The role of durables replacement and second-hand markets in a business-cycle model∗

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Abstract

Transactions of used durables are large and cyclical, but their interaction with purchases of new durables has been neglected in the study of business cycles. I fill in this gap by introducing a new model of durables replacement and second-hand markets. The model generates a discretionary replacement demand function, it nests a standard business-cycle model of durables, and it verifies the Coase conjecture. The model delivers three conclusions: markups are smaller for goods that are more durable and more frequently replaced; markups are countercyclical for durables, resolving the comovement puzzle of Barsky, House, and Kimball (2007); and procyclical replacement demand amplifies durable spending.

Keywords: Durables; Business cycles; Countercyclical markup; Second-hand markets.

JEL Classification Numbers: E20, E30, E32.

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I Introduction

Standard models of business cycles and monetary policy abstract from durables and assume that agents consume only nondurables and services.\(^1\) While there have been many attempts to introduce durables in these models, none of them included an active second-hand market where used durables can be sold. The models assume that used-durable transactions play no value-added role in the market and in equilibrium, households consume their durables until they fully depreciate.\(^2\)

This paper introduces a new model for durable consumption with an active second-hand market, and shows that this feature significantly enhances the model to fit the data on new-durable purchases. Section II starts by documenting statistics for used durables, focusing on car sales data which is the largest and the most cyclical component of durable purchases. Used-durable purchases are a high portion of durable spending in the U.S. data. For example, about half of the size of new-durable purchases are net purchases of used durables in this category. Moreover, they are cyclical and highly volatile compared to both nondurable and durable consumption expenditures. In response to a 25 basis point increase in the interest rate, used car sales fall by 0.4 percent, compared to only 0.1 percent for nondurables. Finally, used car sales are quite procyclical, despite the general presumption that second-hand purchases rise during recessions.\(^3\)

Based on these observations, sections III and IV embed household resale of used durables and second-hand markets in an otherwise standard business-cycle model of durables as in Barsky, House, and Kimball (2007, BHK henceforth). On the household side, the model connects to the old idea of “discretionary replacement demand” for durables which has been claimed as a better empirical fit over the traditional stock-adjustment model (Westin (1975); Smith (1974)). On the firm side, the model generalizes the “Coase conjecture” by deriving a negative relation between markup and durability.

Section V highlights some of the key features that the model predicts: time-varying markups, negative relation between markups and durability, and the price-elasticity effect of cyclical replacement.

Sections VI and VII show that these features provide answers to two long-standing questions with regards to durables in business cycles. First, BHK point out that with relatively flexible prices in the durable sector, a standard new Keynesian model with durables does not generate comovement of durable and nondurable consumption expenditures with regards to a monetary shock. Second, Baxter (1996) notes that a standard business-cycle model with durables does not generate the high volatility of durable spending that we observe in the data.

\(^1\)Woodford (2003) gives a thorough analysis of these type of models.
\(^2\)Parker (2001) and Caplin and Leahy (2006) are exceptions since they consider durables replacement, but the models are partial equilibrium and the replaced goods are assumed to be scrapped.
\(^3\)“In tough times, auto-parts firms receive a countercyclical boost - Scrapped consumers buy used or fix the old.” Wall Street Journal, Feb 20, 2009.
In my model, cyclical replacement is the key channel. Replacements are procyclical due to a positive wealth effect and replacing durables provides future utility benefit to households. Since procyclical replacement results in higher second-hand market transactions during booms, there is higher competition in the market for durable production. Hence desired markups for durables are countercyclical, resolving the comovement puzzle. Moreover, procyclical replacement amplifies the volatility of durable spending.

Transactions in the second-hand market have expanded over time. Applying my model to the Great Moderation, I show that the expansion of the second-hand market would lead to a decline in the persistence of durable spending, which is consistent with Ramey and Vine (2006).

The other prediction of my model is the negative relation between price markups and durability. In section VIII, I test the validity of this prediction by conducting an industry-level empirical exercise using the NBER manufacturing productivity database. The data do not reject the prediction of my model. Lastly, section IX concludes.

That durability opens up households’ dynamic considerations and affects firms’ behavior in the presence of second-hand markets have been at the heart of the industrial organization literature. Hendel and Nevo (2006) explain the observed high elasticity of storable goods to temporary sales as evidence of consumers holding inventories. Schiraldi (2011) studies the consumer behavior of automobile replacement and show that transaction costs play a key role. For durable goods firms in the presence of second-hand markets, Swan (1980) analyzes the 1945 Alcoa case. More recently, Esteban and Shum (2007) analyze firm behavior in a used car market.

In the macroeconomic literature, however, second-hand markets have been mostly ignored since they were claimed to not affect the general equilibrium when all agents are identical. My model shows that this is only true when second-hand markets are assumed to have no value-added role when used goods are sold back to households, implying a zero margin for dealers. To the contrary, the data supports that this margin is not only high in levels, but also cyclical and highly volatile in its movements.

Parker (2001) is closest to my motivation where he studies the case when sellers have market power and buyers can time their purchases subject to search costs. This leads him to generate countercyclical markups with regards to demand-driven movements in sales. Besides being a partial equilibrium model, the economy is limited in two dimensions: consumers are not allowed to purchase nondurables and importantly, consumers cannot re-sell their used goods and must discard them when they make a replacement. My model is a general equilibrium allowing for consumer behavior along these dimensions.

Carlstrom and Fuerst (2006) suggest an alternative solution to the comovement puzzle by assuming equal

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4 Although specific to his model, Bernanke (1983) notes the following: Second-hand markets are important only if beliefs or preferences are so heterogeneous that there is no agreement on what constitutes good or bad news for a particular investment.
wage stickiness in both sectors.\textsuperscript{5} Although being a plausible channel, the sectoral symmetry of wage rigidities is not well established in the literature. For example, a recent survey by Klenow and Malin (2011) explain that wage and price adjustments are likely to be synchronized, citing survey evidence showing a correlation between wage and price flexibility. Hence without further empirical evidence on the degree of wage stickiness in the two sectors, this story remains debatable. Moreover, due to price flexibility in the durables sector, their model predicts a countercyclical response of real wages, which contrasts the mildly procyclical response documented in empirical studies.\textsuperscript{6}

II Some facts on used durables

This section documents business-cycle facts on the movements in the second-hand market for durables. Given its large second-hand market, I will focus on the motor vehicle industry.

A Used durable transactions are large

Time series for motor vehicle consumption are taken from the quarterly personal consumption expenditure (PCE) data in NIPA (Table 7.2.4B. and 7.2.5B.). Within PCE, durable spending consists of 4 categories: Motor vehicles and parts; Furnishing and durable household equipment; Recreational goods and vehicles; Other durables.\textsuperscript{7} The category ‘Motor vehicles and parts’ is historically the largest component of durable spending, consisting of 40\%, followed by furnishing (26\%), recreational goods (23\%), and the others (12\%). ‘Motor vehicles and parts’ consists of three subcategories: New vehicles; Net purchases of used vehicles; Motor vehicle parts and accessories. Net purchases of used vehicles consist of both dealers’ margin and net transactions from business and government to households. Figure 1 compares the composition of new and net used motor purchases relative to the overall durable spending. We observe that net used vehicle purchases are large. Since 1990, net used motor purchases are on average 11 percent of durable spending, while new motor purchases are 22 percent.\textsuperscript{8}

Looking into the actual number of sales of new and used vehicles, the high level of second-hand transactions is even more apparent. Figure 2 plots the annual sales of new and used passenger vehicles from 1990

\textsuperscript{5}Carlstrom and Fuerst (2006) also have a section with credit constraints to solve the puzzle, but argue that wage stickiness is a more plausible resolution. Monacelli (2009) addresses the comovement puzzle by adding credit-constrained households into the model and making these households borrow from the patient savers. However, Sterk (2010) points out that Monacelli (2009)’s result is not robust to the degree of the price flexibility in the durable sector. In some cases, Monacelli (2009) might actually exacerbate the comovement puzzle compared to the model without credit frictions.

\textsuperscript{6}See p.220 of Woodford (2003) for the empirical response of real wages as well as other references.

\textsuperscript{7}Other durables includes 5 components: Jewelry and watches; Therapeutic appliances and equipment; Educational books; Luggage and similar personal items; Telephone and facsimile equipments.

\textsuperscript{8}Net purchases of used goods besides motor vehicles are not separately available in the PCE category, since they are combined together with new goods.
Transactions are much higher in the second-hand market compared to new vehicle sales.\textsuperscript{10}

\section*{B Replacement is procyclical}

Many observed purchases of consumer durables are motivated by the replacement of used goods. Based on the Survey of Consumer Finances, Aizcorbe, Starr, and Hickman (2003) argue that most of the demand for motor vehicles comes from replacement demand.\textsuperscript{11} Marketing studies also show that for many other consumer durables, the observed sales are mostly due to replacement purchases. Bayus (1988) presents evidence that replacement sales of refrigerators and washers are 88\% and 78\% respectively in the 1980s, and other items also show high levels of replacement. He further shows statistical results to claim that replacements are discretionary and not merely forced by product failure.

The recognition that postponable replacement of consumer durables might be an important source of business-cycle fluctuations goes back to the Great Depression (George (1939); Tippetts (1939)). However, there are limited studies in measuring the cyclicality of replacements. The most relevant is Greenspan and Cohen (1999) where they study the cyclical scrappage of automobiles. Netting out the physical or “built-in” scrappage of vehicles, Greenspan and Cohen (1999) construct a measure of cyclical scrappage and show that this measure is procyclical.

Although scrappage is one measure of replacement, cautions must be taken on it as well. Households need not scrap their vehicles when making a replacement, in particular if their automobile is relatively new and in good shape. They can rather sell the vehicle to a used auto dealer. At the same time, Aizcorbe, Starr, and Hickman (2003) argue that when vehicle replacements occur, more than half of the new purchases are used cars.\textsuperscript{12} Hence the cyclicality of replacement could also be measured by transaction activities in the second-hand market. In the next two sections, I check whether movements in the second-hand market are also procyclical, unconditionally and conditional on a monetary shock.

\section*{C Used durables are procyclical and volatile}

It is well recognized that the spending of durables is procyclical and highly volatile. Here I show that movements in the second-hand market for vehicles are procyclical and volatile as well.

Using the quarterly PCE and GDP data from NIPA, I HP-filter the series and provide business-cycle facts in table 1. The first column shows the share of each variable, and the second column provides the

\begin{itemize}
  \item \textsuperscript{9}This data is taken from the National Transportation Statistics 2011, issued by the Department of Transportation.
  \item \textsuperscript{10}Transactions of used vehicle sales include sales from franchised dealers, independent dealers, and casual sales. In 2011, the shares of each category are 35.5\%, 35.5\%, and 29\%, respectively.
  \item \textsuperscript{11}They show that the share of households owning or leasing vehicles are 89 percent between 1989 and 2001, and that the average number of vehicles per household remained below 2 during this period.
  \item \textsuperscript{12} Even amongst the top 25 household income percentile, they show that the share of used car purchases is above 40 percent.
\end{itemize}
correlation with output as a measure of cyclicality. The third and fourth columns are the standard deviation relative to output and the first order autocorrelation, respectively.

We confirm here that motor vehicles are highly volatile compared to overall durable spending. Importantly, net purchases of used motor vehicles are also highly volatile compared to overall durables. Focusing on its correlation with output, we observe that net purchases of used vehicles are also procyclical. Hence the notion that used vehicle sales are high during recessions does not hold in an absolute sense.

Within net purchases of used vehicles, the margin of used autos reflects more closely the value-added component of second-hand market transactions.\textsuperscript{13} The business-cycle features also remain for each type of margins. Margins for used vehicle transactions from household to business and business to households are both procyclical and highly volatile.

D Used durables decline with a contractionary monetary shock

To understand the conditional response of durables and motor vehicles to a monetary policy shock, I estimate a 5 variable VAR analysis for the period 1967Q1-2007Q4.\textsuperscript{14} The sectoral variables are decomposed from PCE, which include the logarithms of new motor, net purchases of used motor, durables net of motor, and lastly nondurables and services.\textsuperscript{15} To identify monetary policy shocks, I include the level of the end-of-quarter federal funds rate. VAR analysis is conducted with 4 lags and a constant, and monetary policy shocks are identified with a standard Cholesky decomposition.

The result is depicted in figure 3. With respect to a 25 basis-point innovation to the federal funds rate, we observe that all the sectoral consumption variables contract. New motor is the most sensitive, exhibiting a contraction twice as large compared to other components of durables and ten times large compared to nondurables and services. Moreover, new motor exhibits lower persistence relative to the other variables, since within 20 quarters it quickly returns back to trend. Importantly, net purchases of used motor also show a cyclical pattern that is similar to the other components of durables. Hence there seems to be less aggregate evidence that the high sensitivity of new motor purchases is due to the increasing demand for used motors during a bust. On the other hand, a monetary contraction implies both a reduction in new motor and net purchases of used motor.\textsuperscript{16}

\textsuperscript{13}Due to data availability, I neglect the margin related to government. Since government purchases of motor vehicles are on average only 4 percent of the total motor vehicle sales, this will not affect the analysis.
\textsuperscript{14}The most recent recession is excluded since monetary policy has been unconventional during this period.
\textsuperscript{15}‘Durables net of motor’ and ‘Nondurables and services’ are constructed by the Tornqvist method.
\textsuperscript{16}The qualitative results remain when using auto margins instead of net purchases of used motor as the variable for used car transactions. For robustness, I have also conducted analysis with different lags, different time periods, and different control variables. The results are not sensitive to different specifications along these three dimensions.
III Model: Consumers

This section extends a standard new Keynesian model with durables as in BHK by introducing household resale of used durables that is capable of reproducing the above business-cycle facts on second-hand market transactions.

