SYLLABUS

IEOR E4721 – Topics in Quantitative Finance:
Modeling Equity Derivatives in Java

Term: Summer 2007
Department: Industrial Engineering and Operations Research (IEOR)

Instructor: Iraj Kani
TA: Wayne Lu

References:


Requirements: Basic knowledge of financial derivatives and derivative valuation methodologies is strongly recommended. Knowledge of programming and object-oriented concepts is also recommended.

Assignments and Grading: There will be 4-5 assignments during the course, generally involving programming by groups composed of 2 to 4 students, depending on the class size. Grading is based on performance on the assignments, overall learning and general level of participation in the course.

Office Hours: There will be 2 office hours by the instructor (in the adjuncts office) and 4 office hours by the TA (in the computer laboratory).

Course Website: http://www.columbia.edu/~ik2133
Overview

Martingale methods and Partial Differential Equations are among the most commonly used techniques in financial engineering. This course will cover numerical methods used for implementing these techniques, and their application to pricing and hedging of equity exotic options and equity structured products. We will discuss the conceptual and mathematical principals underlying these techniques, and go over practical issues that arise in programming and testing their implementations in a Java programming environment. We will review various contractual provisions commonly embedded in equity-related structures and attempt to incorporate volatility smile and other real-world nuances of the underlying price processes into our implementation framework. We will cover fundamentals of application development and learn how to apply them in context of risk and portfolio management of equity derivatives.

Aims

- To develop a conceptual and practical understanding of the pricing methodologies using lattice techniques and finite difference solutions of partial differential equations.

- To highlight the strengths and shortcomings of different numerical methods, and to gain an understanding of where a specific numerical method can or should be applied.

- To understand the accuracy, stability, convergence and efficiency of various numerical methods.

- To develop overall competency with computing environments and use of numerical techniques.

- To gain exposure to different types of equity derivatives, exotic options, and structured products, and to become familiar with their practical treatment in the context of risk and portfolio management.

- To understand how the real-world features of the underlying price process such as discrete dividends and volatility smile affect pricing and hedging of equity derivative products.

- To grasp fundamentals of application development, data management, and user interface design.

- To highlight the importance of testing, parsimony, structure and reusability in the programming environment.
Objectives

The main objective of the course is to bridge the gap between the theoretical and mathematical concepts that belie derivatives pricing and the practical considerations that arise in implementation of numerical techniques, incorporation of commonly encountered contractual features and the nuances of the real-world markets, and the development of software applications used for pricing and managing risk of portfolios of equity derivatives.

Our objectives in this course are:

- To gain familiarity with different types of equity derivative products, including commonly known equity exotic options, warrants and convertible bonds, and equity-linked structured products.

- To develop competency with lattice methodologies and implementation of numerical solutions to partial differential equations related to the valuation of equity derivative products.

- To enhance students’ familiarity with commonly embedded contractual provisions such as callability/puttability, lookback and reset features, and with the real-world nuances of the underlying price processes such as discrete dividends, periodic settlement, and volatility smile.

- To introduce students to software development concepts for developing applications that can be used for pricing and managing risk of these products on a daily basis.

Course Outline

Part I – Fundamentals

Fundamentals of Java Programming

Java Programming Language

- Java Platform and Java Virtual Machine
- Java Language Constructs (Packages, Classes, Methods, Fields, and Interfaces)
- Encapsulation, Overwriting, Inheritance, and Polymorphism
- Java Exception Handling
- Syntactic and Semantic Features
- Fundamentals of Object-Oriented Design
Developing Simple Programs

- Integrated Development Environments (IDEs)
- Defining Simple Classes and Interfaces
- Implementing Analytical Solutions for Standard Options and Barrier Options
- Execution, Testing and Debugging of Java Programs

Fundamentals of Derivatives Pricing

Stochastic Processes and Differential Equations

- Continuous Markov Processes and Stochastic Differential Equations
- Arbitrage Pricing Theory and Martingale Representations
- Ito’s Formula for Continuous and Discontinuous Processes
- Chapman-Kolmogorov and Feynman-Kac Equations
- Numeraires, Change of Measure and Girsanov’s Theorem

Contingent Claims Valuation

- No-Arbitrage Conditions
- European Options Pricing
- American Options and Linear Complementarity Formulation
- Valuation of Path-Dependent Options
- Pricing Equations for Discontinuous Processes

Part II – Numerical Solutions: Trees and Finite Difference

Discretization and Numerical Integration

Binomial and Trinomial Trees

- Backward Equations and Tree Implementations
- Binomial and Trinomial Discretizations of Weiner Processes
- Tree Implementations for Mean-Reverting Processes
- Convergence, Speed and Accuracy of Tree Techniques
- Extension to Multinomial Trees and Other Lattice Techniques

Finite Difference Solutions

- Constructing Finite Difference Approximations
- Explicit, Implicit, Crank-Nicolson and Theta Methods
- Stability, Convergence, Speed and Accuracy of Finite Difference Methods
- Implementing Boundary Conditions
- Early Exercise Features and SOR Method
Alternative Price Processes and Payoff Features

Discontinuous Price Processes

• Implementing Jump Conditions
• Discrete Dividends and Predictable Jumps
• Pure Jump Processes
• Jump-Diffusion Processes

Volatility Smile

• Implied and Local Volatility Surface
• Local and Stochastic Volatility Models of the Smile
• Parametrization of Implied and Local Volatility Surfaces
• Implied Tree Implementations

Payoff Features

• Discontinuous Payoffs and Barrier Conditions
• Exercise-Exercise and Conversion Features
• Callability and Puttability Features
• Discrete and Continuous Cashflows

Part III - Equity Derivatives and Structured Products

Valuation of Standard and Path-Dependent Equity Options

Standard Equity Options

• Standard European Options
• American and Bermudan Options
• Equity Index Options
• General Payoffs and Exercise Conditions

Path-Dependent and Exotic Equity Options

• General Formulation of Path-Dependent Options
• Single and Double Barrier Options
• Lookback and Asian Options
• Exotic Equity Derivatives
Equity Structured Products

- Warrants and Convertible Bonds
- Equity-linked Notes
- LYONs, PERCs and Synthetic Convertible Bonds