FOUNDATIONS OF METALOGIC

Justin Clarke-Doane Associate Professor Department of Philosophy 712B Philosophy Hall Phone: 212-854-3246 jc4345@columbia.edu Office Hours: By Appointment	<u>Course Details</u> GR9941 (Logic & Mathematics Foundations) <u>Meeting Information</u> Wed 6:10pm-8:00pm, 716 Philosophy Hall <u>Requirements</u> Participation counts for 20% of your grade. A paper of ≈ 20 pages counts for 80%.

Primary Reference

My Lecture Notes will serve as the primary reference material. Handouts on Incompleteness, Undefinability & Undecidability, the ZFC Axioms, and Constructability & Forcing, available at my website, may also be useful. We will, however, review this technical material as we go.

Other References

An Introduction to the Philosophy of Logic (Cohnitz & Estrada-Gonzàlez)
Computability and Logic (Boolos, Jeffrey, & Burgess)
Fast Track to Forcing (Džamonja)
Philosophy and Model Theory (Button & Walsh)

Week 1 (Jan. 19): Logical Form

Iacona, "Two Notions of Logical Form"
Pietroski, "Logical Form"
Russell, *Our Knowledge of the External World* (Lecture II)

Optional: Lepore & Ludwig, "What is Logical Form?"

Week 2 (Jan 26): Logical Constants

MacFarlane, "Logical Constants"
Tarski, "A Philosophical Letter of Alfred Tarski"
Tarski, "What are Logical Notions?"

Optional: Varzi, "On Logical Relativity"

Week 3 (Feb 2): Ascendence of First-Order Logic

Eklund, "How Logic Became First-Order"
Goldfarb, "Logic in the 20s: The Nature of the Quantifier"
Shapiro, "Do Not Claim Too Much: Second-Order Logic and First-Order Logic"

Optional: Brady, *From Peirce to Skolem: A Neglected Chapter in the History of Mathematical Logic*
Optional: Ewald, "The Emergence of First-Order Logic"

Week 4 (Feb. 9): Types and Tokens

Hilbert, "On the Infinite"
Kreisel, "Hilbert's Prograamme"
Wetzel, "Expressions vs. Numbers"

Optional: Bourbaki, *Theory of Sets* (Introduction & Chapter 1)
Optional: Clarke-Doane, "Platonic Semantics"
Optional: Wetzel, "What are Occurrences of Expressions?"

    [Visit from Linda Wetzel]

Week 5 (Feb. 16): Strict Finitism

Alston, "Ontological Commitments"
Quine & Goodman, "Steps Toward a Constructive Nominalism"
Weir, "A Neo-Formalist Approach to Mathematical Truth"

Optional: Friedman, *Philosophical Problems in Logic*
Optional: Rossberg & Cohnitz, "Logical Consequence for Nominalists"

Week 6 (Feb 23): Soundness & Completeness

Crossley et al., *What is Mathematical Logic* (Chapter 2)
Dummett, "The Justification of Deduction"
Kaye, "Circularity in Soundness and Completeness"

Optional: Haack, "Dummett's Justification of Deduction"

Optional: Kreisel, "Informal Rigor and Completeness Proofs"

Week 7 (March 2): Compactness & Löwenheim–Skolem

Benacerraf, "Skolem and the Skeptic"
Crossley et al., *What is Mathematical Logic* (Chapter 3)
Skolem, "Some Remarks on Axiomatic Set Theory"

Optional: Field, "Are Our Mathematical and Logical Concepts Highly Indeterminate?"
Optional: Väänänen, "Second-Order and Higher-Order Logic"

Week 8 (March 9): The Purpose of Formalization

Azzouni, "The Derivation-Indicator View of Mathematical Practice"
Fallis, "Intentional Gaps in Mathematical Proofs"
Marfori, "Informal Proofs and Mathematical Rigor"

Optional: Macbeth, "Formal Proofs in Mathematical Practice"
Optional: Rav, "Why do we Prove Theorems?"

Week 9 (March 16): NO CLASS (Spring Break)

Week 10 (March 23): Automated Theorem Proving

[Visit from Michael Harris]

Avigad, "Computers in Mathematical Inquiry"
Bassler, O. Bradley, "The Surveyability of Mathematical Proof: A Historical Perspective"
Harris, "Do Androids Prove Theorems in Their Sleep?"

Optional: Ornes, "How Close Are Computers to Automating Mathematical Reasoning?"
Optional: Thurston, "On Proof and Progress in Mathematics"

Week 11 (March 30): First Incompleteness Theorem

Benacerraf, "God, the Devil and Gödel"
Crossley et al., *What is Mathematical Logic?* (Chapter 5)
Feferman, "Transfinite Recursive Progressions of Axiomatic Theories"

Optional: Lacey & Joseph, "What the Gödel Formula Says"

Optional: Smith, *An Introduction to Gödel's Theorems, 2nd Ed.* (Chapter 29, Sec. 30.4, & *MRDP Theorem* supplement)

Week 12 (April 6): Undefinability of Truth

Shapiro, "Proof and Truth: Through Thick and Thin"
Smith, *An Introduction to Gödel's Theorems, 2nd Ed.* (Chapter 27)
Tarski, "The Semantic Conception of Truth"

Optional: Woleński, "Godel, Tarski and Truth"

Week 13 (April 13): Second Incompleteness Theorem

Detfleson, *Hilbert's Program* (Chapter 3: The Gödelian Challenge)
Smith, *An Introduction to Gödel's Theorems* (Chapters 31-36 & Sec. 37.5)

Optional: Hilbert, "Knowledge of Nature and Logic"
Optional: Zach, "Hilbert's Program"

Week 14 (April 20): Inner Models & Forcing

Crossley et al., *What is Mathematical Logic?* (Chapter 6)
Kennedy, *Gödel's Theorems* (Chapters 6-7)
Ciesielski, *Set Theory for the Working Mathematician* (Chapter 9)

Optional: Honzik, "A Quick Guide to Independence Results"

Week 14 (April 27): Pluralism, Incommensurability, & Physics

[Visit from Sean Carroll]

Carroll, "Real Physics without Real Math"
Clarke-Doane, *Mathematics and Metaphilosophy* (Chapters 2 & 4)
Shapiro, *Varieties of Logic* (Chapter 7)

Optional: Arenhart, "The Received View on Quantum Non-Individuality: Formal and Metaphysical Analysis"
Optional: French & Krause, *Identity in Physics: A Historical, Philosophical, and Formal Analysis* (Chapter 9: The Logic of Quanta)