OUTLINE OF QUANTUM MECHANICS

(1) THE <u>STATE</u> OF A QUANTUM SYSTEM IS GIVEN BY A <u>NORMALIZED</u> VECTOR, $|\psi\rangle$, IN A <u>COMPLEX VECTOR SPACE</u>.

$\boxed{\langle\psi|\psi\rangle = 1}$

"WAVEFUNCTION"
→ FOR POSITION: $\langle x|\psi\rangle$
→ FOR MOMENTUM: $\langle p|\psi\rangle$

"STATE SPACE"

$\boxed{\hat{O} = [\hat{O}^T]^* = \hat{O}^\dagger}$

(2) <u>MEASURABLE QUANTITIES</u> ARE REPRESENTED BY <u>HERMITIAN</u> <u>LINEAR OPERATORS</u>, $\hat{O}$.

EIGENVALUES ARE REAL

EIGENVECTORS FORM AN ORTHONORMAL BASIS

$\boxed{\hat{O}|\lambda_i\rangle = \lambda_i|\lambda_i\rangle}$

"QUANTUM"

(3) THE <u>POSSIBLE RESULTS</u> OF A MEASUREMENT OF THE PHYSICAL QUANTITY REPRESENTED BY $\hat{O}$ ARE THE <u>EIGENVALUES</u>, λ_i, OF $\hat{O}$.

(4) THE <u>PROBABILITY</u> OF OBTAINING λ_i WHEN MEASURING THE PHYSICAL QUANTITY REPRESENTED BY $\hat{O}$ WHEN THE SYSTEM IS IN STATE $|\psi\rangle$ IS

"BORN'S RULE" → $\boxed{P_i = |\langle\lambda_i|\psi\rangle|^2}$

EIGENVECTOR

$= |c_i|^2$ IN THE <u>EXPANSION</u> $\sum_i c_i|\lambda_i\rangle = |\psi\rangle$

NORMALIZED EIGENVECTOR, $|\lambda\rangle$

$= \boxed{c^* c}$

COMPLEX CONJUGATE

$\boxed{|c_1|^2 + \ldots |c_j|^2 + \ldots = 1}$

(5) IF WE <u>OBTAIN</u> λ_i WHEN MEASURING THE PHYSICAL QUANTITY REPRESENTED BY $\hat{O}$, THEN $|\psi\rangle = |\lambda_i\rangle$ IMMEDIATELY <u>AFTER</u> THE MEASUREMENT.

"COLLAPSE" OF THE WAVE FUNCTION

(6) BETWEEN MEASUREMENTS, THE STATE VECTOR OF THE SYSTEM, $|\psi\rangle$, EVOLVES ACCORDING TO THE TIME-DEPENDENT SCHRÖDINGER EQUATION:

$$\left.\boxed{i\hbar \frac{\partial |\psi\rangle}{\partial t} = \hat{H}|\psi\rangle}\right\} \begin{array}{l} \leftarrow \text{DETERMINISTIC} \\ \leftarrow \text{UNITARY} \\ \leftarrow \text{LINEAR} \end{array}$$

$\boxed{\frac{h}{2\pi}}$ ($\hbar$)

QUANTUM HAMILTONIAN ($\hat{H}$)

UNCERTAINTY

- WE CAN MEASURE TWO PHYSICAL QUANTITIES WITH CERTAINTY ONLY IF THERE IS A COMMON BASIS OF SIMULTANEOUS EIGENSTATES OF THE CORRESPONDING OPERATORS, $\hat{A}$ AND $\hat{B}$
  - NOTE: IF THE COMMUTATOR OF $\hat{A}$ AND $\hat{B}$, $[\hat{A},\hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$ IS ANYTHING OTHER THAN 0, THEN WE CANNOT.