A Household

An infinitely-lived representative household chooses \( \{C_t, B_t, D_t, D^N_t, s_t, H_t\} \) to maximize

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, D_t, H_t), \quad \text{subject to} \tag{1}
\]

\[
P_{c,t}C_t + N_t + \frac{1}{R_t} B_t \leq B_{t-1} + W_t H_t + R_t^K K + \Phi_t,
\]

\[
N_t = P_{d,t} D^N_t - P_{u,t} s_t (1 - \delta_{t-1}) D_{t-1} - ADJ_t,
\]

\[
D_t = (1 - s_t)(1 - \delta_{t-1}) D_{t-1} + D^N_t,
\]

and also a borrowing limit, taking as given the processes \( \{P_{c,t}, P_{d,t}, P_{u,t}, R_t, W_t, R^K_t, \Phi_t\} \), and initial values \( B_{-1}, D_{-1}, D^N_{-1} \).

Households derive utility from consuming nondurables \( (C_t) \) and a service flow proportional to the stock of durables \( (D_t) \), and disutility from hours worked \( (H_t) \). Hence the period utility function \( U \) is strictly increasing in the first and second arguments, and strictly decreasing in the third argument. It is also twice continuously differentiable and strictly concave. \( E_t \) is an expectations operator conditional on information available at time \( t \), and \( \beta \in (0, 1) \) is the subjective discount factor.

Equation (2) shows the household budget constraint. Income for households consists of maturing one-period nominal bonds \( B_{t-1} \), labor income \( W_t H_t \), rental payment of capital \( R^K_t K \) where capital is assumed to be fixed as in a standard new Keynesian model, and lump-sum nominal profits from firms \( \Phi_t \). Households purchase consumption goods \( P_{c,t}C_t \) and durable goods net of resales \( N_t \). Asset markets are complete so households can buy one-period nominal risk-free bonds \( B_t \) at a discounted price \( 1/R_t \) per unit.

The main focus of this paper is equation (3) which indicates net purchases of durable goods \( N_t \). It equals the difference between purchases of new durable goods \( P_{d,t} D^N_t \) and the income from selling a fraction \( s_t \) of the undepreciated used durable goods \( (1 - \delta_{t-1}) D_{t-1} \) at a price \( P_{u,t} \). I allow the depreciation rate \( \delta_{t-1} \) to be time-varying. There is also a convex adjustment cost of reselling used goods \( ADJ_t \). Contrary to standard models where households mechanically run down all their durable goods \( (D_t) \) once they make a purchase, in this model they are allowed to resell a portion of their undepreciated used durable stock every period. I
call \( s_t \) the replacement rate of used durable goods. When \( s_t = 0 \), households do not replace any of their durables which is equivalent to the set-up in BHK.

The durable stock accumulation equation (4) also reflects the fact that some portion of the stock is being replaced. Durable stock evolves as the sum of the unreplaced durable stock \((1 - s_t)(1 - \delta_{t-1})D_{t-1}\) and the purchase of new durable goods \(D^N_t\).

Finally, nondurable consumption \(C_t\) and purchases of new durables \(D^N_t\) are CES aggregates of varieties \(\{c_t(i)\}_{i \in [0,1]}\) and \(\{d^N_t(i)\}_{i \in [0,1]}\):

\[
C_t = \left( \int_0^1 c_t(i) \frac{\theta_c - 1}{\theta_c} \, di \right)^{\frac{\theta_c}{\theta_c - 1}} , \quad (5)
\]

\[
D^N_t = \left( \int_0^1 d^N_t(i) \frac{\theta_d - 1}{\theta_d} \, di \right)^{\frac{\theta_d}{\theta_d - 1}} . \quad (6)
\]

The assumption that households purchase and resell the bundled durable good is not crucial. I could assume alternatively that households purchase and resell each variety of the durable good. However, in that case, I assume that the replacement margin is identical for all varieties. The focus of this paper is on the cyclical properties of replacement rates, and allowing for heterogeneous replacement rates across durable varieties will be an interesting extension although not explored in this paper.

The two major departures from the standard model of durables are the non-constant depreciation schedule and adjustment costs for replacement. I discuss these separately in the following sections.

**B Replacement adjustment cost**

Adjustment costs for durable replacement enters the budget constraint. This cost captures the transaction cost of resales for used durables following the spirit of Stolyarov (2002). I assume the following quadratic adjustment-cost form:

\[
ADJ_t = P_{u,t} \xi s_t \left( \frac{s_t}{s_{t-1}} - 1 \right)^2 .
\]

In this form, \( \xi \) governs the quantitative degree of durable replacement. With \( \xi = 0 \), the model is assumed to have no adjustment cost of resales.

By adding adjustment costs of this form, I nest the case without replacement dynamics by setting \( \xi \) at infinity. In this case replacements occur at the steady state, but they are constant over time.
C Depreciation acceleration and replacement

I assume that the depreciation rate of a durable good depends on its vintage. New goods and used goods follow different depreciation schedules. This implies that the effective (or average) depreciation rate of the overall durable stock is not constant and depends on the average vintage. The effective depreciation rate at time $t$ is denoted as $\delta_{t-1}^{\text{eff}}$ and is a function of all the previous history of replacement and new durable purchases:

$$\delta_{t-1}^{\text{eff}} \equiv \delta(s_{t-1}, D_{N,t-1}; D_{-1}),$$

(7)

where $s_{t-1} = \{s_{t-1}, s_{t-2}, \cdots, s_0\}$, $D_{N,t-1} = \{D_{N,t-1}, D_{N,t-2}, \cdots, D_{N,0}\}$.

Importantly, the non-constant effective depreciation function is not an assumption but an equilibrium result due to different depreciation rates for vintages and endogenous replacement rates in the model.

For further analysis, I work with a quasi-geometric depreciation acceleration assumption studied extensively in Hassler et al. (2008). Following their argument, assume that new durables depreciate at a rate $\rho \delta_d$ where $\rho < 1$. After one-period usage, the new durable is labeled as used and follows a higher depreciation rate of $\delta_d$. Denoting $D^N_t$, $D^U_t$, and $D_t$ as the new, used, and total durables respectively, the following law of motion holds:

$$D^U_t = (1 - s_t)(1 - \delta_d)D^U_{t-1} + (1 - \rho \delta_d)D^N_{t-1},$$

$$D_t = D^U_t + D^N_t.$$

By netting out $D^U_t$, we can summarize the law of motion as the following stock accumulation function for durables:

$$D_t = (1 - s_t)(1 - \delta_{t-1}^{\text{eff}})D_{t-1} + D^N_t,$$

(8)

where $\delta_{t-1}^{\text{eff}} = \delta_d - \frac{\delta_d(1 - \rho)D^N_{t-1}}{D_{t-1}}$.

We verify that the effective depreciation rate $\delta_{t-1}^{\text{eff}}$ is a function of all previous histories of $s_t$ and $D^N_t$, by

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17See Hassler et al. (2008) for their references on empirical evidence of depreciation acceleration. The biggest problem of geometric depreciation for capital is that it implies a rapid decline in productive capacity at the beginning. Penson, Hughes, and Nelson (1977) argue that this is not the case and rather shows that depreciation is concave at the beginning. For automobiles in particular, see figure 2 of Greenspan and Cohen (1999) where they plot the aggregate stock of automobiles for each vintage. A very small reduction in stock is observed for the first several years, while in the later years the reduction in stock resembles a geometric depreciation assumption. If law of large numbers holds for aggregate scrappage and idiosyncratic depreciation of vehicles, then this shows that for automobiles a quasi-geometric depreciation rate is reasonable. On the other hand, Hassler et al. (2008) discuss that measuring depreciation based on price data is problematic since prices for used products reflect information as well as technology issues. These results tend to conclude that the highest depreciation occurs at the beginning, since prices fall immediately as soon as the durable good become used.
plugging in the previous history of $D_t$ recursively. For example, plugging in $D_{t-1}$ renders the following expression:

$$\delta_{t-1}^{\text{eff}} = \delta_d - \frac{\delta_d(1-\rho)D_t^N}{(1-s_{t-1})(1-\delta_d)D_{t-2} + \delta_d(1-\rho)D_{t-2}^N} + D_{t-1}^N.$$ 

By mathematical induction, the depreciation rate depends on all the lagged values of $s_t$ and $D_t^N$, which confirms (7). Moreover, it is easy to verify from the above expression that $\frac{\partial \delta_{t-1}^{\text{eff}}}{\partial s_{t-1}} < 0$ and $\frac{\partial \delta_{t-1}^{\text{eff}}}{\partial D_{t-1}^N} < 0$.

With quasi-geometric depreciation rates, the benefit of replacing used durables by new durables is defined as the lower depreciation of the aggregated durable stock that the households face in the future. This assumption makes it meaningful to introduce the replacement of durable stocks within a representative-agent framework. By replacing 1 unit of used durable with a new one, households maintain the same stock today, but obtain higher benefit tomorrow captured by the lower depreciation rate. This gain in utility by replacement decisions is distinguished from that of new purchases since new purchases also increase today’s utility.

### D Link to discretionary replacement demand

The model of replacement demand above is related to the literature on discretionary replacement demand. In the traditional stock-adjustment model with constant depreciation $\delta$, demand for durables ($D_t^N$) is written as follows:

$$D_t^N = \underbrace{(D_t - D_{t-1})}_{\text{stock adjustment}} + \underbrace{\delta D_{t-1}}_{\text{normal replacement demand}}. \quad (9)$$

Normal replacement demand is the demand for durables that makes up for the physical depreciation of used durables, while the stock adjustment term accounts for the net addition to stock. This equation is estimated by assuming the existence of some desired stock $D_t^*$ and partial adjustment towards the desired stock. Hence, in any given period, the stock-adjustment term is assumed as

$$D_t - D_{t-1} = k(D_t^* - D_{t-1}), \quad (10)$$

where $k \in [0, 1]$ is the parameter governing the partial adjustment towards it. The desired stock is typically assumed as a function of current income and prices of durables.
Equation (4) can also be expressed in this form:

\[
D^N_t = (D_t - D_{t-1}) + \delta_{t-1} D_{t-1} + s_t(1 - \delta_{t-1})D_{t-1}.
\] (11)

Under the standard case of no replacements \((s_t = 0)\) my model nests the stock-adjustment framework. However, with depreciation acceleration and positive replacements \((s_t > 0)\), there is also an additional term on the right hand side which represents discretionary replacement demand.

This extra term distinguishes the discretionary replacement demand model from the stock-adjustment framework. Using passenger car sales data, Westin (1975) shows that estimating (9) with additional discretionary terms (e.g. change in unemployment rate) on the right hand side fits better than the stock-adjustment model. Moreover, the contemporaneous terms included have a positive sign, whereas the lagged terms have a negative sign.

Deaton and Muellbauer (1980) discuss that this last property is the essential feature of the discretionary replacement model. In detail, the model explicitly considers the case when durable replacement is postponed or advanced due to economic conditions. For example, when there is an economic boom, the estimated result forecasts that beyond the pure demand for new durables and normal replacement, consumers also advance their replacement of used durables. The model also predicts that the advancement of replacement leads to a depressing effect on the purchase of durables in the near future.

Equation (11) has the same property as in Westin (1975) when the stock-adjustment term (10) is plugged in. In particular, when \(D^*_t\) is exogenous, a pure change in the replacement demand \(s_t\) that does not change the overall durable stock \((\Delta D_t = 0)\) leads to an increase in durable purchases today, but a decrease in the demand for durables in the next period. The important implication of discretionary replacement demand is that it lowers the persistence of durable spending. This feature will be further explored in section VII.

E  Optimal consumer behavior

Solving for the household optimization problem with quasi-geometric depreciation, we obtain equilibrium conditions for the processes \(\{C_t, H_t, B_t, D_t, D^N_t, s_t\}\) and a transversality condition for bonds. The full optimality conditions are standard and provided in the appendix. Here I focus on the following three optimal

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18A detailed discussion of the comparison between the stock adjustment model and the discretionary replacement model is found in Westin (1975), Smith (1974), and Deaton and Muellbauer (1980).

19This aspect also connects to Mian and Sufi (2010)’s recent findings where they show that the “cash-for-clunkers” program increased car replacements but due to this fact, car purchases were below-normal for the next several periods.

20The expressions are:

\[
\frac{\partial D^N_t}{\partial s_t} = (1 - \delta_{t-1})D_{t-1} > 0, \quad \frac{\partial D^N_{t+1}}{\partial s_t} = (1 - s_{t+1})D_t \frac{\partial \delta_t}{\partial s_t} < 0.
\]
choice of durables (stock/spending/replacement).

Denoting the shadow value of the durable stock as $\nu_t$ and the marginal utility of durables and nondurables as $U_d(t)$ and $U_c(t)$ respectively, the first condition comes from the optimal choice of durable stock:

$$
\nu_t = U_d(t) + \beta (1 - \delta_d) \mathbb{E}_t \left[ U_c(t + 1) \frac{P_{c,t+1}}{P_{c,t+1}} s_{t+1} + \nu_{t+1}(1 - s_{t+1}) \right].
$$

This condition states that the value of the durable stock is the combination of both the current marginal utility derived from the durable stock, and the expected future gain from the undepreciated durable stock. In turn, the expected future gain from the undepreciated durable stock consists of two parts: the market value of reselling the fraction $s_t$ of the durable stock at a price $P_{u,t+1}$, and the future shadow value of the durable stock that is not replaced.

The second condition comes from combining the optimal choice of new durable purchases with the hours choice:

$$
-U_h(t) \frac{P_{d,t}}{W_t} = \nu_t + \beta \delta_d (1 - \rho) \mathbb{E}_t \left[ U_c(t + 1) \frac{P_{u,t+1}}{P_{c,t+1}} s_{t+1} + \nu_{t+1}(1 - s_{t+1}) \right].
$$

The left hand side is the utility cost of purchasing an additional new durable by working an additional hour. The right hand side is the value of purchasing a new durable, which consists of two terms. The first term is the shadow value of the durable stock. By purchasing a new durable, there is an increase in the overall stock of durables and this benefits the household by the shadow value discussed above. The second term states the additional gain of purchasing a new durable good due to its low depreciation rate. Note that in a standard model with geometric depreciation, $\rho = 1$ and this term does not exist.

