Quantum Field Theory[1]

- Relativity and quantum mechanics each raise philosophical problems. However, their combination raises new problems. The most obvious is how to marry the collapse of the wavefunction with the relativity of simultaneity. But the combination of quantum mechanics and relativity raises many other issues too, such as: whether the Schrodinger picture is more accurate than the Heisenberg picture (or whether some alternative, like the Dirac's interaction picture is superior to both), what makes for a particle, how to understand virtual particles and Feynman diagrams, the vacuum, and gauge invariance.

Field Equations

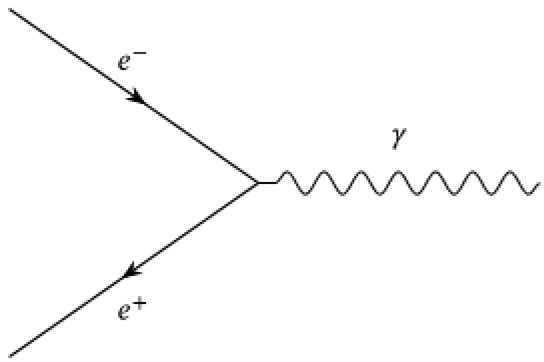
- Quantum Field Theory (QFT) is standardly formulated from Lagrangians (Lagrangian densities) by demanding that they satisfy certain symmetries. By Noether's Theorem, every symmetry corresponds to a conserved quantity. The key (free) field equations are:
    - Klein-Gordon Equation (Spin 0 particles): $(\partial_\mu \partial^\mu + m^2)\Phi = 0$.
    - Dirac Equations (Spin ½): $(i\gamma_\mu \partial^\mu - m)\Psi = 0$ (for particles) & $(i\partial^\mu \bar{\Psi}\gamma_\mu + m\bar{\Psi}) = 0$ (for antiparticles)
        - *Note*: One can write this as one equation, letting be Ψ the four-component **Dirac Spinor**. The equation then has positive and negative energy solutions, whose states are represented by a two-component **Weyl Spinor**.
    - Proca Equation (Spin 1 particles): $m^2 A^\rho = (½)\partial_\sigma(\partial^\sigma A^\rho - \partial^\rho A^\sigma)$ (which is just the Inhomogeneous Maxwell Equation when $m = 0$).
- *Example*: The Lagrangian $\mathcal{L} = (½)\,(\partial_\mu \Phi \partial^\mu \Phi - m^2\Phi^2)$ gives the Klein-Gordon Equation.
- Canonical Commutation Relations: $[\Phi(x), \pi(y)] = \Phi(x)\pi(y) - \pi(y)\Phi(x) = i\delta(x-y)$ (where $\delta(x-y)$ is the Dirac Delta Distribution and $\pi(y) = \partial\mathcal{L}/\partial(\partial_0\Phi)$ is the conjugate momentum).
    - *Note*: Conjugate momentum is the conserved quantity that is implied by Noether's Theorem from the following translational symmetry of $\Phi(x)$: $\Phi(x) \to \Phi(x + \epsilon)$.

[1] Thanks to Eric Majzoub and Porter Williams for comments.

- *Observation*: Numbers commute, but quantum fields do not. Fields are, accordingly, now represented by operators. However, it turns out that the field operators, $\Phi(x)$, can be written in terms of so-called **creation** and **annihilation** operators, $a^\dagger(k)$ and $a(k)$ (so $[a^\dagger(k), a(k)] = i\delta(x-y)$). These operators are used to construct the **Fock space** of non-interacting particle states in QFT. Everything else in the solutions is just a number.
    - The operator $a(k)$ *lowers the energy* of the system by the amount ω_k, while $a^\dagger(k)$ *raises the energy* of the system by that amount. Indeed, if $|0\rangle$ is a state with no 'particles', then $a^\dagger(k)|0\rangle$ is a state with one particle with energy ω_k, written $|1_k\rangle$
        - *Note*: In light of **Pauli's Exclusion Principle**, one requires that $a^\dagger(k)a^\dagger(k)|0\rangle$ gives 0 for **fermions** (such as electrons, which have half odd integer spin) but not for **bosons** (such as photons, with integer spin).
- 'Operator-valued' fields contrast with traditional fields, like Maxwell's. The latter assign definite physical quantities to each spacetime point. By contrast, operators represent a range of quantities. We get a physically significant formula by supplying a state vector.
- The operators, not the state vectors, carry the time-dependence. But this should not be mistaken for a deep fact of metaphysics. It reflects our decision to formulate QFT using the Heisenberg picture. If we make certain assumptions (like that the universe is *folliable*, as is Minkowski spacetime), then we can switch between it and the Schrödinger Picture.
- *Observation 1*: Despite being operators, quantum fields are not generally *themselves* **observables** and need not be gauge invariant or satisfy spacelike commutation relations.
- *Observation 2*: Hamiltonians are now constructed from quantum fields. So, Hamiltonians remain operators. One computes them also from the Lagrangians.