  → UPSHOT: IF $\hat{A}$ AND $\hat{B}$ DO NOT COMMUTE, WE GET UNCERTAINTY

- GENERALIZED UNCERTAINTY PRINCIPLE: $\boxed{\Delta\hat{A}\,\Delta\hat{B} \geq \frac{1}{2}\left|\langle\psi|[\hat{A},\hat{B}]|\psi\rangle\right|}$

- HEISENBERG'S: $\boxed{\Delta x\,\Delta p \geq \frac{\hbar}{2}}$

POSITION (x) MOMENTUM (p)

ENTANGLEMENT

- WHEN $|\psi\rangle$ IS A STATE LIKE:

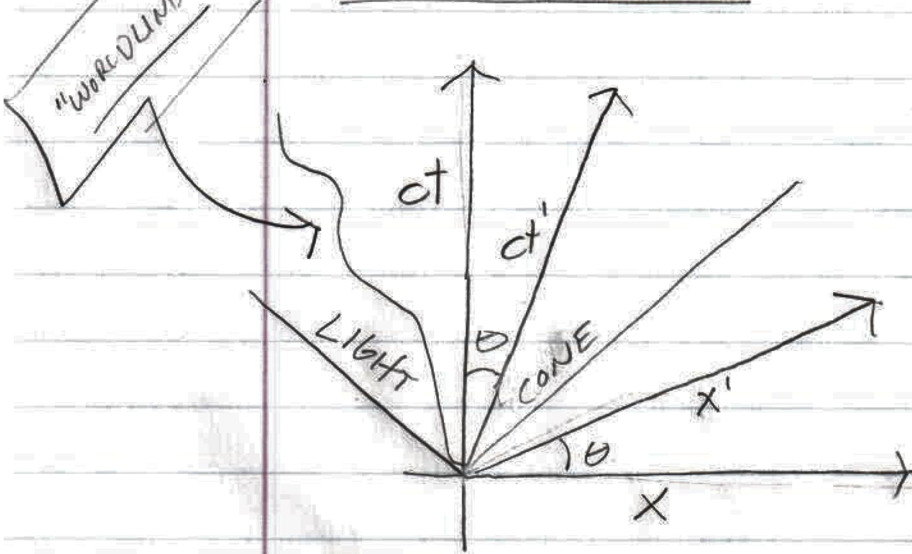
PARTICLE 1 PARTICLE 2

$$\boxed{|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\text{SPIN}\uparrow\rangle|\text{SPIN}\downarrow\rangle - |\text{SPIN}\downarrow\rangle|\text{SPIN}\uparrow\rangle\right)}$$

NO MATTER HOW FAR APART THE PARTIC[LES] ARE!

NEITHER PARTICLE HAS A DETERMINATE SPIN. BUT WHATEVER SPIN, PARTICL[E 1] IS MEASURED TO HAVE, PARTICLE 2 MUST THEN HAVE THE OPPOSITE!

RELATIVITY NOTES



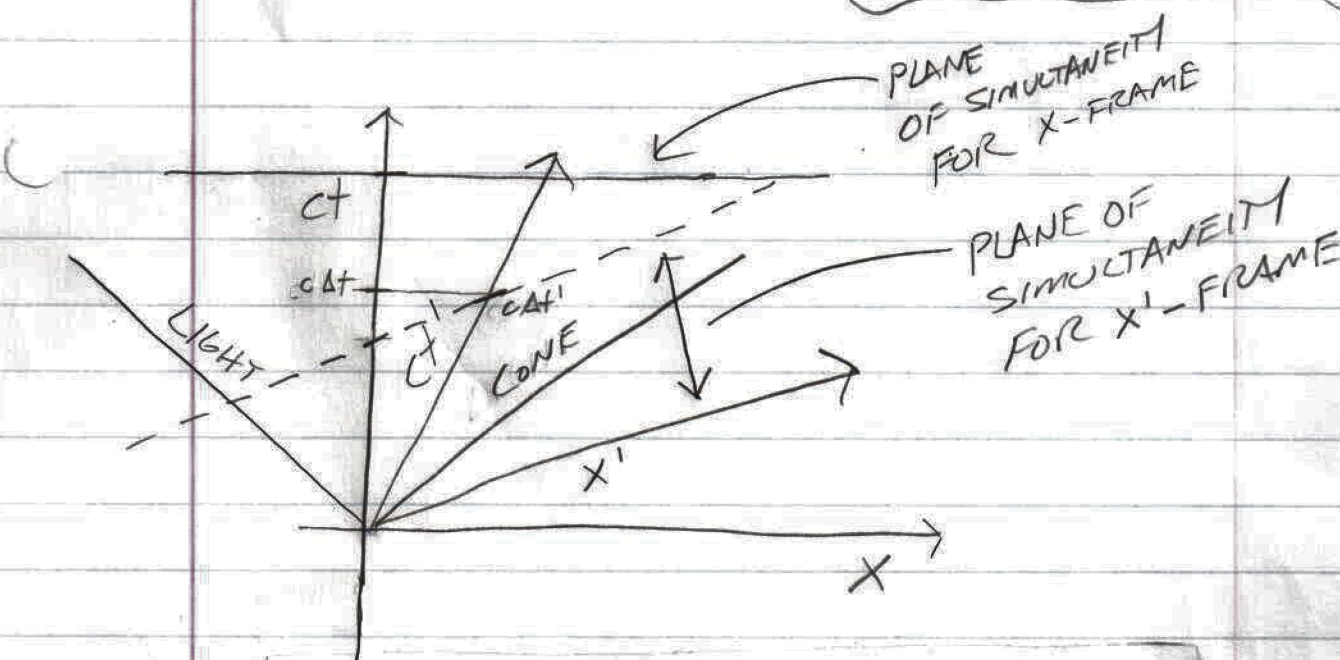
MINUS!

