

## Research Statement

### **I. Framing of central questions**

Knowledge comes in two forms. First, there is knowledge that we acquire at least in part by means of observation and experiment. This is empirical knowledge. Knowledge that there is a piece of paper before you, that aspirin is protective against gastric cancer, and that a spin  $\uparrow$  electron has a 50% chance of being measured to be spin  $\rightarrow$  is empirical. Second, there is knowledge that we at least appear to acquire by reasoning alone, without reliance on observation or experiment. This is a priori or, as I shall call it, *armchair*, knowledge. Mathematical knowledge (e.g., that there are infinitely-many prime numbers), logical knowledge (e.g., that either grass is green or it is not the case that grass is green), modal knowledge (e.g., that there could have been exactly one more word on this page), and basic evaluative knowledge (e.g., that we ought to treat others as we would wish to be treated) are all varieties of armchair knowledge.

Whether the empirical-armchair divide is one of kind or merely one of degree is controversial. However we conceive of the distinction, though, armchair knowledge is opaque in a way that empirical knowledge is not. We have an emerging sketch of how humans acquire empirical knowledge. Facts about paper, aspirin, and electrons make causal marks on the world -- marks to which our nervous systems respond. But armchair knowledge remains mysterious. We do not causally interact with the likes of numbers, possibilities, or values. And while empirical science is increasingly illuminating our *beliefs* about these things, it has been conspicuously silent on their subject matter -- i.e. on the nature of numbers, values, and possibilities themselves.

This is a striking lacuna. Armchair commitments pervade our worldview. Our best theories of the world refer to *numbers*, are closed under *logical consequence*, tell us how things *would have been different* had initial conditions been, and are selected for their *theoretical virtues*. We cannot understand the world, or our place in it, absent a clear account of armchair knowledge.

My work investigates armchair knowledge, focusing on the following questions.

- Are there armchair facts that are independent of human minds and languages?
- If there are such facts, how can we know them?
- In what sense, if any, might independent armchair facts be conventional?

While a great deal of excellent work has been done on mathematical, logical, modal, and evaluative knowledge individually, my work is animated by the conviction that progress on these questions is facilitated by deep interaction between traditionally insulated areas of philosophy, like ethics and the philosophy of mathematics.

In what follows I describe the projects that have organized my thinking, plans for future research, and other research interests. Although each project is self-standing, the projects are unified by the ambition to understand, as Wilfrid Sellars put it, “how things in the broadest possible sense of the term hang together in the broadest possible sense of the term” [2007/1962, 369]. My work juxtaposes two ideas that have long been associated -- realism (the idea that there is an independent reality) and objectivity (the idea that in a disagreement, only one of us can be right). It explores these ideas across disparate domains, with implications for ethics, science, metaphysics and epistemology. One theme of my work is that *naturalism*, understood as the view that there are independent scientific facts, but not independent evaluative ones, is untenable. Another theme is that certain intractable theoretical questions can be traded for practical questions. (For a unified treatment of the above questions, see my [2020A].)

## **II. Metaphysics: The existence of independent armchair facts**

To what extent are the subjects of our thought and talk real? This is the question of *realism*. Realism about a subject, such as ethics or mathematics, contrasts with views like relativism and constructivism. Realism says that there are facts about the subject, and these facts hold independently of our beliefs about it. For example, physical realism is the view that there are facts about planets, electromagnetic fields, and so on, and these facts obtain whether or not any of us says or believes that they do.

Is armchair realism true? The logical positivists, inspired by the 18th century empiricist, David Hume, argued that it is not. Armchair facts are just the products of our linguistic conventions. “ $1 + 1 = 2$ ”, for example, is merely true because we decided to mean what we do by the symbols “1”, “2”, etc. W.V.O. Quine ([1951]) pointed out, however, that even if we get to decide that we will express the fact that one plus one equals two with the symbols “ $1 + 1 = 2$ ”, it is hard to see how we could get to decide *that the fact that “ $1 + 1 = 2$ ” actually expresses obtains*. Even if we had all decided to use “1” to mean zero and “0” to mean one, it still would have been the case that the fact we *actually* use “ $1 + 1 = 2$ ” to express was true. We simply would have expressed it with the symbols “ $0 + 0 = 2$ ”.

Considerations like the above have led many philosophers to be realists about mathematics, logic, and modality. But when it comes to evaluative realism -- i.e., realism about moral, intellectual, aesthetic, or other values -- philosophers (and scientists) have balked. A widespread naturalist position combines realism about the sciences, including mathematics, logic, and modality, with anti-realism about value ([Crisp 2006, 17], [Carroll 2010], [Gibbard 2003, Ch. 13], [Greene 2013], [Mackie 1977, Ch. 1], [Leiter 2009, 1], [McGrath 2010], [Joyce 2006, 182], [Rachels 1998, 3], [Rosenberg 2015], [Singer 1994, 8], [Sinnott-Armstrong 2006, 46], [Sosa 2002], and [Street 2006, 160, fn. 35]). Naturalists believe that there are independent facts about prime numbers, what follows from what, and how the world could have been different. But they deny that there are such facts about what is good or bad, and how we ought to live our lives.

