Handout #9

Bell's Theorem and Empirical Adequacy

- The EPR argument suggests that the correlations observed with **singlet states** are due to unknown variables concerning the individual spin states of electrons 1 and 2, prior to measurement of either particle. It points to a so-called **local hidden variables** theory.
- John Bell realized that any such a theory will make empirical predictions that could in principle be tested. His key insight was to ask what we should expect from such a theory when the measuring devices are oriented differently, rather than along the same axis.
- What he showed is that, given plausible assumptions, the predictions of any such theory disagree with those of standard (non-local) quantum mechanics. But the predictions of standard quantum mechanics are confirmed. So, one of the assumptions must be wrong.
- *Note*: This is not to say that (mere) hidden variables theories are incompatible with the confirmed predictions of quantum mechanics (contra the 'proof' of von Neumann). The **de Broglie-Bohm ('Pilot Wave') theory** has long been a counterexample. (*A fortiori* it is not to refute realism about quantum mechanics, whatever that could mean! It actually all but rules out the most simple instrumentalist interpretation of the wavefunction on which it is a predictive device with superpositions just representing our ignorance.) The upshot is that, barring 'retrocausality', 'many-worlds', and similar exotic options, any empirically adequate hidden variables theory -- because any theory at all -- is non-local.
- *Bell*: "The paradox of Einstein, Podolsky, and Rosen was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore the theory causality and locality….[However] that idea [is] incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality...that creates the...difficulty."

Bell's Theorem

- The phrase 'Bell's theorem' is now a kind of catch-all for so-called **No-Go theorems**. We follow Barrett [2019] and Wigner [1970, 1976] below. We begin by assuming:
  - (1) **Determinacy**: Particles 1 and 2 in a **singlet** state, $|\Psi\rangle = \sqrt{(1/2)}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2)$, have unique spin properties along every axis prior to measurement. So, in particular, they have spin values along 60°, 0°, and -60°, at **Location$_A$** and

Location$_B$, respectively, which we suppose are very far apart. We can symbolize this situation as follows: <A(+60°), A(0°), A(-60°)>, <B(+60°), B(0°), B(-60°)>.

- *Note*: In the formalism introduced last class, each of these spin axes corresponds to a <u>different orthonormal basis</u> in the <u>spin state space</u>.
  - *Notation*: We write the **probability** that their spins exhibit a particular sequence of ups and downs along the axes at **Location**$_A$ and **Location**$_B$, like so: **P(<**$\mathbf{a_{60+}}$**,** $\mathbf{a_{600}}$**,** $\mathbf{a_{60-}}$**->**$_A$**, <**$\mathbf{b_{60+}}$**,** $\mathbf{b_{600}}$**,** $\mathbf{b_{60-}}$**->**$_B$**)**, where each of $\mathbf{a_{60+}}$, $\mathbf{a_{600}}$, $\mathbf{a_{60-}}$ and $\mathbf{b_{60+}}$, $\mathbf{b_{600}}$, $\mathbf{b_{60-}}$ is either + or -.
    - *Example*: **P(<+, -, +>**$_A$**, <-, +, ->**$_B$**)** is the probability that particle 1 at **Location**$_A$ is spin up along the +60° axis, spin down along the 0° axis, and spin up along the -60° axis, while 2 at **Location**$_B$ is spin down along the +60° axis, spin up along the 0° axis, and spin down along the -60° axis.
- *Recall*: Taken at face-value, the particles in Singlet state $|\Psi\rangle = \sqrt{(1/2)}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2)$ are such that <u>neither possesses a state of its own</u>, much less a property that might <u>distinguish</u> it from the other. (One can even arrange that the particles have the same state more generally!)
- *Note*: We can understand <u>probability</u> here on a <u>frequency</u> interpretation.

○ (2) **Complementarity**: The individual spin properties of particles 1 and 2 are always <u>opposite</u> along an axis -- in particular, along the +60°, 0°, and -60° axes.

