividend series, the conditional price $p^e(\Omega_t)$ of an equity security is recursively computed according to the Lucas' asset pricing equation:

$$
p^e(\Omega_t) = \beta E(\frac{\lambda_{t+1}^s}{\lambda_t^s}[p^e(\Omega_{t+1}) + d(\Omega_{t+1})] \mid \Omega_t),
$$

where $\Omega_t = \{k_t, n_t; z_t\}$ is the aggregate state of economy and $\lambda_t^s = u_c(c^s(\Omega_t), h^s(\Omega_t))$ is the shareholder-worker's marginal utility due to limited participation in the stock market.

Using these prices, the time series of equity returns is computed in the conventional way:

$$R^e_{t,t+1} = \frac{p^e(\Omega_{t+1}) + d(\Omega_{t+1})}{p^e(\Omega_t)} - 1.$$

In a similar fashion, the price of a one-period risk-free real bond is given by

$$p^f(\Omega_t) = \beta E(\frac{\lambda_{t+1}}{\lambda_t} \mid \Omega_t)$$

where $\lambda_t = u_c(c^s(\Omega_t), h^s(\Omega_t))$ or $\lambda_t = v_c(c^n(\Omega_t), h^n(\Omega_t))$. Note that the risk-free bond is available to all households. The risk-free rate of return, R^f_t, is computed using

$$R^f_t = \frac{1}{p^f(\Omega_t)} - 1.$$

Also, given the aggregate state $\Omega_t = \{k_t, n_t; z_t\}$, the conditional term structure $\{R^f_{t,n}\}$ can be derived:

$$R^f_{t,n} = [\frac{1}{p^f_n(\Omega_t)}]^{1/n} - 1,$$

where $p^f_n(\Omega_t) = \beta^n E(\frac{\lambda_{t+n}}{\lambda_t} \mid \Omega_t)$.

2.8 Elasticity of Intertemporal Substitution

To see how the elasticity of intertemporal substitution (EIS) is identified in our preference specifications, first note that the equations (7) and (13) can be rewritten as

$$\frac{1}{1+R^f_t} = \beta E(\frac{\lambda^s_{t+1}}{\lambda^s_t} \mid \Omega_t) \tag{28}$$

$$\frac{1}{1+R^f_t} = \beta E(\frac{\lambda^n_{t+1}}{\lambda^n_t} \mid \Omega_t) \tag{29}$$

where $\lambda^s_t = u_c(c^s(\Omega_t), h^s(\Omega_t))$, $\lambda^n_t = v_c(c^n(\Omega_t), h^n(\Omega_t))$ and $\frac{1}{1+R^f_t} = p^f_t$.

The period utility function of the representative insider-stockholder is postulated as

$$u(c^s_t - X_t - H(h^s_t)) = \frac{(c^s_t - X_t - H(h^s_t))^{1-\gamma_s} - 1}{1-\gamma_s}$$

while the preference of the representative outsider-nonstockholder is postulated as

$$v(c^n_t - L(h^n_t)) = \frac{(c^n_t - L(h^n_t))^{1-\gamma_n} - 1}{1-\gamma_n}.$$

Here γ_s and γ_n are the insider-stockholder's coefficient of risk aversion and the outsider-nonstockholder's coefficient of risk aversion, respectively; X_t is the exogenous habit stock, evolving according to

$$X_t = \phi X_{t-1} + (1-\phi)\chi c^s_{t-1}$$

where c^s_{t-1} denotes the aggregate average level of the insider-stockholder group's consumption last period, χ is the habit parameter of the insider-stockholder group and $\phi = 0$.

Under the above specifications of the each agent's preference, we log-linearize and rearrange the equations (28) and (29) in order to obtain the each agent's EIS (the bar "$-$" represents the steady state value of variables):

$$\frac{1}{\gamma_s}\frac{[\bar{c}^s(1-\chi) - H(\bar{h}^s)]}{\bar{c}^s}\hat{r}^f_t = E_t[\log\frac{c^s_{t+1}}{c^s_t} + [\text{remainder terms}]] \quad (30)$$

$$\frac{1}{\gamma_n}\frac{[\bar{c}^n - \bar{n}L(\bar{h}^n)]}{\bar{c}^n}\hat{r}^f_t = E_t[\log\frac{c^n_{t+1}}{c^n_t} + [\text{remainder terms}]]. \quad (31)$$

From the equations (30) and (31), we identify the insider-stockholder's EIS with

$$\frac{1}{\gamma_s}\frac{[\bar{c}^s(1-\chi) - H(\bar{h}^s)]}{\bar{c}^s}$$

while the outsider-nonstockholder's EIS can be identified with

$$\frac{1}{\gamma_n}\frac{[\bar{c}^n - \bar{n}L(\bar{h}^n)]}{\bar{c}^n}.$$

Note also that

$$\begin{aligned}\log\frac{c_{t+1}}{c_t} &= \log(1+\frac{\mu_s(c^s_{t+1}-c^s_t)+(c^n_{t+1}-c^n_t)}{c_t}) \\ &\approx \mu_s\frac{(c^s_{t+1}-c^s_t)}{c_t} + \frac{(c^n_{t+1}-c^n_t)}{c_t} \\ &\approx \mu_s\log\frac{c^s_{t+1}}{c^s_t}\cdot\frac{c^s_t}{c_t} + \log\frac{c^n_{t+1}}{c^n_t}\cdot\frac{c^n_t}{c_t}.\end{aligned} \quad (32)$$

Therefore, taking the expectation operator E_t in (32) and using the equations (30) and (31), we derive the formula:

$$\begin{aligned}& E_t[\log\frac{c^s_{t+1}}{c^s_t} + [\text{remainder terms}]] \\ = \; & [\mu_s\frac{1}{\gamma_s}\frac{[\bar{c}^s(1-\chi) - H(\bar{h}^s)]}{\bar{c}^s}E_t\frac{c^s_t}{c_t} + \frac{1}{\gamma_n}\frac{[\bar{c}^n - \bar{n}L(\bar{h}^n)]}{\bar{c}^n}E_t\frac{c^n_t}{c_t}]\hat{r}^f_t.\end{aligned}$$

We identify the aggregate EIS with

$$\mu_s \frac{1}{\gamma_s} \frac{[\bar{c}^s(1-\chi) - H(\bar{h}^s)]}{\bar{c}^s} E_t \frac{c_t^s}{c_t} + \frac{1}{\gamma_n} \frac{[\bar{c}^n - \bar{n}L(\bar{h}^n)]}{\bar{c}^n} E_t \frac{c_t^n}{c_t}.$$

Abstracting from uncertainty our identified EIS is reduced to

$$\mu_s \frac{1}{\gamma_s} \frac{[\bar{c}^s(1-\chi) - H(\bar{h}^s)]}{\bar{c}} + \frac{1}{\gamma_n} \frac{[\bar{c}^n - \bar{n}L(\bar{h}^n)]}{\bar{c}} \tag{33}$$

which must be the true value of the aggregate EIS in our economy when estimated from the generated data. Under our benchmark calibration, the aggregate EIS is predicted to be close to zero (0.0307). This predicted number is consistent with Hall's findings: consumption growth is completely insensitive to changes in interest rates and thus EIS is close to zero. It is also consistent with the conclusion found in Binsbergen et al. (2008); in their estimated DSGE model, they find that a low elasticity of intertemporal substitution (around 0.06) is estimated from upward-sloping (nominal) yield curve data and macro data.

3 Calibration

In this paper, the business cycle is defined by standard deviations from a Hodrick-Prescott filtered trend. The time unit of the models is three months. We calibrate the process for aggregate productivity shocks to match the quarterly AR(1) process found by Cooley and Prescott (1995) to match the US Solow residual. The productivity shock z_t evolves according to the law of motion:

$$\log z_{t+1} = 0.95 \log z_t + \epsilon_{t+1}$$

where ϵ is distributed normally, with mean zero and standard deviation σ_ϵ; in what follows, the standard deviation of technology shock σ_ϵ will be chosen by a procedure of "hyperparameter search."

For all simulation runs, the production function employed is the customary Cobb-Douglas function

$$z_t f(k_t, h_t^s \cdot 1, h_t^n \cdot n_t) = z_t M k_t^\alpha ((h_t^s \cdot 1)^\mu (h_t^n \cdot n_t)^{1-\mu})^{1-\alpha}$$

where $\mu \equiv \frac{\mu_s}{1+\mu_s}$.

The parameter M serves as a scale parameter, while μ and $1-\mu$ respectively are the normalized measure of insider-stockholders and the normalized measure of outsider-nonstockholders. To allow for debt-financing and to impose the constraint that corporate debt is risk-free, however, we scale our production technology by setting $M = 1.25$ as in Danthine and Donaldson (2002). This makes the average output high enough to guarantee a uniformly positive dividend in all states of nature for reasonable calibrations of any debt level. Following Guvenen (2003), the stock market participation rate is set to be 20 percent, so that μ and μ_s are 0.2 and 0.25, respectively.

The parameter α is typically calibrated to reproduce the observed share of capital in total value added. We adopt the most commonly used value, 0.36. The subjective discount factor β is fixed at $\beta = 0.99$, corresponding to a steady state return on capital of 4%. Following Kydland and Prescott (1982), the quarterly capital depreciation rate δ is 0.025.

The model economy assumes that search and matching frictions characterize the labour market only for outsider-nonstockholders. Therefore, we calibrate the labour market for outsider-nonstockholders using the standard parameters for labour market search and matching.

The suggestions of the empirical literature vary with several measures of the US worker separation rate. We follow Davis, Haltiwanger and Schuh (1996) and fix the quarterly separation rate ρ as 8 percent. According to Petronglo and Pissarides (2001), the elasticity of matches to unemployment of outsiders $1-\sigma$ falls within the range of plausible values of 0.5 to 0.7. We set $1-\sigma$ to be 0.5. The mean quarterly unemployment rate of the model economy is set to 6%, which can be found in the literature (e.g. Merz (1995) and Christoffel and Kuester (2008)). The steady state value of the vacancy-filling probability $\bar{q}$ is set to be 0.7, following the literature (e.g. Cooley and Quadrini (1999)). The existing literature mostly suggests that the bargaining power parameter η is equal to 0.5; we follow suit.

The choice of the unemployment benefit b is controversial. In Shimer (2005), the unemployment benefit b is set to 0.40 so that average "replacement rates," i.e. the ratio of benefits to average wages is 0.41. This value implies that the matching model cannot account for the observed fluctuations of unemployment. In Hagedon and Manovskii (2006), the unemployment benefit b is set to 0.95 so that the average replacement rates are 0.98, which contributes to the *opposite* conclusion that the standard search model is consistent with the data. The main reason behind these conflicting conclusions is that higher unemployment benefits b make workers indifferent to the substitution between working and not working. This indifference significantly dampens the variations of the standard period-by-period Nash bargaining wage over the business cycle. In sum, the Shimer's critique is not extremely robust to the choice of the parameter b.

To avoid the above controversy, we put more restrictions on the choice of b. In fact, the OECD (1996) computes the average replacement rates across countries, and finds that average replacement rates are at most 0.20 in the United States (Hornstein et al, 2005). For this reason, we choose b to be consistent with this empirical evidence; $\frac{b}{\bar{w}^n \ \bar{h}^n} = 0.1988$. Table 1 summarizes the present discussion.

Table 1: Unemployment benefit

	Us Data	This paper	Shimer	Hagedon & Manovskii	Hall
$\frac{b}{wh}$	0.2	0.1988	0.41	0.98	0.41

The vacancy cost κ is chosen so that the steady state ratio of adjustment costs to output $\frac{\frac{\kappa}{2}\ \bar{x}^2\ \bar{n}}{\bar{y}}$ is 0.01. This ratio is a widely accepted upper bound in the business cycle literature. The period utility function of the representative insider-stockholder is postulated as

$$u(c_t^s - X_t - H(h_t^s)) = \frac{(c_t^s - X_t - B_s(h_t^s)^{\psi_s})^{1-\gamma_s} - 1}{1-\gamma_s}$$

where γ_s is the coefficient of insider-stockholder's relative risk aversion, ψ_s is the insider-stockholder's disutility parameter of labour hours, which control the Frisch elasticity of labour, and $H(h_t^s) = B_s(h_t^s)^{\psi_s}$. The disutility parameter B_s is obtained from the steady state calculation.

