

What is the Benacerraf Problem?

- *Benacerraf*: “[S]omething must be said to bridge the chasm, created by...[a] realistic... interpretation of mathematical propositions, between the entities that form the subject matter of mathematics and the human knower ([1973], 675).”
- *Question*: What exactly is the problem?

Original Formulation

- *Benacerraf*: “I favour a causal account of knowledge on which for X to know that S is true requires some causal relation to obtain between X and the referents of the names, predicates, and quantifiers of S.....[But]...combining *this* view of knowledge with the “standard” view of mathematical truth makes it difficult to see how mathematical knowledge is possible....[T]he connection between the truth conditions for the statements of number theory and any relevant events connected with the people who are supposed to have mathematical knowledge cannot be made out ([1973], 671-3).”
- *Problem*: Causal accounts of knowledge in the relevant sense are highly implausible.

Hart’s Addendum

- *Hart*: “[I]t is a crime against the intellect to try to mask the problem...with philosophical razzle-dazzle. Superficial worries about...causal theories of knowledge are irrelevant...for the problem is not so much about causality as about the very possibility of natural knowledge of abstract objects ([1977], 125 – 6).”

Field’s Improvement

- *Field*: “Benacerraf formulated the problem in such a way that it depended on a causal theory of knowledge. The [following] formulation does not depend on *any* theory of knowledge in the sense in which the causal theory is a theory of knowledge: that it, it does not depend on any assumption about necessary and sufficient conditions for knowledge. Instead, it depends on the idea that we should view with suspicion any claim to know facts about a certain domain if we believe it impossible in principle to explain the reliability of our beliefs about that domain ([1989], 232 – 3).”

- “We start out by assuming the existence of mathematical entities that obey the standard mathematical theories; we grant also that there may be positive reasons for believing in those entities. These positive reasons might involve...initial plausibility...[or] that the postulation of these entities appears to be indispensable.... But Benacerraf’s challenge...is to...explain how our beliefs about these remote entities can so well reflect the facts about them...*[If it appears in principle impossible to explain this, then that tends to undermine the belief in mathematical entities, despite whatever reason we might have for believing in them ([1989], 26).]*”
- *Question Becomes:* What would count as an explanation in the relevant sense?

Answer 1: Safety

- An explanation in the relevant sense would block the worry *that had our mathematical beliefs been different, our mathematical beliefs would have been false.*
- *Problem (Balaguer):* “[If] all [logically] possible mathematical objects exist.... then all we have to do in order to attain [mathematical] knowledge is...think about, or even “dream up”...a mathematical object. Whatever we come up with, so long as it is consistent, we will have formed an accurate representation of *some* mathematical object, because...all [logically] possible mathematical objects exist ([1995], 303).”
 - *Field:* “There are...answers [to the Benacerraf Problem] that seem satisfactory. Some of these...(Balaguer 1995)...solve the problem by articulating views on which, though mathematical objects are mind-independent, any view we had had of them would have been correct ([2005], 77 – 8).”
- *Deeper Problem:* The worry that had our mathematical beliefs been different, they would have been false is only compelling to the extent that it seems a fluke *that we came to have the mathematical beliefs that we came to have.* But at least our “core” mathematical beliefs were plausibly to be expected on broadly evolutionary grounds.

Answer 2: Sensitivity

- An explanation in the relevant sense would block the worry *that had the mathematical truths been different, our mathematical beliefs would have been false.*
 - *Field:* “The Benacerraf problem...seems to arise from the thought that we would have had exactly the same mathematical...beliefs even if the

mathematical...truths were different; [so] it can only be a coincidence if our mathematical...beliefs are right, and this undermines those beliefs ([2005], 81).

- *Problem* (Lewis): “[I]f it is a necessary truth that so-and-so, then believing that so-and-so is an infallible method of being right. If what I believe is a necessary truth, then there is no possibility of being wrong. That is so whatever the subject matter...and no matter how it came to be believed ([1986], 114-115).”
 - *Field*: “If the intelligibility of talk of ‘varying the facts’ is challenged...it can easily be dropped...[T]here is still the problem of explaining the *actual* correlation between our believing ‘p’ and...p ([1989], 238).”
 - *Problem*: If it is no fluke that we have the mathematical beliefs that we have *and* the mathematical truths could not have been different, then how could any remaining mystery undermine our beliefs? We were *bound* to get it right.¹
- *Deeper Problem*: In whatever sense one *can* motivate the above worry, it would seem to generalize to relevantly uncontroversial truths. Had the necessary truths that link the instantiation of *being a chair* to the instantiation of subvenient properties been different, our beliefs about chairs would have been false. However, there is not supposed to be a Benacerraf Problem with respect to our knowledge of chairs.
 - *Korman*: “[W]e would have believed that there are baseballs even if it were false that atoms arranged baseballwise compose baseballs ([Forthcoming], 23).”