In my [2014], I argue that naturalism, in the above sense, is incoherent. In particular, the reasons to be a moral antirealist are equally reasons to be a mathematical antirealist. This means, surprisingly, that we must choose between moral realism and mathematical anti-realism.

It is natural to suppose that there are deep differences between morality and mathematics. For example, the persistence of apparently intractable moral disagreement is often cited as a reason to doubt that there are independent moral facts (Mackie [1977, 37]). By contrast, mathematical propositions admit of the most universal confirmation there is -- proof (Rachels [1998, 3]). However, what is called a “mathematical proof” is really just a deduction (or deduction-sketch) from some claims labeled “axioms”. We could equally prove moral theorems in this sense (as Baruch Spinoza’s *Ethics* attests). Gather together some moral claims from which the others follow and call them “axioms”! What matters is how mathematical axioms compare to alleged “moral axioms”. It turns out that there are enduring and apparently intractable disagreements over mathematical axioms too -- from recondite axioms of set theory (Forster, [2018], Fraenkel, Bar-Hillel, & Levy [1973]), to the characteristic axiom of analysis (Kilmister [1980, 157]), to such elementary axioms of arithmetic as the first-order induction schema (Nelson [1986, 1]), and the principle that every natural number has a successor (Zeilberger [2004, 32-3]).<sup>1</sup> Unlike typical scientific disagreements, these do not admit of empirical resolution. Indeed, as in the moral case, it is not clear that they admit of resolution at all. And while such disagreements are paradigmatically “academic”, limited as they are to a small number of informed specialists, the same is true of many disagreements in ethics. What matters is not the raw number of participants on each side of a debate, but whether knowledgeable theorists can converge on a view.

My arguments for parity point to fascinating problems that I hope to explore in the future.<sup>2</sup> If we cannot believe in independent mathematical, logical, or modal facts without believing in independent evaluative facts, then we might be tempted to reject independent armchair facts altogether. But this is easier said than done. Again, taken at face-value, our best theories of the physical world rely on mathematical, logical, and modal facts. So, anti-realism about, say, mathematics would seem to engender physical anti-realism. For example, the (time-dependent) Schrödinger equation of quantum mechanics tells us how the state vector of a physical system changes with time. How could this express an independent fact, given that there are not really any independent facts about vectors? There is even a sense in which our physical theories presuppose evaluative facts. We pick them on the basis of evaluative criteria, such as elegance and beauty. Is there a way to “factor out” the armchair commitments of physical science, and believe in only the remainder? What precisely is the role of armchair facts in empirical theories anyway? What is the connection between, e.g., vectors in a Hilbert space and the quantum states we use them to describe? I have recently begun to dive into these stimulating questions in detail.

Another question suggested by my parity arguments that I look forward to tackling is the bearing of theoretical disagreement on our everyday beliefs about the world. Bertrand Russell once quipped that “the point of philosophy is to start with something so simple as not to seem worth stating, and to end with something so paradoxical that no one will believe it” [1918, 514]. His idea was that when you submit any theory -- whether mathematical, physical, moral or historical -- to philosophical scrutiny, there turns out to be ample room for disagreement. A rigorous statement of an area’s findings will generally be up to its ears in controversial philosophy. Is there a way to quarantine the questions that generate the puzzles? Can we be said to “know” that

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<sup>1</sup> Edward Nelson sometimes demurs from asserting that every natural number has a successor as well, at least in connection with “actual” (or “genetic”) numbers. See his [1986, 176] and [2013].

<sup>2</sup> I discuss some of these other arguments in subsequent sections.

there are infinitely-many prime numbers, for example, while failing to know whether there are really any numbers? Clarifying these issues could illuminate the nature of philosophy itself.

### III. Epistemology: Our knowledge of independent armchair facts

Let us set aside the prospect of global armchair anti-realism. Assuming that there are independent facts about prime numbers, what follows from what, how we ought to live, and so forth, how could we know those facts? In the case of everyday empirical facts, such as the fact that there is a piece of paper before you, we have at least the start of an explanation: we causally interact with those facts. The piece of paper deflects photons that hit your retinas, stimulating your optic nerve. The problem is that nothing remotely like this could work for armchair facts.