The preference of the representative outsider-nonstockholder is postulated as

$$v(c_t^n - n_t L(h_t^n) - (1-n_t)L(0)) = \frac{(c_t^n - n_t B_n(h_t^n)^{\psi_n})^{1-\gamma_n} - 1}{1-\gamma_n}$$

where γ_n is the coefficient of outsider-nonstockholder's relative risk aversion, ψ_n is the outsider-nonstockholder's disutility parameter of labour hours and $L(h_t^n) = B_n(h_t^n)^{\psi_n}$. The disutility parameter B_n is obtained from the steady state calculation. We assume that γ_s is equal to γ_n and ψ_s is equal to ψ_n. So hereafter, we use γ as the coefficient of relative risk aversion in this economy (i.e. $\gamma = \gamma_s = \gamma_n$) and ψ as the disutility-of-labour parameter in this economy (i.e. $\psi = \psi_s = \psi_n$).

It is well known that empirical studies do not offer much precise guidance when it comes to calibrating the habit formation parameter χ, the capital adjustment cost ξ and the coefficients of relative risk aversion γ. It is also widely known that the innovation standard deviation of the technology shock, σ_ϵ, is difficult to measure from available data since this number, usually identified with the direct estimate of the volatilty of Solow residual for the post war period, is significantly affected by measurement error. Furthermore, the higher number, σ_ϵ, highlights the danger of implying that probabilities of technological regress are implausibly large. Lastly, we add the disutility-of-labour parameter ψ to our list of free parameters. Although it is believed to be less than 0.5 (e.g., McCurdy 1981), the estimate of the Frisch elasticity of labour supply is not conclusive. Indeed, Imai and Kean (2004) recently estimate the Frisch elasticity of labour supply as 3.8, which is much higher than what is believed. For this reason, we conduct a "hyperparameter search" for the parameters that are free at this point (χ, ξ, γ,ψ, σ_ϵ) to match a set of empirical targets of interest. For the baseline calibration, we choose the free parameters (χ, ξ, γ,ψ σ_ϵ) to match four empirical targets: (i) the relative standard deviation of unemployment (a ratio of unemployment volatility to to output volatility) (ii) a risk-free rate volatility (iii) the mean risk-free rate and (iv) the equity premium. Practically, we restrict our hyperparameter search to a grid of values for $\chi \in [0, 0.9]$, $\xi \in [0.23, \infty)$, $\sigma_\epsilon \in [0.0037, 0.00712]$, $\psi \in [1, 2]$ and $\gamma \in [1, 7]$. Then, we minimize an equally weighted quadratic criterion function written in the deviation from each empirical target. For the baseline calibration, the minimum is achieved for $\sigma_\epsilon = 0.006$, $\chi = 0.9$, $\xi = 0.23$, $\psi = 1.4$

and $\gamma = 3.6$. The model is solved using the linearization methods and undetermined coefficients widely employed in the business cycle literature and then log-normal formulae are applied to price the relevant asset returns (see e.g. Uhlig (1999) or Jermann (1998))[10].

4 Results

4.1 Model Results

Reassessing Shimer's critique Before we report the quantitative results for the baseline model, we raise several issues on how Shimer's critique should be applied to (real) business cycle models with labour-market search, and modify it accordingly. In his seminal paper, Shimer claims that the incorporation of the standard search model into a real business cycle framework with intertemporal substitution of leisure, capital accumulation, and other extensions such as the Merz model (1995) or the Andolfatto model (1996) does not invalidate his critique. In his words, "Neither paper can match the negative correlation between unemployment and vacancies, and both papers generate real wages that are too flexible in response to productivity shocks" (p.45). Indeed, the Andolfatto model cannot pass a litmus test for the unemployment volatility puzzle Shimer raises: the model allows for real wage that is too flexible in response to productivity shocks and the volatility of job vacancies is too moderate to match its empirical counterpart. The Merz model, however, is hard to reject on this *ad hoc* basis. Table 2 in her paper shows that the model with fixed search intensity can account "well" for basic stylized facts of the labour market volatility; the wage is indeed rigid in terms of the relative standard deviation ($\frac{\sigma_w}{\sigma_y} = 0.34$) and the job vacancies are reasonably volatile ($\frac{\sigma_\nu}{\sigma_y} = 6.38$). Both models generate the negative correlation between unemployment and vacancies, although that correlation is only weakly negative. Furthermore, it can be shown that the Merz model with fixed search intensity is isomorphic to the Andolfatto model with inelastic labour supply of hours (up to a first-order approximation). The relative success of the Merz model (with fixed search intensity) in generating realistic labour market statistics rides not only on wage stickiness, however, but also on the absence of variations at the intensive margin. If the Merz model were to allow for variations at the intensive margin, its ability to explain labour market volatility might be significantly compromised; the representative firm now could substitute between hours per incumbent and hiring new workers. This substitution effect is not negligible over the business cycle. This is why the Andofatto model performs so poorly on the dimensions of the labour market business cycles: it allows both variations. This means that the model's ability to resolve the unemployment volatility puzzle can depend upon the extent to which the labour supply of hours is elastic. To see if the (quarterly) business cycle models with labour-market search can pass a litmus test for the resolution of the unemployment volatility puzzle, the consideration of both variations at the intensive margin and at

[10] Log-normal formulae can be found in the Appendix A.

the extensive margin is required.

We propose the following modification of Shimer's critique: (i) a quarterly business cycle model with labour-market search must generate the absolute amplitude of the standard deviations of key variables in the labour market activities as well as the relative standard deviations to the standard deviation of output; (ii) the model should allow for variations at the intensive margin and at the extensive margin simultaneously; and (iii) the negative correlation between unemployment and vacancies should be substantially consistent with the data[11].

Model simulations First of all, recall that we minimize an equally weighted quadratic criterion function for a grid of values for $\chi \in [0, 0.9]$, $\xi \in [0.23, \infty)$, $\sigma_\epsilon \in [0.0037, 0.00712]$, $\psi \in [1, 2]$ and $\gamma \in [1, 7]$ for the baseline model. The minimum is achieved for $\sigma_\epsilon = 0.006$, $\chi = 0.9$, $\xi = 0.23$, $\psi = 1.4$ and $\gamma = 3.6$.

$\psi = 1.4$ implies that the Frisch elasticity of labour supply in this economy is $\frac{1}{1.4-1} = 2.5$ as in Jaimovich and Rebelo (2008). Our Frisch elasticity of labour supply is higher than its traditional estimate but is less than the Imai-Kean estimate of 3.8.

The value of the innovation standard deviation, σ_ϵ, is 0.6%, which is much smaller than the values used by major macro-asset pricing models proposed in the existing literature, that is to say, Boldrin, Christiano and Fisher (2001), Danthine and Donaldson (2002), and Guvenen (2003). These models value the innovation standard deviation per quarter at close to 2%. For instance, Boldrin, Christiano and Fisher (2001) use permanent shocks with a standard deviation of 1.8% per quarter. This leads to the interpretation that the exogenous driving source in this class of models may encompass more than technology shocks; this again highlights the danger of implying that probabilities of technological regress are implausibly large. Indeed, our value is even smaller than the direct estimate of the volatility of Solow residuals for the post war period, which is about 0.7%.

Table 2 reports the second moments of endogenous variables as implied by the model, namely unconditional standard deviations and the contemporaneous correlation with output, alongside the moments implied by the data. Table 3 also reports the associated financial statistics implied by the model alongside the financial statistics implied by the data (Mehra and Prescott, 1985). These results are discussed below.

Labour market volatility The model reproduces the substantial fluctuations in the key variables of labour market activity found in the data and emphasized by Shimer (2005) and Hall (2005). In particular, in terms of the (absolute) volatility, the model comes remarkably close to the (absolute) volatilities of the key labour market variables including unemployment u, vacancies ν, and the market

[11] The Merz model (with fixed search intensity) cannot pass this litmus test for the resolution of the unemployment volatility puzzle. For instance, the amplitude of the standard deviation of vacancies is 6.85% while it empirical counterpart is around 13.15%; it violates the condition (iii); the correlation between unemployment and vacancies (-0.15) falls short of its realism (-0.89); and the Merz model allows only for variations at the extensive margin.

tightness measure $\theta \equiv \frac{\nu}{u}$, although the volatility of output falls short of its empirical counterpart (1.47 versus 1.59). This indicates that the propagation mechanism in this model economy is more powerful in the class of business cycle models with search; note also that the magnitude of productivity shock required to produce the observed variations in the labour market activity of interest is a standard deviation of 0.006, smaller than the direct estimate of the volatility of Solow residuals from the post war data (about 0.007).

A distinguishing feature of our analysis is that we can disentangle the variations at the intensive margin from the variations at the extensive margin. The model comes close to matching both the relative volatility of total hours (0.91 versus 0.95 in the data) and hours per worker (0.44 versus 0.43 in the data), although the correlation of hours per worker with output is too procyclical; the model captures the reality of the relevant labour market activity found in the data. As a consequence, the statistical behavior of employment also comes reasonably close to its counterpart in the data.

Along the wage dimensions, however, the model overstates or understates its empirical magnitude: real hourly wage is not volatile enough and the contemporaneous correlation of hourly wage with output is too procyclical. The departure of hourly wage volatility from its empirical magnitude is in a way predictable. The Nash bargaining wage (wage per outsider) in this model economy is significantly affected by the countercyclicality of endogenous distribution risk or Nash bargaining power shock. This added wage shock will dampen the variations of the Nash bargaining wage over the business cycle. Indeed, the endogenous distribution risk is highly volatile and strongly countercyclical, and thus the wage is less volatile over the business cycle. Nevertheless, the correlation of wage per outsider with output is still procyclical. Wage per insider is also less volatile, but its root cause is quite different. That is, the insider's wage is determined by marginal product of labour. This mechanism for wage determination usually displays the low volatility and the strong procyclicality of wage. For instance, in the indivisible RBC model of Hansen (1985), the relative standard deviation of the real wage is 0.28 and the correlation of wage with output is 0.88 if the wage is equal to marginal product of labour.

Financial statistics and conventional business cycle On the financial front, the performance of the baseline model is reasonably promising: the model delivers an equity premium of 4.28 percent and a mean risk-free rate of 1.24 percent. The volatilities of the equity return and risk-free bond only slightly exceed their empirical counterparts. As Table 2 shows, however, these results are delivered at the cost of the volatilities of consumption and investment: total consumption has the 95 percent volatility of output and investment is one and half as volatile as output. As well documented in Jermann (1998), capital adjustment costs make it more costly to smooth consumption through changing the capital stock, resulting in a lower volatility of investment while consumers end up taking more consumption risk (higher volatility of total consumption).

Table 2: Business cycle: the baseline model

Business Cycle							
Variable	Meaning	Std		Std. to σ_y		Corr. with y	
		Data	Model	Data	Model	Data	Model
y	output	1.59	1.47	-	-	-	-
c	consumption	1.23	1.39	0.77	0.95	0.83	0.94
i	investment	4.87	2.22	3.06	1.51	0.91	0.86
th	total hours	1.51	1.34	0.95	0.91	0.92	0.90
h	hours per worker	0.69	0.65	0.43	0.44	0.62	0.90
h^s	hours per insider	-	1.05	-	0.71	-	1.00
h^n	hours per outsider	-	0.56	-	0.38	-	0.87
w	wage	0.70	0.37	0.44	0.25	0.68	0.88
w^s	wage per insider	-	0.42	-	0.29	-	1.00
w^n	wage per outsider	-	0.23	-	0.16	-	0.87
n	employment	1.02	0.90	0.64	0.61	0.78	0.98
u	unemployment	11.01	10.36	6.92	7.05	-0.87	-0.84
ν	vacancy	13.15	13.42	8.27	9.13	0.91	1.00
θ	tightness	21.66	22.52	13.62	15.32	0.90	0.98

Table 3: Financial statistics: the baseline model

Financial Statistics					
Variable	Meaning	Mean		Std	
		Data	Model	Data	Model
R^f	risk-free bond return	0.80	1.24	5.67	6.48
R^e	equity return	6.98	5.48	16.54	17.71
R^p	equity premium	6.18	4.28	16.67	17.40

Term structure It is well documented that the standard RBC model (e.g., the indivisible RBC model of Hansen) with persistent technology shocks generates a downward-sloping real term structure, as shown in the 4th column of Table 4 (the "RBC model"). Several empirical studies (Mishkin (1990)) suggest that nominal rates and real rates move in tandem, which implies that a corrected model must generate an upward-sloping real term structure. Indeed, the baseline model produces an upward-sloping real yield curve. However, it is difficult to assess the plausibility of this property without a long sample on real yields; in the United States, the trading of indexed bonds called TIPS has just started recently from 1997 in the United States. Nevertheless, the real yield curve has been mostly upward sloping based on the raw TIPS data during this period (McCullogh Data in Table 4).

The model-implied average real curve seems to be broadly consistent with the McCullogh data. Taking into account that the mean annualized inflation rate is around 4.5% from the period 1971:3 to 2005:4 (Gallmeyer et al., 2008), the model also favors the hypothesis that nominal rates and real rates move in tandem. However, its implied volatilities are more volatile than its empirical counterparts, although the very volatilities rapidly die out.

