

PHILOSOPHY OF MATHEMATICS

<i>Justin Clarke-Doane</i> 712B Philosophy Hall Phone: 212-854-3246 Office Hours: By Appointment jc4345@columbia.edu	<u>Meeting Details</u> Friday 4:10pm-6pm in 716 Philosophy Hall <u>Requirements</u> Participation counts for 20% of your grade. A paper of ≈ 20 pages counts for 80%.
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Summary

Mathematics is often regarded as the most successful discipline. But like most -- maybe all -- disciplines, the picture is complicated on inspection. Disagreements persist over axioms and standards of proof. Independence results threaten the objectivity and determinacy of the subject. And the question of how human beings could acquire anything approximating the knowledge of mathematics we take ourselves to have acquired remains wide open. All of this might be thought to recommend some kind of “anti-realism” about mathematics. But, to add insult to injury, our best -- and only! -- comprehensive theories of the mind-independent world are up to their ears in abstract mathematics. In this seminar, we will try to figure out what to make of this situation.

Useful References

Computability and Logic (Boolos, Jeffrey, and Burgess)

Set Theory: An Introduction to Independence Proofs (Kunen)

Logical Foundations of Mathematics and Computational Complexity Theory (Pudlak)

A Tour Through Mathematical Logic (Wolf)

Note: All readings will be available electronically. Selections TBD.

Week 1: Logicism, Intuitionism, and Formalism

Carnap, “The Logicist Foundation of Mathematics” (in Benacerraf and Putnam)

Heyting, “The Intuitionist Foundation of Mathematics” (in Benacerraf and Putnam)

von Neumann, “The Formalist Foundation of Mathematics” (in Benacerraf and Putnam)

Optional: Putnam, “Philosophy of Mathematics: Why Nothing Works”

Optional: Kreisel, “Mathematical Logic: What Has It Done for the Philosophy of Mathematics?”

Week 2: Set-Theoretic Foundations

Bar-Hillel, Fraenkel, and Levy, “The Antinomies” (Chapter 1, *Foundations of Set Theory*) and “Axiomatic Foundations of Set Theory” (Sections 2.1 - 2.5 & 2.7 *Foundations of Set Theory*)
Boolos, “The Iterative Conception of Set”

Optional: Forster, [The Axioms of Set Theory](#)

Optional: Kreisel, “Two Notes on the Foundations of Set Theory”

Optional: Marquis, “Category Theory and the Foundations of Mathematics”

Optional: Quine, *Set Theory and Its Logic (Revised Edition)*, Part III

Optional: Russell, “On Some Difficulties with the Theory of Transfinite Numbers and Order Types”

Week 3: Metalogical Considerations

Bar-Hillel, Fraenkel, and Levy, “Metamathematical and Semantic Approaches” (Chapter 5, *Foundations of Set Theory*)

Dummett, “The Philosophical Significance of Godel’s Theorem”

Optional: Smullyan, *Godel’s Incompleteness Theorems*

Week 4: Independence

Bar-Hillel, Fraenkel, and Levy, “Questions Unanswered by the Axioms” (Section 2.6, *Foundations of Set Theory*)

Cohen, “Independence Results in Set Theory”

Kennedy and van Atten, “On Godel’s Modernism: Set-Theoretic Incompleteness Revisited”

Optional: Chow, “A Beginner’s Guide to Forcing”

Optional: Clarke-Doane, “What is Absolute Undecidability?”

Optional: Easwaran, “A Cheerful Introduction to Forcing”

Optional: Weaver, *Forcing for Mathematicians*

Week 5: Pluralism and Universalism

Barton, “Independence and Ignorance: How Agnotology informs Set-Theoretic Pluralism”

Godel, “What is Cantor’s Continuum Hypothesis?” (in Benacerraf and Putnam)

Hamkins, “The Set-Theoretic Multiverse”

Maddy, “Set-Theoretic Foundations”

Optional: Arrigoni, *What is Meant by V?*

Optional: Clarke-Doane, “Set-Theoretic Pluralism and the Benacerraf Problem”

Optional: Koellner, [“Hamkins on the Multiverse”](#)

Optional: Reitz, “From Geometry to Geology: An Invitation to Mathematical Pluralism through the Phenomenon of Independence”

Optional: Woodin, “The Continuum Hypothesis, I & II”

Week 6: Determinacy

Field, “Are our Logical and Mathematical Concepts Highly Indeterminate?”