It might be thought that there is nevertheless a scientific story to tell. Is there not an obvious evolutionary explanation of our knowledge of numbers? It would have benefited our ancestors to have true rather than false arithmetic beliefs ([Pinker 2002, 192])! As Richard Joyce writes, “There is...evidence that the distinct genealogy of [arithmetic] beliefs can be pushed right back into evolutionary history: that natural selection has provided humans with an inbuilt faculty for simple arithmetic (Butterworth 1999)...[But] we have no grasp of how [belief in  $1 + 1 = 2$ ] might have enhanced reproductive fitness independent of assuming its truth” [2006, 182]. Indeed, it might be thought that this breaks the parity between moral and mathematical realism mentioned in Section II. Unlike our mathematical beliefs, “The evolutionary explanations [of our moral beliefs] work even if there are no moral facts at all” [Sinott-Armstrong 2006, 46].

However, in my [2015 B] and [2016 B], I show that this reasoning is fallacious. First, that “[t]he truth of ‘ $1 + 1 = 2$ ’ is a background assumption to any reasonable hypothesis of how this belief might have come to be innate” does not show that it would have benefited our ancestors to track the arithmetic truths. Every logical truth is a background assumption to every explanation whatsoever, and so is certainly a background assumption to any explanation of how belief in that logical truth might have come to be innate. But it is not that easy to show that we were selected to track the logical truths. Moreover, as I argue in my [2012 A], there is an alternative explanation of the selective advantage of having true arithmetic beliefs. Consider a creature, A, who believes that  $1 + 1 = 2$ , and a conspecific, B, who believes that  $1 + 1 = 0$ . Let us grant that in the presence of a lion to the left and a lion to the right, say, A will be less likely than B to walk out into a meadow and get eaten by two lions. This could be because  $1 + 1 = 2$ . But it also could be because of the *logical truth* that if there is “exactly one” lion to the left, and there is “exactly one” lion to the right, and no lion to the left is identical to any lion to the right, then there are “exactly two” lions to the left or to the right (where the phrases “exactly one” and “exactly two” here abbreviate logical constructions out of symbols like “ $\forall$ ” and “ $=$ ”, and do not refer to numbers). In other words, A may have had a selective advantage over B because her belief appropriately coincided with logical truths about her surroundings, not because her belief was literally true. And while it might be hoped that we were at least selected to have *the mathematical beliefs that we do have, which are, in fact, true*, not even belief in Peano Arithmetic appears to have been evolutionarily inevitable (Pantsar [2014, 4219], Relaford-Doyle & Núñez [2018]).

Of course, showing that the truth of our beliefs is a background assumption to some explanation of our having them might be *necessary* for vindicating their status as knowledge (even if it is not sufficient). If so, then our moral beliefs would indeed be on worse footing than our mathematical ones. This is the idea behind Genealogical Debunking Arguments. They say that our beliefs are *undermined* when we realize that the explanation of our coming to have them does not assume their truth (Greene [2013], [Joyce 2006, 182], [Ruse, 1986], [Street 2006, 160, fn. 35])). Undermining evidence contrasts with rebutting evidence. Rebutting evidence is direct reason to believe the negation of the proposition that we believe, while undermining evidence is reason to give up our belief that fails to be direct evidence for its negation. Consider Joe's belief that torture is always wrong. If Joe were to hear a compelling argument that torture is sometimes permissible, then he would get rebutting evidence against the content of his belief. But if he were instead to learn that his belief that torture is always wrong is the product of hypnosis, then he would get undermining evidence. Intuitively, in the latter case, he would not get evidence about torture. He would get evidence about the reliability of the method he used in forming his beliefs about torture. Debunkers claim that learning that the causal etiology of our beliefs makes no appeal to their truth is like learning that they are the result of hypnosis. However, in a series of articles ([2016 A], [2017 A]), I develop, and with [Dan Baras](#) refine ([Forthcoming C]), a principle that I call "Modal Security" governing undermining evidence. This principle, which enjoys independent plausibility, implies that Genealogical Debunking Arguments *must* be unsound. The problem is that, while these arguments fail to tell directly against the truth of our beliefs (since they are not rebutting), they also give us no reason to suspect a mismatch between our beliefs and the truths in other circumstances. The result is very general, and applies to all arguments with the structure of Genealogical Debunking Arguments meeting two conditions.