The intuitive explanation behind the positive slope is that although our preference specification does not belong to a class of "generalized expected utility" preferences, the baseline model conveys the sense that there is strong preference for late resolution of uncertainty among agents in the model economy: in other words, $CRRA = \gamma = 3.6 \ll \frac{1}{EIS} = \frac{1}{0.0307} = 25.25$. Here EIS is understood as the model-implied aggregate EIS, equal to (33). When there is preference for a late resolution of uncertainty, agents prefer to buy short maturity bonds and roll them over time instead of buying long maturity bonds, which make them get paid only in a particular yield. As a result, the demand for short term bonds is high while the demand for long term bonds is low; that is, the prices of the short term bonds are high and the prices of the long term bonds are low.

In fact, the same intuition is found in Binsbergen et al. (2008); in their estimated DSGE model with fully specified Epstein-Zin preferences, they find that a low elasticity of intertemporal substitution (around 0.06) is estimated from upward-sloping (nominal) yield curve data and macro data. Their estimates also satisfy the condition that $CRRA \ll \frac{1}{EIS}$, which supports the hypothesis that there is strong preference for a late resolution of uncertainty.

Table 4: Term structure: the baseline model

	US Data				Model			
	Nominal Data		McCullogh Data		This paper		Hansen	
Maturity	Mean	Std	Mean	Std	Mean	SD	Mean	SD
4	5.60	2.93	1.06	1.61	1.39	6.32	4.0387	0.31
8	5.81	2.89	1.39	1.37	1.76	5.72	4.0375	0.27
12	5.98	2.82	1.69	1.23	2.10	4.98	4.0370	0.25
16	6.11	2.79	1.95	1.15	2.37	4.30	4.0368	0.23
20	6.19	2.74	2.16	1.09	2.57	3.72	4.0366	0.22

Notes: Maturity in quarters. Nominal data is from Wachter (2006).
McCullogh data is drawn from Piazzesi and Schneider (2006).

Shimer's Statistics The baseline model also can capture the observed business-cycle-frequency fluctuations in unemployment and job vacancies in the US statistics as reported in Shimer (2005). In his statistics, Shimer chooses a much smoother trend component, corresponding to an HP smoothing parameter of 10^5. The baseline model now is simulated with an HP smoothing parameter of 10^5 and the same parameters as our benchmark except the innovation standard deviation of technology shock σ_ϵ[12]. Table 5 shows that the model statistics of the baseline model is significantly consistent with the

[12] The innovation standard deviation of technology shock σ_ϵ ($\sigma_\epsilon = 0.812\%$) is chosen to match the volatility of real GDP, which is around 2% for an HP parameter of 10^5.

statistics reported in Shimer (2005); values in the data are given in brackets.

Table 5: Labour market volatility: the baseline model

		u	ν	θ
Standard deviation		17.3 (19.0)	22.3 (20.2)	37.6 (38.2)
Quartely autocorrelation		0.94 (0.936)	0.88 (0.940)	0.94 (0.941)
	u	1	−0.80 (−0.894)	−0.94 (−0.971)
Correlation matrix	ν	-	1	0.96 (0.975)
	θ	-	-	1

4.2 Model Evaluation: Comparisons with the Literature

Unemployment volatility puzzle We begin by comparing the results of our baseline model with those of two other leading business cycle models with search and matching frictions, namely, Gertler and Trigari (2005) and Christoffel and Kuester (2008). We also compare our model with the existing benchmark business cycle models with search such as Andolfatto (1996) and Merz (1995) and the standard RBC model of Hansen.

Table 6 shows that our model and two models with staggered Nash bargaining wage can account very well for the observed volatility in key labour market variables emphasized by Shimer (2005) and Hall (2005). In addition, our model and the Christoffel-Kuester model capture variations at the intensive margin as well as at the intensive margin. They especially account well for the amplitude of the volatilities of the key labour market indicators including unemployment and vacancies.

Table 6: Comparison: labour market volatility

	σ_y	σ_u/σ_y	σ_ν/σ_y	σ_θ/σ_y	σ_n/σ_y	σ_{th}/σ_y	σ_h/σ_y	σ_w/σ_y	$\rho(w,y)$	$\rho(u,\nu)$
Data	1.59	6.92	8.27	13.62	0.64	0.95	0.43	0.44	0.68	−0.88
This paper	1.47	7.05	9.13	15.32	0.61	0.91	0.44	0.25	0.88	−0.79
Gertler-Trigari	–	5.68	7.28	12.52	0.44	–	–	0.48	0.55	–
Christoffel-Kuester	1.91	5.74	7.23	–	–	1.09	0.78	0.22	0.09	–
Merz	1.07	4.63	6.38	1.67	–	–	0.51	–	0.95	−0.15
Andolfatto	1.45	0.68*	3.20*	2.64	0.51	0.59	0.22	0.39	0.95	−0.19
Hansen	1.76	–	–	–	–	0.77	0.77	0.28	0.87	–

Notes: The statistics from Gertler and Trigari (2005) are from their model with the staggeredness λ =11/12, i.e 4 quarters. The statistics from Merz (1995) are from her model with fixed search intensity. *numbers in the Andolfatto model are the ones reproduced by Costain and Reiter (2008).

The gist of our model is that the Nash wage bargaining between capital owners (insiders) and workers (outsiders) makes it possible for capital owners to provide workers with partial insurance against their labour income variations as measured by countercyclical distribution risk ϕ_t. This endogenous distribution risk is exactly identified with the Nash bargaining power shock Shimer (2005) proposed, which is a critical factor to reproduce the observed volatilities of the key indicators of labour market activity. This countercyclical distribution risk dampens the variation in the Nash bargaining wage for outsider-nonstockholders and thus the variation in hours per outsider-nonstockholder is severely moderated; note that the relative standard deviation of labour hours per outsider is 0.38 (See Table 6). Capital owners (insider-stockholders) who face a high level of volatility of their marginal rate of substitution are prevented from smoothing their consumption due to frictional reallocations of capital stock and labour hours: namely, costs of adjusting capital and low-amplitude labour hours resulting from low-amplitude Nash bargaining wage. Capital owners then try to smooth their consumption by building up more labour stock; in other words, capital owners try to do so via more variations at the extensive margin. As a result, a high-productivity realization increases job vacancies and employment dramatically and vice versa. This sequence of events resolves the unemployment volatility puzzle raised by Shimer. Equivalently, note also that as Figure 1 indicates, this risk-sharing mechanism in the form of endogenous distribution risk generates less risky shares of income going to labour in the short run, resulting in variations in labour's income share found in the data. In other words, the wage bills vary less than output, falling proportionately less in recessions and increasing less during expansions and as a result, the risk of the firm's free cash flow and derived dividends tends to increase substantially. Naturally, this channel, the "operating leverage channel," plays a key role in generating a sizable equity premium.

Additional insight into the resolution of the unemployment volatility puzzle can be obtained by examining the model's impulse response functions or by estimating how a positive 1% productivity shock affects the key decision variables in the baseline model. Using the method of undetermined coefficients proposed by Campbell (1994), the key detrended endogenous variables are expressed as a linear function of the state variables (in logs). For instance, consumption in the baseline model can be expressed as:

$$\hat{c}_t = \eta_{cz}\hat{z}_t + \vec{\eta}_{cs} \cdot \vec{s}_t.$$

Here η_{xy} denotes the elasticity of endogenous variable "x" with respect to state variable "y"; $\vec{s}_t$ is the vector of state variables and $\vec{\eta}_{cs}$ is the corresponding vector of the elasticities of endogenous variable "x" with respect to the vector $\vec{s}_t$. Table 7 summarizes the elasticities of the endogenous variables of interest with respect to productivity shock z.

In response to a positive 1% productivity shock, our Nash wage bargaining between capital owners (insiders) and workers (outsiders) makes it possible for capital owners to provide workers with some insurance against their labour income variations. indeed, the distribution risk, namely worker's bargaining power ϕ_t immediately drops down by 10.23%. This dramatic decrease of worker's bargaining

Figure 1: Operating leverage: baseline model

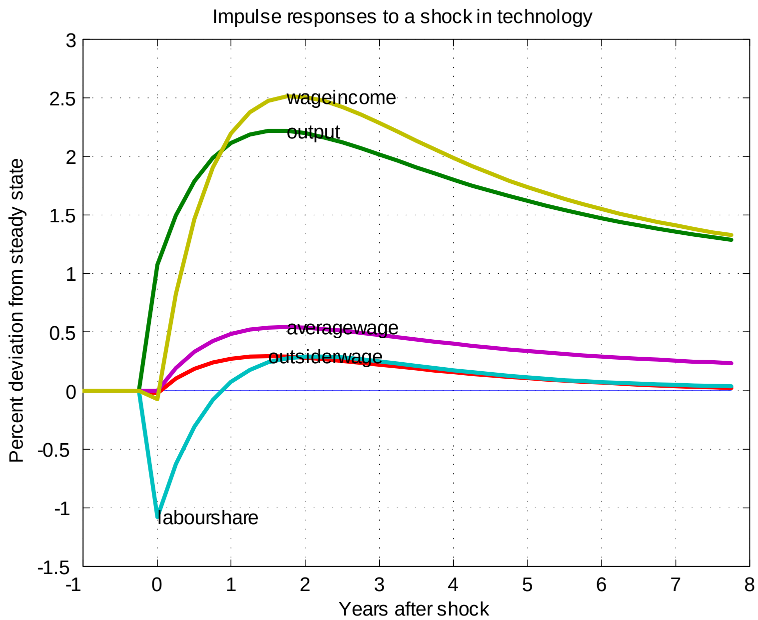


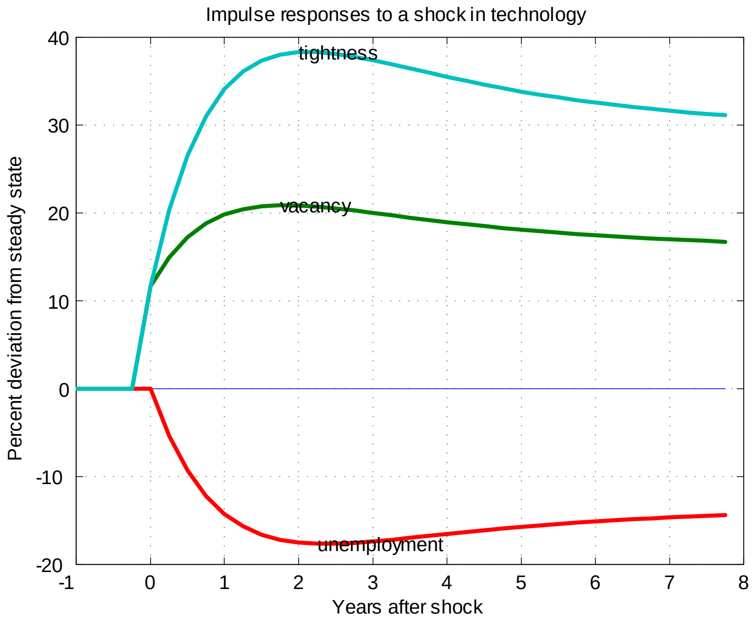
Table 7: Elasticities for the baseline model

	η_{yz}	$\eta_{\lambda^s z}$	η_{wz}	$\eta_{w^n z}$	η_{hz}	$\eta_{\nu z}$	η_{wiz}	η_{lsz}	η_{dz}	$\eta_{\phi z}$
Model										
Baseline	1.07%	-19.84%	-0.001%	-0.02%	0.12%	11.706%	-0.07%	-1.08%	3.78%	-10.23

power dampens the volatility of Nash bargaining wage (outsider-nonstockholder's wage) w_t^n and thus the bargaining wage slightly decreases by 0.02%. In turn, the overall average wage in this economy is almost acyclical; in response to a positive 1% productivity shock, the average wage w_t varies by -0.001%. Note that 80 percent of the workforce in this economy comes from the outsider-nonstockholders. Since outsider's labour hours are dependent on the determination of Nash bargaining wage, they are also dampened and the overall hours supplied merely increase by 0.12%. As a result, the overall wage income $wi_t \equiv \mu_s w_t^s h_t^s + w_t^n h_t^n n_t$ is nearly acyclical, i.e. it drops by 0.07% and the labour income share drops significantly by 1.08%. This fixed wage income effect or operating leverage effect amplifies the firm's free cash flow and derived dividends; indeed, the dividends increase substantially by 3.78%.