TECHNICALLY: $\Delta x^2 + \Delta y^2 + \Delta z^2$

• MINKOWSKI INTERVAL:

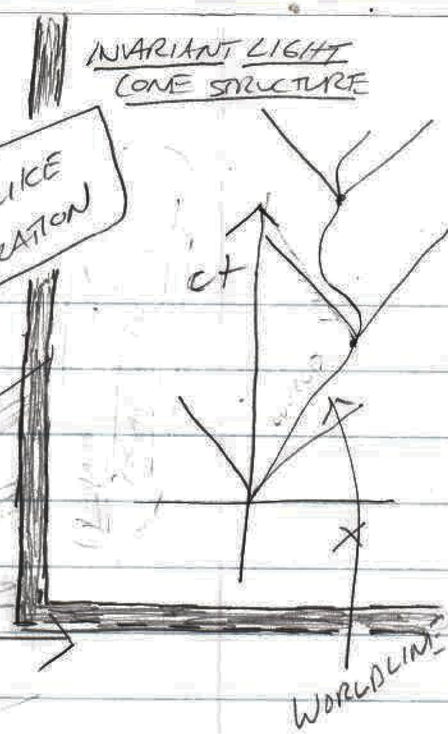
$$\underbrace{\Delta s^2}_{\text{THIS IS FRAME INVARIANT}} = \underbrace{c\Delta t^2 - \Delta x^2}_{\text{THESE VARY FROM FRAME TO FRAME}}$$

