Biform Games:
Additional Online Material*

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These appendices supplement Brandenburger and Stuart [1, 2006], [2, 2006] ("Biform Games"). Appendix C uses the efficiency results (Propositions 5.1 and 5.2) in the main text to frame some of the ideas in the corporate-strategy literature. Appendix D contains further comments on the efficiency results.

Appendix C: A Connection to Corporate Strategy

Corporate (multibusiness) strategy is the study of when 'the whole is worth more than the sum of the parts.' Alternatively put, corporate strategy is about the opportunities that arise because individual entities (divisions of a corporation, potential alliance partners, etc.) may each be maximizing the value they can capture, but appropriate joint action by the entities could capture more value in total.

The literature usually distinguishes two kinds of corporate strategy. The "market power" route involves joint action that leads to capturing a greater slice of the existing pie. A good example is when different divisions (or businesses) engage in joint purchasing of an input, and thereby negotiate a lower unit cost. The "efficiency" route involves joint action that leads to creating a larger pie, and capturing at least a part of the increase. The focus of the literature is on the second kind of corporate strategy, and we follow that here.

Our observation is that the efficiency results in the main text (Section 5) give us a way to classify different kinds of efficiency-oriented corporate strategies. Think of a corporate strategy as a way of addressing an inefficiency. Then, by the results in Section 5, we can then break this down into addressing a failure of Adding Up, or No Externalities, or No Coordination. (Of course, a given strategy could address more than one failure.) Here is a sketch of how this scheme works.

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a. No Adding Up A big question in corporate strategy is that of the appropriate vertical scope of the firm. Within this topic, the Holdup problem has, of course, been central (e.g., Collis and Montgomery [3, 1998, pp.108-111]). Here is a simple biform model of Holdup. There are two players–call them the upstream firm U and the downstream firm D. If U makes an upfront investment of \$4, then U and D can transact and together create \$7 of (gross) value. Calculate added values: The overall pie is \$3, D creates \$0 on its own, U creates $-\$4$ on its own. The added value of D is thus \$7, the added value of U is \$3. In the Core, the \$3 pie will be divided so that D gets between \$0 and \$7, and U gets between $-\$4$ and \$3. (Of course, these calculations simply reflect the fact that the game is a bilateral monopoly between U and D.) Will U make the efficient decision to invest? Not necessarily–which is the Holdup problem. (In the biform game formalism, it will if and only if its confidence index is greater than $4/7$.)

Our analysis says that one way to understand the source of potential inefficiency in Holdup is as a failure of AU. (NE and NC are clearly satisfied.) Solutions to Holdup can be understood similarly. Vertical integration, bringing in a downstream competitor, etc. are ways of getting AU or closer to it. (For example, in its ideal form, vertical integration creates a one-player game, in which AU is automatically satisfied. A second downstream competitor would reduce the gap between the sum of the added values and the pie, ensuring the upstream firm more value.) But we are not suggesting any new solutions to Holdup, only pointing out that Holdup can be classified in our scheme.

Another area of corporate strategy where failures of AU come up is in horizontal relationships, e.g., in strategic alliances. Here, again, there may be inefficiency–usually, underinvestment–if a party isn't assured its added value.

b. Externalities Typical corporate-strategy issues that fit in here are the problem of transferring knowledge across divisions of a corporation and managing a corporate brand. The externality is that one division could take an action to share its knowledge with another division that would increase the value created by the second division (together with its customers and suppliers). Or, it is that one division might make an investment in a common brand that is suitable for that division's 'image' but unsuitable for another division's image–i.e., lowers willingness-to-pay for the second division's product, and so decreases the value created by that division, its customers, and its suppliers. In both cases, NE might fail, and inefficiency could result. Corporate-level knowledge and brand management are often discussed as appropriate strategies to overcome such potential inefficiencies.

c. Coordination Ideas around scale and scope would fit here. Consider several divisions where each has to decide which of two suppliers to source from. This could be analyzed as a biform game, similar to Example 5.2 in the text, where the *No* and *Yes* labels correspond to choosing one or other supplier. The efficient profile of choices would be when each division chooses the supplier corresponding to the *Yes* label. But each choosing the other supplier (the *No* label) would also be a Nash equilibrium. We would again have a coordination game, and a failure of NC. The gains from coordination would come from the usual reasons of scale and scope–opportunities to share in defraying the fixed costs of the *Yes* supplier, to speed this supplier's movement down the learning curve, etc. But with a failure of NC, there is no guarantee that the divisions, acting on their own, will make the efficient choice. Solutions to this kind of potential inefficiency are discussed in the literature under precisely the heading of coordination of the multibusiness firm (Collis-Montgomery [3, 1998, p.156]).