To smooth their consumption, capital owners (insider-stockholders) who already face a high level of volatility of marginal rates of substitution ($\eta_{\lambda^s z} = -19.84\%$) and the frictional reallocation of capital due to the *a priori* specified cost of adjusting capital now must deal with the additional frictions of

Figure 2: Employment fluctuations: baseline model



reallocating labour inputs due to the distribution risk and the resulting bargaining wage for any given degree of the employment size of workers (outsider-nonstockholders). Therefore, in the last resort, capital owners end up with increasing employment in the next period, n_{t+1}, by posting job vacancies enormously; in other words, expecting trading frictions due to the job matches in the labour market for outsider-nonstockholders, capital owners (firms) increase job vacancies up by 11.706% and as they build up the employment size of workers in the following period, market tightness also increases dramatically while the unemployment decreases persistently (See Figure 2). As capital owners build up the labour stock of workers, however, wage income gets more risky than output within one year and after one year, the rise of wage income exceeds that of output; in other words, the operating leverage effect or the fixed wage income effect is completely destroyed after one year. We conclude that our operating leverage channel is a short-run mechanism for shifting workers' labour income risk onto the capital owners.

In sum, we argue that the short-run operating leverage channel is the key mechanism for resolving the unemployment puzzle. In particular, it should be emphasized that the distribution risk plays a key role in generating this short-run operating leverage channel; the countercyclical distribution risk (workers' bargaining power) dampens the bargaining wages significantly, creating the rigid wage income effect.

Gertler and Trigari (2005) embed the standard Nash bargaining wage contracting into the framework of (Calvo-type) staggered multiperiod wage contracting. Hence, the wage contract ends up with taking the form of a fixed wage per period over an exogenously given horizon. The Gertler-Trigari model is quite successful in accounting for the overall volatility in the data when average wage contract length is assumed to be four quarters (the fourth row of Table). The model, however, is silent about how variations at the intensive margin affects its quantitative validity; in other words, the model completely abstracts from variable labour hours. As observed in the Andolfatto model and the Merz model, we cannot exclude the possibility that the Gertler-Trigari model is extremely sensitive to variations of labour hours.

Christoffel and Kuester (2008) incorporate search frictions in the labour market into a New Keynesian framework characterized by price rigidities in the goods market. Building on right-to-manage Nash wage bargaining proposed by Trigari (2006), the Christoffel-Kuester model can account for the observed variations of key indicators of labour market activity, including vacancies and unemployment, as well as the variations at the intensive margin (the fifth row of Table 6). To reproduce the empirically pronounced fluctuations of unemployment over the business cycle, the model must rely on (i) multiple shocks including productivity shocks, monetary policy shocks, government spending shocks, and a risk premium shock, (ii) the exogenously specified duration of the wage contract (five months) and (iii) exogenously specified fixed costs of maintaining an existing job which amplify profit fluctuations for any given degree of wage fluctuations. A notable feature of the Christoffel-Kuester model is that it requires the same operating leverage channel to give a satisfactory replication of the pronounced fluctuations of unemployment. Without an exogenous risk premium shock or *ad hoc* fixed costs of job maintenance, profit fluctuations for any given degree of wage fluctuations are completely destroyed, i.e. the operating leverage is completely absent. In contrast, our model completely endogenizes the important source of operating leverage channel.

Table 8: Comparative Model Specifications

	This paper	Gertler-Trigari	Christoffel-Kuester
Productivity shock	Yes	Yes	Yes
Government spending shock	No	No	Yes
Monetary policy shock	No	No	Yes
Risk premium shock	No (endogenous risk premium)	No	Yes
Duration of Nash bargaining wage contract	one quarter	3-4 quarters	5 months
Extensive margin	Yes	Yes	Yes
Intensive margin	Yes	No	Yes
Class of each model	RBC model with real rigidites	RBC model	NK model

Equity premium puzzle In this section, we begin by comparing the output of our model with that of several leading macro-asset pricing models proposed in the existing literature: namely, Boldrin, Christiano and Fisher (2001), Danthine and Donaldson (2002), Guvenen (2003) and Jermann (1998).

To maximize the model's ability to match stylized financial statistics, we now set up a hyperparameter search slightly different from the baseline calibration. Assuming that $\chi = 0.9$ and $\xi = 0.23$, we conduct a hyperparameter search for a smaller number of the parameters that are free at this point (γ, ψ, σ_ϵ) to match a smaller number of empirical targets of interest, which focuses more on financial statistics. In other words, we choose the free parameters (γ,ψ, σ_ϵ) to match three empirical targets: (i) a risk-free rate volatility (iii) the mean risk-free rate and (iv) the equity premium. Practically, we restrict our hyperparameter search to a grid of values for $\sigma_\epsilon \in [0.0037, 0.00712]$, $\psi \in [1, 2]$ and $\gamma \in [1, 7]$. Then, we minimize an equally weighted quadratic criterion function written in the deviation from each empirical target except using a weight of 10 for the risk-free rate volatility. The minimum is achieved for $\sigma_\epsilon = 0.0052$, $\psi = 1.26$ and $\gamma = 4.5$. The coefficient of relative risk aversion jumps to 4.5 from the benchmark case, 3.6 and $\psi = 1.26$ implies that the Frisch elasticity of labour supply in this economy is $\frac{1}{1.26-1} = 3.846$ which is similar to the Imai-Kean estimate of 3.8. Our value of the innovation standard deviation, σ_ϵ, is 0.52%, which is even smaller than the value of our baseline model, 0.6%. Again, this value of the innovation standard deviation tightly identifies technology shocks with the single exogenous driving source in the model economy.

Table 9 and Table 10 display the statistics from the simulated models along with their empirical counterparts from US data. The model optimized by the above method of hyperparameter search (hereafter the optimized model) generates an equity premium of 4.89%, which still falls short of its empirical counterpart, 6.18% but is higher than the baseline model. The standard deviation of excess returns is 17.00%, which is broadly consistent with the empirical magnitude of the standard deviation of excess returns found in the US data—- the standard deviation of excess returns is estimated to be 16.67%. The volatility of the average risk-free rate is 5.98%, which achieves a satisfactory replication of its empirical counterpart, 5.67%. The average risk-free rate, however, is 0.11%, which is slightly off its empirical target, 0.80% in the US data.

Table 9: Comparision: financial statistics

	$E[R_t^f]$	$\sigma[R_t^f]$	$E[R_{t,t+1}^e]$	$\sigma[R_{t,t+1}^e]$	$E[R_{t,t+1}^e - R_t^f]$	$\sigma[R_{t,t+1}^e - R_t^f]$
Model						
Baseline	1.24	6.48	5.48	17.71	4.28	17.40
Optimized	0.11	5.98	4.96	17.47	4.89	17.00
Danthine-Donaldson	2.46	4.05	5.92	22.20	3.46	22.34
Guvenen (2003)	1.98	5.73	5.30	14.10	3.32	14.70
Boldrin-Christiano-Fisher	1.20	24.6	7.83	18.4	6.63	–
Jermann	0.82	11.64	7.00	19.86	6.18	–
Hansen	4.05	0.46	4.04	0.48	0.01	–

Table 10: Comparison: business cycles

	σ_y	σ_c/σ_y	σ_i/σ_y	σ_{th}/σ_y	σ_h/σ_y	$\rho(c,y)$	$\rho(i,y)$	$\rho(th,y)$	$\rho(h,y)$
Data	1.59	0.77	3.09	0.95	0.43	0.83	0.91	0.92	0.62
Model									
Baseline	1.47	0.95	1.51	0.91	0.44	0.94	0.86	0.90	0.94
Optimized	1.28	0.96	1.73	1.02	0.23	0.89	0.82	0.88	0.86
Danthine-Donaldson	1.77	0.82	1.72	–	–	0.96	0.93	–	–
Guvenen (2003)	2.40	0.96	1.13	–	–	–	–	–	–
Boldrin-Christiano-Fisher	1.97	0.69	1.67	0.51	0.51	0.95	0.97	0.86	0.86
Jermann	1.76	0.49	2.64	–	–	–	–	–	–
Hansen	1.76	0.29	3.24	0.77	0.77	0.87	0.99	0.98	0.98

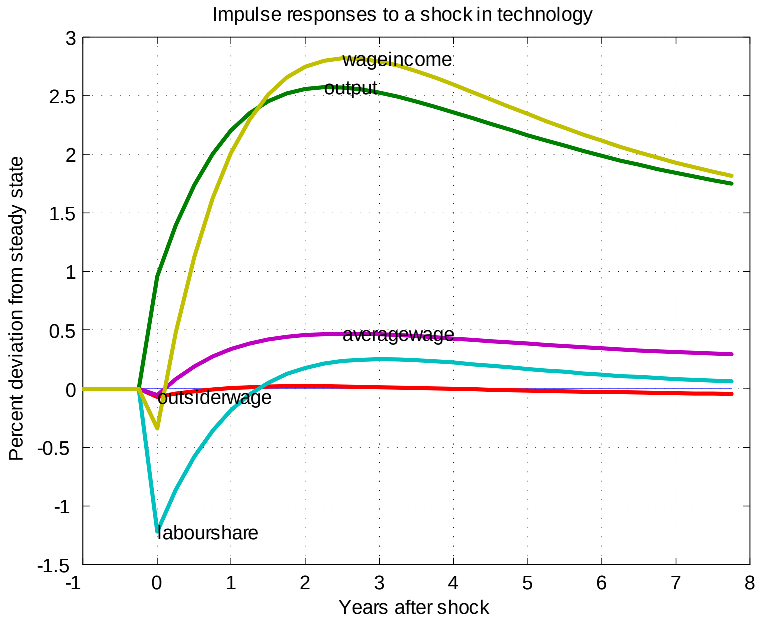
Most of the finance-cum-production models require that the capital owner display a strong desire to smooth his consumption intertemporally while simultaneously acting in a context that makes it difficult to reallocate labour or capital to that same end. These frictions of reallocating labour or capital essentially substitute for some form of market incompleteness: in either case, agents are prevented from smoothing their consumption across states and dates. These ingredients are the key to generate a high equity premium. Our baseline model (or the optimized model) follows suit: capital owners who already face a high level of MRS volatility and the costly reallocation of capital due to the capital adjustment cost now must deal with the additional frictions of reallocating labour inputs due to the shift of added income risk onto capital owners caused by the distribution risk to smooth their consumption.

A distinguishing feature of our mechanism is that the model still can achieve a satisfactory replication of stylized financial statistics while also allowing variations at both the intensive margin and the extensive margin. In other words, the model not only can generate the observed wedge between the volatility of total hours and the volatility of hours per worker but also resolve the unemployment puzzle popularized by Shimer and Hall. Most of the macro-asset-pricing models abstract from this wedge; they assume that the labour supply is inelastic or, when endogenous labour supply choice is incorporated, the models usually display low volatility of labour hours, which comes close to the observed volatility of hours per worker rather than that of total hours. Therefore, variations at the extensive margin are ignored in this class of the models.

The gist of our mechanism for generating a substantial equity premium, the partial risk-sharing mechanism in the form of endogenous distribution risk, generates less risky shares of income going to labour in the short run, resulting in variations in labour's income share found in the data. The wage bills vary less than output, falling proportionately less in recessions and increasing less during expansions and as a result, the risk of the firm's free cash flow and derived dividends tends to increase substantially.

As in the previous section, additional insight into where the equity premium comes from can be

Figure 3: Operating leverage:optimized model



obtained by examining the model's impulse response functions. In what follows, we show that our operating leverage effect can be the key to generating a high equity premium. Table 11 summarizes the elasticities of the endogenous variables of interest with respect to productivity shock z. As reference points, we summarize the elasticities of the endogenous variables from the baseline model and the standard RBC model of Hansen.

Table 11: Elasticity for the model economy: comparison

Model	η_{yz}	$\eta_{\lambda^s z}$	η_{wz}	$\eta_{w^n z}$	η_{hz}	$\eta_{\nu z}$	η_{wiz}	η_{lsz}	η_{dz}	$\eta_{\phi z}$
Baseline	1.07%	−19.84%	−0.001%	−0.02%	0.12%	11.706%	−0.07%	−1.08%	3.78%	−10.23%
Optimized	0.96%	−27.05%	−0.05%	−0.07%	−0.05%	17.05%	−0.34%	−1.21%	0.33%	−20.07%
Hansen	1.94%		0.47%	–	1.37%	–	1.94%	–	−8.62%	–

Figure 3 presents the impulse response functions from the optimized model. As in the baseline model, the optimized model highlights the significance of this operating leverage effect. In response to a positive 1% productivity shock, the optimized model displays the huge decrease of the distribution risk ϕ_t by 20.07%, dampening the variations of the wage income by the decrease of 0.34%. Furthermore, Figure 3 clearly indicates that this operating leverage effect is more persistent than in the baseline model: approximately after one and half year, the wage income gets more risky than output (the rise of wage income exceeds that of output).