The upshot is that whether the truth of our armchair beliefs is a background assumption to some explanation of our having them is *independent* of our claim to armchair knowledge. What, then, does it take to vindicate our claim to armchair knowledge? "Knowledge" is a technical term that means justified and non-coincidentally true belief. One's belief fails to count as knowledge if it is false, if one lacks good reason to believe its content, or if one's belief is true, but only coincidentally so. There is no obvious obstacle to explaining the justification of our armchair beliefs. What, after all, explains the justification of your empirical belief there is a piece of paper before you? A natural answer is that it seems to you to be so, and you have no reason to doubt appearances. The same thing can be true of our armchair beliefs. As Kurt Gödel [1947] infamously put it, we can just "see" -- or "intuit" -- that, e.g., given a prime number, we could construct a larger one. The problem with Gödel's suggestion is that it does nothing to explain the *non-coincidental truth* of our armchair beliefs. In the empirical case, appearances causally co-vary with the facts. So, barring unusual hallucinations, had you not had an experience as of a piece of paper, a piece of paper would not have been before you. The problem is that numbers, possibilities, and values are causally inert. So, in this case, we cannot use the explanation of the justification of our beliefs to generate an explanation of their non-coincidental truth. Had our armchair beliefs been different, the armchair facts would have been unaffected. In order to vindicate our claim to armchair knowledge we need to undercut the appearance that we could have easily had systematically false beliefs insofar as we could have easily had different ones.

In my [2015 A], [2020 A, Ch. 5-6] and [2017 B], which includes a reply from [Thomas Scanlon](#), I argue that we can. The key idea is to give up on the *objectivity* of armchair areas but not on their independence. Very roughly, objective questions are those which only admit of a single answer. By contrast, in a disagreement over a non-objective question, we can both be right. Constructivists in the tradition of Immanuel Kant have long denied that objectivity implies realism. But the converse implication is not often questioned. *Pluralism*, as I will call realism plus the view that the facts are not objective, rejects this latter implication. Pluralism says that while there is an independent fact as to whether, e.g., the Axiom of Choice (a principle of standard mathematics) is true, armchair reality is so rich as to witness any answer to the question that we might have given. The picture is similar to the one most realists already accept about pure (rather than physical) geometry. Along with Euclidean lines, which satisfy the Parallel Postulate, there are hyperbolic lines, which satisfy its negation. Similarly, the pluralist says, along with sets, which satisfy the Axiom of Choice, there are related objects -- “shhets” -- which satisfy its negation. Along with possibility, which satisfies standard modal principles, there is “shpossibility” which satisfies different ones. And so on. Pluralism says that, although the logical positivists were wrong to think that we can literally *stipulate* facts into existence, they were right about the methodological moral. “The conflict between the divergent points of view... disappears... [B]efore us lies the boundless ocean of unlimited possibilities” [Carnap 1937/2001, XV]. There is enough *mind-and-language independent* reality to go around.

The foundations of armchair pluralism are as beguiling as they are exciting. There are two key questions. First, with respect to what subjects exactly is pluralism tenable? Pluralism itself would seem to be an armchair theory. It is a product of philosophy, which we have arrived at by reasoning. Moreover, our views on pluralism seem to share the contingent profile of other armchair beliefs. So, the above argument would seem to show that whether pluralism is true itself lacks an objective (even if not independent) answer (because there are subtly different notions of pluralism giving intuitively opposite verdicts). Is that right? Or is pluralism itself objectively (and independently) true? Second, how liberal should our pluralism about a subject be? It is natural to be as liberal as we can consistently be. But Gödel’s Second Incompleteness Theorem tells us that it is consistent to say false things about consistency. Is a more stringent criterion needed? Even if one is not, logical pluralism generates different versions of the consistency requirement, depending on the logic assumed. So, in order to know how rich armchair reality is, we first must know how rich logical reality is. In my [2019 A], I argue that logical reality is *indefinitely extensible*. Given a logic, we can always intelligibly weaken it. But as the locution “*can...weaken it*” suggests, indefinite extensibility is normally taken to be a modal notion. I cannot take it that way, because I believe that modal reality -- the way the world *could have been* -- is itself indefinitely extensible! Developing a new account of indefinite extensibility that makes sense even in the modal case is one aspect of this fascinating problem. My PhD student, [William McCarthy](#), and I tackle it in our work-in-progress [In Progress F].

#### **IV. Metaethics: The conventionality of independent armchair facts**

I have been discussing my work on armchair reality and how we know it. I have described my work arguing that realism about armchair facts stands or falls as a package, and that armchair questions fail to be objective, if we have armchair knowledge. But, as I said in Section II, there is an influential tradition of taking armchair facts to be conventional -- a reflection of our

practical decisions. The fact that, e.g., every set has a Choice function (which is equivalent to the Axiom of Choice) does not *seem* to be a substantial fact about the world in the way that force = mass × acceleration does. Can we do justice to this appearance, as armchair realists?

In my [2019 A], [2020 B], and [2020 D], I argue that we can. We do not make up the facts, whether they are about gravitons or sets. Armchair facts would exist even if we did not. But armchair facts are conventional in a way that empirical facts are not. There is one physical world to which our empirical theories answer. So, in trying to determine, e.g., whether there are gravitons, we can certainly go wrong. But if pluralism is true, then armchair reality is as rich as can be. So, in trying to determine, e.g., whether every set has a Choice function (i.e., whether the Axiom of Choice is true), we cannot help but get it right. We simply get it right of one part of mathematical reality rather than another part depending on the answer we proffer (much like the case of the Parallel Postulate in pure geometry). The question of truth factors out.