The third condition comes from the optimal choice of replacement rates. For simplicity of discussion, I abstract from replacement adjustment costs and set $\xi = 0$:

$$
U_c(t) \frac{P_{u,t}}{P_{c,t}} = \nu_t.
$$

In this condition, households set their replacement rate to equate the gain of reselling the good on the left hand side and the cost of giving up their durable stock on the right hand side.

Using these conditions, we can derive the following two equilibrium pricing dynamics of new and used
durables:

\[ P_{d,t} = P_{c,t} \frac{U_d(t)}{U_c(t)} + (1 - \rho \delta_d) \mathbb{E}_t \Lambda_{t,t+1} P_{u,t+1}, \]  

(15)

\[ P_{u,t} = P_{d,t} - \delta_d (1 - \rho) \mathbb{E}_t \Lambda_{t,t+1} P_{u,t+1}. \]  

(16)

The nominal stochastic discount factor is \( \Lambda_{t,t+1} = \beta U_c(t+1) P_{c,t} / U_c(t) P_{c,t+1} \).

Equation (15) shows the equilibrium pricing for new durable goods \( (P_{d,t}) \). The gain of purchasing a new durable good is the current marginal utility of durable consumption relative to nondurable consumption plus the future discounted price of this good net of depreciation. Note that depreciation is low for the new durable good. However, the good becomes used in the future, therefore the market valuation of this good is the used goods price \( P_{u,t} \).

Equation (16) describes the resale price of a used durable good as the price of the new durable good discounted by tomorrow’s benefit of the new durable good in terms of reducing the overall depreciation. If the degree of depreciation acceleration is high (when \( \rho \) is close to 0), then households are more willing to replace their used durables which drives its price down relative to new durables. In the case of a constant depreciation (\( \rho = 1 \)), I nest the standard model where the selling and buying price are equalized \( (P_{u,t} = P_{d,t}) \).

The underlying mechanism that generates optimal behavior for replacements can be easily seen by a perturbation argument. Suppose that the household sell 1 unit of used durable and replace it with 1 unit of new durable. Without convex adjustment costs, the cost of this transaction is \( P_{d,t} - P_{u,t} > 0 \). There is no current benefit in this transaction since this is a pure replacement \( (\Delta U_d(t) = 0) \). However, there is a future benefit of this transaction since the depreciation of the durable stock becomes lower \( (\Delta U_d(t+1) > 0) \). Forward-looking agents optimally balance the cost and benefit of replacing their durables.

IV Model: Firms

I move on to a detailed description of the supply side. It consists of two sectors: nondurable goods sector and durable goods sector. The durable goods sector consists of new goods producing sector and a second-hand sector.
A Nondurables

The intermediate nondurable sector is monopolistically competitive with a linearly-homogeneous production technology and facing the CES demand, derived from the consumer problem:

\[
\bar{c}_t(i) = Z_t F(k_{c,t}(i), h_{c,t}(i)),
\]

\[
\bar{c}_t(i) = \bar{C}_t \left( \frac{p_{c,t}(i)}{P_{c,t}} \right)^{-\theta_c}.
\]

For each firm \( i \), \( F(k, l) \) is the production function with capital \( k \) and labor \( l \). \( \bar{c} \) is their production level and \( \bar{C} \) is the aggregate demand for nondurables. \( Z \) is the productivity level that is common across all firms.

The nondurable sector is mostly standard and many aspects are symmetric to the durable counterpart. The steady state markup is:

\[
\frac{p^*_c(i)}{mc^c_{n}} = \frac{\theta_c}{\theta_c - 1},
\]

where \( p^*_c(i) \), \( mc^c_{n} \) are the price and nominal marginal cost of the nondurable sector, respectively. Note that the markup is constant and falls with the elasticity of substitution \( \theta_c \).

There is also a competitive retail sector that bundles the good by the CES aggregator with elasticity of substitution \( \theta_c \) and sells it to the household. With Calvo sticky prices, the new Keynesian Phillips curve is

\[
\hat{\pi}_{c,t} = \beta \mathbb{E}_t \hat{\pi}_{c,t+1} + \frac{(1 - \alpha)(1 - \alpha\beta)}{\alpha} \tilde{m}c_{c,t},
\]

where \( \pi_{c,t} \equiv P_{c,t}/P_{c,t-1} \) is the gross rate of inflation from period \( t-1 \) to \( t \), \( \alpha \) is the price stickiness parameter for nondurables, and \( mc_{c,t} \) is the real marginal cost for nondurables. Hatted variables represent log deviations from the noninflationary steady state.

B Durables: New goods

The durable goods production sector is composed of new goods producing firms and a second-hand sector. I start with the new goods producing firms.

The production function of each variety in the new durable good sector is symmetric to that of the nondurables sector:

\[
x_t(i) = Z_t F(k_{d,t}(i), h_{d,t}(i)).
\]
Since the production function exhibits CRS with perfectly competitive factor markets, the firm’s total cost of production is linear in the level of production. Hence the period-\(t\) nominal profit of each firm \(i\), \(\Phi_{d,t}(i)\), is the following:

\[
\Phi_{d,t}(i) = [p_{d,t}(i) - mc_{d,t}^n]x_t(i),
\]

where \(mc_{d,t}^n\) is the common nominal marginal cost of all new durable producing firms.

Firms set prices and production subject to a demand function. The two key assumptions I impose are that firms are not able to control the second-hand market and that firms are not able to credibly commit to its future production. This leads to a static demand function that depends also on its interaction with the second-hand market, which I now discuss.

C  Durables: Second-hand

The second-hand sector engages in three activities: purchasing used durables from households; refurbishing the used durable into a new one; selling back the used durable varieties. I assume that the second-hand sector consists of second-hand firms and second-hand retailers, where the former engage in the first two activities, and the latter the last one.

C.1  Second-hand firms

Second-hand firms purchase used goods from households and refurbish them to sell to second-hand retailers. In detail, second-hand firms purchase the composite used durable \(M_t\) from households at a price \(P_{u,t}\) per unit. They refurbish this good and sell it back to the retailer at a price \(P_{m,t}\). The overall process of purchasing and refurbishing the used durables bears service activities \(f(M_t)\), which costs \(P_{c,t}\) per unit.\(^{21}\)

Therefore, a second-hand firm’s nominal profit \(\Phi_{u,t}\) is:

\[
\Phi_{u,t} = P_{m,t}M_t - P_{u,t}M_t - P_{c,t}f(M_t).
\]  \(\text{(19)}\)

Each firm chooses \(M_t\) that maximizes its profit. Besides taking \(P_{c,t}\) as given, a firm also takes \(P_{m,t}\) as given under a perfectly competitive output market. The equilibrium purchasing price is determined as:

\[
P_{u,t} = P_{m,t} - P_{c,t}f'(M_t) - \frac{\partial P_{u,t}}{\partial M_t}M_t.
\]

\(^{21}\)These service activities may represent not only the search and marketing activities that the dealer has to provide in purchasing the used durable, but also the cost of refurbishing the goods.
For further analysis, it is convenient to assume a functional form for $f(\cdot)$. I assume that $f(M_t) = \epsilon M_t$. Under this assumption, $\epsilon$ summarizes the value-added role of a second-hand firm in refurbishing its used durable. If $\epsilon$ is high, a second-hand firm is required to consume a large amount of resources to refurbish its purchased used durable. I call this parameter the value-added parameter.

The determination of $P_{u,t}$ depends on the market structure for purchasing used durables. To be robust, I assume two extreme market structures. First, second-hand firms might be perfectly competitive in purchasing used durables. In this case, they take $P_{u,t}$ as given by assuming $\partial P_{u,t} / \partial M_t = 0$.

Second, each second-hand firm might have monopsony power in purchasing used durables. With monopsony power, each firm recognizes that its choice of $M_t$ will also change $P_{u,t}$. In the appendix I show that when the market structure for purchasing used durables exhibits monopsony, a well-defined supply function for used durables is derived by combining the durable accumulation function with the two household equilibrium conditions for durable purchases and resales. This supply function is shown to be well-behaved in the sense that its slope is strictly positive under a general class of utility functions for durables.

Proposition 1 summarizes the pricing of used durables $P_{u,t}$ under the two different market structures assumed.

**Proposition 1 (Equilibrium purchasing price of used durables)** A second-hand firm chooses $M_t$ to maximize (19), taking as given $P_{m,t}$ and $P_{c,t}$, and with $f(M_t) = \epsilon M_t$. If the market for purchasing used durables is competitive, then the equilibrium purchasing price $P_{u,t}$ is

$$P_{u,t} = P_{m,t} - P_{c,t} \epsilon.$$

On the other hand, if the second-hand firm holds monopsony power in the purchasing market by internalizing (8), (15), and (16), then the equilibrium purchasing price is

$$P_{u,t} = P_{m,t} - P_{c,t} \epsilon_{m,t},$$

where $\epsilon_{m,t} = \epsilon - \delta(1-\rho_U) \frac{U_{dd}(C_t,D_t)}{U_c(C_t,D_t)} M_t$. Note that $\epsilon_{m,t} > \epsilon$.

The term $(P_{m,t} - P_{u,t})/P_{c,t}$ is the difference between the selling and purchasing price of used durables in units of nondurable goods. I call this the real margin for second-hand firms. When the market is perfectly competitive, the real margin equates the value-added parameter $\epsilon$. In other words, all the real margin of the second-hand firm is coming from their value-added role in refurbishing the good. On the other hand, if the

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22I assume that each firm is randomly assigned to a partition of households when making purchase but perfectly competitive when selling their refurbished goods into the retail market.
market exhibits monopsony, the real margin $\epsilon_{m,t}$ is strictly larger than the value-added parameter. Hence the price difference not only reflects the value-added role of a second-hand firm, but also its monopsony rent.

C.2 Second-hand retailers

The representative second-hand retailer unbundles $M_t$ into $\{m_t(i)\}_{i \in [0,1]}$ via

$$M_t = \left( \int_0^1 m_t(i) \frac{x_i^{\theta_d-1}}{x_i^\theta_d} \, di \right)^{\frac{\theta_d}{\theta_d-1}}.$$  

Unbundled varieties $\{m_t(i)\}_{i \in [0,1]}$ are provided back into the market. Retailers are perfectly competitive in both the input and output markets, with free entry. Hence they take the buying price $P_{m,t}$ and selling prices $\{p_{d,t}(i)\}_{i \in [0,1]}$ as given. The nominal profit of the second-hand retailer $\Phi_{r,t}$ is:

$$\Phi_{r,t} = \int_0^1 p_{d,t}(i)m_t(i)\, di - P_{m,t}M_t.$$  

However, retailers are assumed to make entry decisions before observing the realized selling prices $\{p_{d,t}(i)\}_{i \in [0,1]}$. Therefore, they base their decision on their expectations for selling prices $\{E_p_{d,t}(i)\}_{i \in [0,1]}$. Retailers enter if expected profits are nonnegative:

$$\int_0^1 E_p_{d,t}(i)m_t(i)\, di - P_{m,t}M_t \geq 0.$$  

Given perfect competition with free entry, retailers that enter are expected to earn zero profit in equilibrium. Hence the equilibrium price for $P_{m,t}$ is the following:

$$P_{m,t} = \frac{\int_0^1 E_p_{d,t}(i)m_t(i)\, di}{M_t}. \quad (20)$$  

D Durables: Market structure

Recall that from the Dixit-Stiglitz setup, the demand for durables for each variety is the following:

$$d_{t}^{N} = D_{t}^{N} \left( \frac{p_{d,t}(i)}{P_{d,t}} \right)^{-\theta_d}.$$  

However, durables of each variety are supplied into the market both by the new firms, and by the second-hand market. Incorporating the market structure in a price leadership model, I assume that the market for each variety consists of a dominant leader (the new durable producing firm) who sets prices, and a price-taking
competitive fringe (second-hand retailers), as in Judd and Petersen (1986). The timing of decisions at each period is as follows:

1. Second-hand retailers observe the purchasing price $P_{m,t}$ of the bundled good and decide whether to enter or not, based on their expectations of the unbundled prices $\{E_p^{d,t}(i)\}_{i \in [0,1]}$.

2. Each new durable producing firm $i$ sets the price of its variety $p_{d,t}(i)$, taking into account both the direct effect on the total demand function, and the indirect effect on the response from the price-taking second-hand retailers who entered the market.

3. The entrant retailers observe the price that the leader sets and choose the supply of the unbundled goods, given their pre-purchased bundled good when they entered the market.

The subgame perfect equilibrium can be solved by backward induction.

D.1 Retailer problem after entry

Solving backwards, assume that a price-taking representative retailer after entry holds $M_t$ of the bundle and sells each unbundled variety $\{m_t(i)\}_{i \in [0,1]}$ at its respective price. As far as the prices are positive, the retailer will always provide its maximum quantity for each variety. Hence the supply for each variety is price-inelastic at a quantity $\{m_t(i)\}_{i \in [0,1]}$, the maximum capacity it holds after entry.

D.2 New durable producing firm

Since new durable producing firm of variety $i$ recognizes that price-taking retailers will always provide $m_t(i)$ into the market at the last stage, the demand function it recognizes is the following residual demand:

$$x_t(i) = D_t^N \left( \frac{p_{d,t}(i)}{P_{d,t}} \right)^{-\theta_d} - m_t(i).$$

It is worthwhile to mention that the new durable producing firm cannot credibly signal its price when a second-hand retailer makes its entry decision. If it could, the new durable producing firm would try to signal a lower price to drive the retailer out of the market. However, if no retailer enters the market, then the firm will now raise its price up to the monopoly level. Because the retailer recognizes this, the signal is not credible and the firm is not able to deter entry.