Intuitively, high-productivity shocks coincide with the situations where the bargaining power of capitalists is high and the bargaining power of workers is low; in turn, the high residual payment to capitalists due to the high-productivity realization is further magnified by the decrease of the labour income shares. As a result, the firm's free cash flow and derived dividends increase during the boom. Similarly, low productivity shocks reduce further the low residual payment to capitalists.

We argue that the persistent procyclicality of the residual payments to capitalists can generate a sizable equity premium. To this end, we present a decomposition of the equity premium that makes it possible directly to relate impulse responses to risk premia, following Jermann (1998). Figure 4 shows the impulse responses of the dividend and the marginal utility of insider-stockholders in the optimized model. As in Jermann (1998), we consider a n-period dividend strip, i.e. the risk premium for holding a security that promises a stochastic dividend n-periods from now; given that a stock is just a claim to an infinite sequence of dividends, the equity premium will be a composite of such strip premia. Denoting $1 + R^{e,n}_{t,t+1}$ by the n-period strip return, the strip premium can be written as:

$$\frac{E_t[1 + R^{e,n}_{t,t+1}]}{1 + R^f_t} = \exp(-cov_t(\hat{\lambda}^s_{t+1}, E_{t+1}\hat{d}_{t+n})) \times \exp(-cov_t(\hat{\lambda}^s_{t+1}, E_{t+1}\hat{\lambda}^s_{t+n} - \hat{\lambda}^s_{t+1})) \tag{34}$$

where $\hat{\lambda}^s_t$ is the log-deviation of the marginal utility of insider-stockholders. From the above equation (34) it is evident that when two covariance terms, $-cov_t(\hat{\lambda}^s_{t+1}, E_{t+1}\hat{d}_{t+n})$ and $-cov_t(\hat{\lambda}^s_{t+1}, E_{t+1}\hat{\lambda}^s_{t+n} - \hat{\lambda}^s_{t+1})$ enter positively, positive strip premia can be obtained, ending up with generating a positive equity premium.

In Figure 4, we look at the impulse responses of the dividend and the marginal utility of insider-stockholders . First, Figure 4 shows that the second covariance term, $-cov_t(\hat{\lambda}^s_{t+1}, E_{t+1}\hat{\lambda}^s_{t+n} - \hat{\lambda}^s_{t+1})$, is positive. The marginal utility λ^s jumps down dramatically and then, it is negatively serially correlated, that is to say, $E_{t+1}\hat{\lambda}^s_{t+n}$ exceeds $\hat{\lambda}^s_{t+1}$. It deserves being mentioned that the high-productivity realization coincides with the situations where the bargaining power of capitalists is high and thus the countercyclical distribution risk $\phi = \frac{\lambda^s}{\lambda^n}$ shifts down the marginal utility of capitalists (insider-stockholders) more. Note also that the optimized model with stronger operating leverage effect shifts down the marginal utility of capitalists more in contrast to the baseline model (Figure 5).

Figure 4 also shows that the first covariance term, $-cov_t(\hat{\lambda}^s_{t+1}, E_{t+1}\hat{d}_{t+n})$, contributes positively. The dividend d is procyclical persistently and thus it is negatively serially correlated with $\hat{\lambda}^s_{t+1}$. Such persistent procyclicality of the dividend (the residual payment to capitalists) originates from the operating leverage effect. In contrast to the baseline model (Figure 5), the dividend is more persistent; the peak of the dividend from the optimized model with the stronger operating leverage effect is reached

Figure 4: Dividend and marginal utility: optimized model

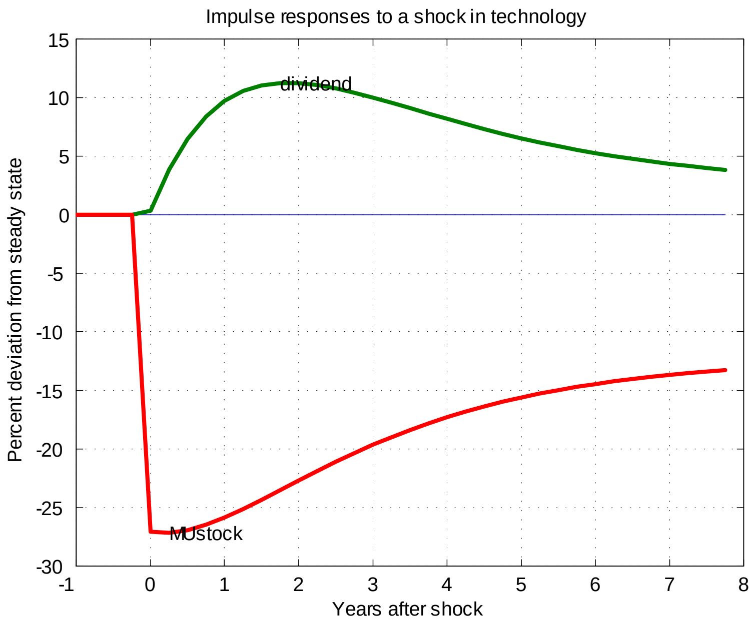


Figure 5: Dividend and marginal Utility: the baseline model

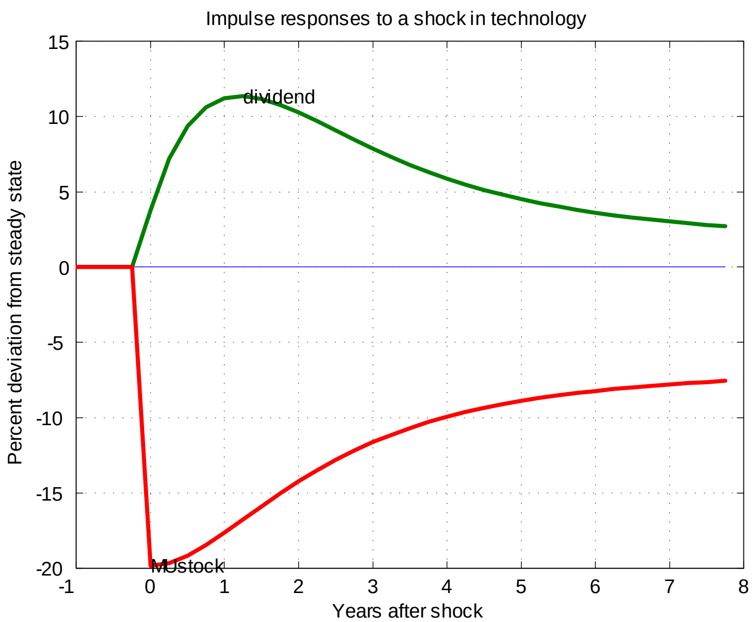
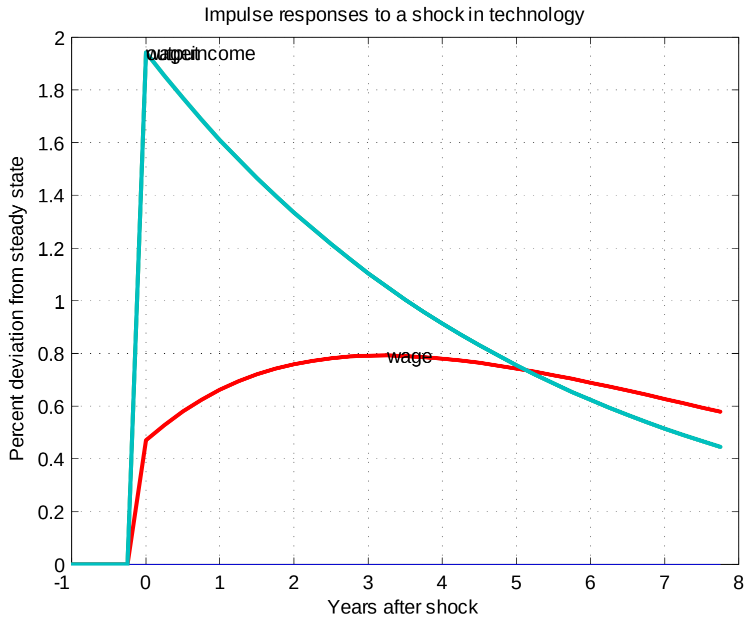


Figure 6: Operating Leverage: Hansen model

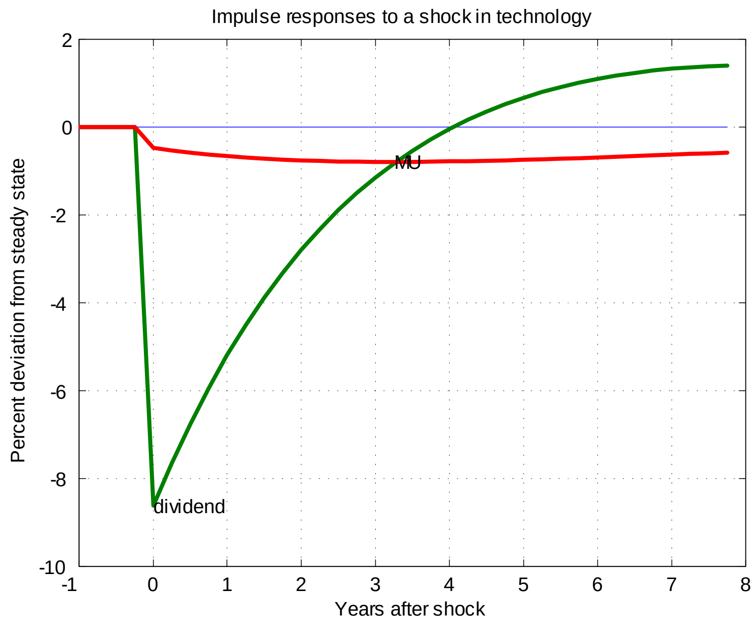


around two years after the shock, while the corresponding peak of the dividend from the baseline model is reached just around less one and half years after the shock. Therefore, the equity premium generated by the optimized model is higher than the baseline model.

The significance of the operating leverage channel as a key mechanism for generating a sizable equity premium also can be confirmed by the comparison with the standard RBC model of Hansen. As Figure 6, the operating leverage effect (the fixed wage income effect) is completely destroyed. The impulse responses of output and wage income are the same: the wage bill now expands and contracts exactly in tandem with output. Table 11 also shows that the output and the wage income also increase by the same amount, 1.94%. This demonstrates that in the Hansen model, the shares of income going to capital and to labour are equally risky and there is no mechanism for shifting risk between capitalists and labourers. Indeed, distribution risk ϕ_t is equal to one in the Hansen model and changes in the balance of the bargaining power between capitalists and labourers are completely absent. The absence of the fixed wage income effect then dampens substantially the residual payment to capitalists both during expansion and in recession; in fact, the dividend decreases significantly by 8.62% in response to a positive productivity shock and it is countercyclical in the end. In other words, the persistent procyclicality of the dividend (the residual payment to capitalists) is completely absent in the Hansen model.

According to the impulse responses of the dividend and the marginal utility of capitalists in the

Figure 7: Dividend and marginal Utility: Hansen model



Hansen model (Figure 7), it is evident that the absence of the operating leverage effect implies the absence of a equity premium. Invoking the decomposition equation of the equity premium (34), Figure 7 shows that the second covariance term, $-cov_t(\hat{\lambda}^s_{t+1}, E_{t+1}\hat{\lambda}^s_{t+n} - \hat{\lambda}^s_{t+1})$, contributes negatively up to more than four years; the marginal utility λ^s jumps down initially but it is positively serially correlated up to more than four years, that is to say, $\hat{\lambda}^s_{t+1}$ exceeds $E_{t+1}\hat{\lambda}^s_{t+n}$. Therefore the overall contribution of the second covariance term may be positive but substantially less than in the model with the operating leverage effect.

Figure 7 also shows that the first covariance term, $-cov_t(\hat{\lambda}^s_{t+1}, E_{t+1}\hat{d}_{t+n})$, contributes negatively up to four years after shock. The dividend d is strongly countercyclical up to four years and thus it is positively serially correlated with $\hat{\lambda}^s_{t+1}$. The overall contribution of the first covariance term may be positive but much less so since the negative contribution of the first covariance up to four years is too strong.

In conclusion, we argue that the operating leverage effect (the fixed wage income effect) is the key to generating a substantial equity premium in our context; the Nash wage bargaining between capital owners and workers makes it possible for capital owners to provide workers with some insurance against their labour income variation in the form of countercyclical distribution risk ϕ_t and this risk-shifting mechanism leads to the operating leverage effect, which can account for both stylized financial statistics and the salient business cycle statistics in the labour market.

There are three main channels to accomplish a high equity premium in the existing literature[13]. First, the operating leverage channel with idiosyncratic distribution risk, as advocated by Danthine and Donaldson (2002) and in the same spirit of our model, postulates that capital owners provide nonshareholder-workers with insurance against the latter group's income variation; this risk-sharing mechanism creates a high level of volatility of shareholder consumption, while the shareholders can insure such high-level volatility only via management of the capital stock. As a result, the risk properties of the residual payments to firm owners are magnified and a substantial risk premium is achieved (the fourth column in Table).