This means that the only live questions in many armchair debates *are* practical. It is as if armchair claims did not admit of truth at all. Instead of asking whether every set has a Choice function, we ask the practical question of *whether to use* a concept of set that satisfies the Axiom of Choice (for some purpose). Instead of asking whether we could have had different parents, we ask whether to use a concept of possibility that satisfies the so-called Necessity of Origins. And so on. All manner of intractable armchair questions get traded for practical ones.

In my [2015 A], [2020 A, Chapter 6], and [2020 C], which includes a response from [Ramon Das](#), and my [In Progress G], which will include a response from [Holly M. Smith](#), I apply this point to evaluative concepts. The result is a radicalization of Hume's dictum that one cannot derive an "ought" from an "is". If evaluative pluralism is true, then there is a property answering to John Stuart Mill's utilitarianism alongside a property answering to Kant's deontology -- and, indeed, a property answering to every other evaluative theory. So, as in the case of mathematics, logic, or modality, there is no useful question of whether, e.g., we "really" ought to kill the one to save the five (in some concrete situation). We ought-utilitarian, and ought-not-deontological! Instead, we need to ask whether to use a utilitarian or deontological concept of ought. However, there is an intimate connection between *this* practical question and a question of fact from the target area. Evaluative facts, unlike mathematical, logical, or modal ones, are supposed to tell us what to do. So, what happens when we replace the practical question of whether to use a given concept with the factual question of whether we *ought* to use it? The argument reapplies! We utilitarian-ought to use utilitarian-ought, and deontological-ought to use deontological-ought (and so on for every notion of ought). The question of *which concept to use* remains open. This cannot be the question of which concept we *ought* to use on pain of triviality. So, the facts, *even the evaluative facts*, fail to settle what to do. (See my [2020 G] for an application of this reasoning to epistemic facts.)

This "pragmatist" position is in the spirit of Rudolf Carnap's formulation of logical positivism [1937/2001]. Carnap argued that the only sensible armchair questions are practical questions, if they are not merely definitional. But, according to him, this is because there are no independent armchair facts. Unfortunately, Carnap failed to address the obvious worry for his view: is the fact that there are no armchair facts an armchair fact? Neither answer "yes" nor "no" seems

tenable. Pluralism explains the conventionality of armchair inquiry while avoiding armchair anti-realism.

In my view, practical inquiry plays an absolutely central role in our theoretical lives. When we argue about basic principles in mathematics, logic, modality, or ethics, we are really having a non-factual practical debate -- a negotiation about coordinated action.<sup>3</sup> So, in future work, I hope to investigate the nature of practical reasoning in detail. A striking feature of it is that it is *reasoning*. We can be criticized for holding incoherent views. We can extend our “knowledge” via deductive, inductive, and abductive inference. And our judgments can be right or wrong. But if what I said above is correct, practical questions cannot be questions of fact. Reality -- even evaluative reality -- does not decide them. They are answered with something like an intention (Gibbard [2003]). But how can we reason with intentions? In what sense can they be incoherent? And how can they be wrong? These questions will occupy me in coming years.

Another point of interest is the bearing of my pragmatism on the method of conceptual analysis. Many philosophers have held that the essence of philosophy, properly conceived, is articulating the concepts we have -- whether they be of personhood, time, freedom, or number. Even those who deny a central role for conceptual analysis tend to agree that it is an important part of the philosophical process. But if pluralism is true, conceptual analysis is almost always out of place. In non-evaluative cases, conceptual analysis at most tells us what we are talking about (e.g., sets as opposed to “shmetts”), rather than telling us what there is. And in evaluative cases, conceptual analysis fails to settle the practical questions that are really at stake. Suppose, for instance, that conceptual analysis reveals that our concept of responsibility entails retribution. Whether to retribute is left open. That depends on whether to hold people responsible rather than responsible\*, for some responsibility-like concept, responsibility\*, according to which responsibility\* does not entail retribution (or retribution\*). As Friedrich Nietzsche might put it, for any evaluative concepts, we can “critique...the value of these values” [1887, Preface, § 6].