Answer 3: Indispensability

- An explanation in the relevant sense would show *that the contents of our mathematical beliefs figure into the best explanation of our having those beliefs*.
- *Problem* (Steiner): “[S]uppose that we believe...the axioms of...number theory...[T]here exists a theory...which can...explain our belief in a causal style. This theory, like all others, *will contain the axioms of number theory* ([1973], 61).”

¹ *Similarly*:

- *Schechter*: “Lewis is correct...that the reliability challenge for mathematics... is subject to a straightforward response, so long as the challenge is construed to be that of [explaining how our mechanism for mathematical belief works such that it is reliable]...[But] [t]here remains the challenge of [of explaining how we came to have a reliable mechanism for mathematical belief] ([2011], 14).”
- *Problem*: If explaining how we came to have a reliable mechanism for mathematical belief is different from explaining how we came to have the mechanism for mathematical belief that we have, then it must make sense to say what would have been the case had the mathematical truths been different.

- *Deeper Problem*: In order for the Benacerraf Problem to *undermine* our belief in mathematical realism, it would seem to have to show that our mathematical beliefs could have easily been false. *But given the (actual) truth of our mathematical beliefs, and given that those beliefs are appropriately safe and sensitive, they could not have easily been false – even if their contents fail to figure into the best explanation of our having them.*
- In general, claims like Steiner’s are relevant to the plausibility of the *Indispensability Argument* – according to which, roughly, we have (defeasible) *empirical justification* for believing in mathematical realism because realistic mathematics figures indispensably into our best empirical scientific theories. However, the Benacerraf Problem is supposed to be *independent* of the Indispensability Argument.
 - *Field*: “Of course...indispensability arguments...still need to be addressed, but the role of the Benacerrafian challenge...is to raise the cost of thinking that the postulation of mathematical entities is the proper solution...([1989], 26).”

Broader Relevance

- The obscurities surrounding the Benacerraf Problem infect any argument with the structure of Field’s aimed at realism about a domain, D, meeting two conditions:
 1. The D-truths would be metaphysically necessary.
 2. There is a plausible explanation of our having the D-beliefs that we have.
- In particular, they carry over to so-called “Genealogical Debunking Arguments” aimed at moral realism (Street, Joyce, Lillehammer, etc.). Richard Joyce writes:
 - “Nativism offers us a genealogical explanation of moral judgments that nowhere...presupposes that these beliefs are true.... My contention...is that moral nativism...might well...render [moral beliefs] unjustified....In particular, any epistemological benefit-of-the-doubt that might have been extended to moral beliefs -- based upon some principle of conservatism, for example -- will be neutralized by the availability of an empirically confirmed moral genealogy that nowhere...presupposes their truth ([2008], 216).”
- Joyce’s Problem could be any of the following.
 - First, Joyce’s Problem could be the problem of blocking the worry *that had our moral beliefs been different, our moral beliefs would have been false.*

- In this case, Joyce’s Problem might generate its own solution – it might show that it is no fluke that we came to have the moral beliefs that we came to have.
- Second, Joyce’s Problem could be the problem of blocking the worry *that had the moral truths been different, our moral beliefs would have been false*.
 - In this case, Joyce’s Problem is either unintelligible or seems to arise equally for realists about relevantly uncontroversial truths.
- Finally, Joyce’s Problem could be that of *empirically justifying* moral realism.
 - In that case, Joyce’s Problem is just Harman’s -- which is analogous to the negation of the key premise in the Indispensability Argument.
 - *Harman*: “Observation plays a part in science it does not appear to play in ethics, because scientific principles can be justified ultimately by their role in explaining observations.... [M]oral principles cannot be justified in the same way ([1977], 10).”
 - It is hard to see how this problem, which has nothing essential to do with the genealogy of our moral beliefs, could *undermine* our belief in moral realism, given that neither the first nor the second problems do.

Conclusion

- The Benacerraf Problem, and like problems for realism in other areas, are obscure.
 - They supposedly differ from the familiar demand for empirical justification by *allowing* both that there is the relevant body of truths that we take there to be and that our belief in it is defeasibly justified.
 - However, it is hard to see how such problems could *undermine* this justification by showing that it must be a fluke that our beliefs are true.

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