Danthine and Donaldson (2002) entertains the hypothesis that observed countercyclical variations in labour income shares are the result of changes in uninsurable distribution risk, conceived as the balance between capital owners and workers in labour relations. By the countercyclical nature of distribution risk, low-productivity shocks coincide with the situations where the bargaining power of capitalists is low; in turn, the low residual payment to capitalists due to the low-productivity realization is further reduced by the increase of labour income shares. In contrast, high productivity shocks increase further the high residual payment to capitalists. Indeed, these distribution risk effects are equivalent to operating leverage, originating in the priority status of wage claims over the business cycle. Although both specifications amount to the same operating leverage effects, our specification of distribution risk ϕ_t differs from that of Danthine and Donaldson (2002) in two details. First, our distribution risk ϕ_t is determined endogenously by aggregate productivity shocks; distribution risk ϕ_t itself represents variations in the degree of imperfect risk sharing between capitalists (insiders) and workers (outsiders) over the business cycle. Second, as conjectured by Danthine and Donaldson (2002), distribution risk ϕ_t not only displays (strong) counter-cyclicality over the business cycle but also readjusts the relative bargaining power of workers in the Nash wage bargaining process.

The second channel originates from limited participation in the stock market and heterogeneity in the elasticity of intertemporal substitution in consumption. Shareholders participate in both stock and bond markets while more risk-averse nonshareholders trade only bonds. Since bond trading is their only mechanism for consumption smoothing, nonshareholders bid up bond prices, resulting in a low risk-free rate. In equilibrium, shareholders end up insuring non-shareholders by increasing their debt holdings exactly when a low-productivity realization reduces both agents' income. Again, bond market events serve to smooth the consumption of non-stockholders and amplify the volatility of shareholder consumption. As a result, stockholders demand a large premium for holding aggregate risk (Guvenen (2003)).

Third, this issue can be approached from the perspective of capital owners' strong smoothing motive (e.g. habit formation) rather than from the perspective of worker income insurance (the "habit cum

[13] As noted earlier, these leading macro-asset-pricing models abstract from the wedge between variations at the intensive margin and at the extensive margin. In other words, variations at the extensive margin are ignored in this class of the models.

cost-of-adjustment" channel). Capital owners who face a high level of volatility of marginal rates of substitution are prevented from smoothing their consumption due to frictional reallocations of capital stock and labour inputs (Jermann (1998) and Boldrin et al. (2001)).

Distribution risk and limited asset market participation Several distinguishing features of our endogenous distribution risk should be noted. First, the variations in our distribution risk ϕ_t over the business cycle is generated by aggregate productivity shocks; as a result, distribution risk ϕ_t itself represents variations in the degree of imperfect risk sharing between capitalists (insiders) and workers (outsiders) over the business cycle. As conjectured by Danthine and Donaldson (2002), distribution risk ϕ_t not only displays (strong) counter-cyclicality over the business cycle but also readjusts the relative bargaining power of workers in the Nash wage bargaining process. In fact, our endogenous distribution risk is exactly identified with the Nash bargaining power shock Shimer (2005) proposed (up to a first order approximation) in the framework of our Nash bargaining wage solution. Therefore, endogenous distribution risk is a key mechanism for resolving the unemployment volatility puzzle Shimer (2005) emphasized. Second, it should be noted that the source of our distribution risk originates from limited asset market participation of agents in this economy. In contrast, any representative agent models with search display the complete absence of our distribution risk since agents in those economies have full access to the financial markets (by definition). The absence of the distribution risk channel, however, will lead to the absence of the fixed wage income effect, resulting in the complete smoothing of the labour market volatility of the key variables including unemployment and job vacancies. Third, as Danthine and Donaldson (2002) emphasized, our endogenous distribution risk captures the idea of the operating leverage (the fixed wage income effect), originating in the priority status of wage claims over the business cycle. Low-productivity shocks coincide with the situations where the bargaining power of capitalists is low or the distribution risk is procyclical; in turn, the low residual payment to capitalists due to the low-productivity realization is further reduced by the increase of labour income shares. In contrast, high productivity shocks increase further the high residual payment to capitalists.

Table 12 illustrate those points discussed so far. First of all, we begin by standing the comparison with the Danthine-Donaldson model. As noted in the previous section, Danthine and Donaldson (2002) entertains the hypothesis that observed countercyclical variations in labour income shares are the result of changes in uninsurable distribution risk, conceived as the balance between capital owners and workers in labour relations. As Table 12 shows, the distribution risk ϕ_t in the Danthine-Donaldson model is calibrated to replicate the time series properties of the labour income share. In other words, the shock process on the idiosyncratic distribution risk is chosen to replicate the correlation between output and the labour income share found in the data; by the specified hypothesis, a negative correlation between idiosyncratic distribution risk and the economy-wide technology shock is chosen *a pirori* ($\rho(\phi_t, z_t) = -0.63$ in the benchmark case). In contrast, the variations of our distribution risk ϕ_t is generated by aggregate productivity shocks and our distribution risk ϕ_t is indeed countercyclical over the business cycle ($\rho(\phi_t, y_t) = -0.89$ in the baseline model). The model also can replicate the target correlation between output and the labour income share found in the data.

Table 12: Robustness: business cycles

	σ_y	σ_u/σ_y	σ_ν/σ_y	σ_θ/σ_y	$\rho(u,\nu)$	σ_{th}/σ_y	σ_h/σ_y	σ_w/σ_y	σ_ϕ/σ_y	$\rho(\phi,y)$	$\rho(ls,y)$
Data	1.59	6.92	8.27	13.62	−0.88	0.95	0.43	0.44	–	–	−0.053*
Model											
Limited part.											
Baseline	1.47	7.05	9.13	15.32	−0.79	0.91	0.44	0.25	5.38	−0.89	−0.03
Optimized	1.28	11.72	14.95	25.47	−0.82	1.02	0.23	0.195	10.51	−0.81	−0.09
No habit	1.45	5.82	7.57	12.66	−0.78	0.75	0.37	0.20	4.77	−0.95	−0.39
DD*	1.77	–	–	–	–	–	–	1.11	–	–	−0.07
Full part.											
SH model	1.20	0.675	2.58	3.01	−0.55	0.69	0.67	0.45	0	–	0.98
Hansen	1.76	–	–	–	–	0.77	0.77	0.28	0	–	–

Notes: DD stands for Danthine and Donaldson (2002). These statistics are drawn from Danthine et al. (2008).

Table 13: Robustness: financial statistics

	$E[R_t^f]$	$\sigma[R_t^f]$	$E[R_{t,t+1}^e]$	$\sigma[R_{t,t+1}^e]$	$E[R_{t,t+1}^e - R_t^f]$	$\sigma[R_{t,t+1}^e - R_t^f]$
Data	0.80	5.67	6.98	16.54	6.18	16.67
Model						
Limited part.						
Baseline	1.24	6.48	5.48	17.71	4.28	17.40
Optimized	0.11	5.98	4.96	17.47	4.89	17.00
No habit	2.205	2.706	3.82	8.33	1.61	7.77
Danthine-Donaldson	2.46	4.05	5.92	22.20	3.46	22.34
Full part.						
SH model	1.93	5.04	4.65	13.93	2.76	14.44
Hansen	4.10	0.37	4.11	0.52	0.01	–

To analyze the effect of our distribution risk and the interaction between our distribution risk and limited asset market participation, we compare our baseline model with a business cycle model with search that allows agents to have the full access to the financial market, i.e. in which there is no limited asset market participation: namely, the search with segmented labour markets (called "SH Model")[14]. This search model is the representative agent (family) model that shares with our baseline model the preference specification of agents (a hybrid of GHH preference and external habit formation), the presence of two segmented labour markets including search-matching labour market and the capital

[14] This search model is analyzed in Keam (2008) but in the appendix we provide the details of the model.

accumulation technology with adjustment cost *except the limited asset market participation*. The model also can generate a sizable equity premium. Therefore, the model is a good reference point to analyze the effect of limited asset market participation in our context[15].

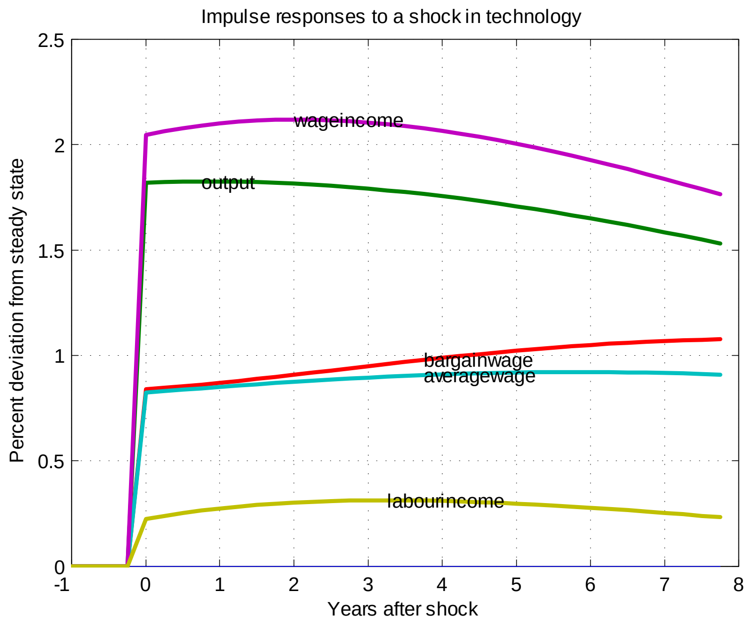
As Table 12 indicates, distribution risk ϕ_t in the baseline model not only displays strong counter-cyclicality but also a high level of volatility over the business cycle; the contemporaneous correlation of distribution risk with output is -0.89 and the relative standard deviation of distribution risk is 5.38. The counter-cyclicality of the distribution risk ϕ_t is translated into low-amplitude wage variation. In addition, extremely volatile variation at the extensive margin is achieved due to capitalists' strong incentive to smooth their own consumption, a fact which leads to the resolution of the unemployment volatility puzzle.

In contrast, note that in the SH model, distribution risk ϕ_t is identical to one since agents have the full access to the financial markets. The SH model in Table 12 shows that if distribution risk ϕ_t is completely absent, labour market volatility is severely diminished: the relative standard deviation of unemployment is 0.675, one-tenth as volatile as its empirical counterpart (6.92). The relative standard deviations of job vacancies and market tightness severely fall short of their empirical counterparts; they just can account for 20-25 % of the volatilities of the corresponding variables found in the data. More interestingly, with full participation to the financial market, the distinction between variations at the intensive margin and at the extensive margin also disappears; the relative standard deviation of total hours is 0.69 while the relative standard deviation of hours per worker is 0.67. The complete absence of the workers' bargaining power shock in the SH model coincides with situations where the fixed wage income effect completely disappears; capitalists now opt for more variations at the intensive margin rather than variations at the intensive margin. In fact, hours per workers are more reliable labour inputs because labour hours can be utilized without delay to produce output; in contrast, variations at the extensive margin are costly in the sense that the hired employment size can be utilized only next period. Moreover, variations at the intensive margin are further amplified by the relatively low-amplitude wage resulting from the Nash wage bargaining process.

Additional insight into the effect of limited participation can be obtained by examining the SH model's impulse response functions or by estimating how a positive 1% productivity shock affects the key decision variables in that model. Figure 8 shows that the operating leverage effect in the SH model completely disappears. The wage income varies even more than output and the labour income share is counterfactually procyclical. Table 14 also highlights the lack of the fixed wage income effect in the SH model. In response to a 1% positive productivity shock, the total wage income increases up by 2.05%, which is even greater than the increase of output by 1.82%. This dramatic disappearance of the fixed

[15] It deserves being mentioned that a direct representation of the baseline model without limited asset market participation is not trivial; this results from the definition of Nash bargaining wage between capitalists and workers, which requires that the firm's crucial intertemporal decisons be all in accord with the intertemporal marginal rate of substitution of the capitalists only.