D.3 Retailer entry problem

At the first stage, the retailer understands the price that each leading firm will set by backward induction. Therefore, $E_p^{d,t}(i) = p_{d,t}(i)$ for all $i \in [0,1]$, and the equilibrium price $P_{m,t}$ in (20) is also consistent with
the following:

\[ P_{m,t} = \frac{\int_0^1 p_{d,t}(i) m_t(i) \, di}{M_t}. \]

Since the retailer realize that they will earn zero profit after entry, they enter the market. We verify that the entry equilibrium is subgame perfect and the retailer enters the market by purchasing \( M_t \) from the second-hand refurbishing firms.

**E Closing the model**

Monetary policy is represented by a simple feedback rule

\[ \hat{R}_t = \tau \pi c_{\hat{\pi}_{c,t}} + (1 - s_c) \hat{\pi}_{d,t} + \epsilon_R^t, \]

where \( s_c \) is the steady state share of nondurables out of total output. The policy shock \( \epsilon_R^t \) is assumed to be i.i.d. Goods markets clear for all \( i \):

\[ \bar{c}_t(i) \equiv c_t(i) + \epsilon M_t = Z_t F(k_{c,t}(i), h_{c,t}(i)), \]

\[ x_t(i) = Z_t F(k_{d,t}(i), h_{d,t}(i)). \]

As discussed, the nondurable goods also incorporate service activity of the dealers in the second-hand market. Both labor and capital markets are mobile across sectors:

\[ H_t = H_{c,t} + H_{d,t}, \quad K = K_{c,t} + K_{d,t}, \quad \text{where} \]

\[ H_{j,t} = \int_0^1 h_{j,t}(i) \, di, \quad K_{j,t} = \int_0^1 k_{j,t}(i) \, di, \quad j \in \{c, d\}. \]

Bond market clears which induces zero bonds in equilibrium (\( B_t = 0 \)). Second-hand market also clears:

\[ M_t = s_t[(1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}]. \]

Lastly, common technology follows an AR(1) process:

\[ \ln Z_t = \rho Z_t \ln Z_{t-1} + \epsilon_Z^t, \quad \epsilon_Z^t \sim i.i.d. \]
Durable sector pricing with second-hand markets

Prices are known to be frequently adjusted in the durable sector.\textsuperscript{23} Consistent with this evidence, I assume that durable prices are flexibly priced and solve the new durable producing firm’s problem.

A The desired durable markup

Under flexible prices, optimal price setting of the new durable producing firm $i$ is the solution to the following problem:

\[
\max_{p_{d,t}(i), x_{d,t}(i)} \left[ p_{d,t}(i) - mc_{d,t}^{n} \right] x_{t}(i),
\]

subject to

\[
x_{t}(i) = D_{t}^{N} \left( \frac{p_{d,t}(i)}{P_{d,t}} \right)^{-\theta_d} - m_t(i).
\]

By the market clearing condition for the second-hand, $m_t(i)$ has the following functional form:

\[
m_t(i) = s_t \left[ (1 - \delta_d) d_{t-1}^{U}(i) + (1 - \rho \delta_d) d_{t-1}^{N}(i) \right],
\]

where $d_{t-1}^{U}(i)$ denotes the stock of used durable of variety $i$ at period $t - 1$.

The demand function for new durable goods (22) consists of two parts: the first term related to the relative price elasticity of the variety and the second term that is price-inelastic.

Taking the first order conditions, we have:

\[
D_{t}^{N} \left( \frac{p_{d,t}(i)}{P_{d,t}} \right)^{-\theta_d} \left( p_{d,t}(i) - \left( \frac{\theta_d}{\theta_d - 1} \right) mc_{d,t}^{n} \right) = - \left( \frac{1}{\theta_d - 1} \right) p_{d,t}(i)m_t(i).
\]

From this equation, we observe that the usual relation between the elasticity of substitution and markup no longer holds even under flexible prices due to the existence of durable stocks of its own and the inability to commit to future production or prices. Rearranging under a symmetric equilibrium ($p_{d,t}(i) = P_{d,t}$, $d_{t-1}(i) = D_{t-1}$):

\[
p_{d,t}^{*}(i) = \frac{D_{t}^{N}}{D_{t}^{N} + \frac{s_t}{\theta_d - 1} \left[ (1 - \delta_d) D_{t-1}^{U} + (1 - \rho \delta_d) D_{t-1}^{N} \right] \left( \frac{\theta_d}{\theta_d - 1} \right) mc_{d,t}^{n}}.
\]

Hence, we now have time-varying markups under flexible prices.

The Coase conjecture

At a symmetric steady state \( ([1 - (1 - s)(1 - \delta_d)] D^U = (1 - s)(1 - \rho \delta_d) D^N) \), the markup of a durable firm is the following:

\[
\frac{p_d^*}{m c_d^m} = \left( \frac{\theta_d}{\theta_d - \frac{\delta_d [1 - s(1 - \rho)]}{\delta_d + s(1 - \delta_d)}} \right).
\]

Note that durability reduces the markup that the firm can charge. When the good becomes nondurable \((\delta_d = 1, \rho = 1)\), we are back to the usual relation between the elasticity of substitution and markup. Similarly, the higher the replacement rate, the lower the market power of these firms.\(^{24}\) In the extreme case where durability is infinite \((\delta_d = 0, \text{ e.g. land})\), the market is perfectly competitive, revisiting the Coase conjecture (Coase (1972)) in a monopolistically competitive framework. For later reference, it is useful to summarize this in a proposition.

**Proposition 2** (Steady state markup of a durable good firm) Under a symmetric equilibrium with the inability to internalize the dynamics of second-hand markets, the markup of durable good firms is negatively correlated with both durability and the replacement rate. When durability is infinite, the markup is zero (Coase Conjecture).

Relation to the deep-habit model

The relation between durability and markup can also be phrased in terms of the deep-habit literature. The model description follows closely Ravn, Schmitt-Grohe, and Uribe (2006, RSU henceforth). There is a continuum of identical households and each household \( j \in [0, 1] \) is assumed to derive utility from the consumption bundle \( \tilde{X}_t^j \) defined by

\[
\tilde{X}_t^j = \left( \int_0^1 [\tilde{c}_t^j(i) - \gamma \tilde{c}_{t-1}(i)] \frac{\theta_d}{\theta_d + s(1 - \delta_d)} di \right)^{\theta_d / (\theta_d + s - 1)},
\]

where \( \tilde{c}_t^j(i) \) is the consumption of variety \( i \) in period \( t \), and \( \tilde{c}_{t-1}(i) = \int_0^1 \tilde{c}_{t-1}(i) dj \) is the average level of consumption of variety \( i \) across all households in period \( t - 1 \). \( \gamma \) is the degree of habit persistence when it is positive. Under external deep habits, households take the average level of consumption for each variety as exogenous. In this case, cost minimization of households and averaging across all \( j \in [0, 1] \) deliver the

\(^{24}\) The steady-state replacement rate \( s \) is discussed in the appendix.
following demand function for each variety $i$:

$$
\tilde{c}_t(i) = \left( \frac{\tilde{p}_t(i)}{\tilde{P}_t} \right)^{-\theta} \tilde{X}_t + \gamma \tilde{c}_{t-1}(i)
$$

(25)

where $\tilde{P}_t = \left( \int_0^1 \tilde{p}_t(i)^{1-\theta} di \right)^{\frac{\theta}{1-\theta}}$ and $\tilde{X}_t = \int_0^1 \tilde{X}_t^j dj$. Firm maximizes its revenue as in (21) subject to this demand function. Solving the problem at the symmetric steady state, we have the following steady state markup expression:

$$
\frac{\tilde{p}^*}{\tilde{mc}_t} = \left( \frac{\theta}{\theta - 1 + \frac{1}{1-\gamma}} \right). 
$$

Note that the markup is an increasing function of $\gamma$. Hence the higher the habit persistence, the higher the markup.

Within this framework, negative habits are interpreted as the durability of the good. This is because with $\gamma < 0$, higher previous stock from the past decreases the demand for that variety today, and this is likely to be the case with durable goods. Under this interpretation, a smaller value of $\gamma$ corresponds to a higher level of durability. In the steady state, therefore, we have the same prediction on the negative relation between durability and the markup. However, comparing (25) with (22), we observe that in the deep-habit model, the price-inelastic term is a constant at period $t$. On the other hand, the price-inelastic term in the durables replacement model is proportional to the period-$t$ replacement rate in (22) by (23).

Thus, although the deep-habit framework delivers similar predictions to the durables replacement model at the steady state, there is an important difference in the dynamics since the price-inelastic term in the demand function changes contemporaneously for the durables replacement model.

### D The price-elasticity effect and dynamics

To understand its dynamics, we define the price markup for durable goods as

$$
\mu_{d,t} = \frac{P_{d,t}}{\tilde{mc}_{d,t}}.
$$

Log-linearizing (24) and expressing it as a function of durable price markup gives the following:

$$
\mu_{d,t} = A \left( \tilde{d}^N_t - \delta_t - B \tilde{d}_{t-1} - (1 - B)\tilde{d}^N_{t-1} \right),
$$

where

$$
A = \frac{s(1 - \rho \delta_d)}{(\theta_d - 1)(\delta_d + s(1 - \delta_d)) + s(1 - \rho \delta_d)}, \quad B = \frac{1 - \rho \delta_d - \delta_d(1 - \rho)(\delta_d + s(1 - \delta_d))}{1 - \rho \delta_d}.
$$
In the same manner, the new durable good supply function can be expressed in a log-linear fashion:

\[
\hat{d}_t^N = s(1 - \delta_d)D \frac{D^N}{D^N} \left( \hat{s}_t + \hat{d}_{t-1} \right) + s\delta_d(1 - \rho) \left( \hat{s}_t + \hat{d}_{t-1} \right) + \frac{X}{D^N} \hat{x}_t.
\]

Combining both terms, the durable price markup can be expressed as follows:

\[
\hat{\mu}_{d,t} = A \left( \frac{\delta_d[1 - s(1 - \rho)]}{\delta_d + s(1 - \delta_d)} (\hat{x}_t - \hat{s}_t) + \text{lagged terms} \right),
\]

where \( D, D^N, X \) represent the steady state values of the stock, purchases, and production of durables, respectively. Note that replacements have a negative impact on markups. Hence if replacement is procyclical, there would be a countercyclical pressure on the markup.

The distinction from RSU is apparent in (22). In the deep-habit demand function (25), an increase in the term representing aggregate demand \( \tilde{X}_t \) leads to a higher weight on the first, price-elastic term. Since the second, price-inelastic term is constant at period \( t \), prices become more elastic and this channel induces countercyclical markups. In (22), the price-inelastic term decreases when replacement is procyclical. Hence the relative weight on the price-elastic term increases and leads to a countercyclical pressure on markups.

In the end, the price-elasticity effect discussed in RSU is also included in the durables replacement model, but for a different reason. In the deep-habit model, the price-elastic term becomes high due to an aggregate demand effect. In the durables replacement model, the price-inelastic term decreases which makes the price-elastic term relatively highly weighted.

Consistent with the argument that higher replacement rate leads to higher price elasticity, a higher replacement rate implies a lower price markup for durables. The exact dynamics will be discussed after calibration.

### VI The comovement puzzle

When durables are flexibly priced relative to nondurables in a two sector new Keynesian model, BHK show that durables and nondurables move in opposite direction with regards to a monetary policy shock. Since durables and nondurables comove with monetary shocks in the data, they claim this as a puzzling feature of new Keynesian models.

In this section, I show that the durables replacement model can resolve the comovement puzzle. Because the results are numerical, the functional form for utility and production as well as parameter calibrations are first discussed and the model impulse responses are displayed next.
A Functional form and calibration

The utility function is set as additively separable with possibly different utility weights. In particular, log utility is assumed for both nondurables and durables, with constant Frisch elasticity of labor supply:

\[ U(C_t, D_t, H_t) = \psi_c \ln C_t + \psi_d \ln D_t - \chi H_t^{1+1/\phi} \frac{1}{1+1/\phi}. \]

The functional form for both the nondurable and durable production are Cobb-Douglas:

\[ F(k_{j,t}, h_{j,t}) = k_{j,t}^{\eta} h_{j,t}^{1-\eta}, \quad j \in \{c,d\}. \]

Calibration is at quarterly frequency and summarized in table 2. Most parameters are within the range of business-cycle studies, with price rigidity for nondurable goods set at 0.50, which implies an average duration of price change for 2 quarters. Steady state markups for nondurables and durables are set at 10 percent each. However, \( \theta_d \) is not the same as \( \theta_c \) since the relation between markup and the elasticity of substitution is no longer the same with replacements.

The nonstandard parameters to calibrate are the used durable depreciation rate \( \delta_d \), new good depreciation discount \( \rho \), and \( \epsilon \) which represents the relative service activity in the second-hand market. Since automobile data is the most reliable source of statistics for used durables, I use these to calibrate the parameters of interest. In the numerical analysis, I assume a perfectly competitive second-hand market. In this case, the margin over used car sales is matched to the size of the value-added component of used car sales in the overall consumption basket. Hence \( \epsilon \), \( \delta_d \), and \( \rho \) are calibrated to match the margin over used car sales, the average depreciation for consumer durables, and the relative transaction for used cars.\(^{25}\) The calibration and numerical analysis for the monopsony second-hand market are in the appendix.

The adjustment cost parameter governs the quantitative dynamics of the model. With perfectly competitive second-hand markets, I consider four benchmarks, \( \xi = [0.001, 0.002, 0.003, \infty] \).\(^{26}\) Infinite adjustment cost corresponds to a model with no discretionary replacement demand.