Scattering, Feynman Diagrams & Virtual Particles

- In practice, Quantum Field Theory is concerned with computing **scattering matrices**, S, (and **cross sections**) . The entries in the S matrix give the amplitude of detecting, e.g., a photon of momentum, k, upon colliding an electron of momentum, p, and positron of momentum, q, written: $\langle\gamma(k)|S|e_-(p), e^+(q)\rangle$



- One approximates the scattering matrix using a **perturbative expansion** in terms of **Feynman diagrams**, each contributing less to the sum (analogous to the expansion of 1/√(1+x) for small x). What is observable is $|S|^2$, where one *sums before squaring*.
- Feynman diagrams <u>do not represent physical processes</u>. They represent <u>formulas</u>.
  - The different diagrams <u>interfere</u>, like terms in the double-slit formula.
  - The <u>time axis</u> has no meaning. Diagrams are individuated by topological identity.
  - Individual diagrams are not <u>gauge invariant</u> (see below). Only the sums are.
- The <u>internal lines</u> of Feynman diagrams represent infamous **virtual particles**. These are the '<u>exchange particles</u>' from popular accounts of particle physics. *If they existed,* then they would be '<u>off-mass shell</u>', violating the relativistic energy-momentum relation, $E^2 = p^2c^2 + m^2c^4$. But it is unclear whether they do exist. Popular accounts notwithstanding, there is no '<u>energy-time uncertainty</u>' relation that could support that standard argument that virtual particles can 'borrow energy from the vacuum as long as they give it back in time'. (Note that energy-momentum, not energy *per se*, is what matters in QFT.) One can avoid reference to virtual particles altogether in *Lattice Quantum Field Theories*.
- *Upshot*: Feynman diagrams are <u>formal artefacts</u> of perturbation theory.

<u>Particles or Fields?</u>

- The obvious ontology to attribute to QFT is one of *fields* (whose states are superpositions of a field configurations given by a *functional* Ψ(φ) assigning amplitudes to states of a classical field). But I alluded to the fact that one can define a 'number operator', $N_k|\Psi\rangle = a^\dagger(k)a(k)|\Psi\rangle$, whose eigenvalues are said to be the *number of particles with energy* ω_k.
- However, talk of 'particles' can be misleading.

- While there is a number operator for **free fields**, there is not for **coupled fields**.
    - *Note*: Scattering amplitudes are calculated using 'In' and 'Out' states, which correspond to the free particle states before and after collision. It is when the particles are far apart that the free field description works well.
- The state, |Ψ>, on which N_k acts are **Fock kets** like |0, 1, 12, 0, 3, 1, 1,...> These are states that aggregate quanta, telling us, in the present case, that there are 0 particles with energy ω_1, one particle with energy ω_2, 12 particles with energy ω_3, and so on. However, they do not pretend to count individual particles in the sense that particles do not carry 'labels', or, more carefully, haecceities. That idea does not make sense in even non-relativistic quantum mechanics. **Identical particles** are really *identical*, and obey demonstrably different statistical laws
    - *Example*: Already non-relativistic quantum mechanics fails to distinguish a situation where particle 1 is in state S and identical particle 2 is in state S* from a one in which particle 2 is in state, S, and particle 1 is in S* (although if the particles are *fermions* the wave function changes sign).
- Given a basis of Fock kets, a generic state, |Ψ>, is a superposition of different numbers of quanta: |Ψ> = c_1 |0, 1, 5, 0, 9, 1, 1, 0,...> + c_2 |1, 0, 0, 0, 0, 1, 1,...> +...
- Particles, in the sense of QFT, can be **created and destroyed**. An electron and a positron, say, can spontaneously annihilate, leaving a photon, or vice versa.
- As in non-relativistic quantum mechanics, particles cannot be assigned **trajectories** through spacetime. However, in QFT, **position** is not even a generally applicable idea! For instance, there is no position operator for photons[2]. Indeed, the **Reeh-Schlieder Theorem** limits talk of localized states generally.
- Although inertial observers agree on the **vacuum**, |0>, a uniformly accelerating observer sees 'Rindler quanta'. This is the **Unruh Effect**. By the Equivalence Principle, observers must also disagree about photons in a gravitational field.