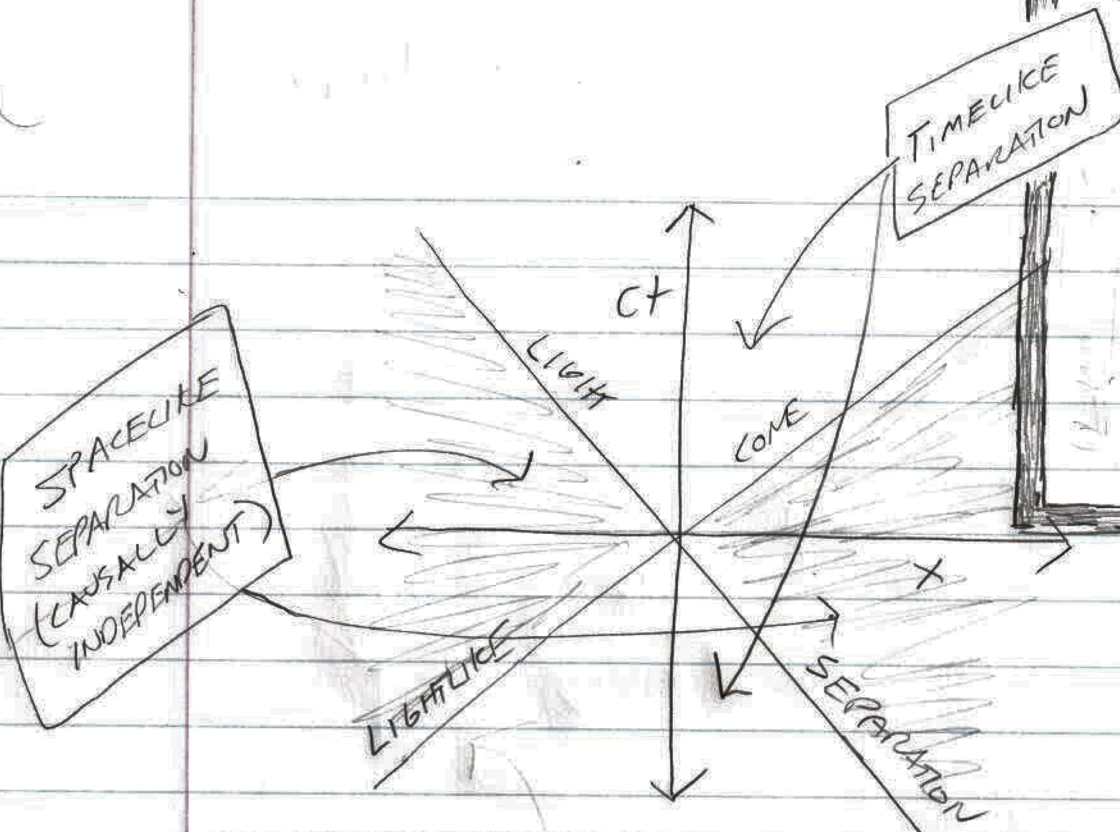
→ WHEN $\Delta x = 0$, WE CALL t "PROPER TIME"



• STRAIGHT LINE (GEODESIC) PATHS MAXIMIZE PROPER TIME.

• HOWEVER, GIVEN AN INTERVAL Δs BETWEEN TWO EVENTS, Δt IS MINIMIZED WHEN t IS PROPER TIME.





- TIMELIKE: $c\Delta t^2 > \Delta x^2$

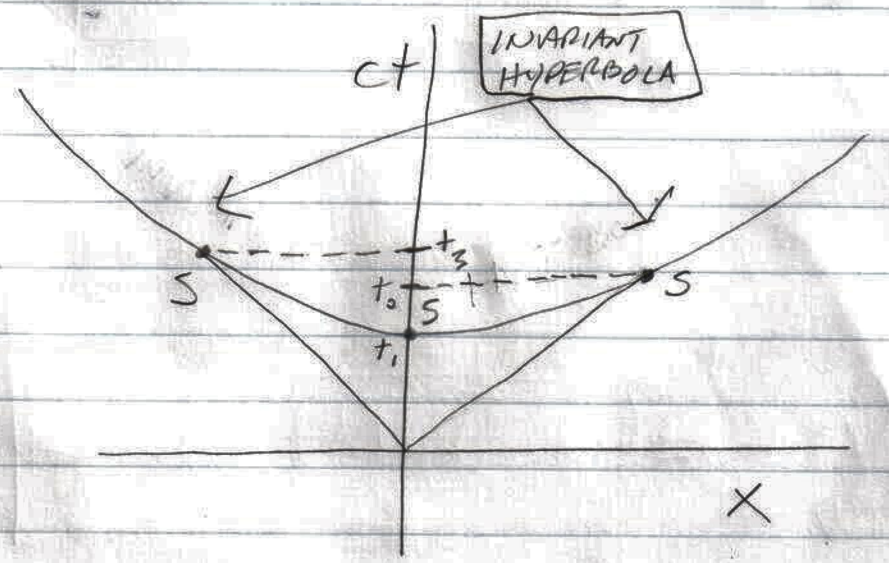
  $\Delta s^2 = \Delta\tau^2 = \Delta ct^2 - \Delta x^2$

- SPACELIKE: $\Delta x^2 > c\Delta t^2$

  $\Delta s^2 = \Delta\sigma^2 = -c\Delta t^2 + \Delta x^2$

- LIGHTLIKE: $c\Delta t^2 = \Delta x^2 \longrightarrow \Delta s = 0 \rightarrow$ PROPER TIME IS 0!

SIGNS FLIP



t_3 = TIME FOR FRAME 3

t_2 = TIME FOR FRAME 2

t_1 = TIME FOR FRAME 1