\(^{25}\)For the relative transaction of used to new cars, I take the average transaction of used and new car sales for 1990-2010 in the National Transportation Statistics published by RITA, BTS. The margin over used car sales is computed from NIPA table 7.2.5U. The depreciation for consumer durables is calibrated on the higher end. In my model, lowering the depreciation rate also limits the role of second-hand markets, since recovering the depreciation rate of the used good is the only value-added role of second-hand markets. That is, by setting a high margin over used car sales, the average depreciation rate is also required to be high. The results in this section are robust to an alternative calibration with low margin over used sales and a low depreciation rate.

\(^{26}\)The qualitative effects are the same for small \( \xi \), but without any adjustment costs, the movement of replacement is very volatile to monetary shocks.
B Monetary policy shock and the comovement puzzle

Figure 4 shows the economy’s response to a 25 basis point increase in the nominal interest rate. First, when $\xi = \infty$, replacements are constant. The impulse response of this model without cyclical replacements exhibits a comovement puzzle of nondurables and durables production. The relative size of production increases in the durable sector and decreases in the nondurable sector adjusted with the relative size of each sector restates the BHK result that monetary shocks have only a mild effect in overall output when the durables sector exhibits flexible prices.

Instead, with a low adjustment cost of $\xi = 0.001$, we observe that the replacement rate drops with an increase in the interest rate. This leads to an increase in the markup for the durable goods sector, resulting in a decrease in the production of durable goods. Overall, consumption expenditures in both sectors decrease, attaining comovement as in the VAR evidence. Hence the model generates procyclical replacement and countercyclical markup with regards to a monetary policy shock.

When higher adjustment costs are considered ($\xi = [0.002, 0.003]$), comovement remains but the relative interest elasticity of durable to nondurable consumption expenditures becomes small compared to the VAR evidence. Hence, adjustment cost around 0.001 and 0.002 delivers an empirically reasonable relative response of durables and nondurables to monetary shocks.

C Inspecting the mechanism

BHK argue that the near constancy of the shadow value of long-lived durables is the reason why we observe the comovement puzzle with flexible prices on durables. By combining (12) and (14), the shadow value of durables in my model can be expressed as

$$\nu_t = E_t \left[ \sum_{i=0}^{\infty} \beta (1 - \delta_d)^i U_d(t + i) \right].$$

where $U_d(t + i)$ is the marginal utility of the durable stock in period $t + i$. Following the argument of BHK, $\nu_t$ is almost a constant with regards to a temporary shock when $\delta_d$ is close to zero. This is because the stock of durables $D_t$ becomes very large when $\delta_d$ is small so that the curvature of the utility function at $D_t$ becomes flat. Hence the marginal utility stays relatively constant. Moreover with small $\delta_d$, $\nu_t$ depends heavily on the remote future. Since with temporary shocks the future marginal utilities should not change much, the change in the shadow value must also be limited. Hence the shadow value of durables remains to be near constant ($\nu_t \approx \nu$).

Next, combining (13) and (14), and using the near constancy of the shadow value of durables, we get the
\[-U_h(t)\frac{P_{d,t}}{W_t} = \nu_t + \beta \delta_d(1 - \rho)E_t \nu_{t+1} \approx \nu[1 + \beta \delta_d(1 - \rho)].\]

Moreover, note also that \(P_{d,t} = \mu_{d,t}mc_{d,t}\) and that with mobile factor markets, the nominal marginal costs in both sectors (with \(Z_t = 1\)) are equated at

\[mc_{d,t}^c = mc_{c,t}^c = \frac{W_t}{F_H(K, H_t)}.\]

Plugging this into the above, we have

\[-U_h(t)\frac{\mu_{d,t}}{F_H(K, H_t)} \approx \nu[1 + \beta \delta_d(1 - \rho)].\]

The left hand side is a function of hours \(H_t\) and the durable markup \(\mu_{d,t}\), while the right hand side is a constant. Hence, the cyclical movement of durable markups are important for hours to move cyclically with monetary shocks. Moreover, plugging in the functional form for marginal utility and the production function, we get:

\[\mu_{d,t}H_t^{1/\phi+\eta} \approx \text{constant}.\]

In sum, countercyclical durable markups imply procyclical aggregate labor response with regards to monetary shocks. Since my model generates countercyclical durable markups even without nominal rigidities, the model is successful in solving the comovement puzzle by increasing aggregate labor supply and hence aggregate output.

**VII Durables in the business cycle**

In this section, I show that the durables replacement model is also capable of delivering two desirable features in explaining the dynamics of durables in the business cycle. First, the model amplifies the spending for durables because durable replacements are procyclical. Second, as the second-hand market expands, the persistence of durable spending declines.

The first point improves on the weak internal propagation of standard business-cycle models of durables. The standard model does not deliver the high volatility of durable spending observed in the data, as pointed out by Baxter (1996).
The second point suggests an explanation of Ramey and Vine (2006)’s findings on the decline in persistence of motor vehicle sales after 1984. The decline in persistence after 1984 is also true in the broader category of durable spending as shown below. At the same time, second-hand markets expanded over time as observed in figure 1. Connecting these two facts, my model shows that an expansion in the second-hand market is one possible explanation of the declining persistence of durable spending.

A  More facts on durables

In the data, durables are highly volatile and less persistent than nondurables. HP filtering the same NIPA data as in section II, table 3 compares the volatility and persistence of durables with that of nondurables including services. In the full sample, the relative standard deviation of durables and nondurables is 5.02.

Focusing on persistence, we observe that durables are less persistent compared to nondurables. Splitting the series into two periods by 1984, we see that the first order autocorrelation of durables is smaller after 1984, whereas that of nondurables in this period remains similar or slightly higher. In particular, the last row shows that the decline in persistence is highly pronounced in the motor vehicle sector, as analyzed in Ramey and Vine (2006).

In figure 5, we observe that the decline in persistence is prominent in higher order autocorrelations as well. For all 5 lags considered, durables and motor vehicle sales are less persistent after 1984. Looking into nondurables and services separately, we observe that the decline in persistence is only true for durables. For service expenditures, persistence increased by a small amount in all lags.

B  Additional calibration

Besides monetary policy shocks, my model also considers technology shocks. Hence persistence of the technology shock process should also be calibrated. Since the persistence of durable spending is an object of interest, I target this statistic in calibrating the persistence of the technology process. By this criterion, $\rho_Z = 0.88$ is chosen which delivers the persistence of durables close to 0.78 conditional on technology shocks.

To compute the unconditional moments, the relative standard deviation of the technology and monetary innovation should also be calibrated. Ireland (2004) estimates it to be around 3.5, while Christiano, Trabandt, and Walentin (2010) estimate the posterior mode around 5. I consider a range of values that cover these estimates.

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27 Ramey and Vine (2006) conduct a statistical test on the changing nature of sales persistence in the motor vehicle sector before and after 1984 and find that there is a significant change.

28 As will be shown in table 4, the persistence of durable spending in the model is not sensitive to the replacement adjustment cost parameter. Even when monetary shocks are considered, the persistence of durables remain similar.
C Volatility of durables is higher with cyclical replacements

Figure 6 shows the impulse responses of durable spending to a negative technology shock with different replacement adjustment costs considered. The black dotted line is the case with no cyclical replacements. In this case, durable spending is 6 times more sensitive than nondurable consumption on impact. The three other lines are cases with households replacing their durables. Since replacements are procyclical to technology shocks, durables decline even more on impact. The relative sensitivity of durable spending is 9 times more sensitive than nondurable consumption for all three replacement adjustment costs considered that are consistent with resolving the comovement puzzle for monetary policy shocks.

This amplification channel is reflected in the second moments of the model. The first panel of table 4 computes the moments conditional on technology shocks. The relative standard deviation of durables to nondurables is only 3.91 without replacements, but as high as 4.93 with replacements which is close to 5.02 observed in the data. Therefore, the amplification of durable spending due to cyclical replacements delivers a 25% increase in the relative standard deviation.

The remaining panels of table 4 present the statistics when monetary shocks are also considered. The second panel is the case when monetary shocks are as volatile as technology shocks. Starting with the last column again, the relative standard deviation is 3.94, far below the data. When replacements are allowed, durables become more volatile and the relative standard deviation becomes as high as 4.95.

The third and fourth panel compute the moments with the relative standard deviation of technology and monetary innovations as 3 and 5. The results again suggest that the amplification channel is robust to different combinations of the two shocks in the model.

Cyclical replacement is not unique in generating amplification for durable spending. Gomes, Kogan, and Yogo (2009) model the durable sector as uniquely holding inventories and also generate highly volatile durable spending. However, in their model, finished-good durable inventories are used as factors of production. Hence the asymmetry between the two sectors comes not only from the durable sector holding inventories, but also from the sector using these inventories as intermediate inputs. The exact mechanism that generates the relative volatility of durables and nondurables is still under question, but cyclical replacement demand is one answer to it outside of the input-output structure.

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29 The decline in nondurable consumption becomes slightly smaller.
30 Note that the relative standard deviation of durables was not the target in calibrating the persistence of the technology shock process.
D  The decline in durable persistence and second-hand markets

As pointed out by Ramey and Vine (2006) and also verified above, the Great Moderation is also a period of lower persistence in durable spending. In section III, I have shown that the discretionary replacement demand model leads to lower persistence of durable spending due to the advancement and postponement of replacement. For example, when replacements are advanced, households in the next period tend to decrease their demand for new durables since the average age of the goods have declined. I argue here that this channel remains true in general equilibrium.

Since figure 1 suggests that the second-hand market expanded during this period, the experiment in this section compares the persistence of durable spending by changing the target value-added of second-hand markets in the model. The key parameter related to the value-added of the second-hand market is the margin of used goods $\epsilon$. I consider two different scenarios in changing $\epsilon$ to target the value-added of the second-hand market.

First, the value-added of second-hand markets could have expanded due to improvements in the durability of new durable goods. In this scenario, higher durability of new durable goods gives more room for second-hand markets to play a value-added role. I call this the supply side expansion of second-hand market. In the model, I hold fixed the steady state replacement rate and change the second-hand market margin parameter $\epsilon$ by changing the new durable goods depreciation discount parameter $\rho$.

Second, the value-added of the second-hand market could have expanded due to higher replacement demand from the households. In this scenario, higher replacement demand of households leads to higher transactions in the second-hand market. Since each transaction involves a value-added role of the second-hand market, I call this the demand side expansion of the second-hand market. In the model, I hold fixed all the parameters and change the margin parameter $\epsilon$ by changing the steady state replacement frequency of durables.

With adjustment cost set at 0.001, the persistence for durables and nondurables conditional on technology shocks are plotted in figure 7. The x axis is the value-added of the second-hand market. Recall that the benchmark value-added of the second-hand market calibrated in the model is 10 percent. Experimenting with different sizes of the second-hand market from 1 to 20 percent, we observe that the persistence of durable spending declines as the second-hand market expands, whether it be due to the supply side or the demand side.\(^{31}\) In particular, a supply side second-hand market expansion from 1 percent to 13 percent induces as high as a 0.035 decline in the persistence of durables. Note that in the data, we observe a 0.07 decline in the persistence of durables before and after 1984. Hence nearly half of the decline in the persistence of durables

\(^{31}\)The supply side expansion does not go up to 20 percent in the model since the new durables discount parameter $\rho$ becomes negative above 13 percent.
could be explained by the expansion of the second-hand market.

The persistence of nondurables only slightly increases. The maximum increase in persistence is 0.01 when the expansion of the second-hand market is purely driven by the supply side. Again, this corresponds to the small increase in the persistence of nondurables observed before and after 1984.

The qualitative results remain with different adjustment costs. As figure 8 shows, adding monetary shocks with the relative standard deviation close to the estimated values do not change the result. Hence, the model suggests that the expansion of the second-hand market is a potential candidate of the factors that lead to the observed decline in persistence of durable spending during the Great Moderation.

VIII Empirical evidence

Section V states the two key mechanisms of the model. First, that when replacements are procyclical, markups for durables are countercyclical. Second, that markup decreases with durability and replacement frequency as in proposition 2. In this section, I look for empirical evidence for both. The first point has been widely discussed and I briefly summarize the literature in the next section. The second point has been less investigated so section C presents new evidence.

A The cyclicality of durable markups

An extensive survey on the cyclicality of the markup for overall goods is presented in Rotemberg and Woodford (1999). They show that markup dynamics measured in various empirical methods leans towards countercyclical markups. Domowitz, Hubbard, and Petersen (1988) as well as Bils (1989) compare markup cyclicality between durables and nondurables and show evidence that markups are more countercyclical for the durable sector. Lastly, Parker (2001) and Bils, Klenow, and Malin (2012) work with industry-level data to show that markups are more countercyclical for infrequently purchased durable goods.