This is obviously not meant to be an exhaustive discussion of corporate strategy. Its purpose is simply to suggest that the efficiency framework of AU, NE, and NC may be a useful way to classify and unify some of the corporate-strategy literature. It does also suggest a 'problem-driven' approach to crafting corporate strategies: try to spot inefficiencies in the organization (or across organizations) and then find ways of correcting them.

Appendix D: Comments on Efficiency Results

i. Efficiency and Rationality We start by establishing a further implication of the AU, NE, and NC conditions.[1]

Proposition D1 *Consider a biform game $(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n)$ satisfying AU, NE, and NC, and that for each $s \in S$, the game $V(s)$ has a nonempty Core. Then each player has a (strongly) dominant strategy.*

Proof. Fix a player i and two strategies $r^i, s^i \in S^i$. Suppose that $V(r^i, s^{-i})(N) > V(s^i, s^{-i})(N)$ for some $s^{-i} \in S^{-i}$. Then NC implies $V(r^i, r^{-i})(N) > V(s^i, r^{-i})(N)$ for all $r^{-i} \in S^{-i}$. Applying NE yields

$$V(r^i, r^{-i})(N) - V(r^i, r^{-i})(N \backslash \{i\}) > V(s^i, r^{-i})(N) - V(s^i, r^{-i})(N \backslash \{i\})$$

for all $r^{-i} \in S^{-i}$. By AU, strategy r^i strongly dominates strategy s^i.

The remaining case is that $V(r^i, s^{-i})(N) = V(s^i, s^{-i})(N)$ for all $s^{-i} \in S^{-i}$. But then NE yields

$$V(r^i, s^{-i})(N) - V(r^i, s^{-i})(N \backslash \{i\}) = V(s^i, s^{-i})(N) - V(s^i, s^{-i})(N \backslash \{i\})$$

for all $s^{-i} \in S^{-i}$. By AU, strategy r^i is payoff-equivalent to strategy s^i for player i. Finiteness of the strategy sets then implies that each player has a strongly dominant strategy (up to repetition of payoff-equivalent strategies). ■

Since a profile of dominant strategies is a Nash equilibrium, we know by Proposition 5.1 in the text that it is efficient. Indeed, we can put Propositions 5.1 and 5.2 in the text, and Proposition D1 above, together in the following way. In a noncooperative game, a player is **rational** if he chooses a strategy that maximizes his expected payoff, under some probability measure on (the product of) the strategy sets of the other players. In a biform game, each player still associates a single

[1] John Sutton prompted us to prove this result.

number with each strategy profile (equal to a weighted average of the Core projection), and so we can define the rational strategic choices of a player correspondingly. With this terminology, we have the following equivalence statement:

Assume AU, NE, and NC (and nonemptiness of the Core for each strategy profile). Then, a profile consists of rational strategies if and only if the profile is efficient.

The proof is immediate. By Proposition E1, under AU, NE, and NC, each player has a strongly dominant strategy (or strategies). This is then the only rational strategy (or strategies) for that player, and we just said that a profile of dominant strategies is efficient. Conversely, Proposition 5.2 says that under AU and NE (even without NC), an efficient profile is a Nash equilibrium. And a Nash-equilibrium strategy is rational–when the player assigns probability one to the other players' strategies that make up the equilibrium.

Viewed this way, our results provide an answer to the question: "When is rationality equivalent to efficiency?" When are these two basic concepts, one from game theory and the other from welfare theory, equivalent? Our answer is that a sufficient condition for this is that AU, NE, and NC hold.

ii. Weak No Coordination Next, we note a possible weakening of the NC condition, and a corresponding counterpart to Proposition D1 (the proof mirrors the one above).