A final upshot of my pragmatist outlook is an *opposition* between realism and objectivity, at least in armchair areas. If we are realists about an armchair subject, then we must give up on the objectivity of the area -- since otherwise we could not know it. The contingency of our beliefs would undercut our claim to knowledge. Conversely, if there is an objective question of what to do, then we must give up on realism about it -- since otherwise it would be like Parallel Postulate question (understood as one of pure mathematics). Realism and objectivity have long been associated, and are commonly even identified. Their opposition raises the question of which should be our focus. In my [2020 A, Conclusion] and my [In Progress A], I argue that objectivity should be. If, e.g., modality fails to be objective, then the question of whether we could have had different parents is like the Parallel Postulate question. We could not have, as a matter of metaphysical possibility (if a famous argument of Kripke [1980, 113] is sound), and we could have as a matter of, say, logical possibility. While we could always ask which notion of possibility is *real* possibility, that is just the question of what “possible” means out of our mouths. What would philosophy look like if the question of objectivity took center stage? I hope to find out.

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<sup>3</sup> Insofar as our disagreements are not misconceived verbal disagreements (about what our words mean), that is!

## V. Other Work

I have described my work, and plans for future research, on armchair knowledge. But my interests extend far beyond armchair inquiry, encompassing moral psychology, the history of analytic philosophy, philosophy of mind, and specialist topics in epistemology and metaphysics.

### *Addiction and Responsibility*

A longstanding interest of mine concerns questions of freedom and responsibility. This is one arena in which I take abstract philosophical reasoning to have important bearing on our day-to-day practices. There are several well-known arguments for incompatibilism, the view, roughly, that our claim to freedom is incompatible with the universe as we find it. But these are abstract arguments that are unlikely to resonate with non-philosophers. In recent years, I have become increasingly interested in how results in psychology and brain science can inform the discussion. [Kathryn Tabb](#) and I have developed an argument for skepticism about moral responsibility which is based on emerging data about addiction ([Forthcoming A]) There is a case to be made that addicts are not very different from the rest of us (Pickard [Forthcoming]). Addicts too respond to incentives. Their behavior is goal-directed. And so forth. Nevertheless, addicts' responsibility for their addictive behavior does seem to be mitigated. They are less responsible for, say, missing work on account of their addiction. Putting these two conclusions together, people's behavior is quite often mitigated because it is relevantly like addicts' behavior. Exactly how often it is mitigated is opaque to us, given our limited knowledge. Hence the "skepticism". But the upshot is similar to what it would be if we had argued, like the incompatibilist, that no one is responsible for anything. It is generally inappropriate to blame.

However, the same reasoning does not show that we are rarely in a position to judge events or actions good or bad. So, we can preserve many of our moral practices, while giving up -- or at least seriously amending -- our practices of blame. What would our moral practices look like if we ceased treating people as blameworthy? In coming years, I hope to investigate this in detail.

### *Gödel and Russell on Axioms*

I mentioned in Section III that Kurt Gödel used intuition to explain our knowledge of pure mathematics. He was actually elaborating an idea put forth by Bertrand Russell in his [1907] Cambridge lecture (and refined, with Alfred North Whitehead, in their [1925/1963]). In that lecture, Russell argued that the order of justification in mathematics is opposite the order of logical implication. We are justified in believing the axioms because they imply intuitive "data" we wish to account for, rather than the other way around -- somewhat like we are justified in believing laws of nature because they predict observational data we wish to account for.<sup>4</sup> The key epistemic difference, on Gödel's view, between mathematics and science is that in mathematics the data is intuitive, while in science it is empirical. However, Gödel's view has been widely dismissed for failing to illuminate the non-coincidental truth of our mathematical beliefs. Again, empirical data causally correlates with empirical facts in a way that intuitions do not. In a work in progress ([In Progress D]), I argue this criticism misunderstands the purpose to which Gödel put intuition. He was trying to account for the *justification* of our mathematical

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<sup>4</sup> There is also a connection between the Russell-Gödel epistemology of mathematics and John Rawls' method of reflective equilibrium (which he developed from Nelson Goodman). See my [2020 A, Ch. 2].

beliefs, not their non-coincidental truth. So, this criticism misses the mark. However, I argue that Gödel's account fails on its own terms. The problem is that there is *disagreement on the data to be accounted for* in the mathematical case that has no analog in the scientific one. For example, some theorists take the fact that the Axiom of Constructability implies the falsity of the hypothesis of a Measurable Cardinal to count as data against it (Woodin [2010, 1]), while others take this to count as data in its favor (Jensen [1995, 401]). Even at the level of justification, then, mathematical knowledge is *not* just like empirical scientific knowledge, but where the data is intuitive rather than empirical. I conclude by discussing the relevance of this difference to analogies between philosophy and science.