B Markup and durability: Empirical strategy and data description

The goal of this section is to provide an empirical exercise on the relation between markup and durability. To estimate markups for each industry, I follow the framework of Hall (1988). Under the assumption of constant returns to scale for three inputs (labor (N), capital (K), and materials (M)), Hall (1988) suggests a method for estimating the average markup of a firm similar to the following equation:

$$\frac{\Delta Y_t}{Y_t} = \mu \left( s_{N,t} \frac{\Delta N_t}{N_t} + s_{K,t} \frac{\Delta K_t}{K_t} + s_{M,t} \frac{\Delta M_t}{M_t} \right) + \zeta_t,$$
where \( \zeta_t \) is the purely exogenous technology component of the firm, \( \mu \) is the average markup, and \( s_{N,t}, s_{K,t}, s_{M,t} \) are the previous period factor share of labor, capital and materials, respectively. Under the constant returns to scale assumption, \( s_{N,t} + s_{K,t} + s_{M,t} = 1 \). Hence we can subtract capital from each side to get

\[
\Delta y_t = \mu (s_{N,t} \Delta n_t + s_{M,t} \Delta m_t) + \zeta_t,
\]

where \( y_t = \ln(Y_t/K_t) \), \( n_t = \ln(N_t/K_t) \), \( m_t = \ln(M_t/K_t) \), \( s_{N,t} = (w_t - 1 N_t - 1)/(p_t Y_t - 1) \), and \( s_{M,t} = (v_t - 1 M_t - 1)/(p_t Y_t - 1) \).\(^{32}\) \( Y, w, p, v \) refers to output, total compensation per worker, output price, and materials cost respectively. To estimate \( \mu \) for each industry, we need to take an instrumental variable approach since the technology component is likely to be correlated with the input variables. Assume that the technology shock of an industry can be decomposed into industry-specific (\( \zeta_i \)) and aggregate (\( \zeta_a \)) components (\( \zeta_t = \zeta_i + \zeta_a \)). The issue is to find an appropriate instrument that is positively correlated with both input and output, while exogenous to technology. Following Hall (1988), I use the difference of real national defence expenditures, domestic crude oil price, and the political indicator when the president is from the Democratic party. These instruments are claimed to be demand shifters without having any significant correlation with the pure technology shock component. Using a similar approach as described and estimating the markup rate at the 1-digit SIC level, Hall (1988) shows that the markup for durable goods (2.058) are lower than the markup for nondurable goods (3.096).\(^{33}\)

Here I use a more detailed 4-digit SIC industry group specified in Parker (2001) and Bils and Klenow (1998), because they have the measure of durability of the goods for 58 consumer goods at the 4-digit SIC level. Since 5 of these are specified as nondurables, I drop them. Also, Parker (2001) and Bils and Klenow (1998) use the 1972 SIC codes, while the data I use are the 1987 SIC codes. Hence 3 additional industries are dropped since they are merged in the 1987 SIC codes.\(^{34}\) This leaves us with 50 industry groups.

I use the NBER manufacturing productivity database.\(^{35}\) For \( Y \) I used the value of shipments. For \( N \), I use production worker hours. Capital is also provided in the dataset. For \( wN \), total payroll is used. For \( vM \) and \( v \), materials cost and price deflator for materials are used. Since the industry is more detailed than what Hall uses, the 3 Hall-Ramey instruments turn out to be only weakly correlated with the input and output variables. Hence I also use the log difference of the aggregate personal consumption expenditure and its lag

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\(^{32}\)Log approximation is used.

\(^{33}\)Hall (1988) does not consider materials as inputs in his basic calculation of average markups.

\(^{34}\)Woven carpets and rugs, tufted carpets and rugs, and carpets and rugs, nec. are all merged to carpets and rugs in SIC87. Similarly, men’s trousers and men’s work clothing are merged to men’s and boy’s trousers and slacks in SIC87.

\(^{35}\)This database contains annual information of over 400 4-digit manufacturing industries from 1958 to 2005. A full description of the dataset can be found in Bartelsman and Gray (1996). The usage of this dataset is similar to Parker (2001), except that I include more recent time series.
as additional instruments to capture demand shifts. If we assume that the technology shock for each industry consists of both industry-specific and aggregate shocks, then it is reasonable to assume that to the industry level (4-digit SIC), the variability of industry-specific shocks dominate aggregate shocks ($\text{std}(\zeta_i^t) > \text{std}(\zeta_a^t)$). Hence, any variation of the aggregate variables will be less correlated with the technology shocks at the industry level. Therefore, including aggregate consumption as an additional demand shift instrument is a compromise that is likely to solve the weak instrument problem while less affecting the exogeneity assumption to the technology shock component.\(^{36}\)

C Markups and durability: Results

Column 2 of table 5 provides the regression result. The t-statistic is 2, hence in line with proposition 1. One critical assumption that I am making is the inability of firms to internalize the interaction with the second-hand market. If the market is concentrated so that firms might be more likely to internalize their activities with the second-hand market, this is unlikely to be the correct assumption. To control this effect, I also add a concentration ratio variable which measures the concentration of 4 big firms. The concentration ratio for 4-digit SIC industries are taken from 1992 Census of Manufacturers Report.\(^{37}\) The regression coefficients indicate that the higher the concentration ratio, the higher the markup, as is expected to be the case. Controlling for the concentration ratio, we still observe that durability is negatively correlated with markup. Including an interaction term between durability and concentration ratio in the next regression, we observe that this coefficient is slightly positive, implying that the negative relation between durability and markup is less pronounced in a highly concentrated market. This could justify the higher possibility of concentrated markets to internalize the second-hand market for used durables. However, the coefficients are less significant to draw any decisive conclusion. As a robustness check due to concerns that an outlier is driving most of the results, I exclude SIC 3914 (silverware) which has a high durability of 27.5 years. The qualitative results still hold, although the significance levels are less dramatic as before. I have also conducted these regressions with different subsample periods and the qualitative results are robust in this dimension illustrating that as for the current empirical strategy, the choice of a sample period is less of a concern. However, given its limited success in statistical significance, future work is required to identify the precise relation between the two variables.\(^{38}\)

\(^{36}\)Including gross domestic output or its lags make small difference.


\(^{38}\)Controlling for the elasticity of substitution for each good might be another direction.
IX Conclusion

This paper proposes a general-equilibrium model with discretionary replacement of durables for households and a well-functioning second-hand market. At the steady state, markup is inversely related to durability and replacement frequency. For the dynamics, the model generates procyclical replacement and countercyclical markup. These features help improve the movements of durables in standard business-cycle models towards the data. Although there seems to be feeble evidence of nominal rigidities in the durable goods production sector, I argue that real rigidities with regards to pricing decisions are important considerations especially at the durable goods sector. As an empirical test of one feature of my model, I also show that durability and markups are negatively correlated, although more work is required to identify their precise relation.

There is a rich avenue of research in this direction. In the recent recession with interest rates hitting the zero lower bound, a relative price distortion policy has been conducted as a way to boost the durable goods sector production. In particular, the “cash-for-clunkers” program was conducted in 2009 and its macroeconomic effectiveness is yet an open question, since no general-equilibrium model with second-hand markets and consumer replacement exists. A further extension of my model would be to analyze this policy. Moreover, I abstracted from the firm’s dynamic considerations when they interact with second-hand markets. In the industrial organization literature, Hendel and Lizzeri (1999) and Porter and Sattler (1999) study the dynamic decision of firms allowing commitment to their own products, while Esteban and Shum (2007) assume that firms follow Markovian strategies. By allowing firms to dynamically interact with the second-hand market, interesting business-cycle dynamics might also arise.
References


Table 1: Business cycle facts of durable consumption expenditures

(Quarterly HP filter, 1967Q1 - 2011Q2)

<table>
<thead>
<tr>
<th>Variable (x)</th>
<th>share (%)</th>
<th>corr(x,y)</th>
<th>std(x)/std(y)</th>
<th>corr(x, x−1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Durables</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Motor vehicles</td>
<td>39.5</td>
<td>0.66</td>
<td>4.62</td>
<td>0.64</td>
</tr>
<tr>
<td>New motor</td>
<td>63.8</td>
<td>0.61</td>
<td>6.36</td>
<td>0.61</td>
</tr>
<tr>
<td>Net used motor</td>
<td>21.6</td>
<td>0.48</td>
<td>3.42</td>
<td>0.44</td>
</tr>
<tr>
<td>(Used car margin 1)(^a)</td>
<td>(7.0)</td>
<td>0.32</td>
<td>3.54</td>
<td>0.62</td>
</tr>
<tr>
<td>(Used car margin 2)(^b)</td>
<td>(0.3)</td>
<td>0.43</td>
<td>4.11</td>
<td>0.67</td>
</tr>
<tr>
<td>(Used truck margin 1)(^c)</td>
<td>(3.3)</td>
<td>0.37</td>
<td>5.72</td>
<td>0.59</td>
</tr>
<tr>
<td>(Used truck margin 2)(^d)</td>
<td>(0.6)</td>
<td>0.40</td>
<td>5.89</td>
<td>0.61</td>
</tr>
</tbody>
</table>

\(^\ast\) y is real GDP. New motor and net used motor do not add up to 100% since auto parts are excluded. Parentheses are share relative to motor vehicles.

\(^a,b,c\) Margin of used vehicles from business (and government) to household

\(^b,d\) Margin of used vehicles from household (and government) to business

\(^c,d\) Series starts from 1983Q1.

Table 2: Quarterly calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta)</td>
<td>0.995</td>
<td>annual discount rate 2%</td>
</tr>
<tr>
<td>(\psi_d/\psi_c)</td>
<td>0.23</td>
<td>durable share 0.15</td>
</tr>
<tr>
<td>(\phi)</td>
<td>1</td>
<td>Frisch elasticity 1</td>
</tr>
<tr>
<td>(\chi)</td>
<td>6.50</td>
<td>hours 0.33</td>
</tr>
<tr>
<td>(\eta)</td>
<td>0.35</td>
<td>labor intensity 0.65</td>
</tr>
<tr>
<td>(\theta_c)</td>
<td>11</td>
<td>nondurable markup 10%</td>
</tr>
<tr>
<td>(\theta_d)</td>
<td>2.97</td>
<td>durable markup 10% (with second-hand)</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.50</td>
<td>nondurable price change duration 2 quarters</td>
</tr>
<tr>
<td>(\tau_{\pi})</td>
<td>1.5</td>
<td>Interest rate feedback coefficient</td>
</tr>
<tr>
<td>(\delta_d)^†</td>
<td>0.057</td>
<td>used good depreciation</td>
</tr>
<tr>
<td>(\rho)^†</td>
<td>0.32</td>
<td>new good depreciation discount</td>
</tr>
<tr>
<td>(\epsilon)^†</td>
<td>0.037</td>
<td>second-hand value-added (competitive)</td>
</tr>
</tbody>
</table>

\(^\dagger\) determined by the following three moments:
1. quarterly durable depreciation: 5\%
2. relative transaction of used vs new cars: 2.70
3. value-added of second-hand over durable spending: 10\%

Table 3: Business cycle facts

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Full sample</th>
<th>Before 1984</th>
<th>After 1984</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_D/\sigma_{ND})</td>
<td>5.02</td>
<td>5.31</td>
<td>4.57</td>
<td>Relative standard deviation</td>
</tr>
<tr>
<td>(\rho_D)</td>
<td>0.784 (0.041)</td>
<td>0.805 (0.058)</td>
<td>0.737 (0.062)</td>
<td>Durable persistence (standard error)</td>
</tr>
<tr>
<td>(\rho_{ND})</td>
<td>0.902 (0.035)</td>
<td>0.897 (0.058)</td>
<td>0.908 (0.040)</td>
<td>Nondurable persistence (standard error)</td>
</tr>
<tr>
<td>(\rho_M)</td>
<td>0.639 (0.060)</td>
<td>0.687 (0.083)</td>
<td>0.525 (0.089)</td>
<td>Motor vehicle persistence (standard error)</td>
</tr>
</tbody>
</table>
Table 4: Model second moments

<table>
<thead>
<tr>
<th>Statistic</th>
<th>$\xi = 0.001$</th>
<th>$\xi = 0.002$</th>
<th>$\xi = 0.003$</th>
<th>$\xi = \infty$</th>
<th>Description</th>
</tr>
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<td>Conditional on technology shock</td>
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<tr>
<td>$\rho_D$</td>
<td>0.77</td>
<td>0.78</td>
<td>0.78</td>
<td>0.79</td>
<td>Durable persistence</td>
</tr>
<tr>
<td>$\rho_{ND}$</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.92</td>
<td>Nondurable persistence</td>
</tr>
<tr>
<td>$\sigma_{D}/\sigma_{ND}$</td>
<td>4.93</td>
<td>4.90</td>
<td>4.88</td>
<td>3.91</td>
<td>Relative standard deviation</td>
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<tr>
<td></td>
<td>Technology and monetary shocks ($\frac{\sigma_{\xi}}{\sigma_{\rho}} = 1$)</td>
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<td></td>
<td></td>
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<tr>
<td>$\rho_D$</td>
<td>0.76</td>
<td>0.78</td>
<td>0.78</td>
<td>0.73</td>
<td>Durable persistence</td>
</tr>
<tr>
<td>$\rho_{ND}$</td>
<td>0.94</td>
<td>0.93</td>
<td>0.93</td>
<td>0.87</td>
<td>Nondurable persistence</td>
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<tr>
<td>$\sigma_{D}/\sigma_{ND}$</td>
<td>4.95</td>
<td>4.89</td>
<td>4.85</td>
<td>3.94</td>
<td>Relative standard deviation</td>
</tr>
<tr>
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<td>Technology and monetary shocks ($\frac{\sigma_{\xi}}{\sigma_{\rho}} = 3$)</td>
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<td></td>
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</tr>
<tr>
<td>$\rho_D$</td>
<td>0.77</td>
<td>0.78</td>
<td>0.78</td>
<td>0.78</td>
<td>Durable persistence</td>
</tr>
<tr>
<td>$\rho_{ND}$</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.91</td>
<td>Nondurable persistence</td>
</tr>
<tr>
<td>$\sigma_{D}/\sigma_{ND}$</td>
<td>4.93</td>
<td>4.90</td>
<td>4.89</td>
<td>3.92</td>
<td>Relative standard deviation</td>
</tr>
<tr>
<td></td>
<td>Technology and monetary shocks ($\frac{\sigma_{\xi}}{\sigma_{\rho}} = 5$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_D$</td>
<td>0.77</td>
<td>0.78</td>
<td>0.78</td>
<td>0.78</td>
<td>Durable persistence</td>
</tr>
<tr>
<td>$\rho_{ND}$</td>
<td>0.94</td>
<td>0.94</td>
<td>0.94</td>
<td>0.92</td>
<td>Nondurable persistence</td>
</tr>
<tr>
<td>$\sigma_{D}/\sigma_{ND}$</td>
<td>4.93</td>
<td>4.90</td>
<td>4.89</td>
<td>3.91</td>
<td>Relative standard deviation</td>
</tr>
</tbody>
</table>