Definition D1 *A biform game* $(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n)$ *is said to satisfy* ***Weak No Coordination (WNC)*** *if for each* $i = 1, \ldots, n$*;* $r^i, s^i \in S^i$*; and* $r^{-i}, s^{-i} \in S^{-i}$*,*

$$V(r^i, r^{-i})(N) \geq V(s^i, r^{-i})(N) \text{ if and only if } V(r^i, s^{-i})(N) \geq V(s^i, s^{-i})(N).$$

Proposition D2 *Consider a biform game* $(S^1, \ldots, S^n; V; \alpha^1, \ldots, \alpha^n)$ *satisfying AU, NE, and WNC, and that for each* $s \in S$*, the game* $V(s)$ *has a nonempty Core. Then each player has a weakly dominant strategy.*

iii. Lack of Necessity. In Section 5 in the text, we mentioned the lack of necessity of the conditions of Propositions 5.1 and 5.2. To see that the lack of necessity in Proposition D1 above, consider the Prisoner's Dilemma (Figure D1), viewed as a biform game satisfying AU. (To see the game this way, note that AU in a two-player game implies that the game is inessential, so that the unique Core allocation gives the players their individually rational payoffs. These are the payoffs shown.) Here, NC holds but NE fails. Yet each player has a dominant strategy.

3, 3	0, 4
4, 0	1, 1

Figure D1

To see the lack of necessity in the equivalence statement in subsection i. above, consider Matching Pennies (Figure D2), viewed as a biform game satisfying AU. Here, as in Figure D1, NC holds but NE fails. Yet each strategy of either player is rational and all profiles are efficient.

1, -1	-1, 1
-1, 1	1, -1

Figure D2

iv. Relationship to Makowski-Ostroy In Section 5, we mentioned the difference between our definition of added value (marginal contribution) and the definition in Makowski-Ostroy ([4, 1994], [5, 1995]). We talk about player i's marginal contribution (defined in the standard game-theoretic way) in each of the cooperative games $V(s)$ induced by different first-stage strategy profiles $s \in S$. Let us now cast the M-O definition in the biform model (recognizing that their model is different). M-O would select a particular strategy $s_0^i \in S^i$ of player i, to be thought of as the strategy of "not participating in the game." They would then say that player i receives his marginal contribution if his payoff from choosing any strategy s^i, when the other players choose strategies s^{-i}, is equal to $V(s)(N) - V(s_0^i, s^{-i})(N\backslash\{i\})$. That is, player i receives the difference between the value created when i is "in the game" (and playing s^i) and the value created by the remaining players when i is "out of the game" (playing s_0^i). By contrast, we say that i receives his marginal contribution if the payoff is $V(s)(N) - V(s)(N\backslash\{i\})$. Note that if NE is assumed, then $V(s)(N\backslash\{i\}) = V(s_0^i, s^{-i})(N\backslash\{i\})$, so that the two definitions then coincide. Thus, seen from the perspective of our formalism, M-O effectively assume NE from the beginning, while we distinguish games that do or do not satisfy NE.[2]

Another difference between M-O and us is in the treatment of coordination. M-O use a condition ("No Complementarities") which, in the formalism of the biform model, says: For each $r, s \in S$,

$$V(s)(N) - V(r)(N) \leq \sum_{i=1}^{n} \left[V(s^i, r^{-i})(N) - V(r)(N) \right].$$

In words, the change in the value of the game when the players switch from strategy profile r to strategy profile s is no more than the sum of the changes caused by the players' switching one at a time. It can be shown that this condition holds if and only if we have: For each $i = 1, \ldots, n$;

[2] As a consequence, M-O would classify some examples differently from us. Thus, we attributed the inefficiency in Example 5.1 to a failure of NE. M-O would attribute it to firm 1's receiving \$1 from negative advertising, different from the marginal contribution of advertising, which M-O would calculate as −\$1. (Take the strategy of "not participating in the game" for firm 1 to be what we called the "status-quo" strategy.) We are grateful to Louis Makowski for much help on the difference between our formalisms.

$r^i, s^i \in S^i$; and $r^{-i}, s^{-i} \in S^{-i}$,

$$V(r^i, r^{-i})(N) - V(s^i, r^{-i})(N) = V(r^i, s^{-i})(N) - V(s^i, s^{-i})(N).$$

By standard arguments, this latter condition holds if and only if the value of the game is additively separable as a function of the players' strategy choices. Our No Coordination condition (Definition 5.2) is weaker than the M-O condition, and, in particular, does not impose additive separability.

References

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[3] Collis, D., and C. Montgomery, *Corporate Strategy: A Resource-Based Approach*, McGraw-Hill, 1998.

[4] Makowski, L., and J. Ostroy, "Is Giving People Their Marginal Products Good for Incentives? Revisiting the Pigovian Tradition," unpublished, Department of Economics, University of California at Davis, 1994.

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