### *Mind-Body Dualism*

Consciousness seems to be a very different kind of thing than brain matter. So-called dualists about consciousness hold that it is. Perhaps the most recognizable contemporary dualist is [David Chalmers](#), who distinguishes the “easy” problem of explaining various cognitive capacities from the “hard” problem of explaining why any such capacity gives rise to felt experience. Chalmers follows a long line of philosophers beginning with René Descartes in arguing that we can coherently imagine physical beings just like us but without consciousness.<sup>5</sup> And, if this is imaginable, he argues, then it must be possible. So, consciousness must be distinct from any physical property. The step from imaginability to possibility is the one that has been widely resisted. Indeed, already Antoine Arnauld objected to Descartes that one could also imagine a difficult theorem of mathematics being false, simply because one was confused. But that would not show that the theorem really could have been false. However, in my [2020 F], to which Chalmers replies in a companion volume, I argue that even if one grants that imaginability is a guide to possibility, the argument for dualism fails. The problem is the last step. It amounts to the assumption that if properties could have come apart, then they are not the same. I argue that this principle is false in general, and that there is no reason to grant that it is true of the specific sense of “possible” invoked in the dualist argument. The paper is part of a larger project seeking to show that modal arguments, from Anselm's to Descartes', cannot establish controversial facts about the actual world -- even granting what are widely supposed to be their suspect premises.

### *Direct and Indirect Evidence* (with [Dan Baras](#))

I mentioned in Section III the distinction between rebutting and undermining evidence, and that Modal Security puts constraints on the latter. But what exactly does the distinction come to? It depends on how we understand the contrast between direct and indirect evidence. Although this contrast is widely taken to be straightforward, in this article Dan Baras and I argue that the distinction is opaque. Consider a cosmological conjecture for which we have no evidence one way or the other. The testimony of a cosmologist as to its truth would then be a paradigmatic example of direct evidence for it, while follow-up evidence that cosmologists tend to be liars would be indirect evidence against it. But why? The answer cannot be that the latter is evidence only given additional background evidence. If we did not have evidence that cosmologists tend to be reliable sources of cosmological truths, then the fact that a cosmologist testified to the truth of the conjecture would not be evidence either. Similarly, both pieces of evidence could rationally lead us to form a positive belief, rather than merely lead us to suspend judgement. For

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<sup>5</sup> Actually, Descartes argued for the converse -- that we can imagine our experience without physicality, and may have even rejected Chalmers' argument as stated ([1637/2016]). But the principles of his reasoning were the same.

instance, if we had background evidence that liars tend to cause false beliefs in others, then the testimony of the cosmologist would actually be good evidence for the negation of the conjecture to which they testified. All manner of traditional philosophical questions presuppose the direct/indirect distinction, and the correlative rebutting/undermining distinction. In coming years, Baras and I hope to clarify the distinction, and the philosophical problems that assume it.

### *Platonic Semantics*

Let us call *platonism* about an area of metaphysical interest, such as mathematics or fiction, the view that apparently true claims from the area are about abstract entities. And let us call the denial of platonism *nominalism*. Then, if anything is taken for granted in contemporary metaphysics, it is that platonism about an area of discourse affords a better account of the semantic appearances than nominalism, other things being equal. Indeed, there is a cliché trade-off harking back to Paul Benacerraf's [1973] between semantic and epistemological desiderata. Platonism affords a plausible semantics, but lacks a plausible epistemology. Conversely, nominalism affords a plausible epistemology, but not a plausible semantics. However, in a work-in-progress ([In Progress E]), I show that not even the platonist can take the semantic appearances at face-value. The problem is that the predicates from areas of metaphysical interest generally express properties that no platonic object can seriously be thought to instantiate, if interpreted at face-value. For instance, some philosophers have hoped to ground mathematical knowledge in knowledge of symbols. Numerals are taken to represent the sequence of strokes 1, 11, 111...  $3 > 2$  means that the latter sequence contains the former [Hilbert 1983/1926, 193]. But "strokes" here must mean stroke types, not tokens, since otherwise, as Gottlob Frege quipped, everyone would have their own numbers! Moreover, types must be understood platonistically, i.e., as existing independent of their instances, since otherwise it would be a doubtful empirical conjecture that every natural number has a successor. But platonic types have no shape, cannot be to the left or right of each other, cannot be written down, and may not have parts. So, e.g., "the sequence of strokes '111' contains three lines next to each other" cannot be true on a face-value interpretation, even according to the platonist. The same is true of discourse about geometrical objects, fictional characters, natural kinds and much more. The lesson is that *no one* can, in general, take the semantic appearances at face-value. They are inconsistent on a systematic scale. This article is part of a long-term project exploring what philosophy looks like when semantic appearances are not afforded even defeasible credibility.

### Books

1. *Objectivity in Mathematics*. Cambridge: Cambridge University Press [In Progress A].
2. [\*Morality and Mathematics\*](#). Oxford University Press [2020 A].<sup>6</sup> Available at Oxford Scholarship Online at:  
<https://oxford.universitypressscholarship.com/view/10.1093/oso/9780198823667.001.0001/oso-9780198823667>

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<sup>6</sup> This is not my dissertation. I liked the title!