Table 5: Linear regression results (dependent variable: estimated markup)

<table>
<thead>
<tr>
<th>(obs=50)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>1.321</td>
<td>1.397</td>
<td>1.329</td>
<td>1.341</td>
<td>1.365</td>
<td>1.296</td>
<td>1.296</td>
</tr>
<tr>
<td></td>
<td>(0.128)</td>
<td>(0.056)</td>
<td>(0.065)</td>
<td>(0.105)</td>
<td>(0.057)</td>
<td>(0.064)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>durability</td>
<td>$-0.004$</td>
<td>$-0.012$</td>
<td>$-0.014$</td>
<td>$-0.016$</td>
<td>$-0.008$</td>
<td>$-0.010$</td>
<td>$-0.018$</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.006)</td>
<td>(0.006)</td>
<td>(0.013)</td>
<td>(0.006)</td>
<td>(0.007)</td>
<td>(0.013)</td>
</tr>
<tr>
<td>concentration</td>
<td>0.195</td>
<td>0.171</td>
<td>0.198</td>
<td>0.027</td>
<td>0.169</td>
<td>0.341</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.171)</td>
<td>(0.319)</td>
<td>(0.198)</td>
<td>(0.027)</td>
<td>(0.169)</td>
<td>(0.341)</td>
<td></td>
</tr>
<tr>
<td>interaction</td>
<td>0.003</td>
<td>0.019</td>
<td>0.003</td>
<td>0.019</td>
<td>0.003</td>
<td>0.019</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.031)</td>
<td>(0.030)</td>
<td>(0.031)</td>
<td>(0.030)</td>
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</tr>
<tr>
<td>R-squared</td>
<td>0.090</td>
<td>0.087</td>
<td>0.119</td>
<td>0.120</td>
<td>0.029</td>
<td>0.065</td>
<td>0.075</td>
</tr>
<tr>
<td>F-statistic</td>
<td>3.970</td>
<td>2.470</td>
<td>2.250</td>
<td>1.360</td>
<td>1.260</td>
<td>1.490</td>
<td></td>
</tr>
</tbody>
</table>

(1): 3 instruments (Hall-Ramey)
(2), (3), (4): 5 instruments (Hall-Ramey + PCE)
(3): regression including concentration ratio (4 big firms)
(4): regression including concentration ratio and its interaction with durability
(5), (6), (7): same regression as in (2), (3), (4) excluding SIC 3914 (silverware)
*Errors are heteroskedasticity robust.
Figure 1: Relative composition of new and net used motor purchases to durable spending

![Graph showing relative composition of new and net used motor purchases to durable spending from 1970 to 2010.](image)

Figure 2: Annual sales of new and used vehicles (Thousands of vehicles)

![Graph showing annual sales of new and used vehicles from 1990 to 2010.](image)
Figure 3: Empirical IRF to a monetary policy shock (1967Q1-2007Q4, 90% centered bootstrap interval)

- **New motor**
- **Used auto margin**
- **Durables net of motor**
- **Nondurables and services**
- **Federal funds rate**

Figure 4: IRF to a 0.25 percent increase in interest rate (with different adjustment cost parameters)

- **Durable consumption**
- **Nondurable consumption**
- **Replacement ratio**
- **Durable markup**
- **Nondurable markup**
- **Interest rate**

\[\text{adj} = 0.001\]
\[\text{adj} = 0.002\]
\[\text{adj} = 0.003\]
\[\text{adj} = \infty\]

In all figures, replacement rate is level changes. All other variables reflect percentage changes. Replacement rate at the steady state is 14.2%. 

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Figure 5: Autocorrelation before and after 1984

Figure 6: IRF to a negative technology shock
Figure 7: Model persistence with technology shocks ($\xi = 0.001$)

Figure 8: Model persistence with monetary and technology shocks ($\xi = 0.001$, $\sigma_{\varepsilon R} = 3$)
Appendix (not for publication)

A.1. Equilibrium conditions

The Lagrangian of the household problem can be written as follows:

\[ \mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(C_t, D_t, H_t) + \frac{\lambda_t}{P_{c,t}} \left[ B_{t-1} + W_t H_t + R^K_t K + P_{u,t} s_t [(1 - \delta_d) D_{t-1} + \delta_d (1 - \rho) D_{t-1}^N] + \Phi_t \right] - P_{c,t} C_t - P_{d,t} D_{t}^N - \frac{B_t}{R_t} - P_{u,t} s_t \frac{\xi}{2} \left( \frac{s_t}{s_{t-1}} - 1 \right)^2 + \nu_t \left[ (1 - s_t) [(1 - \delta_d) D_{t-1} + \delta_d (1 - \rho) D_{t-1}^N] + D_t^N - D_t \right] \right\}. \]

Hence the household equilibrium conditions are the following:

\[ [C_t] : U_c(t) = \lambda_t \]
\[ [H_t] : U_h(t) = -\lambda_t \frac{W_t}{P_{c,t}} \]
\[ [B_t] : \frac{1}{R_t} = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{c,t}}{P_{c,t+1}} \right] \]
\[ [D_t] : U_d(t) = \nu_t - \beta (1 - \delta_d) E_t \left[ \lambda_{t+1} \frac{P_{u,t+1}}{P_{c,t+1}} s_{t+1} + \nu_{t+1} (1 - s_{t+1}) \right] \]
\[ [D_t^N] : \lambda_t \frac{P_{d,t}}{P_{c,t}} = \nu_t + \beta \delta_d (1 - \rho) E_t \left[ \lambda_{t+1} \frac{P_{u,t+1}}{P_{c,t+1}} s_{t+1} + \nu_{t+1} (1 - s_{t+1}) \right] \]
\[ [s_t] : \left( \lambda_t \frac{P_{u,t}}{P_{c,t}} - \nu_t \right) [(1 - \delta_d) D_{t-1} + \delta_d (1 - \rho) D_{t-1}^N] \]
\[ = \xi \left\{ \lambda_t \frac{P_{u,t}}{P_{c,t}} \left[ \frac{1}{2} \left( \frac{s_t}{s_{t-1}} - 1 \right)^2 + \frac{s_t}{s_{t-1}} \left( \frac{s_t}{s_{t-1}} - 1 \right) \right] + \beta E_t \left[ \lambda_{t+1} \frac{P_{u,t+1}}{P_{c,t+1}} \left( \frac{s_{t+1}}{s_t} \right)^2 - \frac{s_{t+1}}{s_t} - 1 \right] \right\} \]
\[ [\nu_t] : D_t = (1 - s_t) [(1 - \delta_d) D_{t-1} + \delta_d (1 - \rho) D_{t-1}^N] + D_t^N \]

\[ TVC : \lim_{T \to \infty} \beta T E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{P_{c,t}}{P_{c,t+1}} \right] B_{t+T} = 0 \]

and also the household budget constraint set at equality.

The nondurable firm equilibrium conditions after aggregation are the following:

\[ mc_{c,t} = \frac{W_t}{Z_t F_H(K_{c,t}, H_{c,t})} = \frac{R^K_t}{Z_t F_K(K_{c,t}, H_{c,t})}, \]
\[ \hat{\lambda}_{c,t} = \beta E_t \hat{\lambda}_{c,t+1} + \frac{(1 - \alpha)(1 - \alpha \beta)}{\alpha} \hat{m}_{c,t}, \]

where \( mc_{c,t} = P_{c,t} mc_{c,t}. \)
The durable firm equilibrium conditions after aggregation are the following:

\[
mc_{d,t}^\alpha = \frac{W_t}{Z_tF_H(K_{d,t}, H_{d,t})} = \frac{P_t^K}{Z_tF_K(K_{d,t}, H_{d,t})},
\]

\[
P_{d,t} = \frac{D_t^N}{D_t^N + \frac{s_t}{\theta_d - 1}[(1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}]} \left( \frac{\theta_d}{\theta_d - 1} \right) mc_{d,t}^\alpha.
\]

The second-hand firm equilibrium condition for each market structure is:

\[
P_{u,t} = \begin{cases} 
P_{d,t} - P_{c,t}\epsilon, & \text{if market is competitive,} \\
P_{d,t} - P_{c,t}\epsilon_{m,t}, & \text{if market is monopsony,} \end{cases}
\]

where \(\epsilon_{m,t} = \epsilon - \frac{\delta_d(1 - \rho)}{1 - \rho \delta_d} \frac{U_{dd}(t)}{U_c(t)} M_t.\)

Monetary policy and market clearing conditions after aggregation are:

\[
\hat{R}_t = \tau_\pi[s_c \hat{\pi}_{c,t} + (1 - s_c) \hat{\pi}_{d,t}] + e^R_t,
\]

\[
C_t + \epsilon M_t \approx Z_tF(K_{c,t}, H_{c,t}) \quad (\text{“=} \text{ up to first order approximation starting at a steady state}),
\]

\[
D_t^N - M_t = Z_tF(K_{d,t}, H_{d,t}),
\]

\[
M_t = s_t[(1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}^N],
\]

\[
H_t = H_{c,t} + H_{d,t},
\]

\[
K = K_{c,t} + K_{d,t}.
\]

Finally, the technology shock process is

\[
\ln Z_t = \rho_Z \ln Z_{t-1} + e^Z_t.
\]

### A.2. Monopsony second-hand market

I assumed in the benchmark that second-hand firms are price-takers in purchasing the used good. In this section, I consider the other extreme where second-hand firms hold monopsony power and derive the second part of proposition 1. Since second-hand firms are competitive in the output market (i.e. they take \(P_{m,t}\) as given), justification is required for the different market structure for inputs and outputs. I assume that second-hand firms are representative and atomic, and each representative second-hand firm gets randomly assigned to a representative household in purchasing their used durables. In this case, second-hand firms
are assumed to hold monopsony power while the output market remains perfectly competitive.

The monopsony second-hand firm now recognizes that the supply of used goods is a function of the price that it sets in purchasing these goods. In particular, the firm should be able to recognize that the higher price they set in purchasing used goods, the more goods households are willing to resell. The supply curve that the firm takes into account should come from the household equilibrium conditions. Assuming zero replacement adjustment costs, household equilibrium conditions for new durable purchases and resales are the following:

\[
P_{d,t} = P_{c,t} \frac{U_d(t)}{U_c(t)} + (1 - \rho \delta_d)E_t \Lambda_{t,t+1} P_{u,t+1},
\]

\[
P_{u,t} = P_{d,t} - \delta_d(1 - \rho)E_t \Lambda_{t,t+1} P_{u,t+1}.
\]

Combining these two,

\[
P_{u,t} = P_{d,t} - \delta_d(1 - \rho) \left( \frac{P_{d,t} - P_{c,t} \frac{U_d(t)}{U_c(t)}}{1 - \rho \delta_d} \right),
\]

or

\[
P_{u,t} = P_{d,t} \frac{1 - \delta_d}{1 - \rho \delta_d} + P_{c,t} \frac{\delta_d(1 - \rho) U_d(t)}{1 - \rho \delta_d U_c(t)}.
\]

Moreover, from the law of motion for durables can be expressed as follows:

\[
D_t = (1 - s_t)[(1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}^N] + D_t^N
\]

\[
= (1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}^N - s_t[(1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}^N] + D_t^N
\]

\[
= (1 - \delta_d)D_{t-1} + \delta_d(1 - \rho)D_{t-1}^N - M_t + D_t^N.
\]

For convenience, I summarize the above law of motion as the following function:

\[
D_t = D(M_t, D_t^N, D_{t-1}, D_{t-1}^N).
\]

I assume that household utility is additively separable with respect to the three arguments. Hence, the
The household pricing function for used durables can be written as follows:

\[
P_{u,t} = P_{d,t} \frac{1 - \delta_d}{1 - \rho \delta_d} + P_{c,t} \delta_d (1 - \rho) \frac{U_d(D(M_t, D^N_t, D_{t-1}^N, D^N_{t-1}))}{U_c(C_t)}.
\]

This is an inverse supply curve for used durables that the monopsony recognizes as far as the monopsony takes \( P_{d,t}, P_{c,t}, C_t, D_t^N \) as exogenous at time \( t \). For this to hold, it is sufficient to assume that the monopsony firm cannot internalize the household budget constraint. The following lemmas show that this inverse supply curve is well-defined.

**Lemma 1** The inverse supply curve is upward sloping from the monopsony’s point of view.

\[
\begin{align*}
\frac{\partial P_{u,t}}{\partial M_t} &= P_{c,t} \frac{\delta_d (1 - \rho) U_{dd}(D_t)}{1 - \rho \delta_d} \frac{\partial D(M_t, D^N_t, D_{t-1}^N, D^N_{t-1})}{\partial M_t} \\
&= -P_{c,t} \frac{\delta_d (1 - \rho) U_{dd}(D_t)}{1 - \rho \delta_d} U_c(C_t) > 0 \quad [: U_{dd}(D_t) < 0]
\end{align*}
\]

**Lemma 2** The slope of the inverse supply curve is increasing, if the utility function for durables exhibits prudence \((U_{ddd}(D_t) > 0)\).

\[
\begin{align*}
\frac{\partial^2 P_{u,t}}{\partial M_t^2} &= P_{c,t} \frac{\delta_d (1 - \rho) U_{dd}(D_t)}{1 - \rho \delta_d} U_c(C_t) > 0
\end{align*}
\]

For example, when the utility function is CRRA for durables, households are prudent in their consumption for durables.