Figure 8: Operating leverage: SH model



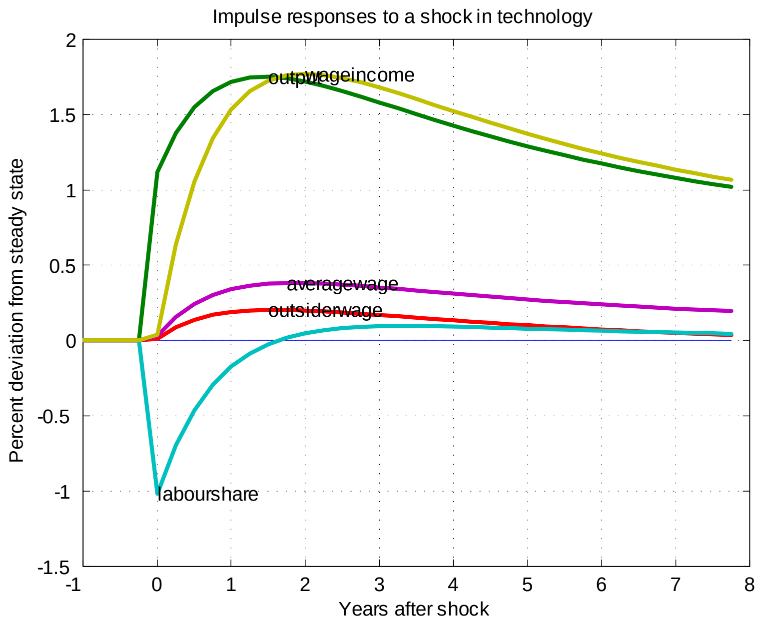
wage income effect is mainly due to a higher increase of hours per worker, not due to more flexible changes of wage; hours per worker in the SH model increase by 1.225%, which is 10 times greater than the increase of hours per worker in the baseline model. The greater variations at the intensive margin finally dampens variations at the extensive margin; indeed, job vacancies merely increase by 4.65%, which is in stark contrast to the increase of job vacancies by 11.706% in the baseline model.

Table 14: Robustness: elasticities

Model	η_{yz}	$\eta_{\lambda^s z}$	η_{wz}	$\eta_{w^n z}$	η_{hz}	$\eta_{\nu z}$	η_{wiz}	η_{lsz}	η_{dz}	$\eta_{\phi z}$
Baseline	1.07%	−19.84%	−0.001%	−0.02%	0.12%	11.706%	−0.07%	−1.08%	3.78%	−10.23%
Optimized	0.96%	−27.05%	−0.05%	−0.07%	−0.05%	17.05%	−0.34%	−1.21%	0.33%	−20.07%
No habit	1.12%	−13.44%	0.03%	0.01%	0.19%	8.41%	0.04%	−1.02%	0.83%	−7.54%
SH model	1.82%	−5.62%	0.825%	0.84%	1.225%	4.65%	2.05%	0.225%	−0.18%	0%
Hansen	1.94%	−0.47%	0.47%	–	1.37%	–	1.94%	–	−8.62%	0%

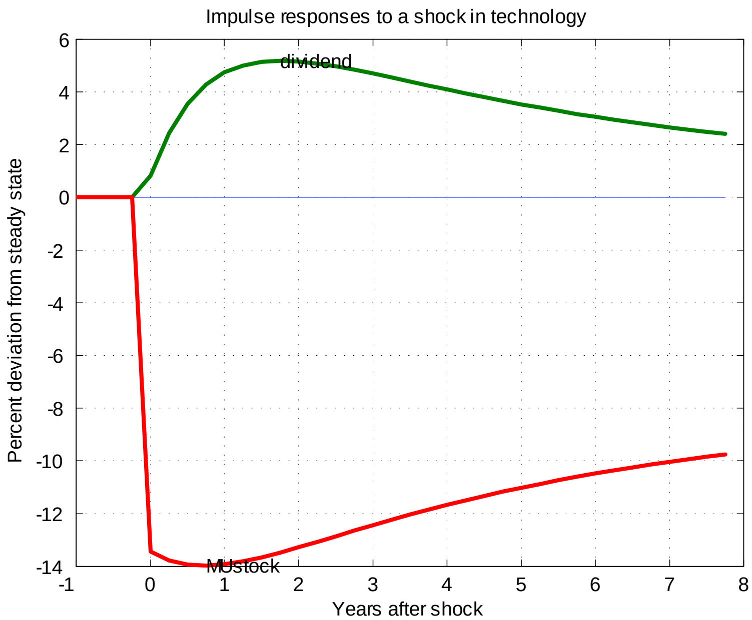
This confirms that the Nash bargaining power shock (distribution risk) is a powerful channel to resolve the unemployment volatility puzzle as Shimer (2005) indicated. It also highlights the importance of limited asset market participation; with full participation in the financial market, the impact of the distribution risk is severely damped, resulting in the counterfactually smooth labour market variables.

Figure 9: Operating leverage: the baseline model without habit



Last, to illustrate how this asymmetry in consumption smoothing opportunities due to limited asset market participation alone affects the volatilities in the labour market, we eliminate the habit formation specification of the insider-stockholder group (i.e. $\chi = 0$). As the impulse response functions from the baseline model without habit indicate (Figure 9), the operating leverage effect is still preserved; the wage bills vary less than output, falling proportionately less in recessions and increasing less during expansions. The risk of the firm's free cash flow and derived dividends tends to be more procyclical. As a result, the baseline model without habit still can capture the excessive volatility of the labour market activity found in the data, although the model only accounts for 90% of the labour market volatility. As Table 14 shows, in response to a 1% positive productivity shock, the distribution risk just falls by 7.54%, which is 30% smaller than the corresponding variations of the distribution risk in the baseline model and the marginal utility of capitalists just falls by 13.44% while in the baseline model, the corresponding marginal utility falls by 19.84%. This means that the fixed wage income effect via the Nash bargaining wage with the countercyclical distribution risk (the workers' bargaining power shock) is weaker and the capitalists' incentive to smooth their consumption is slightly compromised due to the absence of the habit formation. Capitalists now prefer slightly weaker variations at the extensive margin, building up the labour stock of outsiders slightly less than they do in the baseline model. The overall consequences are the peak of output is at most 1.7% deviations from the steady state of output and the derived residual payment to capitalists is much smoother as Figure 10 shows; the peak of the derived dividend is

Figure 10: Dividend and marginal Utility: the baseline model without habit



reached at about 6% deviations while the corresponding peak is reached at more than 10% deviations. In contrast, the peak of output in the baseline model is 2.25% deviations from the steady state of output and output is more persistent and the residual payment to capitalists is also more persistent.

In sum, we conclude that the effect of the habit formation of capitalists on the labour market volatility of interest is limited. Then the remaining question is whether the habit formation is indeed a necessary ingredient for our operating leverage effect. The answer can be found in Table 13. The baseline model without habit formation can achieve only a relatively modest replication of a equity premium: 1.61% per annum. As observed in the previous analysis, the effect of much smoother dividend on the labour market volatility may be limited but its impact on the financial front cannot be ignored; the baseline model can replicate a equity premium of 4.28%. More interestingly, the incorporation of the habit formation feature makes the aggregate EIS implied from the model consistent with Hall's empirical findings: Hall (1988) estimates the aggregate EIS close to zero. Indeed, the model-implied aggregate EIS in the baseline model with habit is 0.0396. This low EIS seems to be consistent with an upward (real) term structure as Table shows. In contrast, when the habit formation feature is eliminated all else equal, the model-implied aggregate EIS is 0.1380, slightly bigger than in the baseline model, but its implication for the term structure is quite different; now the real term structure is U-shaped (Table 15). As noted earlier, it is difficult to assess the plausibility of the positive slope of the real term structure without a long sample on real yields for the United States. Nevertheless, the real yield curve has been mostly upward sloping based on the raw TIPS data recently.

Table 15: Robustness: term structure

Model	$CRRA$	EIS	Maturity						
			2	3	4	8	12	16	20
Baseline	3.6	0.0396	1.26	1.32	1.39	1.76	2.10	2.37	2.57
No habit	3.6	0.1380	2.156	2.125	2.106	2.125	2.19	2.27	2.36
Hansen	1	1	4.0387	4.0385	4.0383	4.0375	4.0370	4.0368	4.0366

Note: maturity is in quarters.

5 Concluding Remarks

In response to a mediocre empirical performance by the conventional model of unemployment dynamics due to Mortensen and Pissarides, a recent body of studies has emphasized the importance of wage rigidity in accounting for observed volatility in key variables characterizing labour market activity over the business cycle. In this paper, we extend the Mortensen and Pissarides model to the environment where the asset market is incomplete and perfect risk-sharing between capital owners and workers cannot be guaranteed. We develop period-by-period Nash wage bargaining between capitalists and workers in a macro model with two key features, namely, limited participation in the stock market and labour-force heterogeneity (insider-outsider labour relation). What emerges from this consideration is that the (short-run) operating leverage effect, a concept known in the traditional business cycle literature, is the key factor in overcoming the unemployment volatility puzzle emphasized by Shimer (2005) and Hall (2005). Specifically, the fully endogenized Nash bargaining power shock, called distribution risk, plays a key role in generating the operating leverage effect in our context; indeed, our distribution risk is exactly the same as the Nash bargaining power shock considered by Shimer without invoking any sources of it (Shimer, 2005). Hence, our model can be viewed as an attempt to provide a microfoundation for the Shimer's *ad hoc* Nash bargaining power shock. Moreover, our operating leverage effect is the very same mechanism for generating a sizable equity premium. We then show that a reasonable calibration of the resulting model, which takes into account the stylized financial statistics, accounts well not only for aggregate fluctuations in unemployment and vacancies but also for the observed wedge between variations at the intensive margin (hours per worker) and at the extensive margin (total hours) over the business cycle. In turn, the model also achieves a satisfactory replication of the major financial stylized facts: a low risk-free rate, an upward-sloping real term structure, and a substantial equity premium. In contrast to existing leading macro-asset-pricing models, the model is unique in the sense that without compromising the overall performance on the financial front, it can fully reproduce the stylized business cycle facts of the labour market activity.

The next research avenue is to incorporate "money" into our baseline model; indeed, our model has some similarities with Cooley and Quadrini (1999), although their model abstracts completely from investment decisions and the financial front. The incorporation of money into our model will makes it possible to analyze important qualitative and quantitative features of labour markets and the Phillips curve relation. It also will clarify the more fundamental role of risk premium in generating monetary business cycles as hinted by Smets and Wouters (2003).

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A Model Solution: Asset Pricing

Asset Pricing Let $1+R_{t+1}$ be the gross return on an asset held from period t to period $t+1$. If the price and the cash flow of the asset in period t are denoted by P_t and F_t, respectively, then

$$1+R_{t+1} = \frac{P_{t+1}+F_{t+1}}{P_t}. \tag{35}$$

The Arrow-Lucas-Rubinstein asset pricing equation requires that any asset with (35) must satisfy

$$1 = E_t[\beta \frac{\lambda_{t+1}}{\lambda_t}(1+R_{t+1})]. \tag{36}$$

Equivalently, we can rewrite (36) as follows:

$$0 = \log\beta + \log E_t[\exp(\hat{\lambda}_{t+1} - \hat{\lambda}_t + r_{t+1})] \tag{37}$$

where $\hat{\lambda}_t$ is the log-deviation of marginal utility of consumption from its steady state value and $r_{t+1} \equiv \log(1+R_{t+1})$. Assuming that λ_{t+1} and $1+R_{t+1}$ are jointly lognormally distributed and using the standard formula for the expectation of lognormally distributed variables, equation (37) can be written as:

$$0 = \log\beta + E_t[\hat{\lambda}_{t+1} - \hat{\lambda}_t] + E_t[r_{t+1}] + \frac{1}{2}[\sigma^2_{\lambda t} + \sigma^2_{rt} + 2\rho_{\lambda rt}\sigma_{\lambda t}\sigma_{rt}] \tag{38}$$

where $\sigma^2_{\lambda t} \equiv Var_t[\hat{\lambda}_{t+1} - \hat{\lambda}_t] = E_t[(\hat{\lambda}_{t+1} - E_t\hat{\lambda}_{t+1})^2]$, $\sigma^2_{rt} \equiv Var_t[r_{t+1}] = E_t[(r_{t+1} - E_t r_{t+1})^2]$, and $\rho_{\lambda rt}$ is the conditional correlation, i.e. $\rho_{\lambda rt}\sigma_{\lambda t}\sigma_{rt} \equiv Cov_t[(\hat{\lambda}_{t+1} - \hat{\lambda}_t), r_{t+1}]$.