Gaifman, “On Ontology and Realism in Mathematics”

Putnam, “Models and Reality”

Skolem, “Axiomatized Set Theory” (in van Heijenoort)

Warren and Waxman, “A Metasemantic Challenge to Mathematical Determinacy”

Week 7: Indefinite Extensibility

Dummett, “What is Mathematics About?”

Meadows, “Sets and Supersets”

Shapiro and Wright, “All Things Indefinitely Extensible”

Soysal, “Why is the Universe Not a Set?”

Optional: Clarke-Doane, “Modal Objectivity” (Section 8)

Optional: Scambler, “Can All Things Be Counted?”

Optional: Hamkins and Linnebo, “The Modal Logic of Set-Theoretic Potentialism”

Week 8: Predicativism, Finitism, and Ultrafinitism

Feferman, “The Significance of Weyl’s *Das Kontinuum*”

Gandy, “Limitations to Mathematical Knowledge”

Hilbert, “On the Infinite” (in Benacerraf and Putnam)

Wright, “Strict Finitism”

Optional: Bernays, “On Platonism in Mathematics” (in Benacerraf and Putnam)

Optional: Bremer, “Varieties of Finitism”

Optional: Nelson, “Completed vs. Incomplete Infinity in Arithmetic”

Optional: Kitcher, “Hilbert’s Epistemology”

Optional: Levine, *Understanding the Infinite*

Week 9: Justification

Gödel, “Russell’s Mathematical Logic”

Parsons, *Mathematical Thought and its Objects*

Putnam, *Philosophy of Logic*

Russell, “The Regressive Method for Discovering the Premises of Mathematics”

Optional: Heck, “Frege’s Theorem: An Introduction”

Optional: Quine, “[Two Dogmas of Empiricism](#)”

Week 10: Reliability

Benacerraf, “Mathematical Truth” (in Benacerraf and Putnam)

Clarke-Doane, “What is the Benacerraf Problem?”

Field, *Realism, Mathematics, and Modality* (Introduction)

Week 11: Indispensability

Artzenius and Dorr, “[Calculus as Geometry](#)”

Burgess and Rosen, *A Subject with No Objects: Strategies for a Nominalistic Interpretation of Mathematics*

Field, *Science without Numbers*

Leng, *Mathematics and Reality*

Pincock, *Mathematics and Scientific Representation*

Optional: Marcus, *Autonomy Platonism and the Indispensability Argument*

Optional: Suppes and Zinn, *Basic Measurement Theory*

Optional: Ye, “Indispensability Arguments and Anti-Realism in Mathematics”

Optional: Weaver, “[Is Set Theory Indispensable?](#)”

Week 12: Pythagoreanism

Maudlin, *New Foundations of Physical Geometry: The Theory of Linear Structures*

Quine, “Ontological Reduction and the World of Numbers”

Segal, “Pythagoreanism: A Number of Theories”

Tegmark, “The Mathematical Universe”

Optional: Heller and Raine, *The Science of Space-Time*

Week 13: Evolution and Cognition

Butterworth, *What Counts? How Every Brain is Hardwired for Math*

Clarke-Doane, [Morality and Mathematics](#), Section 5.5

Daheane, *The Number Sense*

De Cruz, “Numerical Cognition and Mathematical Realism”

Leng, “Debunking, Supervenience, and Hume’s Principle”

Pantsar, “An Empirically Feasible Approach to the Epistemology of Arithmetic”

Optional: Relaford-Doyle and Nunez, “Beyond Peano: Looking into the Unnaturalness of Natural Numbers”

Week 14: Intelligibility, Disagreement, and Quietism

Carnap, “Empiricism, Semantics, and Ontology” (in Benacerraf and Putnam)

Clarke-Doane, *Morality and Mathematics*, Chapter 2

Kreisel, “Observations on Popular Discussions of the Foundations of Mathematics”

Tait, “Truth and Proof: The Platonism of Mathematics”

Wang, “The Concept of Set” (in Benacerraf and Putnam)

Wittgenstein, *Remarks on the Foundations of Mathematics*

Optional: Kreisel, Review of Wittgenstein’s *Remarks*

Optional: Maddy, “Thin Realism” (Chapter 3, *Defending the Axioms*)