1 Characteristics of the large scale extra-tropical circulation.

We will be concerned with large scale motions in the extra-tropics. That is motion on the synoptic scale (length scale ∼1000m, timescale ∼1 week) and planetary scale (length scale ∼10 000km, time scale ∼months). Large scale motions in the extra-tropics have certain characteristics that allow us to make some simplifying approximations to the equations of motion.

- **Rotation is important**: The relative importance of the Earths rotation to the motion is determined by the **Rossby number**. If the motion has a characteristic length scale (L) and velocity (U) then the Rossby number is given by $R_o = U/fL$ where f is the coriolis parameter. This is the ratio of the inertial forces to rotational forces or the ratio of the rotational timescale to the advective timescale. For large scale extra-tropical motions $R_o < 1$. Note that at the equator $f \rightarrow 0$ and R_o is never small.

- **The motion is shallow**: Let H be the vertical scale of the motion. For the atmosphere $H \sim 10km$, for the ocean H ∼1 to 5km. Given our typical length scale of 1000km for atmospheric motions, typical aspect ratios are of the order $\delta \sim H/L \sim 10^{-1} \rightarrow 10^{-2}$.

- **The motion is highly stratified**: On the large scale the atmosphere and ocean are strongly stably stratified. This inhibits vertical motion and reinforces small δ. For stable stratification in a density conserving fluid the density decreases with height (See Fig. 1). A measure of the stratification is the **Brunt-Vaisäla** or **Buoyancy frequency** given by

$$N^2 = -\frac{g}{\rho}\frac{\partial \rho}{\partial z}. \tag{1}$$

  In the atmosphere we are normally concerned with adiabatic motion of an ideal gas. In that case potential density rather than density is conserved and the Brunt-Vaisäla frequency is often written in terms of potential temperature as follows (see Vallis 2.9.1).

$$N^2 = \frac{g}{\theta}\frac{\partial \theta}{\partial z} \tag{2}$$

  If $N^2 > 0$ then a parcel that is displaced upward is heavier than its surroundings, there is a stable stratification. If $N^2 < 0$ then the density profile is unstable and the parcel continues to ascend and convection occurs. The **Richardson Number** is a measure of the effects of stratification. It is the ratio of the stability to the vertical shear i.e. $Ri = N^2/(\partial U/\partial z)^2 = N^2H^2/U^2$. If Ri is small (less than $\sim 1/4$ then the vertical shear is more important relatively and can overcome the tendency of a stratified fluid to remain stratified. You get wind shear induced overturning of a stratified fluid (Kelvin-Helmholtz instability). When Ri is large then turbulent mixing across the stratification is generally suppressed.

So, large scale motions in the extra-tropics can be characterised by three dimensionless parameters:(1) The Rossby number, $R_o << 1$, (2) the aspect ratio, $\delta << 1$ and (3) The Richardson number, $Ri >> 1$.

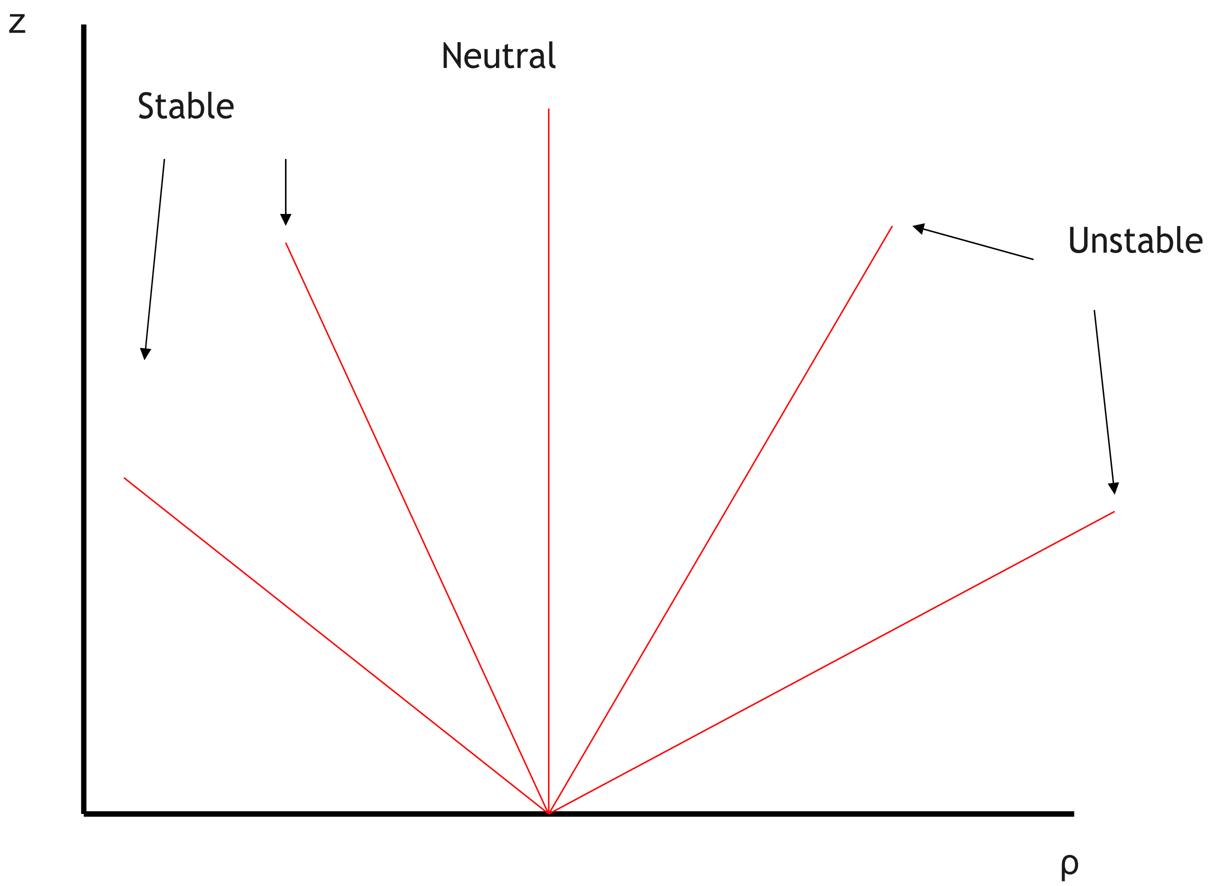

Figure 1: Schematic of stable,unstable and neutral density profiles.