Risk-free Rate A risk-free asset (one quarter real bond) with the risk-free rate $r^f_t \equiv \log(1+R^f_t)$ can be priced in much simpler way. Since $\sigma^2_{rt} = E_t[(r^f_{t+1} - E_t r^f_{t+1})^2] = 0$, we have

$$r^f_t = -\log\beta - E_t[\hat{\lambda}_{t+1} - \hat{\lambda}_t] - \frac{1}{2}\sigma^2_{\lambda t}. \tag{39}$$

Then the simple risk-free rate is given by

$$1+R^f_t = \exp r^f_t.$$

The unconditional moments of the simple risk-free rate can be calculated using the log-normal formula for the unconditional expectation:

$$\begin{aligned} E[1+R^f_t] &= \exp(E[r^f_t] + \frac{1}{2}Var[r^f_t]) \\ Var[R^f_t] &= Var[1+R^f_t] = \exp(2E[r^f_t] + 2Var[r^f_t]) - \exp(2E[r^f_t] + Var[r^f_t]). \end{aligned}$$

The unconditional moments of r_t^f are given by

$$\begin{aligned} E[r_t^f] &= -\log\beta - \frac{1}{2}Var[\hat{\lambda}_{t+1} - E_t\hat{\lambda}_{t+1}] \\ Var[r_{t,n}^f] &= Var[E_t[\hat{\lambda}_{t+1} - \hat{\lambda}_t]]. \end{aligned}$$

Term Structure For the calculation of the conditional term structure $\{R_{t,n}^f\}$, we can apply the same log-linear and log-normal framework as the risk-free rate case. Note that the conditional term structure $\{R_{t,n}^f\}$ can be represented by

$$1 + R_{t,n}^f = [\beta^n E_t[\frac{\lambda_{t+n}}{\lambda_t}]]^{-1/n}.$$

Denote the quarterly continuously-compounded yield of a n-period real bond by $r_{t,n}^f \equiv \log 1 + R_{t,n}^f$. Again using the standard formula for the expectation of lognormally distributed variables, we have

$$\begin{aligned} r_{t,n}^f &= -\frac{1}{n}\log[\beta^n E_t[\frac{\lambda_{t+n}}{\lambda_t}]] \\ &= -\log\beta - \frac{1}{n}\log E_t[\exp(\hat{\lambda}_{t+n} - \hat{\lambda}_t)] \\ &= -\log\beta - \frac{1}{n}[E_t[\hat{\lambda}_{t+n} - \hat{\lambda}_t] + \frac{1}{2}\sigma_{\lambda t}^2]. \end{aligned}$$

The simple bond yield is given by

$$1 + R_{t,n}^f = \exp r_{t,n}^f.$$

The unconditional moments of the simple bond yield can be calculated using the log-normal formula for the unconditional expectation:

$$\begin{aligned} E[1 + R_{t,n}^f] &= \exp(E[r_{t,n}^f] + \frac{1}{2}Var[r_{t,n}^f]) \\ Var[R_{t,n}^f] &= Var[1 + R_{t,n}^f] = \exp(2E[r_{t,n}^f] + 2Var[r_{t,n}^f]) - \exp(2E[r_{t,n}^f] + Var[r_{t,n}^f]). \end{aligned}$$

The unconditional moments of $r_{t,n}^f$ are given by

$$\begin{aligned} E[r_{t,n}^f] &= -\log\beta - \frac{1}{2n}Var[\hat{\lambda}_{t+n} - E_t\hat{\lambda}_{t+n}] \\ Var[r_{t,n}^f] &= \frac{1}{n^2}Var[E_t[\hat{\lambda}_{t+n} - \hat{\lambda}_t]]. \end{aligned}$$

Equity To calculate the equity returns, we adopt a slightly different strategy. The Arrow-Lucas-Rubinstein asset pricing equation tells us that the period t equity price p_t^e must equal the present value of all future dividends discounted by the pricing kernel:

$$p_t^e = E_t[\sum_{k=1}^{\infty} \beta^k \frac{\lambda_{t+k}}{\lambda_t} d_{t+k}]$$

where $\frac{\lambda_{t+k}}{\lambda_t}$ is the stochastic discount factor of insider-stockholders due to the presumed limited participation in the stock market.

Note that equivalently, the period t equity price p_t^e can be written as:

$$\begin{aligned} p_t^e &= \sum_{k=1}^{\infty} E_t[\beta^k \frac{\lambda_{t+k}}{\lambda_t} d_{t+k}] \\ &= \sum_{k=1}^{\infty} E_t[\beta^k \frac{\lambda_{t+k}}{\lambda_t} \frac{d_{t+k}}{d_t} d_t] \\ &= \sum_{k=1}^{\infty} E_t[\beta^k \exp(\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k} - \hat{d}_t) d_t] \end{aligned} \tag{40}$$

where $\hat{d}_t$ is the log-deviation of dividend from its steady state value.

Using equation (40), the simple quarterly equity return is given by

$$\begin{aligned} 1 + R_{t,t+1}^e &= \frac{p_{t+1}^e + d_{t+1}}{p_t^e} \\ &= \frac{[\sum_{k=1}^{\infty} E_{t+1}[\beta^k \exp(\hat{\lambda}_{t+1+k} - \hat{\lambda}_{t+1} + \hat{d}_{t+1+k} - \hat{d}_{t+1}) d_{t+1}]] + d_{t+1}}{\sum_{k=1}^{\infty} E_t[\beta^k \exp(\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k} - \hat{d}_t) d_t]} \\ &= \frac{[\sum_{k=1}^{\infty} E_{t+1}[\beta^k \exp(\hat{\lambda}_{t+1+k} - \hat{\lambda}_{t+1} + \hat{d}_{t+1+k})]] + \exp(\hat{d}_{t+1})}{\sum_{k=1}^{\infty} E_t[\beta^k \exp(\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k})]} \end{aligned}$$

Applying the standard log-normal formula to the random variables $\{ \hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k}\}_{k=1}^{\infty}$, each conditional expectation term can be written as:

$$\begin{aligned} & E_t[\exp(\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k})] \\ = \; & \exp[E_t[\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k}] + \frac{1}{2} Var_t[\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k}]. \end{aligned}$$

Both terms, $E_t[\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k}]$ and $Var_t[\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k}]$, respectively can be computed and then we approximate $1 + R^e_{t,t+1}$ by

$$\frac{[\sum_{k=1}^{n} E_{t+1}[\beta^k \exp(\hat{\lambda}_{t+1+k} - \hat{\lambda}_{t+1} + \hat{d}_{t+1+k})]] + \exp(\hat{d}_{t+1})}{\sum_{k=1}^{n} E_t[\beta^k \exp(\hat{\lambda}_{t+k} - \hat{\lambda}_t + \hat{d}_{t+k})]}$$

for sufficiently large number n.

B SH Model

As noted in the main text, the SH model is the representative agent (family) model that shares with the baseline model the preference specification of agents (a hybrid of GHH preference and external habit formation), the presence of two segmented labour markets including search-matching labour market and the capital accumulation technology with adjustment cost *except the limited asset market participation*; the model also can generate a sizable equity premium. The SH model is one of the most tractable presentations of the baseline model in the case of full asset market participation. Greater details can be found in Kim (2008a) so we will be brief here.

Households Following Merz (1995), each household is viewed as a large extended family which contains a continuum of family members uniformly distributed on a set of Lebesgue measure 1. Each family consists of primary workers of Lebesgue measure μ_p and outsiders of Lebesuge measure 1, whose employment status can vary according to matching friction; employed or unemployed. Primary workers are tenured workers who participate in a segmented market and make the leisure-labour choices; they only face variations at the intensive margin. Outsiders will participate in the Mortensen-Pissarides labour market with search. The family pools their financial and labour incomes before choosing per-capita consumption and asset holdings. Accordingly, given its information set $\Omega^h_t = \{z_t, n^d_t, n^w_t, k_t\}$, the representative family solves:

$$\begin{aligned} & V^h(\Omega^h_0) \qquad (41) \\ = & \max_{\{c_t, h^p_t, h^n_t, b_{t+1}, e_{t+1}\}} E_0 \sum_{t=0}^{\infty} \beta^t [v(c_t - X_t - \mu_p L(h^p_t) - n^w_t L(h^n_t) - (1 - n^w_t) L(0))] \\ & \text{s.t.} \end{aligned}$$

$$\begin{aligned} c_t + p^e_t e^s_{t+1} + p^f_t b^s_{t+1} &\leq w^p_t h^p_t + (p^e_t + d_t) e_t + w^n_t h^n_t n^w_t + b(1 - n^w_t) + b_t + T_t. \\ n^w_{t+1} &= (1 - \rho) n^w_t + s_t (1 - n^w_t) \\ n^d_{t+1} &= (1 - \rho) n^d_t + q_t \nu_t \end{aligned}$$

In the above problem, $v(\cdot)$ denotes their period utility function, $L(\cdot)$ is the disutility of labour function, and h_t^p and h_t^n are the period t labour hours supplied of primary workers and outsiders, respectively. The period utility function v is given by

$$v(c_t - X_t - \mu_p L(h_t^p) - n_t^w L(h_t^n) - (1-n_t^w)L(0))) = \frac{(c_t - X_t - \mu_p B_p (h_t^p)^\psi - n_t^w B_n (h_t^n)^\psi)^{1-\gamma} - 1}{1-\gamma}.$$

Hence the preference is a hybrid of external habit formation and GHH preference. X_t is the exogenous habit stock, evolving according to

$$X_t = \xi X_{t-1} + (1-\xi)\chi \bar{c}_{t-1}$$

where χ and ξ are some (habit) parameters of the representative family. Here, $\bar{c}_{t-1}$ denotes the average consumption level of the whole agents in the previous period:

$$\bar{c}_{t-1} \equiv \frac{1}{\int 1 d\varkappa} \int c_{t-1} d\varkappa$$

where $\varkappa$ stands for the measure of all households. d_t denotes the period t dividend payment by the firm to its stockholders; e_t and b_t, respectively, their period t stock and bond holdings; and the corresponding period t prices of these securities are p_t^e and p_t^f. w_t^p is the wage rate of primary workers, equal to their marginal products of labour. w_t^n is the outsider's wage determined through the contracting process in the labour market for outsiders; b represents unemployment benefits and T_t is lump sum transfers from the government. n_t^w is the number of employed outsiders and n_t^d is the firm's demand for workforce. in equilibrium, however, n_t^w and n_t^d are identical.

Firm The firm's problem is identical to the baseline model except some small changes of notations. Given its information set $\Omega_t^f = \{z_t, k_t, n_t^d\}$, the firm's problem reads as:

$$\max_{\{i_t, h_t^s, x_t\}} d_t + p_t^e \equiv d_t + E(\beta \Lambda_{t,t+1}^h (p_{t+1}^e + d_{t+1}) \mid \Omega_t^f) \tag{42}$$

$$\begin{aligned}
\text{s.t. } d_t &\equiv f(k_t, h_t^p, h_t^n n_t^d) z_t - i_t - w_t^p h_t^p - w_t^n h_t^n n_t^d - \frac{\kappa}{2} x_t^2 n_t^d - \varphi \bar{k} + p_t^f \varphi \bar{k} \\
k_{t+1} &= (1-\delta) k_t + G(\frac{i_t}{k_t}) k_t \\
n_{t+1}^d &= (1-\rho) n_t^d + q_t \nu_t
\end{aligned}$$

where $\Lambda_{t,t+1}^h \equiv \frac{\lambda_{t+1}^h}{\lambda_t^h}$ is the marginal rate of substitution of shareholders.

Nash bargaining The Nash wage bargaining between the firm and the outsiders can be formulated similarly. Note that the firm's crucial intertemporal decisions are all in accord with the intertemporal marginal rate of substitution of the shareholders; the agency problem between firm owners and managers is negligible. In the environment where there will be no corporate governance problem present, we can define the Nash bargainin g problem in the same way as in the baseline model:

$$\max_{w_t^n} (V_{nt}^s)^{1-\eta} \cdot (V_{nt}^n)^{\eta}$$

where η is the bargaining power parameter of outsiders viewed as exogenously given. V_{nt}^s and V_{nt}^n are given by

$$V_{nt}^s \equiv \frac{\partial V_t^h}{\partial n_t^d}$$
$$V_{nt}^n \equiv \frac{\partial V_t^h}{\partial n_t^w}$$

where $V_t^h \equiv V^h(\Omega_t^h)$.

The bargained hours of outsiders are determined by the same condition (23). The firm's shadow value J_t and the outsider's shadow value $W_t - U_t$ are exactly the same as those in the baseline model. It is readily shown that

$$V_{nt}^s = \lambda_t^h J_t$$
$$V_{nt}^n = \lambda_t^h (W_t - U_t)$$

and this implies that the Nash bargaining wage solution in the SH model is given by

$$w_t^n = (1-\eta)\frac{[L(h_t^n) + b - F_t^n]}{h_t^n} + \eta \frac{[h_t^n f_3(k_t, h_t^p, h_t^n n_t^d) z_t + \frac{\kappa}{2} x_t^2 + F_t^s]}{h_t^n}.$$

Note that the distribution risk ϕ_t completely disappears in the SH model.

Equilibrium The equilibrium concept of SH model is identical to that of the baseline model except the additional condition which requires that in equilibrium

$$n_t^w = n_t^d.$$