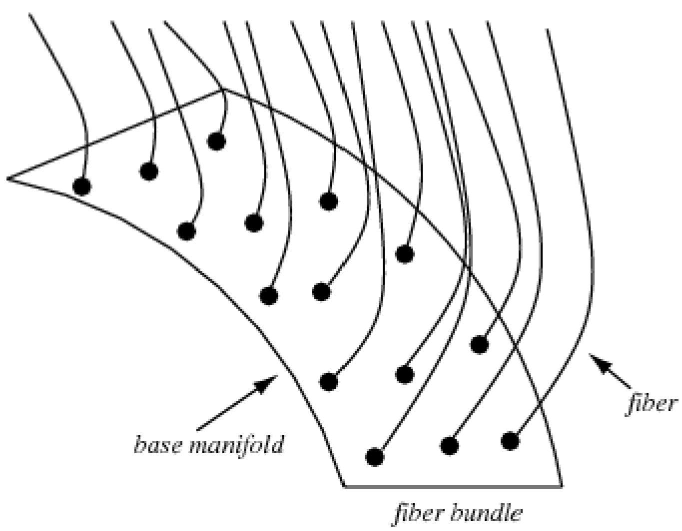
Local Gauge Theories & the Standard Model

[2]For details, see: https://physics.stackexchange.com/questions/492711/whats-the-physical-meaning-of-the-statement-that-photons-don t-have-positions

- A guiding idea of contemporary physics is that the Lagrangians giving the laws of physics should be **locally gauge invariant** (symmetric). Local invariance contrasts with **global invariance**, like that of space, time, or rotation (giving momentum, energy, and angular momentum as conserved quantities, by Noether's Theorem). The demand that Lagrangians be locally gauge invariant requires the existence of **gauge bosons** (like photons for the Electromagnetic force or gluons for the Strong force). The recipe is:
    - Begin with a non-gauge invariant free field Lagrangian, $\mathcal{L}$.
    - Replaces the ordinary derivative, ∂_μ, with the covariant derivative, $D_\mu = \partial_\mu - iA_\mu$ (A_μ will be the gauge boson field or potential).
    - Expand the result to get a locally gauge invariant sum of the free Lagrangian, $\mathcal{L}$, and Interaction Lagrangian, $\boldsymbol{I}$: $\mathcal{L}^* = \mathcal{L} + \boldsymbol{I}$ (where $\mathcal{L}^* = \mathcal{L}$ with no coupling).
- *Upshot*: Free field Lagrangians are not locally gauge invariant, but interacting ones are.
- *Note*: The resulting Lagrangian is *not*, in general, unique. One must also invoke considerations of 'simplicity' and renormalizability, in addition to Lorentz invariance. The demand for renormalizability, at least, can be explained on the ground that non-renormalizable interactions would be insignificant at currently accessible energies.
- *Why* should the Lagrangians giving the laws of physics be locally gauge invariant? The argument originally sketched by Yang and Mills [1954] suggests that performing a global gauge transformation would involve superluminal causation. But that supposes that the transformation is active. The standard view is that the transformations are passive (mere coordinate relabelings). If so, then nothing physical is changed by the transformation.
- Perhaps the best answer is: because it works! But *why* it should work is not obvious.
- The **Standard Model** results from imposing **three local gauge invariances** (symmetries) on the fermion field operators -- namely, U(1), corresponding to neutral gauge bosons, SU(2), corresponding to three vector bosons, and SU(3), corresponding to eight gluons (blurring over the important fact of **chirality**, and the seminal **electroweak unification**).
- U(1) 'rotates' charge singlets (γ) in **phase space** (the electromagnetic force), SU(2) rotates weak isospin doublets (W^+, W^-, Z^0) in **flavor space** (the weak interaction), and SU(3) rotates color charge triplets (the 'color octet') in **color space** (strong interaction).

- *Recall*: U(n) is the set of complex matrices with non-zero determinant such that $U^* = U^{-1}$, and SU(n) is the Special Unitary subgroup of U(n).

- There is a tantalizing analogy between local gauge theories of the Standard Model and General Relativity (GR), notwithstanding the failure to unify General Relativity and the Standard Model. Charged particles 'curve' internal spaces, and are affected by the spatial curvature in turn. Potentials, A_μ, are **connections** giving the curvature of the space. Each **fiber** is a copy of the space and has the symmetry of the gauge group. The **fiber bundle** is the collection of all fibers. Just as we are free to choose coordinates in GR, we are free to choose a phase in charge space, which axes to call the electron and neutrino axes in isospin space, and which axes to call 'red', 'green', and 'blue' in color space. *As in GR, the objective facts are those which are indifferent to our local conventions, i.e., 'gauges'*.