With this inverse supply curve, the monopsony’s problem is the following:

\[
\max_{M_t} P_{m,t} M_t - P_{u,t}(M_t) M_t - P_{c,t} f(M_t),
\]

which delivers the equilibrium condition:

\[
P_{m,t} - P_{u,t} - P_{c,t} f'(M_t) \frac{\partial P_{u,t}}{\partial M_t} M_t = 0.
\]

With \( f(\cdot) \) being linearly homogeneous as above, the price of used durables becomes:

\[
P_{u,t} = P_{m,t} - \epsilon P_{c,t} - \frac{\partial P_{u,t}}{\partial M_t} M_t,
\]
which leads to proposition 1. Given their monopsony power, second-hand firms are able charge a lower purchase price for used durables compared to the perfectly competitive market.

With log separable utility function, the equilibrium condition can be written in a handy manner:

\[
P_{u,t} = P_{m,t} - \left( \epsilon + \frac{\delta_d (1 - \rho)}{1 - \rho \delta_d} \frac{\psi_D C_t M_t}{D_t^2} \right) P_{c,t}
\]
\[
= P_{m,t} - \epsilon_{m,t} P_{c,t}.
\]

A.3. Calibration details

There are three nonstandard calibration targets: \( \delta_d, \rho, \epsilon \). For these targets, we aim at the three moments: the average depreciation rate for consumer durables, the steady state transaction rate of used and new durables, and the rate of used durable margin to durable spending. Note that the model’s steady state relative transaction of used to new durable goods is

\[
s[(1 - \delta_d)D + \delta_d (1 - \rho)D^N] : X.
\]

Note that the following holds:

\[
D^N = \frac{\delta_d + s(1 - \delta_d)}{1 + \delta_d (1 - s)(1 - \rho)} D, \quad X = \frac{\delta_d (1 - s(1 - \rho))}{\delta_d + s(1 - \delta_d)} D^N.
\]

This leads to the effective depreciation rate \( \delta \):

\[
\delta = \frac{X}{D} = \frac{\delta_d (1 - s(1 - \rho))}{1 + \delta_d (1 - s)(1 - \rho)}.
\]

Moreover, the analytical rate between used and new durable goods is:

\[
\frac{M}{X} = \frac{s(1 - \rho \delta_d)}{\delta_d (1 - s(1 - \rho))}.
\]

Lastly, the nominal margin for used durable sales is assumed to be \((P_d - P_u)M\). I define the real margin for used durable sales by deflating this by the price index \(P_c\), so that the real cost that the dealers pay for dealership and refurbishment \(f(M)\) does not include a price term. Under the linear homogeneity assumption \((f(M) = \epsilon M)\), the rate of real margin to new durables is the following:

\[
\frac{f(M)}{X} = \epsilon \frac{M}{X}.
\]
Hence, it is straightforward to calibrate $\epsilon$: take the rate of the empirical real margin over new durables, over the empirical used to new durable transactions. After calibrating $\epsilon$, we have two parameters $\delta_d, \rho$ to calibrate. However, we have 3 unknowns, due to the existence of $s$, the steady state replacement rate. Hence we need to come up with a third equation by utilizing the following household durable replacement decision

$$1 = Q_r - \delta_d(1 - \rho)\beta,$$

where $Q_r(= P_d/P_u)$ is the relative price of new durables over used durables. It remains to express $Q_r$ as a function of the 3 unknowns. This comes from the second-hand pricing condition, which depends on the market structure. We consider 2 polar cases: perfect competition and monopsony.

### A.3.1. Perfect competition

Since the second-hand pricing equation is $P_u = P_d - \epsilon P_c$ in a perfectly competitive market, we have the following condition:

$$\epsilon = Q \left(1 - \frac{1}{Q_r}\right) = Q \frac{\delta_d(1 - \rho)\beta}{1 + \delta_d(1 - \rho)\beta},$$

where $Q(= P_d/P_c)$, the relative price of durables over nondurables, comes straightforwardly from the markup of durables. Hence, we have the following three equations with three unknowns $\rho, \delta_d, s$:

$$\delta_{\text{empirical}} = \frac{\delta_d(1 - s(1 - \rho))}{1 + \delta_d(1 - s)(1 - \rho)},$$

$$\left(\frac{M}{X}\right)_{\text{empirical}} = \frac{s(1 - \rho \delta_d)}{\delta_d(1 - s(1 - \rho))},$$

$$\epsilon_{\text{calibrated}} = Q \frac{\delta_d(1 - \rho)\beta}{1 + \delta_d(1 - \rho)\beta}.$$

### A.3.2. Monopsony

Under monopsony, $P_u = P_d - \epsilon P_c - (\partial P_u/\partial M)M$. The monopsony firm recognizes the following supply relation:

$$\frac{\partial P_u}{\partial M} = -P_c \frac{\delta_d(1 - \rho)}{1 - \rho \delta_d} \frac{U_{dd}}{U_c} (> 0).$$
Plugging this in the pricing equation:

\[ P_u = P_d - \left( \epsilon - \frac{\delta_d(1 - \rho)}{1 - \rho \delta_d} \frac{U_{dd}}{U_c} \right) P_c \]

\[ = P_d - \epsilon_m P_c. \]

With our utility specification, \( U_{dd} = -\psi_D / D^2 \), and \( U_c = \psi_C / C \). Plugging this in:

\[ \epsilon_m = \epsilon + \frac{\delta_d(1 - \rho)}{1 - \rho \delta_d} \frac{\psi_D CM}{\psi_C D^2}. \]

Since \( \epsilon_m = Q(1 - 1/Q_r) \), plugging in the household condition gives the following equation:

\[ \epsilon = Q \frac{\delta_d(1 - \rho) \beta}{1 + \delta_d(1 - \rho) \beta} - \frac{\delta_d(1 - \rho)}{1 - \rho \delta_d} \frac{\psi_D C M}{\psi_C D D}. \]

Hence we need to figure out \( \psi_D C / \psi_C D \) and \( M / D \).

The household optimality condition for new durable purchases pins down \( \psi_D C / \psi_C D \). Recall:

\[ P_d = P_c \frac{\psi_D C}{\psi_C D} + (1 - \rho \delta_d) \frac{P_d - P_u}{\delta_d(1 - \rho)}. \]

Hence,

\[ \frac{\psi_D C}{\psi_C D} = \frac{P_d}{P_c} - \frac{1 - \rho \delta_d}{\delta_d(1 - \rho)} \frac{P_d - P_u}{P_c} = Q - \frac{1 - \rho \delta_d}{\delta_d(1 - \rho)} \left( Q - \frac{Q}{Q_r} \right) = Q \left[ 1 - \frac{1 - \rho \delta_d}{\delta_d(1 - \rho)} \left( 1 - 1/Q_r \right) \right], \]

where \( Q = P_d / P_c \) and \( Q_r = P_d / P_u \). From the household optimality condition for replacement with zero adjustment costs

\[ Q_r = \frac{P_d}{P_u} = 1 + \delta_d(1 - \rho) \beta. \]

Therefore,

\[ \frac{\psi_D C}{\psi_C D} = Q \left[ 1 - \frac{(1 - \rho \delta_d) \beta}{1 + \delta_d(1 - \rho) \beta} \right]. \]

For \( M / D \), note that

\[ \frac{M}{D} = \frac{M X}{X D}. \]
implying that this is given by the two targets (relative used to new transaction rate and the average depre-
ciation rate). The analytical form is:

\[
\frac{M}{D} = \frac{s(1 - \rho \delta_d)}{1 + \delta_d(1 - s)(1 - \rho)}.
\]

Plugging these values back into the margin equation:

\[
\epsilon = Q \frac{\delta_d(1 - \rho)\beta}{1 + \delta_d(1 - \rho)\beta} - \frac{\delta_d(1 - \rho)}{1 - \rho \delta_d} Q \left[ 1 - \frac{(1 - \rho \delta_d)\beta}{1 + \delta_d(1 - \rho)\beta} \right] \frac{s(1 - \rho \delta_d)}{1 + \delta_d(1 - s)(1 - \rho)}.
\]

Hence we are back to three equations and three targets which solves the system.

However, margin in the data is no longer directly linked to the pure value-added component when dealers
hold monopsony power. Hence instead of matching the margin, I take an alternative calibration strategy to
fix \( \delta_d, \rho, s \) calibrated from the perfectly competitive second-hand market. With this strategy, depreciation
and relative transaction of used and new durables remain identical for both market structure. Since the
steady state replacement rate \( s \) is pinned down, we can obtain \( \epsilon \) and the implied value-added component of
second-hand markets. In this case, the implied value-added margin of the second-hand market is less then
what is assumed in the perfectly competitive benchmark.

A.4. Numerical analysis with a monopsony second-hand market

Most of the calibrations follow section VI, but when second-hand markets are assumed as monopsony, I
calibrate \( \epsilon \) differently as explained above (appendix A.3). For clear comparison, all other parameters and
the steady state replacement rate is held to be the same as in the perfectly competitive environment. The
parameter \( \epsilon \) is now smaller than the perfectly competitive case since some portion of the margin in data
should also reflect firms’ monopsony rent.

Figure A.1 shows the impulse response of monetary shocks when the second-hand market is assumed as
a monopsony. For quantitative comparison, the two cases with competitive second-hand markets are plotted
again. We observe that the dynamics for the monopsony second-hand market are similar to the case of
a competitive second-hand market with adjustment costs. Hence the second-hand market structure is not
crucial in resolving the comovement puzzle. Moreover, similarity of the the quantitative dynamics in the
monopsony market without adjustment costs and the perfectly competitive market with adjustment costs
suggests that adjustment costs in the perfectly competitive case may reasonably reflect market frictions.
A.5. Calibration of the counterfactual second-hand market in section VI

In the model, value-added of the second-hand market at the steady state is $\epsilon M$. Since the value-added of the new durable market is $X$, the rate of the value-added in the second-hand market to the new durable market is $\epsilon M/X$. Note that the followings hold in the model (see appendix A.3.):

$$\epsilon = Q \frac{\delta_d(1 - \rho) \beta}{1 + \delta_d(1 - \rho) \beta},$$
$$\frac{M}{X} = \frac{s(1 - \rho \delta_d)}{\delta_d(1 - s(1 - \rho))},$$

where $Q = P_d/P_c$. Expressing $Q$ in structural parameters and the steady state replacement rate $s$:

$$Q = \frac{P_d}{mc_d^d P_c} = \frac{mc_d^d}{P_c} = (1 + \mu_d) \frac{mc_c^c}{P_c} = (1 + \mu_d) \frac{\theta_c - 1}{\theta_c} \frac{\theta_c(\delta_d + s(1 - \delta_d) - \delta_d(1 - s(1 - \rho)))(\theta_c - 1 \theta_c)}{\theta_c(\delta_d + s(1 - \delta_d)) - \delta_d(1 - s(1 - \rho))}.$$

Hence, the relative value added of the second-hand market is expressed as follows:

$$\epsilon \frac{M}{X} = \frac{\theta_d(\delta_d + s(1 - \delta_d))}{\theta_d(\delta_d + s(1 - \delta_d)) - \delta_d(1 - s(1 - \rho))} \left( \frac{\theta_c - 1}{\theta_c} \right) \left( \frac{\delta_d(1 - \rho) \beta}{1 + \delta_d(1 - \rho) \beta} \right) \frac{s(1 - \rho \delta_d)}{\delta_d(1 - s(1 - \rho))} = g(s, \rho, \delta_d, \theta_d, \theta_c, \beta).$$

That is, the value-added is a function $g(\cdot)$ of 5 parameters $\{\rho, \delta_d, \theta_d, \theta_c, \beta\}$ and the steady state replacement rate $s$. Hence, changing the calibration of the value-added component amounts to changing these 6 parameters/variable. I fix $\theta_d, \theta_c, \beta$ since these are not directly linked to the structure of the second-hand market. Moreover, since both $\rho$ and $\delta_d$ govern the depreciation of the durable good, I fix $\delta_d$ and vary $\rho$. Hence the two parameter/variable that I allow to change is $\rho$ and $s$.

1. **Supply side:** Fixing $s$, all changes in the value-added is mapped into changes in the depreciation discount of new durables $\rho$. In this scenario, all the change in size of the second-hand market comes from the improvement of durability of new durable goods in the supply side. For example, if $\rho$ shifts downwards, new goods are less depreciated then before, and hence second-hand markets expand. In this case, new durable markup declines and the relative transaction of second-hand market to new durable market $M/X$ increases. Lastly, the steady-state depreciation of the durable goods declines.
The numerical strategy is to set $\epsilon M/X$ and obtain $\rho$ by holding other parameters/variable fixed. Compute $M/X$ with the newly obtained $\rho$ ($M/X$ is a function of $s, \rho, \delta_d$). Obtain $\epsilon$ from this. Compute $Q$, the net markup $\mu_d$, and the steady state depreciation rate. Using these newly computed statistics, obtain the steady state of the other variables in the model and conduct analysis based on these.

2. **Demand side:** Fixing $\rho$, all changes in the value-added is mapped into changes in the steady state replacement frequency of the households $s$. In this scenario, all the change in the size of the second-hand market comes from the households replacing their durables more often. This could be due to a better opportunity of second-hand market transactions or a higher preference for replacements not specified. In the model, these are all loaded on changes to $\epsilon$.

   In this case, a higher value-added corresponds to a higher replacement frequency. The relative transactions itself increases and the average depreciation also declines.

   The numerical strategy is similar to the above case: set $\epsilon M/X$ and obtain $s$ by holding the other parameters fixed. Compute $M/X$ with the newly obtained $s$ and $\epsilon$ from this. The rest is the same.

3. **Mixture:** The two motivations can also be mixed and I can also consider that. For example, using the value obtained in 1, I can also take the average value of $\rho$ before and after the change in value-added, and use the average $\rho$ to compute $s$. Using this, I repeat the same step to basically get the effect of the change in the value-added component of second-hand due to both the supply and the demand side. These turn out to deliver expected results which is a combination of the results from both motivations.
Figure A.1: IRF to a 0.25 percent increase in interest rate (with monopsony)