- *Recall*: It is an <u>empirical fact</u> that when the spins of the electrons in a singlet state are measured they are opposite. The hidden variable theorist <u>explains</u> this by supposing that <u>the electrons already had opposite spins</u>.
- *Upshot*: Electrons 1 and 2 <u>must thus exhibit exactly one of the following eight sequences of spins</u> at at **Location**$_A$ and **Location**$_B$: **(<+, +, +>**$_A$**, <-, -, ->**$_B$**), (<+, +, ->**$_A$**, <-, -, +>**$_B$**), (<+, -, +>**$_A$**, <-, +, ->**$_B$**), (<+, -, ->**$_A$**, <-, +, +>**$_B$**), (<-, +, +>**$_A$**, <+, -, ->**$_B$**), (<-, +, ->**$_A$**, <+, -, +>**$_B$**), (<-, -, +>**$_A$**, <+, +, ->**$_B$**), (<-, -, ->**$_A$**, <+, +, +>**$_B$**).**

○ (3) **Independence (**alternatively: **Definition of 'Measurement')**: The outcome of an accurate spin measurement on either particle depends only on its spin properties (not on the axis along which we measured it, a future event, and so on).

○ *Question*: What should we mean by "depends" here? Is it a counterfactual idea?

- *Recall*: Given (1) and (2), if we could change the spin property of particle 1 by measuring it, then we could instantaneously change that of particle 2. Hence, premises (1) -- (3) together entail the locality assumption of EPR.
- *Caveat*: The Kochen-Specker version assumes even less than (1) -- (3)!
- *Note*: **Independence** means that the probabilities of a measurement equals the probabilities of the particles having the corresponding spin properties.
  - *Illustration*: $\mathbf{P(+_A \,\&\, +_B \mid A_{+60^\circ}) = P(+_{A+60^\circ} \,\&\, +_B)}$, where $\mathbf{+_B}$ means that a spin measurement at B along the 0° axis resulted in spin up, and $\mathbf{A_{+60^\circ}}$ means that the apparatus at **Location**$_\mathbf{A}$ is turned to +60°.

- With (1) -- (3) in hand, we can now argue as follows:
  - Suppose a composite system of electrons is described as per the EPR argument by the **singlet** state, $\mathbf{|\Psi\rangle = \sqrt{(1/2)}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2)}$, where electrons 1 and 2 are at locations **Location**$_\mathbf{A}$ and **Location**$_\mathbf{B}$, respectively, which are very far apart.
  - If on each trial the measurement device at **Location**$_\mathbf{A}$ or **Location**$_\mathbf{B}$ is used to measure the spin properties of particle 1 or particle 2, respectively, along one of only two of the +60°, 0°, and -60° axes, then, by the classical probability calculus:
    - $\mathbf{P(+_A \,\&\, +_B \mid A_{+60^\circ}) = P(\langle +, -, +\rangle_A, \langle -, +, -\rangle_B) + P\langle +, -, -\rangle_A, \langle -, +, +\rangle_B)}$
    - $\mathbf{P(+_A \,\&\, +_B \mid B_{-60^\circ}) = P(\langle +, +, -\rangle_A, \langle -, -, +\rangle_B) + P\langle -, +, -\rangle_A, \langle +, -, +\rangle_B)}$
    - $\mathbf{P(+_A \,\&\, +_B \mid A_{+60^\circ} \,\&\, B_{-60^\circ}) = P(+_A \,\&\, +_B \mid A_{+60^\circ}) + P(+_A \,\&\, +_B \mid B_{-60^\circ}) - P(\langle +, -, +\rangle_A, \langle -, +, -\rangle_B) - P\langle -, +, -\rangle_A, \langle +, -, +\rangle_B)}$
  - Consequently,
- **Bell's Inequality:** $\mathbf{P(+_A \,\&\, +_B \mid A_{+60^\circ} \,\&\, B_{-60^\circ}) \leq P(+_A \,\&\, +_B \mid A_{+60^\circ}) + P(+_A \,\&\, +_B \mid B_{-60^\circ})}$.
- **Problem**: In the spin state space, the 0°-spin basis is at a 30° angle to the +60°-spin basis. So, quantum mechanics says that $\mathbf{P(+_A \,\&\, +_B \mid A_{+60^\circ} \,\&\, B_{-60^\circ}) = (1/2)\sin^2(60^\circ) = 3/8}$ while $\mathbf{P(+_A \,\&\, +_B \mid A_{+60^\circ}) + P(+_A \,\&\, +_B \mid B_{-60^\circ}) = \sin^2(30^\circ) = 1/4}$, violating Bell's Inequality!
- *Upshot*: Assumptions (1) -- (3) make the wrong predications about the empirical world!
- This is a serious problem if EPR locality is supposed to be parcel to Special Relativity!