[3]

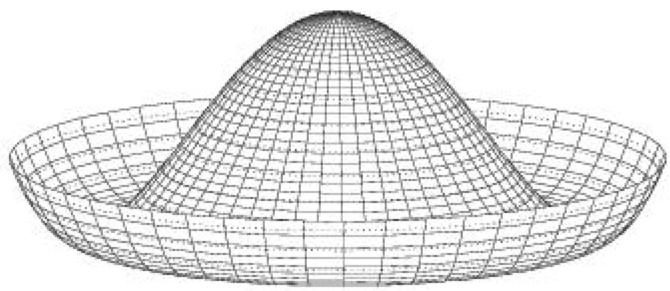
Quantum Vacuum and the Higgs

- The quantum vacuum is the ground state, |0>, i.e., the state of lowest energy. This is, famously, *not* nothing. For a *free* field, Φ, the **expectation value** $\langle\Phi\rangle_0 = \langle 0|\Phi(x)|0\rangle = 0$. However, the field exhibits **vacuum fluctuations** given by the *square* $\langle 0|\Phi^2(x)|0\rangle \neq 0$.
    - *Note*: Nothing is *dynamically fluctuating*. We get vacuum fluctuations of an observable whenever the vacuum state fails to be an eigenstate of an operator to which the observable corresponds. The vacuum is an eigenstate of the number

[3] Image taken from https://mathworld.wolfram.com/FiberBundle.html

operator, but not of complementary operators (the vacuum state of the universe need not be an eigenstate of there being no particles at a given point in it).

- For an *interacting* field system, even $<\Psi>_0$ may be nonzero. The ground state, $|0>$, of the scalar **Higgs field**, ϕ (which is inserted by hand -- i.e., does not 'follow' from the mere demand for local gauge invariance) is positive and degenerate. The fact that the Lagrangian of the system is *rotationally symmetric*, although its ground states are not, is an example of **spontaneous symmetry breaking**, which triggers the **Higgs mechanism**.



Potential of the Higgs Field

- *Note*: The Higgs field can only be nonzero everywhere as a (spin 0) scalar field. If it were a vector field, like the electromagnetic one, then it would give a preferred direction!
- The point of the Higgs is to hide the symmetry of the weak interaction. (Transforming electrons into neutrinos does not *appear* to be a symmetry!) It gives mass to the W^+, W^-, Z^0 (at all but very high temperatures), which the gauge symmetry would have precluded.
- The Higgs also turns out to be responsible for the masses of all known basic fermions. This makes it philosophically interesting insofar as it shows that mass is not an 'intrinsic' property. However, it does not contribute much else to philosophical discussion of mass.
- The Higgs does *not* tell us what mass is, why particles have the masses they do, or, arguably, even why they have masses. Particles that fail to interact with the weak force could have mass absent interaction with a Higgs field. Dark matter may be an example.

Collapse, Entanglement & Relativity

- Notwithstanding the fact that QFT 'marries quantum mechanics and Special Relativity', it remains to say how it affords an explanation of the appearance of state vector collapse.

- Recall that the collapse of the state vector of two particles in a singlet state, $|\Psi\rangle = \sqrt{(1/2)}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2)$, happens instantaneously. But there are no frame-invariant facts about what happens instantaneously, according to Special Relativity. So, if collapse is physically real, then it is at best Lorentz-covariant, like length or order.
- There is a way to formulate a Lorentz-covariant theory of collapse. On such a theory, relative to any frame containing one of the measurements, collapse is instantaneous.
- However, different frames will disagree dizzyingly about the history of the system. According to a frame in which particle 1 is measured first, particle 2 is not entangled when it gets measured, and the result of its measurement is a deterministic event. But according to a frame where particle 2 is measured first, exactly the opposite story is true!
- On this account, there is no frame-invariant fact as to a particle's spin at an event. There is not even such a fact as to whether a measurement of it is deterministic at an event. The Lorentz-invariant facts are that **relative to hyperplane**, H, particle 1 is, say, spin ↑ at event, E, and its measurement at an immediately subsequent event is deterministic.
- *Upshot*: On the present view, the **state vector of a system is relative to a hyperplane**!
- *Aharonov & Albert*: "The state reduction occurs separately along every spacelike hypersurface which passes through the emeasurement event; if one hypersurface is continuously deformed into another, the reduction occurs as [it] crosses that event."
- It is tempting to reply that properties like spin are intrinsic and non-relational, unlike *being to the left of*. At the very least, one might think that the state of the world supervenes on such properties! But given the 'modal pluralism' I have advocated, intrinsicness, supervenience, and so on may not be useful notions. The question of whether a property is intrinsic, or whether some properties supervene on others, is like that of whether two parallel lines will remain the same distance apart. It is not objective.
- There does remain the problem of explaining, or physically characterizing, collapse. But this does not seem hopeless given the work of Ghirardi, Grassi, Pearle, Tumulka, etc.