3. “Précis” and “Reply to Critics” for an author-meets-critics book symposium, *Philosophy and Phenomenological Research* [In Progress B].
4. “Précis” and “Reply to Critics” for an author-meets-critics book symposium, *Analysis* [In Progress C].

## Articles

5. “Russell’s Regressive Method in Mathematics and Philosophy” [In Progress D].
6. “Platonic Semantics” [In Progress E].
7. (with [William McCarthy](#)) “Modal Pluralism and Higher-Order Logic,” *Philosophical Perspectives: Metaphysics* (supplement to *Noûs*) [In Progress F].
8. (with [Kathryn Tabb](#)) “From Addition to Skepticism,” Matt King and Josh May (eds.), *Responsibility and Mental Disorders*. Oxford: Oxford University Press [Forthcoming A].
9. “From Usability to Non-Factualism,” for an author-meets-critics symposium on [Holy M. Smith’s](#) *Making Morality Work*, with replies from Smith, *Analysis* [In Progress G].
10. “Metaphysical and Absolute Possibility,” *Synthese (Special Issue: New Directions in Modal Epistemology)* [Forthcoming B]. Online first at: <https://link.springer.com/content/pdf/10.1007%2Fs11229-019-02093-0.pdf>
11. (with [Dan Baras](#)) “Modal Security,” *Philosophy and Phenomenological Research* [Forthcoming C]. Online first at: <https://onlinelibrary.wiley.com/doi/epdf/10.1111/phpr.12643>
12. “Set-Theoretic Pluralism and the Benacerraf Problem,” *Philosophical Studies*. Vol. 177. 2013-2030 [2020 B]. Online at: <https://link.springer.com/content/pdf/10.1007/s11098-019-01296-y.pdf>
13. “Objectivity and Evaluation.”, with a response from [Ramon Das](#), Christopher Cowie and Richard Rowland (eds.), *Companions in Guilt Arguments in Metaethics*. Oxford: Routledge [2020 C]. Preview online at: <https://www.routledge.com/Companions-in-Guilt-Arguments-in-Metaethics-1st-Edition/Cowie-Rowland/p/book/9781138318335>
14. “Realism, Objectivity, and Evaluation” in David Kasper (ed.), *Explorations in Ethics*. Palgrave MacMillan [2020 D]. Preview electronic version at: <https://www.palgrave.com/gp/book/9783030480509>
15. “The Ethics-Mathematics Analogy.” *Philosophy Compass*. Vol. 15. 1-12 [2020 E]. Online at: <https://onlinelibrary.wiley.com/doi/epdf/10.1111/phc3.12641>

16. “Undermining Belief in Consciousness,” for an [author-meets-critics symposium](#) on [David Chalmers](#)’ “The Meta-Problem of Consciousness,” with replies from Chalmers, *Journal of Consciousness Studies*. Vol. 26. 34-47 [2020 F]. Online at: <https://www.ingentaconnect.com/contentone/imp/jcs/2019/00000026/f0020009/art00003#>
17. “Epistemic Non-Factualism and Methodology.” in Michael Klenk (ed.), *Higher-Order Evidence and Moral Epistemology*. New York: Routledge [2020 G]. Preview online at: <https://www.routledge.com/Higher-Order-Evidence-and-Moral-Epistemology/Klenk/p/book/9780367343200>
18. “Modal Objectivity,” *Noûs*. Vol. 53. 266-295 [2019 A]. Online at: <https://onlinelibrary.wiley.com/doi/pdf/10.1111/nous.12205>
19. “Debunking Arguments: Mathematics, Logic, and Modal Security.” Robert Richards and Michael Ruse (eds.), *Cambridge Handbook of Evolutionary Ethics*. Cambridge: Cambridge University Press [2017 A]. Preview at: <https://www.cambridge.org/core/books/cambridge-handbook-of-evolutionary-ethics/F61CC795F8DC662B0177EA90EA7EBE46>
20. “Objectivity and Reliability,” for an author-meets-critics symposium on [Thomas Scanlon](#)’s *Being Realistic about Reasons*, with replies from Scanlon, in the *Canadian Journal of Philosophy*. Vol. 47. 841-855 [2017 B]. Online at: <https://www.tandfonline.com/doi/full/10.1080/00455091.2017.1315289>
21. “What is the Benacerraf Problem?,” Fabrice Pataut (ed.), *New Perspectives on the Philosophy of Paul Benacerraf: Truth, Objects, Infinity (Logic, Epistemology, and the Unity of Science, 28)*. Dordrecht: Springer [2016 A]. Preview online at: <https://www.springer.com/gp/book/9783319459783>
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