Finding a Loophole?

- Since the assumptions above entail empirically disconfirmed predictions, one of them has to go. The only choice is what. We should examine what exactly we have supposed.

- Assumptions (1) -- (3) entail but are stronger than the EPR locality assumption. A form of this assumption far predates EPR. Newton took it to be beyond serious doubt.
    - *Newton*: "It is inconceivable that...matter should, without the Mediation of something else...operate upon and affect other matter without mutual contact."
- However, it is not an option to merely conclude, with Bohr, that the world is **contextual**. This raises the question of how the 'context' of a system determines its physical features.
    - *Bohr*: "Naturally, in this case no mechanical disturbance of the system under examination can take place in the crucial stage of the process of measurement. But even in this stage there arises the essential problem of an influence on the precise conditions which define the possible types of prediction which regard the subsequent behaviour of the system…*[The quantum] description [is] a rational use of the possibilities of an unambiguous interpretation of the process of measurement compatible with the finite and uncontrollable interaction between the object and the instrument of measurement in the context of quantum theory."*
    - *Question*: Might **Superdeterminism**, according to which the experimenter's choices and the spin states of the electrons have a common cause capture this idea in a coherent way? How could we get evidence for such a view insofar as the experimental results on the basis of which we would believe it would seem to assume that Superdeterminism is false -- i.e., that there is not a global conspiracy?
- Perhaps the most natural proposal is to deny **(1) Determinacy**. Maybe there is no bijection between **Hermitian operators** and **properties**. In that case, there may be no independent spin properties along a given axis. Perhaps what explains the measurement outcomes are independent position properties. The **de Broglie-Bohm** theory (in which spin is not an intrinsic property of a particle) is an influential version of this proposal.[1] But this proposal alone does not explain the correlations between the measurements at **Location$_1$** and **Location$_2$**. Theories like **de Broglie-Bohm** theory are still non-local.
- A different way to deny **(1) Determinacy** would be to deny uniqueness. What could that mean? Maybe a measurement of the spin of an electron not in an eigenstate of the corresponding operator results in both of the possible results, each in its own 'branch' of the wavefunction. This explains the correlations insofar as the singlet state vector 'includes' one component with the first particle being spin up and the second being spin

[1] The Ghirardi–Rimini–Weber (GRW) theory rejects (1) Determinacy in a similar way.

down and another with the first particle being spin down and the second being spin up. This theory would retain locality in configuration space, but not in ordinary spacetime. But this *seems* to be an artifact of the **nonrelativistic Schrödinger Equation** (we will revisit this). A deeper problem is: if anything that physically can happen does happen (on some branch of the wavefunction), how can Born's Rule be *epistemically* justified?

- Finally, one could try to give up on **(3) Independence**. A way to implement this idea is to postulate **retrocausality** so that future measurements can influence past spin states [Price 1994, Goldstein & Tumulka 2003]. There is then no action-at-a-distance because effects propogate locally backwards in time from the moment after measurement. However, it would remain to develop a retrocausal hidden variables theory in any detail. The project does not look trivial because any such theory must incorporate causal loops!
- There remain options in logical (or counterlogical!) space, however unmotivated. We have said nothing about (2) **Complementarity**, for instance. More radically, we have not questioned either **classical logic** or **standard probability theory** -- and, indeed, setting aside the peculiarly probabilistic axioms, the argument is by *reductio ad absurdum*, which is invalid in most paraconsistent logics. But while such principles remain technically open to criticism, it is hard to think of an account resulting from their denial.