SYLLABUS

IEOR E4728 – Topics in Quantitative Finance: Inflation Derivatives

Term: Summer 2007
Department: Industrial Engineering and Operations Research (IEOR)

Instructor: Iraj Kani
TA: Wayne Lu

References:


*Inflation-Indexed Securities, Bonds, Swaps and Other Derivatives, 2nd Edition*, Mark Deacon, Andrew Derry and Dariush Mirfendereski, Wiley 2004


*Option Pricing, Mathematical models and computation*, Paul Wilmott, Jeff Dewynne, Sam Howison, Oxford Financial Press, 1993

Requirements: Basic understanding of interest rate derivatives and derivative valuation methodologies is strongly recommended. Knowledge of Microsoft Excel environment and basic programming concepts is also recommended.

Assignments and Grading: There will be 4-5 homework assignments during the course, involving implementation in Microsoft Excel / VBA or Java (MATLAB and other standard programming environments are also admissible) by groups of 2-4 students depending on the class size. Grading is based on performance on the assignments, overall learning and general level of participation in the course.

Office Hours: There will be 2 office hours by the instructor (in the adjuncts office) and 4 office hours by the TA (in the computer laboratory).

Course Website: http://www.columbia.edu/~ik2133
Overview

In recent years there been a substantial increase in the issuance of inflation-linked bonds and a growing interest in inflation-indexed derivative and hybrid securities by the inflation market participants worldwide. In this course we will seek to gain a conceptual and practical understanding of inflation-linked bonds and derivative securities from a pricing, hedging and risk management perspective. We will examine pricing of these securities as interest-rate contingent claims depending on both the real and nominal rates, and using the foreign exchange analogy, prompted by the specific (inflation-indexed) nature of their payoffs, and corresponding change of measure techniques commonly used in pricing foreign exchange derivatives.

This course will also cover numerical methods, and in particular Monte-Carlo techniques, and their application to pricing interest rate and inflation derivatives. We will discuss the conceptual and mathematical principals underlying these techniques, and examine practical issues that arise in their implementations in the Microsoft Excel/VBA and other programming environments. We will review various contractual provisions commonly embedded in interest rate derivatives and inflation-linked securities and attempt to incorporate correlation structure, volatility smile and other features of the underlying interest rate processes into our pricing and implementation framework.

Aims and Objectives

- To develop a conceptual and practical understanding of the (interest rate) derivative pricing methodologies and use of analytical solutions and Monte Carlo techniques in the context of interest rate derivatives and inflation-linked securities.

- To gain familiarity with different aspects of interest rate processes, including short rate models, HJM forward-rate framework, LIBOR and Swap Market Models.

- To examine calibration of interest rate models to the market data, and their extensions and applications to multiple yield curves and inflation-indexed payoffs required for pricing inflation-linked derivative securities.

- To gain exposure to different types of inflation derivatives and hybrid inflation-linked products, and gain familiarity with their practical treatment in the context of pricing, hedging, and risk management.

- To understand how the real-world features of the interest rate processes such as correlation structure, volatility smile, and the manner that they affect pricing and hedging of interest rate and inflation-linked products.
Course Outline

Part I – Fundamentals

Fundamentals of Stochastic Differential Equations

Stochastic Processes and Stochastic Differential Equations

- Deterministic and Stochastic Differential Equations
- Brownian Motion and Stochastic Integrals
- Martingales and Semi-martingales
- Quadratic Variation and Covariation
- Solutions to General SDE
- Interpretation of Coefficients of SDE
- Ito’s Formula and Stochastic Leibnitz rule
- Discretizing SDEs with Monte Carlo
- Feynman-Kac theorem
- Girsanov Theorem and Change of Measure

Examples of Stochastic Differential Equations

- Linear SDEs with Deterministic Diffusion Coefficients
- Lognormal SDEs and Geometric Brownian Motion
- Square-Root Processes and CEV Models

Fundamentals of Interest Rates

Basic Interest Rate Concepts and Notations

- The Bank Account and Short Rate
- Zero-Coupon Bonds, Spot Interest Rates, and Forward Rates
- Fundamental Interest Rate Curves
- Interest Rate Swaps and Forward Swap Rates
- Interest Rate Caps/Floors and Swaptions

No-Arbitrage Pricing

- No-Arbitrage in Continuous Time
- Change of Numeraire Techniques
- Spot and Forward Measures
- Fundamental Pricing Formula
- Foreign Markets and Numeraire Change
Part II – Interest Rate Models

Short Rate Models

One-factor Short Rate Models

- Endogenous and Exogenous Short Rate Models
- Classical Short Rate Models: Vasicek, Dothan, and Cox-Ingersoll-Ross Models
- Hull-White Extended Vasicek Model
- Extensions of CIR Model
- Black-Derman-Toy and Black-Karasinski Models
- Volatility Structure of Short Rate Models

Heath-Jarrow-Morton (HJM) Framework

- The HJM Forward-Rate Dynamics
- No-Arbitrage Forward Rate Drift Restrictions
- Markovian Short Rate Processes
- Ritchken and Sankarasubramanian Framework

LIBOR Market Models

- Lognormal Forward-LIBOR Model (LFM)
- Forward Rate Dynamics under Different Numeraires
- Calibration of LFM to Caps and Floor Prices

Swap Market Models

- Lognormal Forward-Swap Model (LSM)
- Swaption Pricing under LSM
- Structure of Instantaneous Correlations - Full and Reduced Rank Formulations
- Calibration to Market Data

Incorporating Volatility Smile

- Modeling the Smile
- Local Volatility Models
- Stochastic Volatility Models
- Uncertain Parameter Models
Part III – Inflation Derivatives

Pricing Inflation Linked Derivatives

General Formulation

- Inflation-Linked Bonds
- The Foreign Exchange Analogy
- Inflation-Linked Derivative Securities
- Jarrow-Yildrim (JY) Model

Inflation-Indexed Swaps

- Zero-Coupon Inflation-Indexed Swaps (ZCIIS)
- Year-on-Year Inflation-Indexed Swaps (YYIIS)
- Pricing of YYIIS with JY Model
- Pricing of YYIIS with Market Models

Inflation-Indexed Caps and Floors

- Inflation-Indexed Caps and Floors
- Caplet/Floorlet Pricing using JY Model
- Caplet/Floorlet Pricing using Market Models
- Calibration to Market Data

Pricing with Stochastic Volatility

- Heston PDE and Its Solution
- Forward CPI Evolution with Stochastic Volatility
- Pricing Formulas and Their Solutions

Hybrid Inflation Derivatives

- Equity – Inflation Hybrids
- Other Inflation-Linked Hybrid Securities
- Pricing Hybrid Inflation Derivatives using JY Model
- Pricing Hybrid Inflation Derivatives using Market Models